On Accuracy of Third Party Audits

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Abstract

Third party audits of product quality – such as consumer product reviews, financial audits, and court decisions – are common. Prior economic analysis finds that more accurate assessments improve welfare, either by facilitating better consumer choices or encouraging producers to improve quality. This paper shows that, when auditors have binding capacity constraints, information available to consumers and thus welfare is maximized by less than perfectly accurate audits, specifically by audits that mistakenly give high grades to bad types. Capacity constraints ensure that not all good types can obtain an audit, preventing unraveling. False positives encourage bad types to submit to an audit hoping to get mistakenly favorable assessments.

Keywords: Asymmetric information, auditors, courts, accuracy.

JEL Classification Numbers: K10, K40.

1 Introduction

Third party assessments are commonplace. Courts tell us who is guilty and who is innocent; auditors assess the earnings statements by firms; magazines review

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the quality of consumer products; credit rating agencies report bond quality. Consumers take the information generated by the third party and use it in making decisions. If a court says a product causes harm, consumers take precautions. If an auditor verifies earnings, investors are more comfortable buying the stock. And so on.

In practice, third party assessments aren't perfect. Courts occasionally convict the innocent and free the guilty. Auditors let misstatements of earnings slide. Credit rating agencies give AAA ratings when the underlying assets are junk. When mistakes happen, the usual conclusion is that the assessment system failed; that is to say, flawless assessments represents the ideal.

The economics behind this idea is straightforward. The prospect of a perfectly accurate assessment drives a wedge between the payoff for firms that invest in quality and those that do not, firms that follow the law and those that do not. In so doing, accurate assessment creates incentives to invest in quality and, with legal obligations, comply with the law. No matter the context, more accurate third party assessments create beneficial incentives effects. The standard welfare calculation can then be applied: invest in accuracy until the additional benefit from enhanced incentives equals the marginal cost of the accuracy-enhancing procedures (Kaplow 1994; Kaplow and Shavell 1994; Kaplow and Shavell 1996; Posner 2007).

This paper questions the value of accuracy in third party assessments. To focus on the incentive effects of accuracy, we set aside typical concerns about the financial costs of improving accuracy. We do not assume there is a financial cost to making assessments more accurate. Instead, our core assumption is that auditors have capacity constraints so that not every firm that wants a third party assessment gets one. By implication, consumers cannot distinguish firms that chose not to get an audit from ones that did, but were precluded by the auditor's capacity constraints. This is an empirically common and thus attractive assumption. Courts often have backlogs. Top-tier financial auditors cannot audit all companies in the economy. And no reputable movie or restaurant reviewer can review all movies or restaurants.

A number of novel results follow from this assumption. First, perfectly accurate auditors produce a limited amount of information for consumers because they discourage any bad firm from seeking an audit. When welfare depends on the discovery of bad types, perfect auditors are no better for welfare than wholly uninformative auditors.

Second, some types of auditing error are better for welfare than others. Specifically, mistaken judgments that bad products are "good" are more desirable than mistaken judgments that good products are "bad." The reason is that mistaken exonerations make audits more attractive to bad firms. By contrast, mistaken convictions, by reducing the price paid following a favorable assessment, make audits less attractive. This provides an economic justification – not grounded in contestable assertions about preferences – for the ages-old intuition that mistaken convictions are worse than mistaken exonerations. This insight may be found in texts as diverse as Aristotle's Problems and Blackstone's Commentaries (1769).[1]

A caveat to these findings is that they imply only that some inaccuracy is always better than no inaccuracy. They do not imply that reducing accuracy always improves welfare on the margin. However, the reason we give for why perfect assessments are problematic – bad firms avoid audits – is a non-financial cost of accuracy that has not previously been considered. On the margin it will lower the optimal level of accuracy derived in prior models.

We present a model that formalizes and extends these results. To start, we consider the case where some producers make high quality products and others make low quality products. Consumers are uncertain whether specific producers are good or bad. Improving the accuracy of audits has two effects. The first effect is that, conditional on a producer opting for a third party assessment, a more accurate assessment produces more information about firm type. On this count, accuracy provides more information and enhances welfare. There is, however, an offsetting selection effect. The more accurate the assessment is, the more reluctant the bad type is to submit to it in the first place. Instead the bad type has an incentive to mask its quality by pooling with good types that have sought an audit but, due to capacity constraints, did not receive one.

As noted above, with perfectly accurate third party assessment, no bad type seeks a review of their product. In contrast, if third party assessment is flawed,

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[1] The most widely read fomulation is surely Genesis 18: 23-23 ("And the Lord said, If I find in Sodom fifty righteous within the city, then I will spare all the place for their sakes"). More recently the principle has been endorsed by Benjamin Franklin (1785) and Judge Cardozo (1916).

the bad type will occasionally risk a review. The reason is that they hope for a mistaken favorable rating. But the bad types won't always be so fortunate. Even with inaccurate assessments, sometimes the bad types will be found out. And so long as the bad type is more likely than the good type to be found out, that finding will provide consumers useful information. The lesson is that, to get any information whatsoever about the bad types, assessors must exonerate some fraction of them.

Next we allow producers of bad products to make (non-verifiable) investments in product quality, i.e., bad types can transform themselves into good types. In this scenario, more accurate assessment induces the production of better products. Despite this benefit, some amount of inaccuracy is still desirable so long as consumer precautions are relatively more efficient than producer precautions.

Finally, we consider the case where an outsider can demand an audit, but the firm can pay the outsider to drop its demand. This case corresponds to the situation where the outsider is a plaintiff bringing a legal suit and the defendant can settle with the plaintiff to avoid a court judgment. It turns out that the presence of the outsider improves welfare; restrictions on settlement improve welfare; and our intuitions about imperfect assessments remain valid.

This paper relates to three sets of literature. First, it complements literatures that implicitly assume that third party assessments are perfectly accurate. This includes the costly state verification models of Townsend (1979) and Gale and Hellwig (1985). In the standard CSR model, an entrepreneur credibly commits to disclose certain realizations of random cash flows. Those cash flows are audited, but the audit comes at a cost. The CSR literature assumes that if the auditing cost is paid, the audit is perfect. The auditor never makes mistakes. Scholars investigating the efficiency properties of information intermediaries (Biglaiser 1993; Lizzeri 1999) employ the same assumption. These articles focus on what information the intermediary decides to reveal, with any revelation being perfectly verifiable; that is, no mistakes.

Second, this paper adds to a growing literature on mechanisms by which agents can conceal information. One example is Bolton et al. (2009). In that paper, the authors construct a model where bond issuers shop for good credit ratings. If the issuer gets an unfavorable rating, it need not reveal that rating to investors;

the issuer can shop for another rating. They find that a monopolist credit rating agency improves welfare over having two credit rating agencies because the shopping option is unavailable. Unlike Bolton et al. (2009), the bad firm in our model cannot peek at the results of an audit and, if they are poor, hide them. The only choice is whether to submit to the audit in the first place. The framework here maps onto institutions like, say courts, where the audit results are simultaneously made available to the firm and the public.

Other examples of papers on "concealment" may be found in the literature on mandatory disclosure laws. A central result from that literature is that a rule which merely requires companies to disclose information they possess about their product will discourage companies from gathering information about their product (Farrell 1986, Shavell 1994).[2] A normative implication of the mandatory disclosure literature is that disclosure rules should require companies to disclose information they know or *should have known* about their product (Kronman 1978). If this is not possible, voluntary disclosure rules may be superior (Polinsky & Shavell 2006). Our finding is analogous. A perfect audit may be thought of as a mandatory disclosure for any firm that gets an audit: it forces disclosure of a bad firm's type. Declining an audit is equivalent to failure to gather information: it hides information that a perfect audit would reveal.[3]

Finally, this paper contributes to the literature on trial accuracy, which concludes that court errors reduce welfare (Kaplow 1994; Kaplow and Shavell 1994). We obtain a conflicting result: in many circumstances, fostering some inaccuracy substitutes low cost consumer precaution for high cost producer investment – a welfare enhancing move.

The paper has five sections. Section 2 considers the case where producers can-

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[2] A paper from the mandatory disclosure literature that is closely related to Bolton et al. (2009) is Shavell (1994). Shavell shows that, before learning their type, firms would prefer mandatory disclosure over voluntary disclosure. Shavell is concerned that voluntary disclosure encourages excessive acquisition of information about product quality by producers because it provides an option to hide bad information.

[3] Daughety & Reinganum (2005) consider the use of settlement to avoid mandatory disclosure via trial. They do not explore the effect of accuracy. Instead they note that good firms may be able to signal their type by committing to a non-confidential settlement. To the extent that settlement may be supplemented by simultaneous but secret side deals, as is common for example in drug patent litigation (Hemphill 2006), open settlements are not verifiable. Therefore, when we discuss audits triggered by outsiders, we assume settlements with outsiders are confidential.

not invest to improve the quality of their product. The focus is on equilibria in which bad types mix between submitting and not submitting to the assessment, whereas good types always submit. Section 3 incorporates the possibility of investments in quality and derives the condition under which welfare is still higher with inaccurate assessments. Section 4 extends the model to include a triggering of an audit by an outsider. This extension corresponds most closely to a litigation game between a plaintiff and the defendant. Welfare results are derived, including the impact of accuracy on the plaintiff's decision to file suit and settle that suit. Section 5 concludes. The appendix contains proofs for each proposition in the main text.

2 Model Without Investment

2.1 Setup

The market is populated by two types of firms, good and bad. The proportion of good firms is π_g; the proportion of bad firms is π_b. Without loss of generality, we assume both firms have identical marginal costs of zero. The value of a good firm's product is V. The value of a bad firm's product is 0. However, at a cost of c, consumers can take a precaution that raises the net surplus from a bad product to $V - c$. Assume that $V > c$, so that precautions improve welfare when the product is from a bad firm. We assume that consumers do not know and firms cannot verifiably signal their types without an assessment from a third party.

Audits. Each firm can submit its product to a third party auditor. The auditor conducts a test and provides a verification score. For simplicity, the score takes one of two values: high or low. However this test can misfire. A good firm might receive a low score (a mistaken conviction) or a bad firm might receive a high score (a mistaken exoneration). Let

$$\begin{aligned} 1-a &= \Pr\{\text{bad type receives a high score}\} \\ 1-b &= \Pr\{\text{good type receives a low score}\} \end{aligned}$$

In the statistics literature, a is known as the sensitivity of the test and b is the specificity of the test. Without loss of generality, we focus on audits that are minimally information, that is, $a > \frac{1}{2}$ and $b > \frac{1}{2}$. If $a = b = 1$, the audit is perfectly accurate. Firms cannot verifiably signal that they sought an audit until they receive a score. We assume that all parties observe the results of the audit and that the parameters a and b are common knowledge.

Audits are costless for firms and can be made more accurate at zero cost. The deck is thus stacked in favor of a system of perfectly accurate auditing by assuming away the usual reasons given for imperfect accuracy. We replace the assumption that improving accuracy is costly with the assumption that the auditor has limited capacity. Specifically, suppose that resources K are devoted to auditing. The probability of an audit is the ratio, $s = K/M$, where M is the number of firms seeking an audit. The larger the number of firms that seek an audit, the lower the probability that any specific firm receives one.

This probability of an audit is open to two interpretation. First, a capacity constrained auditor might simply turn down a request for an audit because it can't handle more business. Second, imagine that all firms that want an audit get one, but the timing is uncertain. Under that interpretation, the probability of the audit refers to the probability the firm's audit score is released on a given day. Either way, the implication is that, at any point in time, consumers cannot tell the difference between a firm that has attempted to get an audit and not received one and a firm that has refused auditing altogether.

Consumer purchases. Consumers make purchasing decisions based on the results of the auditing process and the equilibrium strategies of the firms. Given the capacity constraint, consumers receive one of three signals: a high score, a low score, or no score. Let τ be the consumer's belief the firm is bad if he observes a high score; let λ be the consumer's belief the firm is bad if he observes low score; and let μ be the consumer's belief that a firm is a bad type if he observes no score.

Consumer beliefs determine what price they are willing to pay for the product and whether they take precautions when using the product. If consumers do not take precautions, they obtain the expected value of the good, given the firm's score and their beliefs about firm type. This value equals the consumer's belief that the firm is a good type times V. If consumers take precautions, they get $V - c$ whether the firm turns out to be good or bad. That cost is well spent if the firm is a bad type, but wasted if the firm is a good type.[4] Consumers will take precautions if

$$V - c > (1 - \beta) V \text{ or } c \leq \beta V$$

where $\beta \in \{\tau, \lambda, \mu\}$ is the consumer's posterior belief that the firm is a bad type.

Firms make take-it-or-leave-it offers that extract all the consumer surplus. The price firms charge anticipates the precaution taken by the consumer. If it is in the consumer's interest to take a precautions in a particular state of the world, the firm charges $V - c$. If it is not in the consumer's interest to take a precaution in a specific state of the world, the firm charges $(1 - \beta) V$, where again $\beta \in \{\tau, \lambda, \mu\}$. Combining the firm's take it or leave it offer with the consumer's rational deployment of precautions, prices can be written as:

$$\begin{aligned} P^H &= \max\{V - c, (1 - \tau) V\} = V - \min\{c, \tau V\} \\ P^M &= \max\{V - c, (1 - \mu) V\} = V - \min\{c, \mu V\} \\ P^L &= \max\{V - c, (1 - \lambda) V\} = V - \min\{c, \lambda V\} \end{aligned}$$

P^H is the price following a high score; P^L is the price following a low score; and P^M is the price following no score.

The timing of the game follows:

1. Nature selects the initial distribution of types.
2. Firms decide whether to submit to an audit.
3. Some fraction of firms submitting to the audit are reviewed and their assets verified.
4. Firms make consumers a take it or leave it offer.

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[4] In the precaution case, the price following a unfavorable score is constant. This simplification eases the notation. If consumer precautions resulted in say $V - \lambda c$ instead of V after a finding of poor quality, price would fall as the consumer's beliefs become more pessimistic. However, our results would not change. Although there is a constant price following an unfavorable audit, the welfare effects of consumer precautions depend on the precision of the beliefs. More specifically, as the precision of the beliefs following an unfavorable score increase, the chance of consumers misfiring – spending resources on precautions when the firm is, in fact, a good type – goes down.

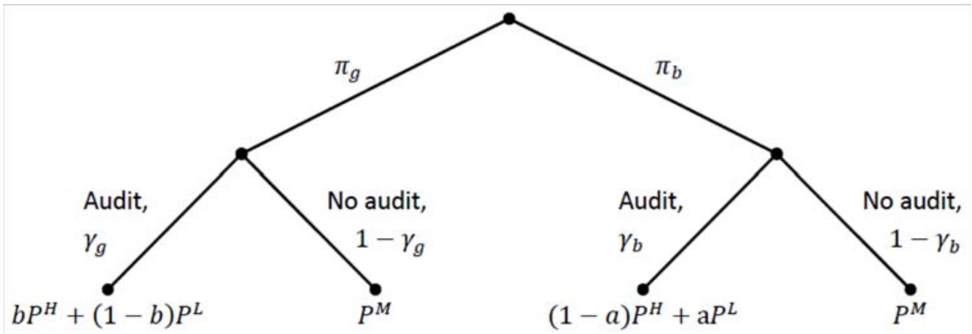

Figure 1: Timing and payoffs.

5. Consumer decide how much to pay for the product and whether to take precautions.

Figure 1 illustrates the moves in the game.

In this game, each firm type chooses the probability of submitting to an audit, denoted by $\gamma_b, \gamma_g \in [0,1]$. A perfect Bayesian equilibrium consists of a set of consumer beliefs β and a strategy profile, $\{\gamma_b, \gamma_g\}$ such that:

(a) no firm type can deviate given the consistent consumer beliefs and the equilibrium strategy of the other firm type and

(b) where possible, beliefs are derived using Bayes rule from the equilibrium strategies and the error rate of the audit technology.

Before analyzing the equilibria and conducting comparative statics, we make the following assumption about the cost of precautions to simplify exposition and focus analysis on interesting equilibria:

(A1) $c > (2\pi_b - \pi_b^2)\, V$

This assumption implies that consumers take precautions, if at all, only after they observe a low score.[5] If consumers took precautions even without a low score, bad

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[5] Consumers take precaution even without observing a low score if $c < \mu V$. In the appendix, we show there are no equilibria in which bad types submit to an audit but good types do not. Thus the upper bound on μ if consumers have consistent beliefs is π_b. Under assumption (A1), however, $c > \pi_b V$ because $2\pi_b - \pi_b^2 > \pi_b$.

types would have no reason to avoid audit. Equilibria in which bad types always and voluntarily seek an audit are empirically not common. Our assumption on c is stronger than required any of our propositions. We offer relaxed but somewhat more complicated sufficient conditions for the propositions in the appendix.

2.2 Equilibria and Selection

To compare welfare under perfect and imperfect audits, the first task is to identify equilibria in both scenarios. The next proposition presents the equilibrium under perfect audits. Following that, attention turns to audits with mistakes.

Proposition 1 *With perfect auditing, there always exists a separating equilibrium where good firms submit to audits and bad firms do not. Formally, we have* $(\gamma_g = 1, \gamma_b = 0)$ *and* $(\tau = 0, \lambda = 1, \mu = \pi_b / [\pi_b + (1 - \pi_b)(1 - s)])$.

Proposition (1) provides benchmark for the welfare analysis. With perfect audits, good types always submit to an audit because they are guaranteed a high score. Bad types never submit because the audit always gives them a low score. By not submitting, the bad type cloaks itself with the good firms that submitted to the audit, but did not receive one due to capacity constraints.

The capacity constraint plays a critical role here. Absent this constraint, every good firm would submit to the audit and receive a high score. The consumers would rationally infer from the absence of a score for a firm that the firm must be bad. In short, without this constraint, the market unravels a la Grossman and Hart (1980).

When mistakes happen in the auditing process, a semi-separating equilibrium exists in which the good type submits to an audit and the bad type randomizes between submitting and not submitting. For bad types to mix, they must be indifferent between audit and no audit. Recall that asking for an audit only yields an actual audit with probability s given the auditor's limited capacity. Thus the indifference condition is

$$s[(1-a)P^H + aP^L] + (1-s)P^M = P^M$$

which can be written as

$$(1-a)P^H + aP^L = P^M \tag{1}$$

Firms' strategies and consumers' consistent beliefs are embedded in the prices charged. The next proposition formally characterizes the semi-separating equilibrium.

Proposition 2 *If auditing is imperfect and assumption (A1) is satisfied, there exists a semi-separating equilibrium where good firms always submit to audits and bad firms randomize between submitting and not submitting. Formally, for any* $(a,b) \in \Theta = (1/2,1) \times (1/2,1)$*, we have* $\left(\gamma_g = 1, \gamma_b = \gamma_b^*\right)$ *and*

$$\tau = \frac{\pi_b \gamma_b^* (1-a)}{\pi_b \gamma_b^* (1-a) + \pi_g b} \qquad \lambda = \frac{\pi_b \gamma_b^* a}{\pi_b \gamma_b^* a + \pi_g (1-b)} \tag{2}$$

$$\mu = \frac{\pi_b (1 - s\gamma_b^*)}{\pi_b (1 - s\gamma_b^*) + \pi_g (1-s)}$$

where γ_b^* *is the solution to equation (1).*

The upside to the audit for the bad type is that they may receive a high score from an error-prone auditor. The high score yields the firm higher revenue as consumers incorrectly infer good attributes to a firm which is, in fact, bad. The downside to an audit is that the firm may receive a correct, low score. Bad types randomize for a chance at the upside. They do not submit with probability one because the upside diminishes as more bad types submit to an error-prone audit. Consumer inferences after a high score depend on the probability of error (a and b) and the probability bad firms submit to an audit (γ_b). As the latter rises, consumers pay less even after a high score. Eventually this revenue boost (which is declining) fails to offset the downside risk of the audit.

Since there is a semi-separating equilibrium for each value of $a, b \in \Theta$, we can explore how improving accuracy affects the probability bad types submit to an audit.

Proposition 3 *In the semi-separating equilibrium of Proposition 2, the probability that bad firms submit to an audit (1) decreases as the sensitivity (a) of an audit* *increases; and (2) increases as the specificity (b) of an audit increases.*

Raising b, the probability that a good type receives a high score, makes the consumer more confident that a high score signals a good type and a low score signals a bad type. As the high score price premium grows, the payoff to a bad type from an accidental high score increases. Thus higher b encourages bad types to seek an audit.

In contrast, raising a, the probability that a bad type receives a low score, has competing effects. The first effect is to reduce the fraction of bad types in among firms that receive a high score, making consumers more confident that a high score signals a good type and a low score a bad type. The resulting increase in the high score price premium encourages bad types to submit to an audit. The second effect, however, is to reduce the probability that bad types will accidentally receive a high score and thus the high score price. Assumption (A1) ensures the second effect more offsets the first and so increasing a discourages bad types from obtaining audits.

2.3 Welfare

Consumers gain from the information provided by the auditor if the information allows them to target precautions at bad types. With perfect auditors, no bad type ever gets an audit, no low scores are posted, and, given Assumption (A1), consumers never employ welfare-improving precautions. Imperfect audits change this dynamic. Some bad types submit, low scores are reported and consumer may take precautions following those reports.

As noted above, prior economics literature suggests the optimal amount of accuracy balances the benefit of a more accurate assessor against the financial and administrative costs of improving accuracy. Because of those costs, perfectly accurate auditors are not optimal. Our model assumes the cost of improving accuracy is zero. Yet perfectly accurate audits remain suboptimal, as the next proposition demonstrates.

Proposition 4 *If consumers take precautions following a low score, the welfare associated with an imperfect auditor (and the semi-separating equilibrium of Propo-*

sition 2) exceeds the welfare associated with a perfect auditor (and the separating equilibrium of Proposition 1).

As previously discussed, inaccurate assessments attract some bad types into the audit pool. Some of those firms are correctly identified by the audit and so consumers will occasionally take efficient precautions against bad types. Of course, assessments may misfire and give good types a low score. In that case, consumers waste precautions on good types. Consumers, however, only take precautions when their expected value is positive after accounting for the possibility of wasted precautions. As a result, inaccurate auditors improves welfare over perfect auditors.

The prospect of mistaken exonerations drives the result in Proposition 4. To see this, write expected welfare with inaccurate audits as

$$W = \pi_g V + \pi_b s \gamma a [V - c] - \pi_g s(1 - b)c \tag{3}$$

A reduction in mistaken convictions – an increase in b – reduces the last term. The last term reflects the costs for wasting precautions on good types. An increase in b also increases the second term because it increases the proportion γ of bad types getting audited according to Proposition 3. Together, these two effects imply that reducing mistaken convictions always increases welfare.

The same cannot be said for reductions in mistaken exonerations. An increase in a improves targeting of precaution after audits, i.e., the probability that a low score predicts a bad type. But it also reduces the probability γ that bad types submit to an audit. In short, whereas the optimal amount of mistaken exonerations is greater than zero, the optimal amount of mistaken convictions is zero

Philosophers, politicians and legal scholars have long suggested that greater effort be devoted to preventing mistaken convictions than to preventing mistaken exonerations. Proposition 4 suggests that this Blackstonian principle applies not just to involuntary audits in criminal law but to any system of voluntary third party assessments. The justification for asymmetric treatment of errors in voluntary audits need not rely on arguments about the "wrongfulness" of imposing an undeserved punishment. Instead, the justification can be that asymmetric treat-

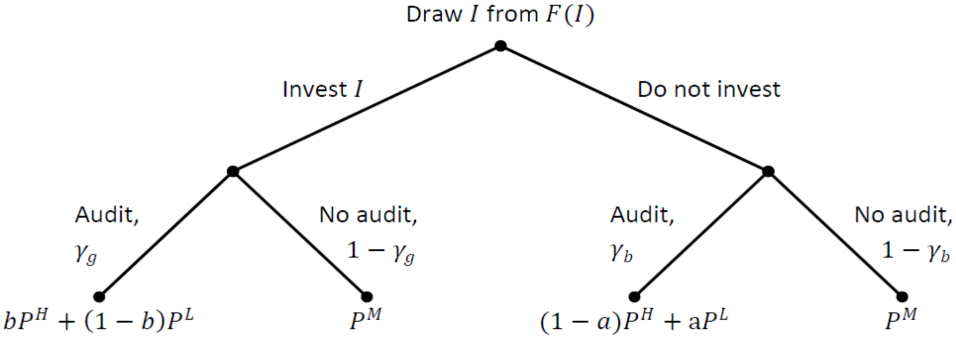

Figure 2: Timing and payoffs with investment

ment induces more audits of bad people and thus provides more information to consumers.

3 Producer Investment

The prior section assumed that firms were either good or bad and could do nothing to change that fact. This section allows firms to invest in improving the quality of their products, i.e., transform from a bad type to a good type. We demonstrate that an increase in the sensitivity (a) of the audit increases investment in quality. Despite this additional benefit from accuracy, under some conditions, the welfare associated with imperfect auditors still exceeds the welfare associated with perfect auditors.

To introduce investment, we alter the game by replacing nature's move in time 1 with a decision by a firm whether to invest to become a good type. Figure 2 illustrates the revised game. Assume that the cost of investment I is a random variable, ranging from 0 to ∞, and distributed according to $F(I)$.

The benchmark is again the case of a perfect auditor. Proposition A1 in the appendix shows that there still exists a separating equilibrium in which only good types submit to the audit and the bad types do not. The payoff to becoming a good type is thus $sV+(1-s)(1-\mu)V-I$, where s is the probability of receiving an audit given capacity constraints and μ is the posterior following no score. The payoff to remaining a bad type is $(1-\mu)V$. The investment level associated with a perfectly accurate auditor is therefore the solution to

$$sV + (1 - s)(1 - \mu)V - I = (1 - \mu)V \tag{4}$$

Define I^o as this solution. Notably, not all firms invest invest even when the auditing process is perfect.

Lemma 5 *With perfect audits,* $0 < I^o < \infty$.

The reason that all firms do not become good types is that the probability that the firm is a good type is capped at one. As the fraction of good types approaches one, the amount upward by which consumers revise their posteriors about the probability of observing a good type diminishes. As a result, the price premium to being a good type falls. By contrast, the cost of investment increases as firms with higher and higher costs attempt to transform themselves into good types. Eventually, the cost of investment swamp the benefits from becoming a good type.

Next we consider the case of imperfect audits. Proposition A2 in the appendix demonstrates that there still exists a semi-separating equilibrium in which the good type always submits to an audit and bad types mix. The payoff to becoming a good type is thus

$$s\left[bP^H + (1 - b)P^L\right] + (1 - s)P^M - I \tag{5}$$

The first term is the good type's expected payoff if the audit is conducted. The second term is the payoff if the audit is not conducted due to the capacity constraint. The last term is the cost of investment. The payoff to remaining a bad type is

$$s\gamma[aP^L + (1 - a)P^H] + s(1 - \gamma)P^M + (1 - s)P^M \tag{6}$$

The first two terms are the payoff if the bad type submits to an audit, which it does with probability γ. The last term is the payoff when it does not. Let I^* be the investment level of the firm that is just indifferent between becoming a good type and remaining a bad type, i.e., where (5) equals (6). Of course this depends on the bad type's mixing probability. That probability is set so that the bad type is indifferent between submitting to an audit and not, as described in equation (1). In equilibrium, both the indifference conditions for investment and bad type mixing must hold.

The main result from the law and economics literature is that, because accuracy increases the gap between the payoff to good behavior and bad behavior, it deters bad behavior (Kaplow 1994; Kaplow and Shavell 1994). The same result obtains in this model with respect to changes in the chance of mistaken exonerations. The deterrence implications of reducing mistaken convictions are not as clear.

Proposition 6 *Assuming some stability conditions, if consumers take precautions after an unfavorable audit, (i) an increase in a increases deterrence (that is,* $\frac{\partial I^*}{\partial a} > 0$*) and (ii) an increase in b has an ambiguous effect on deterrence (that is,* $\frac{\partial I^*}{\partial b} > 0$ *or* $\frac{\partial I^*}{\partial b} < 0$*).*

If one reduces mistaken exonerations, fewer bad types submit to the audit because they are less likely to receive a favorable score. This induces a decrease in the no-score price, which is based on the pool of firms who do not receive an audit score and now includes more bad firms. To maintain the indifference between the submitting and not, the no-score price must reflect the payoff to the bad firm no matter their submission choice. And so, reducing mistaken exonerations reduces the payoff to being a bad type whether they hide by not submitting or "risk it" and submit. In so doing, this change in the error probability increases the gap between the payoff to the good and bad types. Since this gap determines investment in quality, investment rises.

Eliminating mistaken convictions, by contrast, does not necessarily have a similar, salutary effect. Under the same logic as Propostion 3, reducing this type of error increases the proportion of bad types getting the audit. Now there are fewer bad types in the pool of firms who do not receive an audit score, which increases the market price. As a result, the payoff to being a bad type increases. At the same time, the payoff to being a good type also increases because good types are more likely to receive a high score. Since the payoffs to both types increases the impact of a change in this error on the gap between the two is uncertain.

With these comparative statics in hand, we look at welfare. Welfare now depends not just on whether consumers invest in precautions when purchasing from a

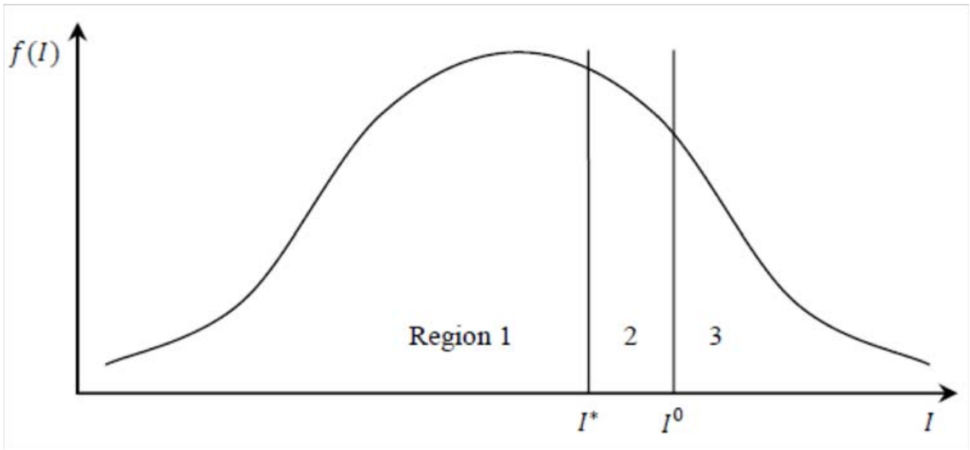

Figure 3: Distribution of investment costs among firms

bad type, but also on how many firms invest in quality. Perfect audits yield a higher amount of investment by firms but no precautions by consumers. Some degree of inaccuracy diminishes the amount of investment by firms but facilitates greater consumer precaution. Which is better? Whether inaccuracy improves welfare depends on two factors: (1) the effectiveness of consumer precautions compared to producer precautions and (2) the fraction of firms that fail to invest in quality when audits are perfect. The next proposition lays out the formal version of this inequality.

Proposition 7 *Consider a semi-separating equilibrium with errors $a < 1, b = 1$, mixing by the bad types of γ^*, and investment I^*. The welfare associated with this equilibrium exceeds the welfare associated with perfect auditors whenever*

$$\left(1 - F\left(I^0\right)\right) s\gamma a \left(V - c\right) > \int_{I^*}^{I^0} \left(V - t\right) f\left(t\right) dt - [F\left(I^0\right) - F(I^*)] s\gamma^* a \left(V - c\right)$$

To understand the condition under which imperfect auditors improve welfare, look at Figure 3, which plots the distribution of investment costs I. Remember that I^* is the investment level associated with the imperfect auditor whereas $I^o > I^*$ is the investment level associated with the perfect auditor. We split the possible values of I into three regions. The first region is investment costs less than I^*.

If the firm draws a cost in this region, it invests in quality whether the audit is perfect or not. The welfare implications of perfect audits and imperfect audits are the same over this region.

The second region is between I^* and I^o. Here accuracy matters. With perfectly accurate audits, the firm makes the investment. The expected value of this investment is $\int_{I^*}^{I^0} (V - t) f(t)\, dt$. With imperfect audits, the firm does not make the investment, and remains a bad type. Yet all is not lost. Given imperfect audits, some fraction of these bad types submit and are found out, which facilitates the deployment of consumer precautions. The expected value of consumer precautions over this range is $[F(I^0) - F(I^*)]s\gamma a (V - c)$. The "net" benefit of perfect audits over this range is the difference between the expected value of producer precautions and the expected value of consumer precautions, discounting this latter value by the probability they are deployed.

Finally, there is the range of investments above I^o. If a firm draws an investment cost in this region, it never makes the investment. Welfare is thus zero with perfect audits. But imperfect audits still trigger consumer precautions, with welfare benefits equal to $(1 - F(I^0))\, s\gamma a (V - c)$. If the welfare gains over region 3 exceed the welfare gains from perfect audits over region 2, imperfect audits increase welfare.

The two factors discussed above determine whether this inequality holds. Region 3 is the fraction of firms that fail to invest with perfect audits. The lower is optimal investment with perfect courts, the higher the gains from imperfect audits because this region is larger. Moreover, the gains from imperfect audits over this region depend on $(V - c)$: the bigger the gains from consumer precautions, the greater the value of imperfect audits. Finally, the "net" benefit over region 2 depends the relative efficiency of producer investments in quality and consumers investments in quality: the more efficient consumers are relative to producers, the smaller is the net benefit from perfect audits in this range.

4 Third Party Triggering

[Note to readers: This section will expand the model to allow an outsider to trigger an audit as, e.g., when a plaintiff sues the producer and a court acts as the auditor. It will examine how accuracy effects the frequency of suit and settlement decisions, showing that inaccurate courts are better for welfare than perfectly accurate courts.]

5 Conclusion

This paper examines mistakes by third party auditors that are capacity constrained, a common problem in the real world. To improve welfare, we want firms to get audits and, conditional on an audit, the result to be accurate. In this paper we show there is a conflict between these two objectives. This conflict implies that mistakes do not necessarily reduce welfare, especially when it is more important to identify which firms are bad than which firms are good. The results do not imply that wholly uninformative auditors are ideal. Rather, they suggest that some degree of imperfection should be tolerated in audits in order to induce some bad firms to reveal themselves via audits. Moreover the imperfection should be of a particular type: mistaken exonerations induce firms to obtain audits but mistaken convictions do not. In other words, if the null hypothesis is that a firm is a bad type, then welfare is maximized by tolerating some Type I error but no Type II error. This is true, even when firms have the ability to make investments to change their type and improve their quality and when outsiders are able to trigger audits.

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6 Appendix

Proof of Proposition 1. Given perfect auditing technology and the equilibrium strategies, the consistent beliefs are

$$\tau = 0 \quad \lambda = 1 \quad \mu = \frac{\pi_b}{\pi_b + (1-\pi_b)(1-s)}$$

For a good type, the expected payoff from submitting to the audit is

$$sV + (1-s)[V - \min\{c, \mu V\}]$$

Or

$$V - \min\{c, \mu V\} + s \min\{c, \mu V\}$$

The payoff from deviating is

$$V - \min\{c, \mu V\}$$

which is less. For a bad type the payoff from not submitting is $V - \min\{c, \mu V\}$. The payoff from submitting is $V - c$, which is strictly lower if $c > \mu V$. The payoff to deviating is the same if $c \leq \mu V$. In either case, the bad type has no profitable deviation. ■

Proof of Proposition 2. Consider the equilibrium where good types always submit. It is optimal for the bad type to mix with probability γ_b if there is a value of $\gamma \in (0,1)$ which induces indifference for all $(a,b) \in \Theta = (1/2,1) \times (1/2,1)$. If we plug prices in equation (1) and rearrange, we can write the indifference condition as

$$h(\gamma) = \min\{c, \mu V\} - a \min\{c, \lambda V\} - (1-a) \min\{c, \tau V\} = 0$$

Under (A1), this becomes

$$h(\gamma) = \mu V - a \min\{c, \lambda V\} - (1-a)\tau V = 0$$

We will demonstrate that there exists some $\gamma \in (0,1)$ for which $h(\gamma) = 0$ in three steps. First, we'll show that $h(0) > 0$ for all $(a,b) \in \Theta$. Second, we'll show that $h(1) < 0$ for all $(a,b) \in \Theta$. The proof is completed by showing that $dh_b/d\gamma < 0$ at fixed (a,b) and hence a fixed point with a unique value of $\gamma \in (0,1)$ must exist.

Step 1. The result follows from

$$h(0) = \frac{\pi_b}{\pi_b + \pi_g(1-s)} V > 0$$

Step 2. For any value of γ, a minimally informative audit impoies that $\tau < \lambda$. We will use this ordering throughout the remainder of the proof. We will also use the following derivatives

$$\frac{\partial \mu}{\partial \gamma} = -\mu\left(1-\mu\right)\frac{s}{1-s\gamma} < 0$$

$$\frac{\partial \tau}{\partial \gamma} = \frac{\tau\left(1-\tau\right)}{\gamma} > 0 \quad \frac{\partial \tau}{\partial a} = -\frac{\tau(1-\tau)}{1-a} < 0 \quad \frac{\partial \tau}{\partial b} = -\frac{\tau(1-\tau)}{b} < 0$$

$$\frac{d\lambda}{d\gamma} = \frac{\lambda\left(1-\lambda\right)}{\gamma} > 0 \quad \frac{d\lambda}{da} = \frac{\lambda\left(1-\lambda\right)}{a} > 0 \quad \frac{d\lambda}{db} = \frac{\lambda\left(1-\lambda\right)}{1-b} > 0$$

With these expressions, we the derivative of $h(1)$ with respect to a may be written

$$\begin{array}{ll} \frac{dh(1)}{da} = -c + \tau(1)V + \tau(1)(1-\tau(1))V < 0 & \text{if } \tau(1)V \leq c < \lambda(1)V \\ \frac{dh(1)}{da} = (2\tau - \tau^2) - (2\lambda - \lambda^2) < 0 & \text{if } c > \lambda(1)V \end{array}$$

which is always negative. This relationship implies that, for any value of b, $h(1)$ takes on its largest value in Θ at $a = \frac{1}{2}$.

The derivative of $h(1)$ with respect to b is

$$\begin{array}{ll} \frac{dh(1)}{db} = (1-a)\frac{\tau(1-\tau)}{b} > 0 & \text{if } \tau(1)V \leq c < \lambda(1)V \\ \frac{dh(1)}{db} = a\frac{\lambda(1-\lambda)}{b} + (1-a)\frac{\tau(1-\tau)}{b} > 0 & \text{if } c > \lambda(1)V \end{array}$$

which is always positive. This relationship implies that, for any value of a, $h(1)$ takes on its largest value in Θ at $b = 1$.

Evaluated at $a = 1/2$ and $b = 1$, we have

$$h(1) = \mu(1)V - \frac{1}{2}c - \frac{1}{2}\tau(1)V$$

At this (a, b) combination, $\mu(1) = \pi_b$, $\lambda(1) = 1$, $\tau(1) = \pi_b/(2 - \pi_b)$. Substituting these into the equation above implies that $h(1) < 0$ if

$$c > \left(2\pi_b - \frac{\pi_b}{2 - \pi_b}\right)V$$

This condition is satisfied by assumption (A1). To see this, note that $(x-1)^2 > 0$ for any x. If we expand the left-hand side, we get $1 > 2x - x^2$ or $x/(2-x) > x^2$. Replacing x with π_b yields

$$\frac{\pi_b}{2 - \pi_b} > \pi_b^2$$

Thus,

$$c > \left(2\pi_b - \pi_b^2\right)V \Rightarrow c > \left(2\pi_b - \frac{\pi_b}{2 - \pi_b}\right)V$$

Step 3. Recall that $s = K/M$, where M is the number of firms seeking an audit. In this equilibrium, $M = \pi_g + \gamma\pi_b$, so $ds/d\gamma < 0$. A few relationships will be helpful in finishing the proof.

$$\frac{d\lambda}{d\gamma} = \frac{\lambda(1-\lambda)}{\gamma} > 0 \quad \frac{d\tau}{d\gamma} = \frac{\tau(1-\tau)}{\gamma} > 0 \quad \frac{d\mu}{d\gamma} = \frac{\pi_b(1-s)\gamma\pi_g\frac{ds}{d\gamma}}{\left(\pi_b(1 - s\gamma_b^*) + \pi_g(1-s)\right)^2} < 0$$

Using these derivatives, we see that

$$\begin{array}{lll} \frac{dh(\gamma)}{d\gamma} & = \frac{d\mu(\gamma)}{d\gamma} - [1-a]\frac{d\tau(\gamma)}{d\gamma} < 0 & \text{if } \mu(\gamma)V \leq c < \lambda(\gamma)V \\ \frac{dh(\gamma)}{d\gamma} & = \frac{d\mu(\gamma)}{d\gamma} - (1-a)\frac{d\tau(\gamma)}{d\gamma} - a\frac{d\lambda(\gamma)}{d\gamma} < 0 & \text{if } \lambda(\gamma)V < c \end{array}$$

For every value of c compatible with assumption (A1), $dh/d\gamma < 0$. ■

Proof of Proposition 3. Denote the bad firm's mixing probability simply by γ. Consistent beliefs are

$$\tau = \frac{\pi_b\gamma_b^*(1-a)}{\pi_b\gamma_b^*(1-a) + \pi_g b} \qquad \lambda = \frac{\pi_b\gamma_b^* a}{\pi_b\gamma_b^* a + \pi_g(1-b)}$$

$$\mu = \frac{\pi_b(1-s\gamma_b^*)}{\pi_b(1-s\gamma_b^*)+\pi_g(1-s)}$$

In equilibrium, after observing a low score, the consumer either (A) takes no precaution or (B) takes precaution. In either case, we show that the sign of the comparative statics are the same for changes in a and different for changes in b

Case A: No Precautions. (1) The bad type's indifference equation is

$$h(\gamma) = \mu V - a\lambda V - (1-a)\tau V = 0$$

Taking the derivative with respect to a yields

$$\frac{\partial \mu}{\partial \gamma}\frac{\partial \gamma}{\partial a}V - (1-a)\frac{\partial \tau}{\partial \gamma}\frac{\partial \gamma}{\partial a}V - (1-a)\frac{\partial \tau}{\partial a}V + \tau V - \lambda V - a\frac{\partial \lambda}{\partial \gamma}\frac{\partial \gamma}{\partial a}V - a\frac{\partial \lambda}{\partial a}V = 0$$

Solving for $d\gamma/da$ we get

$$\frac{\partial \gamma}{\partial a} = \frac{\lambda V - \tau V + a\frac{\partial \lambda}{\partial a}V + (1-a)\frac{\partial \tau}{\partial a}V}{\frac{\partial \mu}{\partial \gamma}V - (1-a)\frac{\partial \tau}{\partial \gamma}V - a\frac{\partial \lambda}{\partial \gamma}}$$

The denominator is negative since $\partial\mu/\partial\gamma < 0$, $\partial\lambda/\partial\gamma > 0$, and $\partial\tau/\partial\gamma > 0$. Define the numerator as N. We have *sign* $\partial\gamma/\partial a = -N$. Plugging in for $\partial\tau/\partial a$ and $\partial\lambda/\partial a$ yields

$$N = \lambda V - \tau V - \tau(1-\tau)V + \lambda(1-\lambda)V$$

$$\Leftrightarrow N = (2\lambda - \lambda^2) - (2\tau - \tau^2)$$

This is positive since $\lambda > \tau$ when audits are minimally informative. Thus, $d\gamma/da < 0$.

(2) Taking the derivative of the indifference equation with respect to b yields

$$\frac{\partial \mu}{\partial \gamma}\frac{\partial \gamma}{\partial b}V - (1-a)\frac{\partial \tau}{\partial \gamma}\frac{\partial \gamma}{\partial b}V - (1-a)\frac{\partial \tau}{\partial b}V - a\frac{\partial \lambda}{\partial \gamma}\frac{\partial \gamma}{\partial b}V - a\frac{\partial \lambda}{\partial b}V = 0$$

Solve for $d\gamma/db$ we get

$$\frac{\partial \gamma}{\partial b} = \frac{a\frac{\partial \lambda}{\partial b}V + (1-a)\frac{\partial \tau}{\partial b}V}{\frac{\partial \mu}{\partial \gamma}V - (1-a)\frac{\partial \tau}{\partial \gamma}V - a\frac{\partial \lambda}{\partial \gamma}}$$

The denominator is negative. Plug in for $\partial\lambda/\partial b$ and $\partial\tau/\partial b$ in the numerator yields

$$N = -(1-a)\frac{\tau(1-\tau)}{b}V + a\frac{\lambda(1-\lambda)}{(1-b)}V$$

which is always positive. As a result, $d\gamma/db < 0$.

Case 2: Precautions After a Low Score. (1) In this case, the indifference equation is

$$\mu V - ac - (1-a)\tau V = 0$$

Take the derivative with respect to a

$$\frac{\partial\mu}{\partial\gamma}\frac{\partial\gamma}{\partial a}V - (1-a)\frac{\partial\tau}{\partial\gamma}\frac{\partial\gamma}{\partial a}V - (1-a)\frac{\partial\tau}{\partial a}V - c + \tau V = 0$$

Solving for $d\gamma/da$ we get

$$\frac{\partial\gamma}{\partial a} = \frac{c - \tau V - \tau(1-\tau)V}{\frac{\partial\mu}{\partial\gamma}V - (1-a)\frac{\partial\tau}{\partial\gamma}V}$$

The denominator is negative since $\partial\mu/\partial\gamma < 0$ and $\partial\tau/\partial\gamma > 0$. Define the numerator as N. We have *sign* $\partial\gamma/\partial a = -N$. Plugging in for $\partial\tau/\partial a$ yields

$$N = c - \tau V - \tau(1-\tau)V$$

In Proposition 2, we demonstrated that assumption (A1) implies $c > f(\pi_b)V$ where $f(x) = 2x - x^2$. Since $f'(x) > 0$ for $x < 1$ and $\pi_b > \tau$ under any separating equilibrium described in Proposition 2, the assumption also implies $N > 0$.

(2) Taking the derivative of the indifference equation with respect to b and solving for $\partial\gamma/\partial b$ yields

$$\frac{\partial\gamma}{\partial b} = \frac{-(1-a)\frac{\tau(1-\tau)}{b}V}{\left[\frac{\partial\mu}{\partial\gamma}V - (1-a)\frac{\partial\tau}{\partial\gamma}V\right]}$$

The numerator is negative. As a result, $\partial\gamma/\partial b > 0$. ■

Proof of Proposition 4. With perfect audits good types submit to an audit and bad types do not. Expected welfare is

$$W^{\text{Perfect}} = \pi_g V$$

With imperfect audits and consumer precautions following a low score, expected welfare is

$$W = \pi_g \left[(1-s) + sb\right] V + \pi_g s(1-b)\left(V - c\right) + \pi_b s \gamma_b^* a (V - c)$$

$$\Leftrightarrow W = W^{\text{Perfect Audits}} + \pi_b s \gamma_b^* a V - \pi_b s \gamma_b^* a c - \pi_g s(1-b)c \tag{7}$$

Since we assume consumer precautions after a low score, we have $\lambda V > c$. Plugging λ from (2) into this inequality and rearranging yields

$$as\gamma_b^* \pi_b V > as\gamma_b^* \pi_b c + \pi_g s(1-b)c$$

Thus the sum of the last three terms in equation (**??**) must be positive. So, $W > W^{\text{Perfect Audits}}$. ■

Proof of Proposition A1. TBD ■

Proof of Proposition A2. TBD ■

Proof of Lemma 5. Rearranging equation (4) gives the following:

$$sV - s(1-\mu)V - I = 0$$

or

$$s\mu V - I = 0 \tag{8}$$

When there are perfect audits, the posterior in the no signal state is

$$\mu = \frac{1 - F(I)}{(1 - F(I)) + F(I)(1-s)}$$

Here $s = \frac{K}{F(I)}$ where s can take on a maximum value of 1. Rewrite equation (**??**) as

$$\frac{K}{F(I)} \left[\frac{1 - F(I)}{(1 - F(I)) + F(I)(1 - \frac{K}{F(I)})} \right] V - I = 0$$

or

$$\frac{K}{F(I)}\left[\frac{1-F(I)}{(1-K)}\right]V-I=0$$

This equation can be written as

$$g(I)=\frac{K}{F(I)}\frac{1}{(1-K)}V-\frac{K}{1-K}V-I=0$$

Notice that $g(\infty)=-\infty$. Further consider $g(\varepsilon)$, where ε is a small positive number. At that point, we have

$$g(\varepsilon)=\frac{K}{F(\varepsilon)}\frac{1}{(1-K)}V-\frac{K}{1-K}V-\varepsilon=0$$

As $\varepsilon\to 0$, we know that $F(\varepsilon)\to 0$, which implies that $\frac{K}{F(\varepsilon)}\to 1$ (because the probability of an audit is bounded at 1). And so, we have

$$g(\varepsilon)=V>0$$

Finally, notice that $g'(I)=-\frac{K}{F(I)^2}\frac{1}{(1-K)}f(I)V-1<0$. As a result, there must be a $0<I^o<\infty$ where $g(I^o)=0$. ■

Proof of Proposition 6. Two equations jointly determine the investment level and the bad type's mixing probability. Those two equations are

$$s\left[bP^H+(1-b)P^L\right]+(1-s)P^M-I=\gamma s[aP^L+(1-a)P^G]+(1-\gamma)P^M$$

$$s[aP^{UF}+(1-a)P^F]+(1-s)P^M=P^M$$

Totally differentiating the initial two equations with respect to a gives

$$H\left[\begin{array}{c}\frac{\partial I}{\partial a}\\ \frac{\partial\gamma}{\partial a}\end{array}\right]=\left[\begin{array}{c}\tau V-c+s(b+a-1)[\frac{\partial\tau}{\partial a}V]\\ c-\tau V+(1-a)\frac{\partial\tau}{\partial a}V\end{array}\right]$$

where $H = [r_1, r_2; r_3, r_4]$ and

$$\begin{aligned}
r_1 &= -s[b+a-1]\left(\frac{\partial \tau}{\partial I}\right)V + \frac{\partial s}{\partial I}[b+a-1][c-\tau V] - 1 \\
r_2 &= -s[b+a-1]\left(\frac{\partial \tau}{\partial \gamma}\right)V \\
r_3 &= \frac{\partial \mu}{\partial s}\frac{\partial s}{\partial I}V + \frac{\partial \mu}{\partial I}V - (1-a)\frac{\partial \tau}{\partial I}V \\
r_4 &= \frac{\partial \mu}{\partial \gamma}V - (1-a)\frac{\partial \tau}{\partial \gamma}V
\end{aligned}$$

In addition to the signs on the derivatives taken in previous proofs, we know

$$\frac{\partial \tau}{\partial I} = -\frac{\tau(1-\tau)f}{(1-F(I)} < 0 \qquad \frac{\partial s}{\partial I} = -\frac{K}{F(I)^2}f < 0$$

$$\frac{\partial \mu}{\partial s} = -\mu\,(1-\mu)\,\frac{\gamma}{1-s\gamma} < 0 \qquad \frac{\partial \mu}{\partial I} = -\frac{\mu(1-\mu)f}{(1-F(I)} < 0$$

[SCOTT: I think we can just assume an internal solution for I and γ. This requires that H be positive semi-definite.] Assume, as a stability condition, that $r_1 < 0$ (this just means that, for any firm, the gains in terms of a better price following investment are not worth the cost of investment plus the reduction in the chance of being audited). Also assume that $r_3 > 0$. The derivatives imply that $r_2 < 0$, $r_4 < 0$. As a result, the $\det H = r_1 r_4 - r_2 r_3$, which is positive.

Applying Cramer's rule to solve the system of equations, we have $sign\ \partial I/\partial a = |A|\,/\,|H|$ where

$$A = \begin{bmatrix} \tau V - c - s(b+a-1)\frac{\tau(1-\tau)}{1-a}V & r_2 \\ c - \tau V - \tau(1-\tau)V & r_4 \end{bmatrix}$$

We know that $\tau V - c - s(b+a-1)\frac{\tau(1-\tau)}{1-a}V < 0$. From the proof to Proposition 2, assumption (A1) guarantees that $c - \tau V - \tau(1-\tau)V > 0$. The determinant of A is thus

$$\left(\tau V - c - s(b+a-1)\frac{\tau(1-\tau)}{1-a}V\right) r_4 - \left(c - \tau V - \tau(1-\tau)V\right) r_2$$

which is positive. So, $\partial I/\partial a > 0$.

(ii) Totally differentiating with respect to b gives

$$H\left[\begin{array}{c} \frac{\partial I}{\partial b} \\ \frac{\partial \gamma}{\partial b} \end{array}\right] = \left[\begin{array}{c} \tau V - c + s(b+a-1)[\frac{\partial \tau}{\partial b}V] \\ -(1-a))\frac{\tau(1-\tau)}{b}V \end{array}\right]$$

Applying Cramer's rule, we have *sign* $\partial I/\partial b = |A|\,/\,|H|$ where

$$A = \left[\begin{array}{cc} \tau V - c - s(b+a-1)\left(\frac{\tau(1-\tau)}{b}V\right) & r_2 \\ -(1-a)\frac{\tau(1-\tau)}{b}V & r_4 \end{array}\right]$$

The determinant of A is

$$\left(\tau V - c - s(b+a-1)\frac{\tau(1-\tau)}{b}V\right) r_4 + \left((1-a)\frac{\tau(1-\tau)}{b}V\right) r_2$$

which has an ambiguous sign. ■

Proof of Proposition 7. Start with the inequality:

$$\int_{I^*}^{I^0} (V-t)\, f(t)\, dt > (1 - F(I^*))\, s\gamma^* a\, (V-c)$$

The benefit of perfect audits occurs with probability $F(I^0) - F(I^*)$. The benefit from imperfect audits occurs with probability $(1 - F(I^*))$, which must be larger than $F(I^0) - F(I^*)$. Conceptually, we get the benefit from imperfect audits over a larger range of the distribution. From the LHS add and subtract $F(I^0)s\gamma^* a\,(V-c)$, which gives us

$$\int_{I^*}^{I^0} (V-t)\, f(t)\, dt > \left(1 - F\left(I^0\right)\right) s\gamma^* a\, (V-c) + [F\left(I^0\right) - F(I^*)]s\gamma^* a\, (V-c)$$

Rearrange and we have

$$\int_{I^*}^{I^0} (V-t)\, f(t)\, dt - [F\left(I^0\right) - F(I^*)]s\gamma^* a\, (V-c) > \left(1 - F\left(I^0\right)\right) s\gamma^* a\, (V-c)$$

what this says is that the "net" gains from perfect audits if $I \in [I^*, I^0]$ must exceed the gains from imperfect audits if $I \in [I^0, 1]$. For any given $s\gamma^* a$, as $(V - c)$ gets bigger this is less and less likely to be true. The LHS is increasing in $(V - c)$ and the RHS is decreasing in $(V - c)$. ■