Bargaining and Dynamic Competition

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Color Figures

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Abstract

In many industries where technological features, such as scale economies or learning-by-doing, may allow a single firm to become dominant, buyers may not be price-takers in the way that the existing literature on dynamic oligopoly competition has assumed. We extend the dynamic "learning-by-doing and forgetting" model of Besanko, Doraszelski, Kryukov, and Satterthwaite (2010) to allow for prices to be set through Nash-in-Nash bargaining, with price-taking and the social planner solution nested as special cases. We show that changing what is assumed about the allocation of bargaining power can have significant and non-monotonic or non-linear effects on market structure, efficiency and the appropriate design of policies, as well as affecting whether multiple equilibria exist.

Keywords: dynamic competition, learning-by-doing, strategic buyers, bargaining power, buyer power, multiple equilibria.

JEL codes: C73, D21, D43, L13, L41.

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1 Introduction

Industries where some feature of demand or supply may allow a firm that achieves an initial advantage to attain a position of lasting market dominance often attract the attention of policy-makers. For example, digital services with network effects have been the focus of recent debates about whether new approaches to antitrust are required, and manufacturing industries with large learning-by-doing effects, such as the manufacture of semiconductors, solar panels and aircraft, have been affected by industrial, trade and national security policies. Even when government objectives are not economic, an understanding of dynamic competition is required to understand what the effects of policies will be.

Almost all existing theoretical or empirical models of dynamic competition, whether in industrial organization (for example, Fudenberg and Tirole (1983), Cabral and Riordan (1994), Benkard (2000), Besanko, Doraszelski, Kryukov, and Satterthwaite (2010), Besanko, Doraszelski, and Kryukov (2014)) or international trade (Dasgupta and Stiglitz (1988), Leahy and Neary (1999),Neary and Leahy (2000)) assume sellers unilaterally set prices or quantities. While sellers in these industries are often larger than buyers, in reality there is often some negotiation over prices especially for industrial products. A second recent literature has shown that allowing for bargaining over prices can matter for our understanding of both horizontal (Ho and Lee (2017), Gowrisankaran, Nevo, and Town (2015)) and vertical issues (Crawford, Lee, Whinston, and Yurukoglu (2018)) in static oligopoly models, using the so-called "Nash-in-Nash" framework (Collard-Wexler, Gowrisankaran, and Lee (2019)). In this framework, a bargaining power parameter captures how the surplus that a bilateral agreement will generate will be split.

We ask whether moving away from the price-setting assumption will significantly affect outcomes and policy conclusions in a model of dynamic competition.[1] Specifically, we extend the well-known duopoly seller "learning-by-doing (LBD) and forgetting" computational model of Besanko, Doraszelski, Kryukov, and Satterthwaite (2010) (BDKS) to allow

[1]We have not identified other attempts to directly address this question. Lee and Fong (2013) consider a dynamic network formation game with bargaining, and use it to consider the effects of hypothetical hospital mergers. In contrast to our paper, the dynamics in Lee and Fong (2013) result from stochastic changes in the network links, whereas, in our paper, dynamics result from supply-side primitives, and cost states which are directly affected by sales reflecting the prices that are negotiated.

for bargaining.

Assuming that there is an atomistic buyer every period, an assumption that we maintain, and that sellers set prices, BDKS compute the Markov Perfect Nash equilibria that exist for different parameters. They show that equilibria often exist where firms price aggressively when they are symmetric and that this can lead to sustained asymmetric market structures with firms at different levels of know-how. These equilibria often co-exist with equilibria where price competition is more accommodative and firms are likely to move down their cost curves roughly in parallel.

We replace the BDKS stage game where sellers set prices with a particular form of Nash-in-Nash bargaining over prices. The Nash bargaining power parameter allows us to vary how much bargaining power sellers have, nesting the BDKS assumption as well as the social planner problem as special cases. We consider what happens with no policies, and under a range of stylized policies, including optimal subsidy/tax schemes for suppliers, and policies that might be used to try to limit the dominance of the market leader.

In the BDKS model, it is the expectation of future profits which drives sellers to price dynamically. It is therefore not surprising that reallocating bargaining power, changing suppliers' share of current and future surplus, will affect prices and welfare. However, what is less intuitive, and might be missed by just trying to extend intuition about how bargaining affects outcomes in static settings, are the ways in which changes in bargaining power interact with the myopic behavior of buyers, and how this feeds back into the dynamic incentives of sellers, prices and market structure.

For example, when the margin of a leader is restricted by giving buyers a limited amount of bargaining power, the leader will become more likely to make sales and leadership will tend to last longer, potentially making it relatively more attractive for a firm to establish a lead. We find that even small shifts away from the price-setting assumption can have very significant qualitative effects. This motivates an ongoing research agenda where we will be looking at a number of empirical applications using models that are more tailored the realities of particular industries than the model that we use here.[2] Our policy analysis also

[2] As an example of the type of setting which we are considering, consider the analysis of aircraft manufacturer mergers considered by An and Zhao (2019), who suggest that some mergers will be efficiency-increasing because they will increase know-how, moving the merged firm efficiently down its cost curve. This conclusion

considers some new questions, such as the design of optimal subsidies and whether it could be advantageous to wait until the leader has made some progress down its cost curve before introducing policies to encourage competition.[3]

We also make two more methodological-related contributions. First, solving even stylized models, like the BDKS model, involves significant computation. For example, with $M = 30$ know-how states for each firm, BDKS's formulation of the symmetric equilibrium has $M^2 = 900$ equations for seller values and $M^2 = 900$ first-order conditions for prices. We develop an alternative formulation where the equilibria are characterized only by buyer choice probabilities in each state. With BDKS's no outside good assumption, the system reduces to a total of $\frac{M(M-1)}{2}$ equations and unknowns. This alternative formulation also allows for a straightforward calculation of the subsidies required to implement the social planner optimum.

Our second (and related) methodological contribution concerns multiple equilibria. Cabral and Riordan (1994), BDKS and Besanko, Doraszelski, and Kryukov (2014) (BDK) document the common existence of multiple equilibria in the type of infinite-horizon dynamic model that we use, reflecting how different expectations about future play can be internally consistent. Given that multiple equilibria can create issues for estimation and interpretation, one reaction has been to design models where uniqueness is guaranteed (e.g., Abbring and Campbell (2010) and Abbring, Campbell, Tilly, and Yang (2018)), for example by imposing symmetry and eliminating seller-specific payoff shocks. This approach can be very useful for studying the effects of aggregate shocks, but it cannot be used to address whether one firm may become dominant. We find that (absent distorting policies) multiple equilibria in the BDKS model are eliminated once buyers have even quite limited bargaining power. In the $M = 30$ model our conclusions are based on the type of homotopy approach used by BDKS, which is not guaranteed to find all of equilibria even when a thorough search is conducted.

may be viewed as consistent with our typical finding (in a model that is much more stylized than their model) that when firms set prices equilibria tend to involve learning that it is too slow. However, when bargaining power is equally split between buyers and sellers we often find that learning is too fast and there is too much concentration from a social perspective. In this case, one might view a merger that increases the leader's advantage more skeptically.

[3]The answer is that we find this can raise efficiency when sellers have all or almost all the bargaining power, but that this is because this tends to increase the asymmetry between firms relative to an industry where a policy is never imposed.

However, we show that we find the same result in an $M = 3$ model where our reformulation of the equilibrium conditions allows us to identify equilibria using a different approach.[4]

We note that Besanko, Doraszelski, and Kryukov (2014), Besanko, Doraszelski, and Kryukov (2019a), Besanko, Doraszelski, and Kryukov (2019b), and Sweeting, Jia, Hui, and Yao (2022) (SJHY) consider a closely-related model, which we will refer to as "BDK". The BDK model also assumes at most two active firms, but differs from the BDKS model in allowing buyers to have an outside option, and for sellers to enter or exit endogenously. Unlike in BDKS, there is no know-how depreciation.

We will analyze some the policies considered by BDK and Besanko, Doraszelski, and Kryukov (2019b) in our counterfactuals, but we deliberately prefer to use the BDKS model in this paper for three reasons. First, in a model with fixed costs (or entry costs and scrap values), transferring bargaining power to buyers may tend to cause market structure to move towards monopoly and the industry to then disappear entirely. While this effect is certainly potentially important, it is distinct from the effects of bargaining on dynamic incentives, and the relative market position of leading and laggard suppliers, that we want to consider in this paper. Second, as discussed in SJHY, competitive dynamics in the BDK model often have the form that, in all, or almost all equilibria, firms will remain in the industry forever once they have made one sale, at which point pricing becomes more accommodative. In contrast, in the BDKS model, the possibility that know-how depreciates can lead to aggressive competition in a wide range of different states.[5] Third, while our reformulation would also reduce the number of equations in the BDK model, the existence of an outside good would mean that the number would only fall from $2M^2$ to M^2, rather than $\frac{M(M-1)}{2}$.[6]

The rest of the paper is structured as follows. Section 2 outlines the model, together with several outcome measures. Section 3 explains the alternative ways of formulating the equations that define a symmetric Markov Perfect Nash equilibrium. Section 4 uses the

[4]The $M = 3$ model also allows us to develop intuition for the effects that we see in the full model in a transparent way. However, we caution that it is unclear exactly how the technology parameters in the $M = 3$ model should be mapped into the $M = 30$ model.

[5]In reality, firms can be accused of anticompetitive behavior because their policies keep rivals, who have had some success, from expanding even if they are not excluded entirely. In this sense, the BDKS model may reflect some enforcement-relevant situations better than the BDK model.

[6]Appendix F shows that assuming an outside good that is as attractive/unattractive as BDK assume does not change outcomes when $\tau = 0$ significantly.

$M = 3$ version of the model to illustrate how bargaining affects incentives and market outcomes, and eliminates multiplicity. Section 5 uses the full model, showing that the effects on outcomes can be substantial, and sensitive to low τ, in this more realistic set-up. Section 6 presents the results of policy counterfactuals, where we primarily focus on a single set of parameters. Section 7 concludes. The Appendices contain methodological details, some (limited) theoretical proofs and additional results that support those reported in the text.

2 Model

In this section we present the model, which follows BDKS except for generalizing the stage game where prices are determined and developing an extension, with a monopsonist long-lived buyer, to represent the social planner problem. Readers should consult BDKS for additional motivation.

2.1 States and Costs.

There is an infinite horizon, discrete time, discrete state game. The common discount factor is $\beta = \frac{1}{1.05}$. There are two ex-ante symmetric but differentiated sellers ($i = 1, 2$), whose marginal costs depend on their past sales. Specifically, each seller has a commonly observed state variable, $e_i = 1, ..., M$, that tracks its "know-how".[7] The state of the industry is $\mathbf{e} = (e_1, e_2)$.

A firm's cost of producing a unit of output is $c(e_i) = \kappa\rho^{\log_2(\min(e_i,m))}$. $\rho \in [0, 1]$ is known as the "progress ratio", and a lower number reflects stronger learning economies. When $\rho = 1$, marginal costs are κ for all e_i. Our two model variants involve setting either $m = 3$ and $M = 3$, or $m = 15$ and $M = 30$ (same as BDKS).[8]

Dynamics arise from the evolution of the know-how states. As described below, in each period one unit will be purchased from one of the sellers. The state of seller firm i evolves,

[7] Asker, Fershtman, Jeon, and Pakes (2020), Sweeting, Roberts, and Gedge (2020) and Sweeting, Tao, and Yao (2023) consider dynamic models where serially correlated state variables are private information.

[8] $M = 3$ is the smallest number where the leader can have both a "small" and a "large" advantage.

except at the boundaries of the state space, according to

$$e_{i,t+1} = e_{i,t} + q_{i,t} - f_{i,t} \tag{1}$$

where $q_{i,t}$ is equal to one if firm i makes the sale, and $f_{i,t}$ is equal to one (0 otherwise) with probability $\Delta(e_i) = 1 - (1-\delta)^{e_i}$ with $\delta \in [0,1)$.[9] The probability of forgetting (Δ) is therefore increasing in both δ and e_i. Equation (1) implies that a firm that makes a sale will either have the same or one more unit of know-how in the next period, whereas a firm that does not make a sale will have either the same or less know-how.

Following BDKS, every period there is a buyer that purchases a single unit from one of the firms. Each period the chosen buyer receives flow indirect utility $v - p_i + \sigma\varepsilon_i$ if it buys from seller i, where p_i is the price paid, and σ parameterizes the degree of product differentiation. The ε_is are private information Type I extreme value payoff shocks, which are i.i.d. across buyers, sellers and periods, and do not depend on a buyer's past purchases. We will assume that $\sigma = 1$, except in Appendix F where we briefly discuss allowing for an outside option and varying differentiation.

Bargaining. In BDKS the sellers compete for the buyer's business by simultaneously setting prices. We now explain how we relax this assumption to allow for bargaining, nesting the BDKS assumption as a special case.

We assume that the buyer sends separate agents to each seller, before the buyer knows the realization of its εs. Each agent-seller pair negotiate the price at which a transaction will happen **if** the buyer chooses to purchase from that seller, with no purchase possible if a price is not agreed. We make the Nash-in-Nash assumption (Collard-Wexler, Gowrisankaran, and Lee (2019)), so that each buyer agent-seller pair takes the price agreed by the other agent-seller pair as given. The buyer's "bargaining power" is equal to a parameter τ. Once both negotiations are completed, the buyer observes its εs and chooses which seller to purchase from given the agreed prices.

There are two benefits of this formulation. First, because information is symmetric

[9] At the boundaries of the state space, the evolution is necessarily restricted. For example, when $e_{i,t} = 1$ and $q_{i,t} = 0$, firm i cannot forget ($f_{i,t} = 0$), and when $e_{i,t} = M$ and $q_{i,t} = 1$, firm i has to forget ($f_{i,t} = 1$).

during negotiations, agreements will always be reached in equilibrium and outcomes will be consistent with BDKS's assumption of trade in every period, even if transaction prices are different.[10] Second, as described below, outcomes equivalent to (i) sellers simultaneously setting prices, and (ii) prices always equalling current production costs are nested as special cases, with $\tau = 0$ and $\tau = 1$ respectively.

In our paper we will look at what happens as τ varies from 0 to 1. However, if $\tau \neq 0$, what values of τ are "reasonable" and how might this affect the interpretation of our results? $\tau = 0.5$, so that buyers and sellers have equal bargaining power, is often assumed when bargaining weights cannot be empirically identified. One interpretation of our results is that strategies and outcomes when $\tau = 0.5$ are often not close to the averages of the outcomes when $\tau = 0$ and $\tau = 1$, whereas, for example, this is usually the case for prices in a static model where outside options are fixed.

However, estimated τs often range quite widely even across similar types of firms (Grennan (2013)), and an alternative view would be that, in many settings where LBD is important, sellers are larger than buyers and an economist might believe that this would give sellers "most of the bargaining power". In this case, the economist might think that a price-setting assumption is a sensible, simplifying assumption. Our finding that outcomes often change very quickly when one moves away from $\tau = 0$ suggests that this type of logic should be viewed skeptically, and that, to the extent buyer bargaining power eliminates multiplicity, it may not really simplify the analysis at all.

Social Planner Problem. SJHY consider a model where buyers are forward-looking and expect to capture some share (b^p) of future buyer surplus. While our analysis of competition in this paper assumes $b^p = 0$ (i.e., that each buyer is short-lived/atomistic), we solve the social planner problem by assuming $b^p = 1$ and $\tau = 1$, so that a monopsonist repeat buyer makes choices that maximize expected total surplus.

[10] The key feature is that, during bargaining, the buyer's agent and the seller have symmetric information about the realization of the εs, which guarantees that prices will be agreed in equilibrium, allowing trade to happen. With asymmetric information during negotiation, there would be some probability that both negotiations would fail and no purchase would happen, creating a potential non-trivial difference to BDKS's model.

3 Equilibrium

This section presents two formulations of the equations characterizing the equilibrium of the model. We then discuss how we find equilibria, and the principal measures that we use to describe equilibrium outcomes.

The equilibrium concept is symmetric and stationary Markov Perfect Nash equilibrium (MPNE, Maskin and Tirole (2001), Ericson and Pakes (1995), Pakes and McGuire (1994)).

3.1 Formulation of Equilibrium Conditions for Prices and Values.

An equilibrium can be expressed as a vector of $p^*(\mathbf{e})$ (negotiated prices), $VS^*(\mathbf{e})$ (beginning of period seller values) and, in the social planner problem, $VB^*(\mathbf{e})$ (value of the monopsonist buyer) that solve the following equations, where symmetry implies that we only need to solve for prices and seller values for firm 1 (i.e., $p_2^*(e_1, e_2) = p_1^*(e_2, e_1)$, $VS_2^*(e_1, e_2) = VS_1^*(e_2, e_1)$). We solve for the buyer's value in states where $e_1 \geq e_2$ (i.e., $VB^*(e_1, e_2) = VB^*(e_2, e_1)$).

Beginning of period value for firm 1 (VS):

$$VS_1^*(\mathbf{e}) - D_1^*(\mathbf{e})(p_1^*(\mathbf{e}) - c(e_1)) - \sum_{k=1,2} D_k^*(\mathbf{e})\mu_{1,k}^S(\mathbf{e}) = 0, \tag{2}$$

where $\mu_{1,k}^S(\mathbf{e})$, is seller 1's continuation value when the buyer chooses to buy from seller k,

$$\mu_{1,k}^S(\mathbf{e}) = \beta \sum_{\forall e'_{1,t+1}|e_{1,t}} \sum_{\forall e'_{2,t+1}|e_{2,t}} VS_1^*(e'_{1,t+1}, e'_{2,t+1}) \Pr(e'_{1,t+1}|e_{1,t}, k) \Pr(e'_{2,t+1}|e_{2,t}, k),$$

and $\Pr(e'_{i,t+1}|e_{i,t}, k)$ is the probability that i's state transitions from $e_{i,t}$ to $e'_{i,t+1}$ when a purchase is made from k ($q_{k,t} = 1$) given the forgetting probabilities. The probability that the chosen buyer purchases from seller k, $D_k(\mathbf{e})$, given negotiated prices, is

$$D_k(\mathbf{e}) = \frac{\exp\left(\frac{v - p_k(\mathbf{e}) + \mu_k^B(\mathbf{e})}{\sigma}\right)}{\exp\left(\frac{v - p_1(\mathbf{e}) + \mu_1^B(\mathbf{e})}{\sigma}\right) + \exp\left(\frac{v - p_2(\mathbf{e}) + \mu_2^B(\mathbf{e})}{\sigma}\right)},$$

where $\mu_k^B(\mathbf{e})$, the buyer continuation value when it purchases from k, is defined below. $D_k^*(\mathbf{e})$

is this demand function evaluated at equilibrium prices and values.

Beginning of period buyer value (VB) (for social planner problem):

Based on a model where there is a pool of symmetric potential buyers who expect to be chosen to be the active buyer with probability b^p in any period

$$VB^*(\mathbf{e}) - b^p \sigma \log \left(\sum_{k=1,2} \exp \left(\frac{v - p_k^*(\mathbf{e}) + \mu_k^B(\mathbf{e})}{\sigma} \right) \right) - (1 - b^p) \sum_{k=1,2} D_k^*(\mathbf{e}) \mu_k^B(\mathbf{e}) = 0 \quad (3)$$

where

$$\mu_k^B(\mathbf{e}) = \beta \sum_{\forall e'_{1,t+1}|e_{1,t}} \sum_{\forall e'_{2,t+1}|e_{2,t}} VB^*(e'_{1,t+1}, e'_{2,t+1}) \Pr(e'_{1,t+1}|e_{1,t}, k) \Pr(e'_{2,t+1}|e_{2,t}, k).$$

In the social planner problem $b^p = 1$. Otherwise b^p, and all VBs and μ_k^Bs, equal zero for the analysis in this paper.

Negotiated prices (p): the assumed simultaneous Nash-in-Nash structure of bargaining implies that the equilibrium price negotiated between the buyer and seller 1, $p_1^*(\mathbf{e})$, will be the solution to the following maximization problem where p_2 is treated as fixed.

$$p_1^*(\mathbf{e}) = \arg\max_{p_1} \left(CS(p_1, p_2, \mathbf{e}) - CS(p_2, \mathbf{e}) \right)^{\tau} \times ...$$
$$\left(D_1(\mathbf{e})(\mu_{1,1}^S(\mathbf{e}) + p_1 - c(e_1)) + (1 - D_1(\mathbf{e}))\mu_{1,2}^S(\mathbf{e}) - \mu_{1,2}^S(\mathbf{e}) \right)^{(1-\tau)}$$

where $CS(p_1, p_2, \mathbf{e}) = \sigma \log \left(\sum_{k=1,2} \exp \left(\frac{v - p_k(\mathbf{e}) + \mu_k^B(\mathbf{e})}{\sigma} \right) \right)$ (i.e., the expected future surplus of the buyer when it is able to choose from both firms) and $CS(p_2, \mathbf{e}) = v - p_2(\mathbf{e}) + \mu_2^B(\mathbf{e})$, which is the buyer's expected surplus when the negotiation with seller 1 fails, and it has to buy from seller 2.

$p_1^*(\mathbf{e})$ is therefore the solution to the first-order condition

$$\tau \frac{\partial CS(p_1^*(\mathbf{e}), p_2, \mathbf{e})}{\partial p_1} \left(D_1^*(\mathbf{e})(\mu_{1,1}^S(\mathbf{e}) + p_1^*(\mathbf{e}) - c(e_1)) + (1 - D_1^*(\mathbf{e}))\mu_{1,2}^S(\mathbf{e}) - \mu_{1,2}^S(\mathbf{e}) \right) + ...$$
$$(1 - \tau) \left(CS(p_1^*(\mathbf{e}), p_2, \mathbf{e}) - CS(p_2, \mathbf{e}) \right) \left(D_1^*(\mathbf{e}) + \frac{\partial D_1^*(\mathbf{e})}{\partial p_1} \left(p_1^*(\mathbf{e}) - c(e_1) + \mu_{1,1}^S(\mathbf{e}) - \mu_{1,2}^S(\mathbf{e}) \right) \right) = 0, \quad (4)$$

where $\frac{\partial CS(p_1^*(\mathbf{e}),p_2,\mathbf{e})}{\partial p_1} = -D_1^*(\mathbf{e})$ and $\frac{\partial D_1^*(\mathbf{e})}{\partial p_1} = -\frac{D_1^*(\mathbf{e})(1-D_1^*(\mathbf{e}))}{\sigma}$. Algebraic manipulation shows that this can be simplified to

$$-\tau D_1^*(\mathbf{e})(p_1^*(\mathbf{e}) - \widehat{c_1}(\mathbf{e})) + (1-\tau)\left[\sigma - (1-D_1^*(\mathbf{e}))(p_1^*(\mathbf{e}) - \widehat{c_1}(\mathbf{e}))\right] \log \frac{1}{1-D_1^*(\mathbf{e})} = 0 \quad (5)$$

where $\widehat{c_1}(\mathbf{e}) = c(e_1) - (\mu_{1,1}^S(\mathbf{e}) - \mu_{1,2}^S(\mathbf{e}))$ is firm 1's "effective" marginal cost of a sale which accounts for dynamic incentives which reflect future profits in different states and the expected evolution of the industry. $\mu_{1,1}^S$ and $\mu_{1,2}^S$ are endogenous and we will see that they can be quite sensitive to small shifts in τ.

When $\tau = 0$ (seller has all of the bargaining power), the FOC reduces to the pricing first-order condition in BDKS. When $\tau = 1$, $p_1^*(\mathbf{e}) = c(e_1)$ in all states, implying that $VS^* = 0$. However, atomistic buyer choices can cause purchases to be socially inefficient when they pay current production costs, which do not reflect the future social benefits from buying from a particular provider.

Note that in our model, the buyer's agent and the seller agree on the price at which trade may happen, not that trade will happen, and that the agreed price affects the probability that a supplier makes a sale.[11] A price increase reduces an agreement's expected surplus, as it is more likely the other seller will make the sale. This effect will tend to lower the markup for a given τ.

3.2 Formulation of Equilibrium Conditions in Terms of Buyer Choice Probabilities.

Assuming $b^p = 0$, the previous formulation gives $2 \times M^2$ equations (18 for $M = 3$ and 1800 for $M = 30$). However, it can be convenient to use an alternative formulation which exploits the fact that buyer and seller values and prices can be written in terms of buyer choice probabilities alone.

[11] In the Nash-in-Nash bargaining applications cited in Lee, Whinston, and Yurukoglu (2021) there is usually a mass of consumers (e.g., a mass of customers for health insurance) so that a known quantity (e.g., patients using a particular hospital) will be transacted at the agreed prices.

Note that

$$D_1(\mathbf{e}) = \frac{\exp\left(\frac{v-p_1(\mathbf{e})+\mu_1^B(\mathbf{e})}{\sigma}\right)}{\sum_{k=1,2}\exp\left(\frac{v-p_k(\mathbf{e})+\mu_k^B(\mathbf{e})}{\sigma}\right)} = \frac{1}{1+\exp\left(\frac{p_1(\mathbf{e})-p_2(\mathbf{e})+\mu_2^B(\mathbf{e})-\mu_1^B(\mathbf{e})}{\sigma}\right)}.$$

so we can formulate the equations that we solve as

$$\sigma \log\left(\frac{1}{D_1(\mathbf{e})}-1\right) - p_1(\mathbf{e}) + p_2(\mathbf{e}) + \mu_1^B(\mathbf{e}) - \mu_2^B(\mathbf{e}) = 0. \tag{6}$$

The key to reformulating the problem is to note that prices and values can be expressed in terms of the buyer choice probabilities D_1 and parameters.

From (5) a seller's markup over its effective marginal cost can expressed solely in terms of buyer choice probabilities

$$p_1(\mathbf{e}) - \widehat{c}_1(\mathbf{e}) = \Phi(D_1(\mathbf{e})) = \frac{(1-\tau)\sigma\log\frac{1}{1-D_1(\mathbf{e})}}{\tau D_1(\mathbf{e}) + (1-\tau)(1-D_1(\mathbf{e}))\log\frac{1}{1-D_1(\mathbf{e})}}. \tag{7}$$

Denote the stacked vector of these markups $\mathbf{\Phi}(\mathbf{D_1})$.

We can stack the equations for seller values and write them as

$$\mathbf{VS_1} = \mathbf{D_1} \circ \mathbf{\Phi}(\mathbf{D_1}) + \beta\mathbf{Q}_2\mathbf{VS_1}, \tag{8}$$

where $\mathbf{Q_k}$ is the state transition matrix conditional on the buyer purchasing from seller k (which only depends on technology, and not endogenous variables), and $\circ$ denotes the element-wise product between two vectors.

$$\widehat{\mathbf{c_1}} = \mathbf{c_1} - \beta(\mathbf{Q_1} - \mathbf{Q_2})\mathbf{VS_1} \tag{9}$$

and

$$\mathbf{VS_1} = (\mathbf{I} - \beta\mathbf{Q_2})^{-1}\left[\mathbf{D_1} \circ \mathbf{\Phi}(\mathbf{D_1})\right]. \tag{10}$$

so that

$$\mathbf{p_1} = \mathbf{\Phi}(\mathbf{D_1}) + \mathbf{c_1} - \beta(\mathbf{Q_1} - \mathbf{Q_2})(\mathbf{I} - \beta\mathbf{Q_2})^{-1}[\mathbf{D_1} \circ \mathbf{\Phi}(\mathbf{D_1})]. \tag{11}$$

If $b^p > 0$, then we also need to substitute in for the choice-specific buyer continuation values. $\mu_{\mathbf{1}}^{\mathbf{B}} - \mu_{\mathbf{2}}^{\mathbf{B}} = \beta(\mathbf{Q_1} - \mathbf{Q_2})\mathbf{V^B}$, and

$$\mathbf{V^B} = b^p \left(\mathbf{I} - \beta \sum_{k=1,2} \mathbf{D_k} \circ \mathbf{Q_k} \right)^{-1} \sum_{k=1,2} \left[\mathbf{D_k} \circ \left(\sigma \log \frac{1}{\mathbf{D_k}} + v - \mathbf{p_k} \right) \right], \tag{12}$$

and we can then use (11) to substitute in for prices.[12]

This gives us M^2 equations in D_1. However, imposing symmetry with no outside good, so that $D_1(e,e) = D_2(e,e) = \frac{1}{2}$ and using $D_2(e,e') = 1 - D_1(e,e') = 1 - D_2(e',e) = D_1(e',e)$, the problem is reduced to $\frac{M(M-1)}{2}$ equations and unknowns, i.e., 435 equations for $M = 30$ and 3 for $M = 3$.

3.3 Unique Equilibria.

One can show the following results.

Proposition 1 *1. If $\tau = 1$, prices will equal marginal production costs in all states, for all b^p, ρ and δ.*

2. when $b^p = 0$, there will be a unique symmetric MPNE when

(a) $\delta = 0$ for all ρ and τ (a result that also holds for intermediate values of b^p not considered in this paper).

(b) $\tau = 1$ for all ρ and δ.

Proof. See Appendix A. ■

The social planner problem will also have a unique solution. These uniqueness results are not surprising. For example, when $\delta = 0$, backwards induction can be applied, as the game must end up in state (M, M) and then stay there forever, and movements through the state space are unidirectional. When buyers are atomistic (completely static), backwards induction can then be applied to prove uniqueness. However, outside of these extreme cases, there may be multiple equilibria. Indeed, the existence of multiplicity has been emphasized

[12]Note that $\log \sum_{k=1,2} \exp\left(\frac{v - p_k(\mathbf{e}) + \mu_k^B(\mathbf{e})}{\sigma}\right) = \sum_{k=1,2} D_k(\mathbf{e}) \log\left(\frac{1}{D_k(\mathbf{e})}\right) + \sum_{k=1,2} D_k(\mathbf{e}) \frac{v - p_k(\mathbf{e}) + \mu_k^B}{\sigma}$.

in much of the existing literature on dynamic competition (BDKS, Cabral and Riordan (1994)).

3.4 Methods for Finding Equilibria and their Classification

When $\delta > 0$, it is always possible for the game to return to states with lower know-how, and backwards induction cannot be used. For a given set of parameters we can find *an* equilibrium by solving the system of equations (whether defined in terms of prices and values or choice probabilities) that defines an equilibrium or by using the iterative algorithm of Pakes and McGuire (1994).

In order to enumerate the set of equilibria in the $M = 30$ model, and to examine what happens to equilibrium strategies and outcomes when we change parameters including τ, we use BDKS's approach of numerical homotopies. Homotopies trace the equilibrium correspondence through the strategy and value space, as a single parameter is varied, using the matrix generalization of the implicit function theorem. A homotopy starts from an equilibrium calculated either by solving the equilibrium equations for a given set of parameters, or as the output from a different homotopy. We will label a homotopy that varies the parameter α as a "α-homotopy".

The homotopy approach can identify equilibria that, because of the failure of local stability conditions, cannot be found using the Pakes-McGuire algorithm (unless one started exactly at the equilibrium). However, there is no guarantee that repeated application of homotopies will identify *all* equilibria and in practice it is quite common that individual homotopies may stall (taking a series of tiny steps).

Our $M = 30$ results are therefore based on an assumption that our implementation (described in Appendix B.1), which runs homotopies in different directions and varying different parameters in turn, is effective. In the context of the BDK model, SJHY show that the results of using this implementation are consistent with the results from a novel backwards induction algorithm that they can show whether particular types of equilibria (defined by the exit and re-entry strategies of the firms) exist. While our experience makes us confident that our broad conclusions from using homotopies are correct, we acknowledge that we could be missing some equilibria for given parameters.

For this reason, we use an alternative approach, detailed in Appendix B.2, for finding equilibria when $M = 3$. This approach uses the reformulated equilibrium conditions. Solving for $D_1(2,3)$ given values of $D_1(1,2)$ and $D_1(1,3)$, the problem then amounts to searching for the solutions to just two continuous nonlinear equations on the unit square.[13]

3.5 Useful Measures.

Given equilibrium strategies, we can calculate a variety of outcome measures. We assume that the industry is in state (1,1) in $t = 1$.

Concentration and Prices. Following BDKS, a summary statistic for expected market structure in period t is HHI^t where

$$HHI^t = \sum_{\forall \mathbf{e}} \mu^t(\mathbf{e}) HHI(\mathbf{e})$$

and

$$HHI(\mathbf{e}) = \sum_{k=1,2} \left(\frac{D_k^*(\mathbf{e})}{D_1^*(\mathbf{e}) + D_2^*(\mathbf{e})} \right)^2 .$$

$\mu^t(\mathbf{e})$ is the probability that a game will be in state $\mathbf{e}$ after t periods. Given two firms, the minimum value of HHI^t is 0.5. We can define measures of expected prices after t periods, P^t in a similar way. P^{PDV} is the expected present discounted value of prices paid.

In the $M = 30$ model, we will focus primarily on HHI^{32} as a measure of *medium-run* market structure, and we will use HHI^{200}, and occasionally HHI^{1000}, as measures of *long-run* market structure. With a discount factor of $\frac{1}{1.05}$, 200 periods is already far into the future from a social planner's perspective. In the $M = 3$ model, we will focus on HHI^4 and HHI^{32}.

We note that one should not assume that the distribution of states will have converged to its steady state distribution after 1,000 periods in the $M = 30$ model, as for many parameters

[13] The procedure involves verifying that there is only one solution for $D_1(2,3)$ given the choice probabilities in other states. We have found that the equation for $D_1(2,3)$ is monotonic in this variable for all of the parameters we have looked at, so that there is a unique solution, although we have not been able to prove monotonicity.

the second firm may only move down its cost curve after a low probability sequence of favorable shocks.[14]

Surplus. As we assume a purchase is always made, v does not affect equilibrium strategies and choices, and is, in this sense, arbitrary. We therefore measure both consumer and total surplus ignoring the vs, so that our reported values may be negative. Our measures of expected consumer surplus (CS, CS^t or CS^{PDV}) are the expected value of the buyers' εs associated with their purchases ($E(\varepsilon)$) less the expected price, even though this measure will (almost always) be negative. We define producer surplus, PS^t or PS^{PDV}, as the expected price less production costs. Total surplus will be defined as $E(\varepsilon)$ for purchased units less production costs (PC^t or PC^{PDV}) (or, equivalently, $CS + PS$). Potentially one could be interested in all of the alternative welfare measures at different points in time. For simplicity, we will emphasize TS^{PDV} (efficiency) as our primary welfare measure.

4 Analysis of a Model with $M = 3$

In this section we analyze the $M = 3$ model. We would not suggest that the $M = 3$ model can provide a useful representation of any real-world industry, but it is computationally convenient, allows the reader to see what happens to all strategies when any parameter changes, and we can identify multiplicity without using homotopies. We first analyze how multiplicity varies with τ across the (ρ, δ) parameter space, before using an example to illustrate how reallocating bargaining power affects the strategies, incentives and the various distortions in the model. We also consider the design of the optimal subsidy scheme.

Bargaining and the Existence of Multiple Equilibria. Although we do not regard the existence of multiple equilibria as being the primary topic of our investigation, the existing literature that has assumed price-setting has emphasized that multiple equilibria

[14]This is true even in the social planner solution. For example, for the illustrative parameters that we will focus on, once two firms get to state (30,30), the assumed forgetting technology implies that they are (virtually) certain to remain at the bottom of their cost curves forever, so that HHI^t will converge eventually to 0.5. However, the social planner's optimal strategy is to get one seller down its cost curve quickly, and then waiting for a low probability sequence of favorable preference shocks before investing in creating a second low cost producer. As a result, HHI^t only converges to 0.5 at around 4,000 periods.

are generic in this type of model. As there is a unique equilibrium when $\tau = 1$, we expect that equilibria will be unique for τ close to 1, but, a priori, it is unclear whether we should expect moving from $\tau = 0$ to intermediate values will eliminate or increase multiplicity. If it caused multiplicity to proliferate then it may be very difficult to present a clear picture about how bargaining power affects outcomes. Equilibria are identified using the (non-homotopy) method described in Appendix B.2.[15]

Figure 1 shows how many equilibria exist for $\tau = 0$, 0.05, 0.1 and 0.15 for a grid of ρ and δ values. As we always identify uniqueness for $\tau \geq 0.2$ or $\delta \geq 0.5$ we do not present these results. The red areas indicate (ρ, δ) parameters where we identify multiplicity for the indicated τ and, for $\tau \geq 0.05$, we also identify multiplicity for $\tau - 0.05$. There are small sets of parameters, marked in green, where there is multiplicity for τ but not for $\tau - 0.05$. The maximum number of equilibria we identify is 3. The general pattern is that increasing τ eliminates multiplicity, and that multiplicity is eliminated entirely well before τ reaches 0.5.

The multiplicity that does exist is associated with strong LBD effects (the first sale lowers costs by at least 40%) and a moderate degree of know-how depreciation. For example, if $\delta = 0.1$, the probability that a firm in state $e = m = M$ experiences depreciation is 0.27. Interestingly, the δs that support multiplicity in the $M = 30$, $m = 15$ model have forgetting probabilities in state m greater than approximately 0.26, although in that model multiplicity can be sustained when this probability rises as high as 0.8.

Polar Cases for Example (ρ, δ). Our primary goal is to understand whether, and why, the re-allocation of bargaining power can have significant effects on market structure and efficiency/welfare. This requires understanding the inefficiencies that exist in the model. We use parameters $\rho = 0.3$ and $\delta = 0.03$, which support a unique equilibrium for all τ considered above, to illustrate.

The left hand panel of Table 1 shows prices, sale probabilities, state distributions and welfare outcomes for the social planner problem. We solve the social planner case using the device of a long-lived buyer who faces prices equal to production costs, although the social planner could, of course, set different prices to transfer surplus in any way that it wants.

[15] *We are currently confirming whether homotopies identify the multiplicity that our non-homotopy method finds.*

Figure 1: Multiplicity of Equilibria in the $M = m = 3$ Model. White = unique equilibrium. Green = multiple equilibria for τ and uniqueness for $\tau - 0.05$. Red = multiple equilibria for τ and (for $\tau \geq 0.05$) also for $\tau - 0.05$. Grid of 0.01 steps from 0 to 1 for ρ, grid of 0.005 steps from 0 to 1 for δ and grid of 0.05 steps for τ. A unique equilibirum is identified for all $\tau \geq 0.2$ for all (ρ, δ) and for all $\delta \geq 0.5$ for all (ρ, τ).

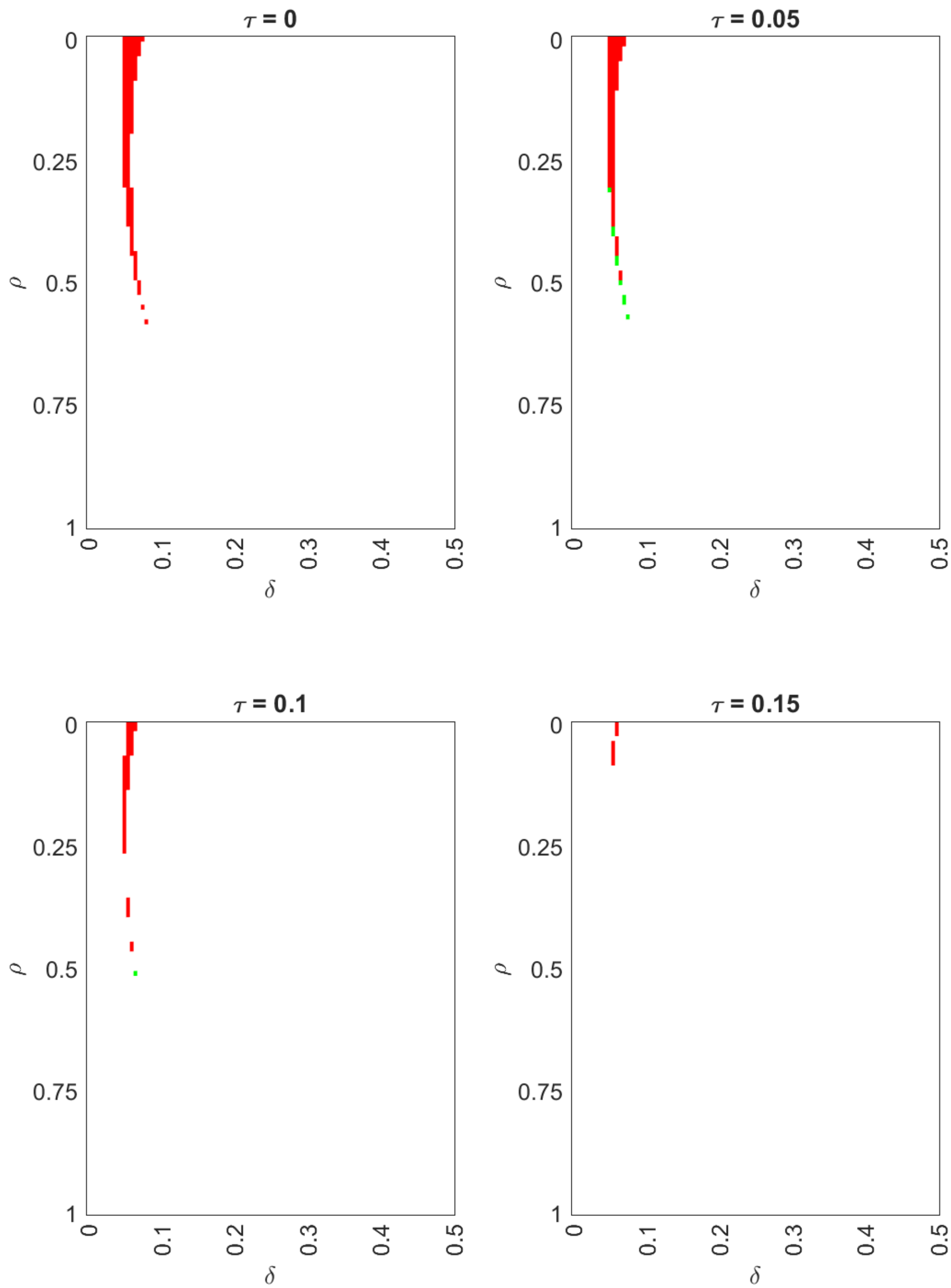


Table 1: $M = 3$ with $\rho = 0.3$ and $\delta = 0.03$: Strategies and Outcomes in Polar Cases. In these tables, firm 2 is assumed to be the leader.

	(a) Social Planner			(b) $\tau = 0$			(c) $\tau = 1$		
	Laggard State Firm 1			Laggard State Firm 1			Laggard State Firm 1		
e_1	1	2	3	1	2	3	1	2	3
c_1	10	3	1.483	10	3	1.483	10	3	1.483
Δ	0.03	0.059	0.087	0.03	0.059	0.087	0.03	0.059	0.087
Prices and Laggard Sale Probability									
$e_2 = 1$	$p_1 = 10$ $p_2 = 10$ $D_1 = 0.5$			$p_1 = 3.378$ $p_2 = 3.378$ $D_1 = 0.5$			$p_1 = 10$ $p_2 = 10$ $D_1 = 0.5$		
$e_2 = 2$	$p_1 = 10$ $p_2 = 3$ $D_1 = 0.076$	$p_1 = 3$ $p_2 = 3$ $D_1 = 0.5$		$p_1 = 4.355$ $p_2 = 3.274$ $D_1 = 0.253$	$p_1 = 2.918$ $p_2 = 2.918$ $D_1 = 0.500$		$p_1 = 10$ $p_2 = 3$ $D_1 = 0.001$	$p_1 = 3$ $p_2 = 3$ $D_1 = 0.500$	
$e_2 = 3$	$p_1 = 10$ $p_2 = 1.483$ $D_1 = 0.102$	$p_1 = 3$ $p_2 = 1.483$ $D_1 = 0.57$	$p_1 = 1.483$ $p_2 = 1.483$ $D_1 = 0.5$	$p_1 = 4.344$ $p_2 = 3.395$ $D_1 = 0.279$	$p_1 = 3.059$ $p_2 = 3.225$ $D_1 = 0.542$	$p_1 = 3.339$ $p_2 = 3.339$ $D_1 = 0.5$	$p_1 = 10$ $p_2 = 1.483$ $D_1 = 0.000$	$p_1 = 3$ $p_2 = 1.483$ $D_1 = 0.180$	$p_1 = 1.483$ $p_2 = 1.483$ $D_1 = 0.5$
4 Period Probability Distribution									
$e_2 = 1$	4.57E-05			8.18E-05			2.72E-05		
$e_2 = 2$	0.0070	0.0190		0.0087	0.050		0.0060	0.0001	
$e_2 = 3$	0.8233	0.1506	0	0.5741	0.3670	0	0.9930	0.0009	0
32 Period State Probability Distribution									
$e_2 = 1$	4.30-09			7.32E-09			1.05E-12		
$e_2 = 2$	2.63E-05	0.0012		1.57E-05	0.00086		6.28E-07	1.97E-05	0
$e_2 = 3$	0.0642	0.1458	0.7888	0.0159	0.1556	0.8277	0.9955	0.0019	0.0026
	TS	**CS**	**PS**	**TS**	**CS**	**PS**	**TS**	**CS**	**PS**
PDV	-38.342	-	-	-38.742	-56.169	17.427	-40.702	-40.702	0
4 period	-2.051	-	-	-2.622	-2.797	0.174	-1.492	-1.492	0
32 period	-1.006	-	-	-0.959	-2.621	1.661	-1.481	-1.481	0

The table reports the probability distribution of states after 4 periods (when the state (3,3) cannot be reached) and after 32 states, as well as the PDV of CS, TS and PS, and the expected values of these welfare measures in the 4^{th} and 32^{nd} periods. The other panels show equilibria when $\tau = 0$ and $\tau = 1$.

Sale probabilities (reported by the probability that the laggard makes the sale, D_1) will always be 0.5 in symmetric states, so everything relevant about endogenous differences in the state transitions is captured by the bottom-left three boxes. In the social planner's problem (left panel), the leader is much more likely to make the sale in state (1,2) ($D_1(1,2) = 0.076$) and in state (1,3) ($D_1(1,3) = 0.102$), but if the game has moved to (2,3), the social planner is more likely to have the laggard produce ($D_1(2,3) = 0.575 > 0.5$), despite its cost-disadvantage. This outcome reflects the future advantage to having a second supplier with the lowest possible cost, and the relatively low probability (with $\delta = 0.03$) that the leader's know-how depreciates to 2 when it does not make a sale. As reflected by the state probability distributions, the industry is likely to evolve with one firm moving very quickly down its cost curve, with the second firm moving with a delay. With $M = 3$, the social planner will likely have two firms in highest know-how state by the 32^{nd} period.

Now consider panel (c) with $\tau = 1$. With no dynamics or seller investments, marginal cost pricing would lead to efficient purchase choices. However, in our dynamic model, even though the total number of sales is unaffected by τ, inefficiencies will arise with marginal cost pricing because myopic buyers will ignore how their choices affect future costs. This could result in buyers being too likely to buy from a laggard (raising the expected lowest cost of future production) or being too likely to buy from the leader (raising the probability that future buyers will buy from a supplier with a lower ε). For our parameters, the cost advantage of a leader is large, and the second type of inefficiency dominates, so that, compared with the social planner solution, the leader is more likely to make the sale in all asymmetric states, and the industry is very likely to be in state (1,3) after 4 and 32 periods. In fact, (1,3) will also be the state after 1,000 periods with probability 0.99 (even if (3,3) is reached at some point, the industry is likely to move to (1,3) once a firm that does not make a sale experiences depreciation). Note, however, that because the know-how state of the firm that sells in the fourth period is more likely to be low, expected fourth period TS is higher than in the social planner solution.

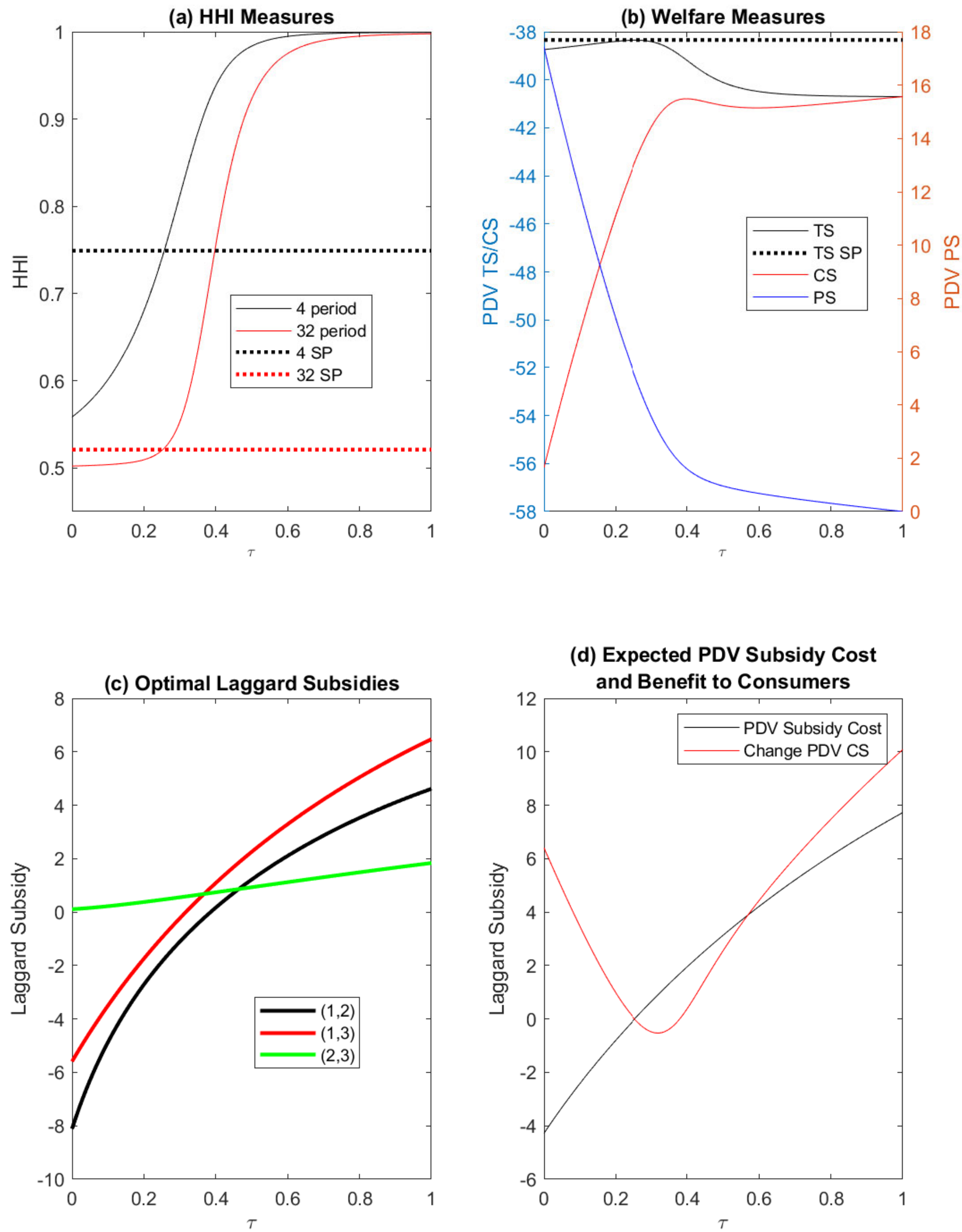
Now consider panel (b) with $\tau = 0$ (corresponding to the BDKS assumptions). In this case, prices do not equal marginal costs, and relative prices, which will reflect seller's opportunity costs and markups, may tend to offset or exacerbate inefficiencies arising from the behavior of myopic buyers. The leader will not account for benefits of lowering its costs that are captured by buyers, and it will not account for how its sales reduce the expected future profits of its rival. For our parameters, the laggard is significantly more likely to make sales in states (1,2) and (1,3) than the social planner would choose. In state (2,3), the laggard is more likely to make a sale than the leader, despite its cost disadvantage, even though it is slightly less likely to make a sale than the social planner would choose. These differences imply that the industry state will likely evolve quite quickly to both firms being in state $e = 2$ or $e = 3$, with HHI^4 and HHI^{32} equal to 0.565 and 0.502. These compare to values of 0.999 and 0.998 respectively when $\tau = 1$, and 0.770 and 0.528 respectively for the social planner.

Varying τ for Example (ρ, δ). Figure 2(a) and (b) shows how welfare and concentration measures vary as τ changes from 0 to 1. As the levels of market concentration in the social planner's solution is between the levels when $\tau = 0$ and $\tau = 1$, it is not surprising that TS^{PDV} is non-monotonic in τ (it is maximized for $\tau \approx 0.3$). CS^{PDV} has a local maximum around $\tau \approx 0.35$ and a global maximum when $\tau = 1$. PS^{PDV} declines monotonically in τ.

Bargaining and Dynamic Incentives. So far the analysis has emphasized how changing τ affects buyer purchase probabilities and seller margins. The directional effects are largely intuitive. However, there are also effects on sellers' dynamic incentives that are more subtle, but which play important roles in determining how quickly changes in τ affect outcomes.

Moving bargaining power to buyers will, holding the distribution of future states and opportunity costs fixed, tend to lower a leader's value and so tend to reduce the incentives to become a leader or preserve a lead. On the other hand, lowering markups may make it more likely that leaders make sales. If so, leadership may tend to last longer, and the difference in value between being a leader and being a follower may increase. This second

Figure 2: Concentration, Welfare and Optimal Subsidies as a Function of τ for $\rho = 0.3$ and $\delta = 0.03$ in the $M = m = 3$ Model.



effect may dominate for low τ as there is a self-reinforcing dynamic: when a lead is likely to last longer, there is more incentive for a leader to invest, through low prices, to preserve the lead.

To illustrate consider state (1,1). Figure 3 panel (a) shows how the price would change if continuation values are held fixed at their $\tau = 0$ values. The effect of changing τ is only to reduce the markup, as in a static model. As expected, the price declines, in an approximately linear way, in τ.

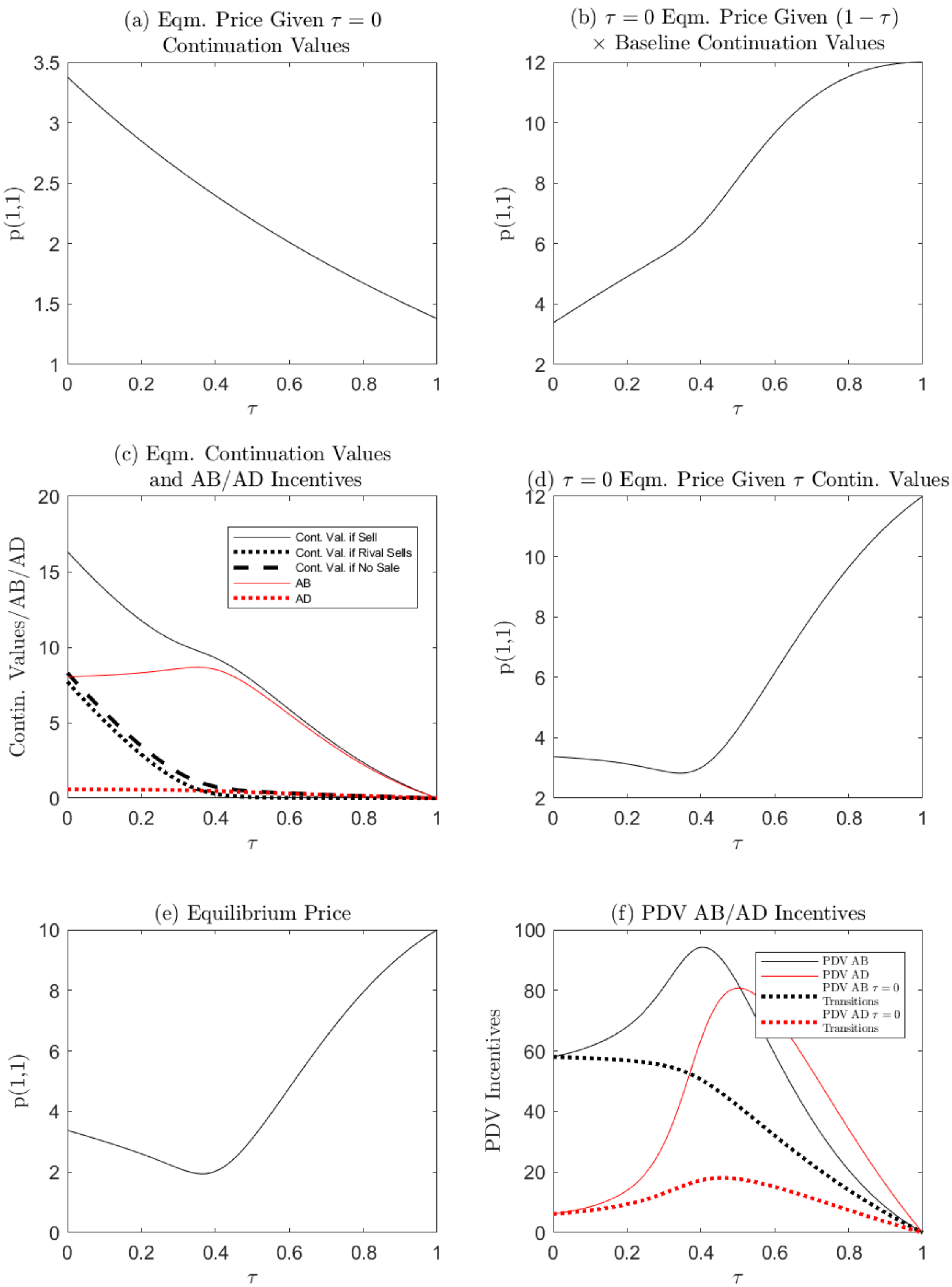
Panel (b) shows what would happen to seller-set equilibrium ($\tau = 0$) prices if $\tau = 0$ continuation values are multiplied by 1 minus the τ value on the horizontal axis. In this case, all dynamic incentives, which lower prices, will be reduced linearly and the price increases monotonically. Panel (c) shows what happens to equilibrium continuation values if a firm sells or if its rival sells. It also shows what happens to the "advantage building" (AB) and "advantage denying" (AD) values that BDK calculate to understand seller incentives, and which are based on the differences between the continuation values and the continuation value if no sale was made (a hypothetical outcome in the BDKS model).

Definition 1 *The firm 1 AB incentive is* $\mu^S_{1,1} - \mu^S_{1,0}$. *The firm 1 AD incentive is* $\mu^S_{1,0} - \mu^S_{1,2}$.

where $\mu^S_{1,0}$ would be seller 1's continuation value if, counterfactually, the buyer was to purchase from *neither* seller. The AB (AD) incentive measures the gain in firm 1's continuation valuation when firm 1 makes a sale (firm 2 does not make a sale). Firm 2's incentives can be defined similarly. Accounting for dynamics, the effective marginal cost of seller 1 in state $\mathbf{e}$ is therefore $\widehat{c_1}(\mathbf{e}) = c(e_1) - (\mathrm{AB}_1(\mathbf{e}) + \mathrm{AD}_1(\mathbf{e}))$.

Increasing τ lowers continuation values, but more slowly for the continuation values associated with making a sale, as a lead is more likely to last longer. This lead preservation effect is substantial: for example, a firm in state (2,1) expects to remain a leader for 5.4 periods when $\tau = 0$, 11.3 periods when $\tau = 0.25$ (which is close to the social planer solution), and 196 periods when $\tau = 0.5$. Therefore, the AB incentive increases, which tends to lower the (1,1) price. Panels (d) and (e) show that for low τ the equilibrium price reflects how τ is changing continuation values (i.e., equilibrium prices are very similar to those that firms would set if $\tau = 0$ and we only change continuation values).

Figure 3: State (1,1) Prices and Incentives as a Function of τ for $\rho = 0.3$ and $\delta = 0.03$ in the $M = m = 3$ Model.



In state (1,1) the effect of τ on the AD incentive is small, because, if there is no sale, the seller knows there will be another period of tough price competition in (1,1). However, in an asymmetric state, the incentive to not allow the laggard to make a sale is significant. The solid lines in panel (f) show that the expected PDV of AB and AD incentives of sellers given equilibrium play. The dotted lines show the values they would have if we computed the PDVs using the distribution of states implied by play when $\tau = 0$. AB and AD incentives show a clear non-monotonic pattern, which is exacerbated by the game spending more periods in asymmetric states where these incentives are higher when $\tau > 0$.[16]

Subsidy. A government could implement the social planner solution using subsidies to firms in different states. These subsidies would vary with τ. Solving for the optimal scheme is straightforward using the reformulated equilibrium conditions. If we fix the choice probabilities at their socially optimal values (D_1^{SP}) and denote the optimal subsidy to firm 1 as s_1 (negative for a tax),

$$p_1(\mathbf{e}) - p_2(\mathbf{e}) = \sigma \log \left(\frac{1}{D_1^{SP}(\mathbf{e})} - 1 \right), \tag{13}$$

for each state, and

$$\mathbf{p_1} + \mathbf{s_1} = \mathbf{\Phi}(\mathbf{D_1^{SP}}) + \mathbf{c_1} - \beta(\mathbf{Q_1} - \mathbf{Q_2})(\mathbf{I} - \beta\mathbf{Q_2})^{-1}[\mathbf{D_1^{SP}} \circ \mathbf{\Phi}(\mathbf{D_1^{SP}})]. \tag{14}$$

If we specify that subsidy/taxes are only applied to the laggard this gives us $M^2 + \frac{M(M-1)}{2}$ equations in the same number of unknowns. The linearity of these equations in p and s, with D_1^{SP} fixed, implies that the subsidy scheme that implements the social planner outcome is unique.[17]

Figure 2(c) and (d) show optimal subsidies in each state, and what this implies for the expected PDV cost of the subsidy scheme to the government (a positive number is a transfer to firms) and the PDV benefit to consumers relative to the equilibrium with no subsidies. For τ close to zero, firms in state (1,2) and (1,3) face a tax, and there is a net gain to the

[16] This effect is more pronounced in the $M = 3$ game where a laggard is never more than one sale from potentially catching up.

[17] This does not rule out the possibility that there will be multiple equilibria for a given set of subsidies.

government, and a gain to consumers reflecting changes in prices. As τ increases, the effect on consumers is U-shaped (with a small loss in CS^{PDV} for $\tau \approx 0.3$), while the optimal scheme subsidizes the laggard in all asymmetric states for $\tau > 0.4$.

Figure 4 shows outcomes if the subsidy scheme that would implement the social planner outcome if $\tau = 0$ is imposed for different values of τ. This is one illustration of how policy might go wrong if $\tau = 0$ is assumed. For $\tau > 0.06$, TS^{PDV} is lower with the subsidy scheme than it would be if no subsidy scheme was implemented at all, while concentration is much higher, reflecting the fact that the subsidy scheme is designed to produce more concentrated outcomes. The scheme also generates less revenue for the government as τ increases, reflecting the decreasing probability that a laggard will pay the tax in states (1,2) or (1,3).

Figure 5 shows that the effect that optimal $\tau = 0$ subsidies can lower efficiency relative to a no subsidy policy if $\tau > 0$ generalizes to at least some other parameters.[18] The red areas indicate technologies where the $\tau = 0$ subsidies increase TS^{PDV}. In the white areas, the subsidies lower efficiency. The similarity in the red areas across the two panels suggests that, at least for this exercise, assuming $\tau = 0.5$ might give more accurate predictions for what happens when sellers have most, but not quite all, of the bargaining power than $\tau = 0$.

[18] With $\tau = 0$ subsidies we have difficulties finding equilibria for $\delta > 0.75$, so we restrict the parameter space considered in this diagram. We consider $\tau = 0.2$ as we know that there is a unique equilibrium when there are no subsidies.

Figure 4: Outcomes in the $M = m = 3$ Model as a Function of τ when the Subsidy Scheme that Would Implement the Social Planner Scheme when $\tau = 0$ is Implemented.

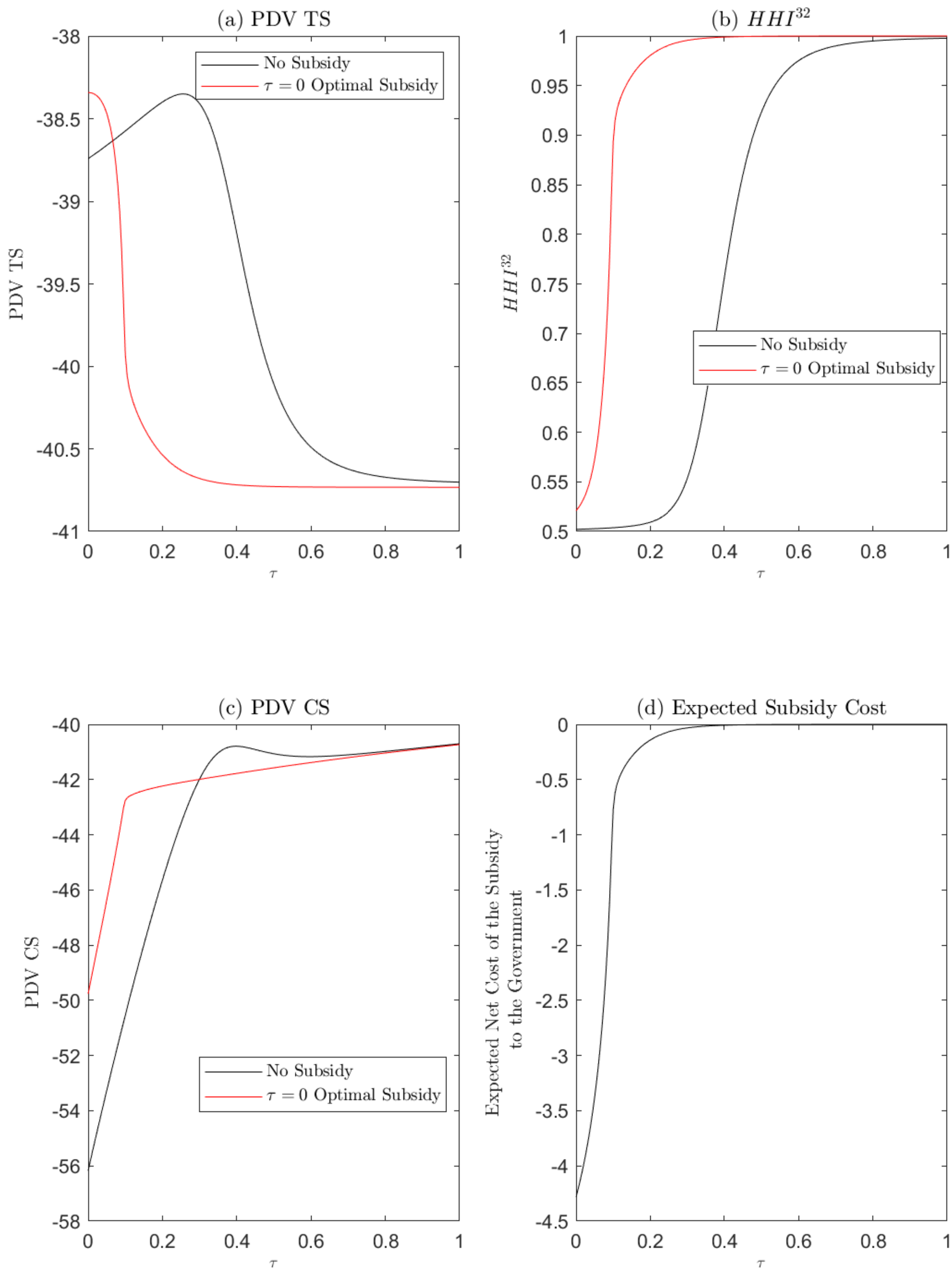
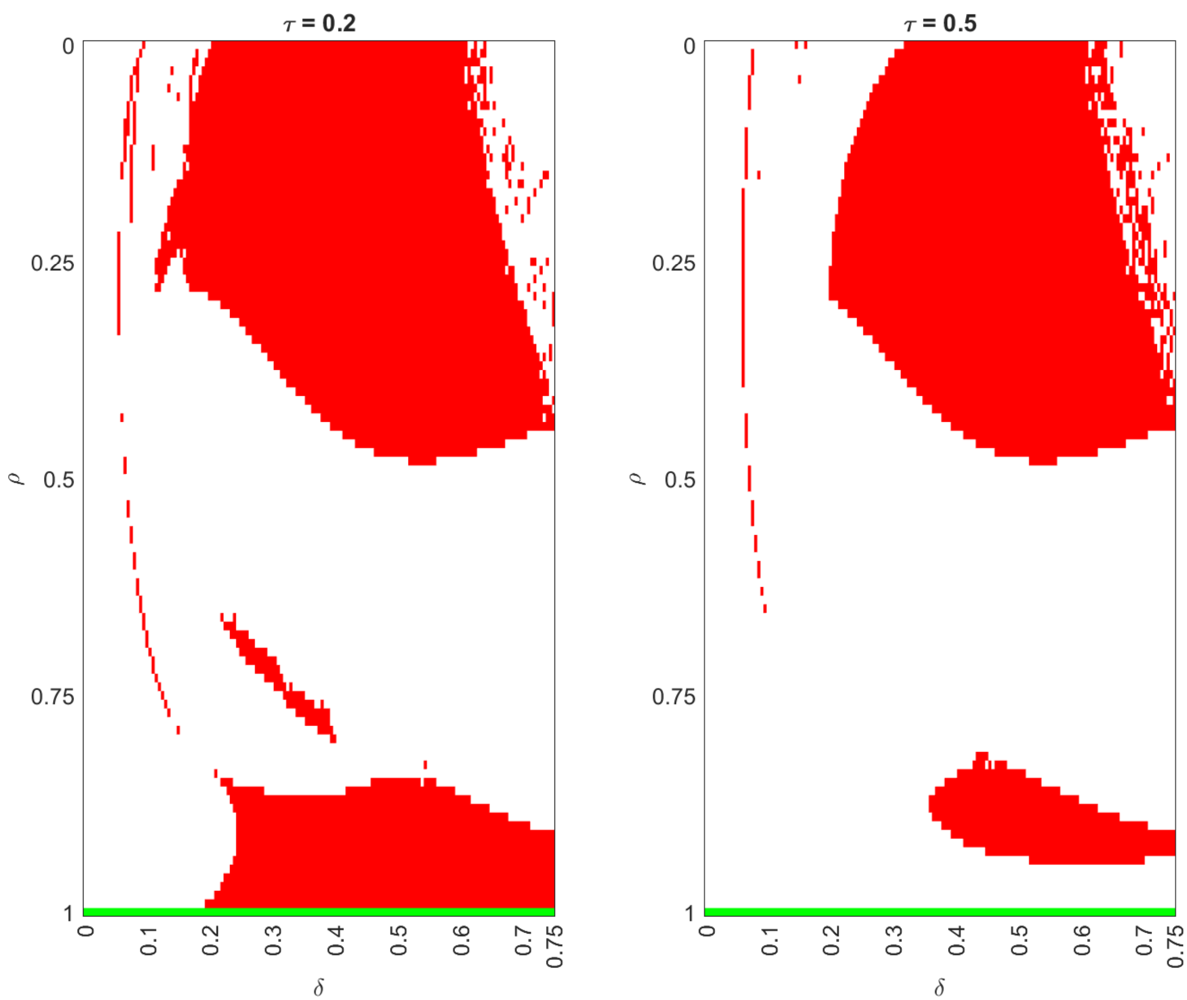


Figure 5: Technology Parameters for which Optimal $\tau = 0$ Subsidies Increase or Reduce TS^{PDV} in $M = m = 3$ Model when $\tau = 0.2$ and $\tau = 0.5$. Changes in TS^{PDV} are measured relative to the unique no subsidy equilibrium, and are based on the most efficient equilibrium supported by the subsidies. White areas indicate efficiency decreases. Green areas are technology parameters where the subsidy does not have efficiency effects (when $\rho = 1$ the costs in all states are equal).



5 $M = 30$ Model

The $M = 3$ model is convenient but it is too simple to represent reality even in a stylized way. We therefore use the full $M = 30$ BDKS model to perform our policy analysis. In this section we will describe the technology parameters that we focus on, and we will show that the qualitative features of outcomes (without policies), and how they vary with τ, resemble the $M = 3$ model. The next section will show that policy effects can be sensitive to τ.

5.1 Technology Parameters and the Social Planner Solution

Ghemawat (1985) reports that the progress ratios (ρ) estimated in 97 academic studies cover a range 0.6 to 1, with the 25$^{\text{th}}$ percentile between 0.75 and 0.8 and the 75$^{\text{th}}$ percentile between 0.85 and 0.9. When we consider a range of ρ we therefore consider $0.6 \leq \rho \leq 1$. Figure 6 shows costs curves implied by ρs within this range.

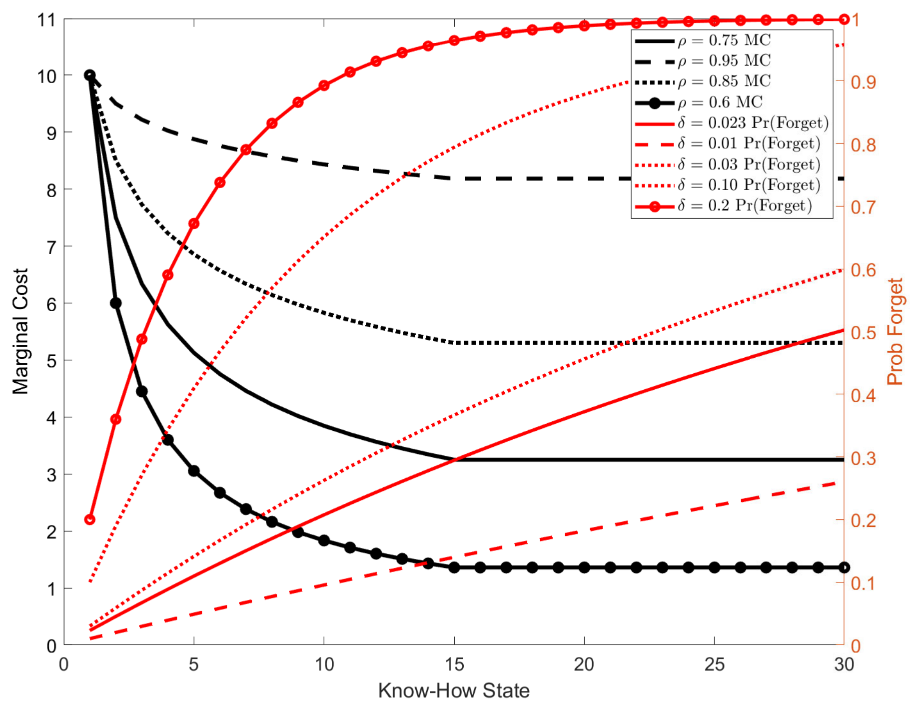
We will also consider $0 \leq \delta \leq 0.2$. For $\delta > 0.1$, it is not possible for both firms to coexist at the bottom of the learning curves ($m = 15$ is the know-how level where costs fall to their minimum values).

We consider $\rho = 0.75$ and $\delta = 0.023$ ("illustrative parameters") in more detail. $\rho = 0.75$ implies quite large LBD effects. $\delta = 0.023$ implies that it is just possible for both firms to remain close to $e = 30$ in the long-run if their prices support this outcome. In Appendix E we will also consider the effects of policies when $\rho = 0.95$ and $\delta = 0.03$. In this case, experience does not lower costs very much, but the dynamics are still important enough to be able to support multiple equilibria for some τ.

Figure 7(a)-(c) show the values of HHI^t for $t = 32$, 200 and 1,000 for different technologies when the social planner controls the industry. The areas that are red in panel (a) and blue in panels (b) or (c), which include the illustrative parameters, correspond to technologies where the social planner prefers one seller to move down its learning curve quickly, and then invests in moving the second firm down its curve once it has, as a result of favorable buyer ε shocks, accumulated a few units of know-how.

Panel (d) compares HHI^{32} and HHI^{200} under the social planner and market equilibria when $\tau = 0$ (market equilibria are found using homotopies, see the next subsection). The

Figure 6: Costs as a Function of ρ and Forgetting Probabilities as Function of δ in the $M = 30$, $m = 15$ Model.



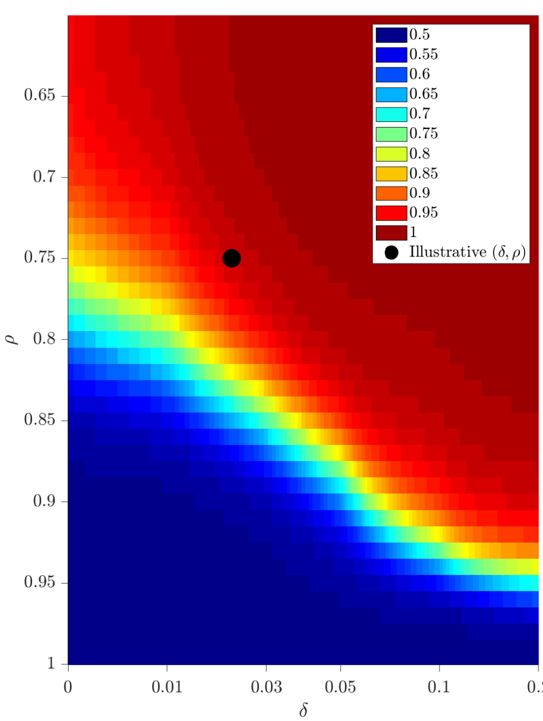
white regions support multiple equilibria which lead to ambiguous comparisons for either the medium-run or long-run or both. For the royal blue region, which covers almost all the area where $\rho < 0.85$ (approximately the median value in the empirical studies considered by Ghemawat (1985)) market equilibria are less concentrated, in both the medium-run and the long-run, than the social planner solution. As noted, Appendix E presents policy effects when market equilibria are more concentrated than the social planner would choose, although the degree of equilibrium concentration in these cases is quite limited.

5.2 Multiple Equilibria

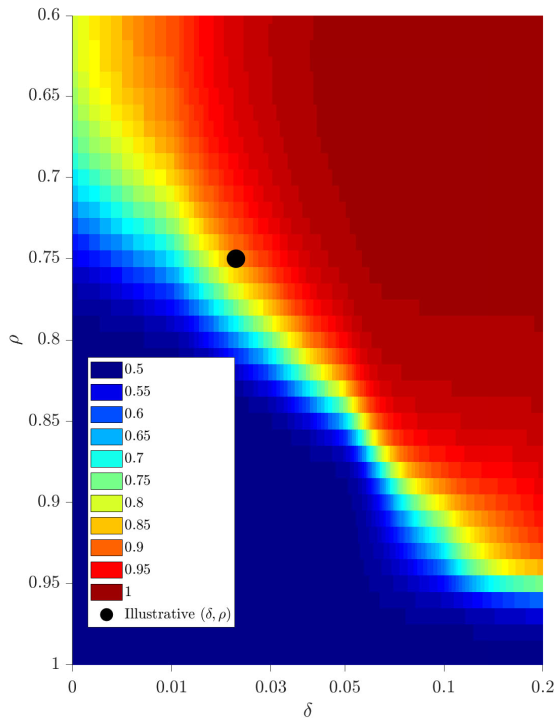
We begin by examining how bargaining affects the existence of multiple equilibria. Figure 8(a) shows the number of equilibria that are identified for different (ρ, δ) combinations when $\tau = 0$. This figure is constructed using the paths created by ρ and δ homotopies run sequentially in different directions (i.e., we may either increase or reduce a parameter) (Appendix

Figure 7: Values of HHI^{32}, HHI^{200} and HHI^{1000} for $(\delta,\ \rho)$ with Social Planner Choices and Comparison with HHI^{32} and HHI^{200} When $\tau = 0$.

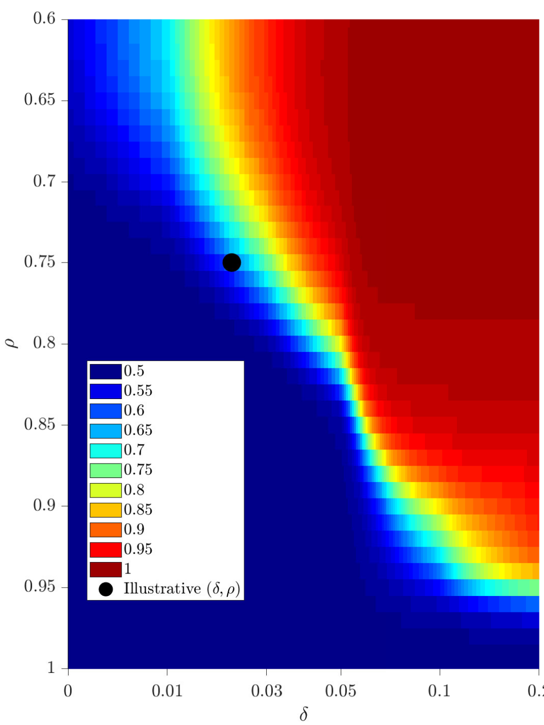
(a) HHI^{32} (Medium-Run)



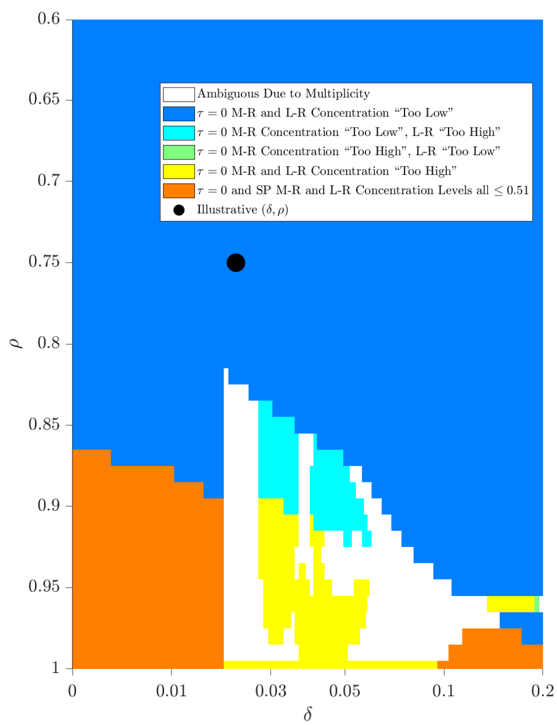
(b) HHI^{200} (Long-Run)



(c) HHI^{1000} (Very Long-Run)



(d) Compare Social Planner and Equilibrium $\tau = 0$ HHI^{32} and HHI^{200}



B.1). It resembles the equivalent figure in BDKS very closely apart from a small region, with ρ close to 1 and $\delta \approx 0.035$ where we identify more equilibria, although our search was naturally affected by knowing that we should at least identify the equilibria that BDKS found. In this richer model, there are parameters where we identify as many as nine equilibria, although, if multiplicity exists, three equilibria are most common.

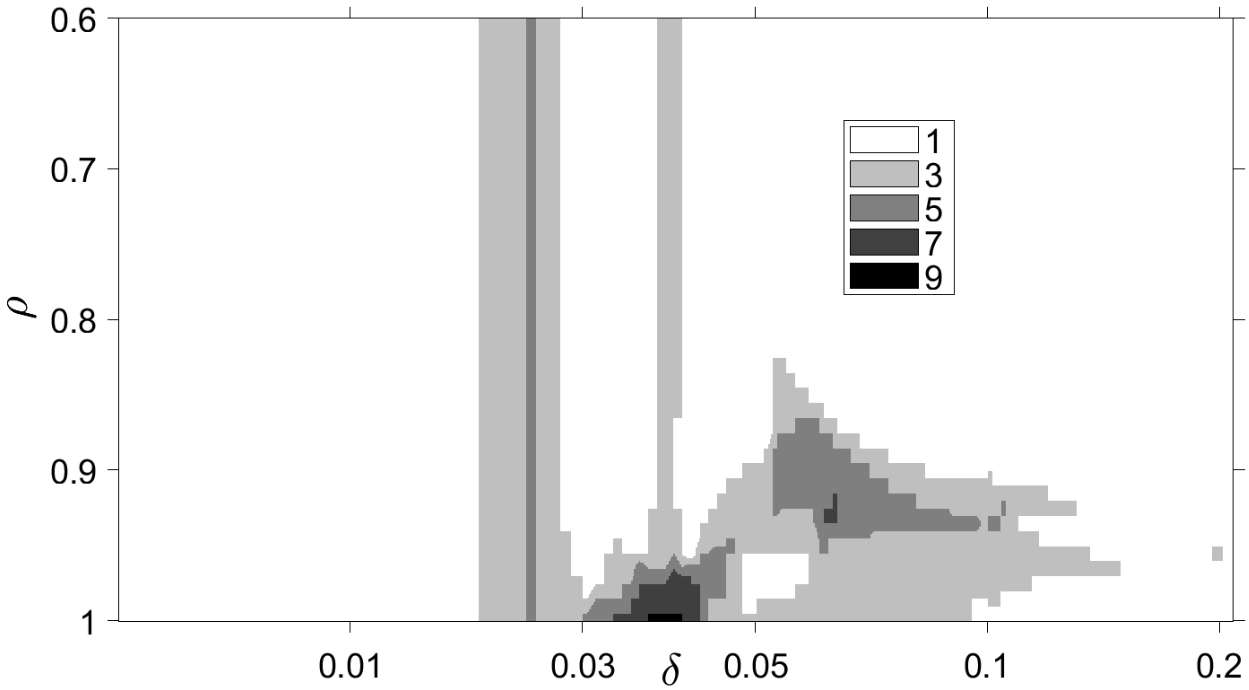
We repeat the analysis for different τ in 0.05 steps. Figure 8(b) shows the *smallest* value of τ for which multiplicity is detected. The white areas identify values where no multiplicity is identified for any τ. There are some small groups of parameters, located around the edges of regions where multiplicity exists for $\tau = 0$, where multiplicity exists when τ has a small positive value, but we do not identify multiplicity when $\tau = 0$ (Appendix E provides an example). We found similar regions when $M = 3$. In this sense, it is not always the case that shifting bargaining power to buyers tends to eliminate multiplicity.

However, as shown in Figure 8(c), we never identify multiplicity for any (δ, ρ) when τ is greater than or equal to 0.25, and, increasing τ at lower values, tends to shrink the combined area of the parameter space where multiplicity exists. No multiplicity exists for $\tau = 0.5$, a commonly assumed value in the bargaining literature.

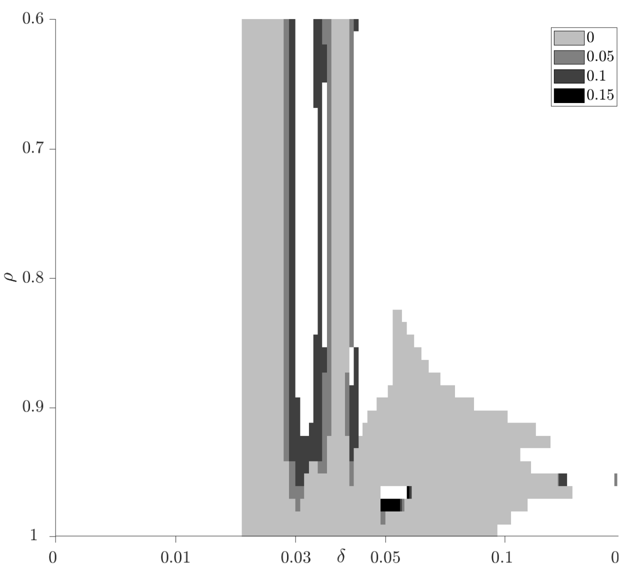
A natural question is whether bargaining selects a particular "type" of equilibrium and, if so, what are the economics that underlie this selection. To understand what an answer might look like, consider the illustrative parameters. Figure 9(a)-(c) show the prices in the three $\tau = 0$ equilibria that we identify. The one with an almost flat price surface has a 200-period expected HHI of 0.5 (the firms are expected to be symmetric by this point), whereas the other equilibria have a trench with lower prices in symmetric states, which tends to mean that the firms will be asymmetric, with expected HHI of 0.517 and 0.526, after 200 periods (although both firms will usually be at or close to the bottom of their cost curves). Panel (d) shows the HHI^{200}s associated by the equilibria on the τ-homotopy paths. The trench equilibria are on a path that does not continue beyond $\tau = 0.067$, whereas the path through the flat equilibrium extends all of the way to $\tau = 1$, although concentration increases and the price surface changes so that all prices equal production costs as $\tau = 1$. This also implies that diagonal trenches (where prices are lower than in a state with less know-how) are eliminated eventually.

Figure 8: Multiplicity in the $M = 30$ Model

(a) Number of Identified Equilibria when Sellers Set Prices ($\tau = 0$).



(b) Smallest Values of τ where Multiple Equilibria Are Identified.



(c) Smallest Values of τ' where Equilibria are Unique For All $\tau \geq \tau'$.

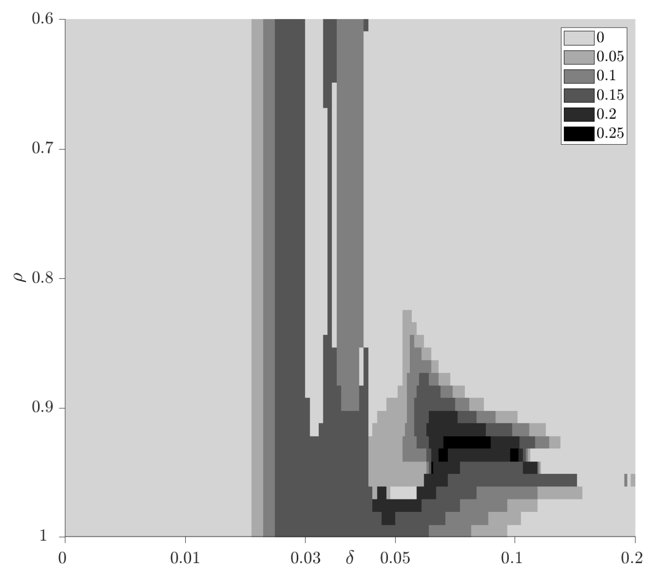
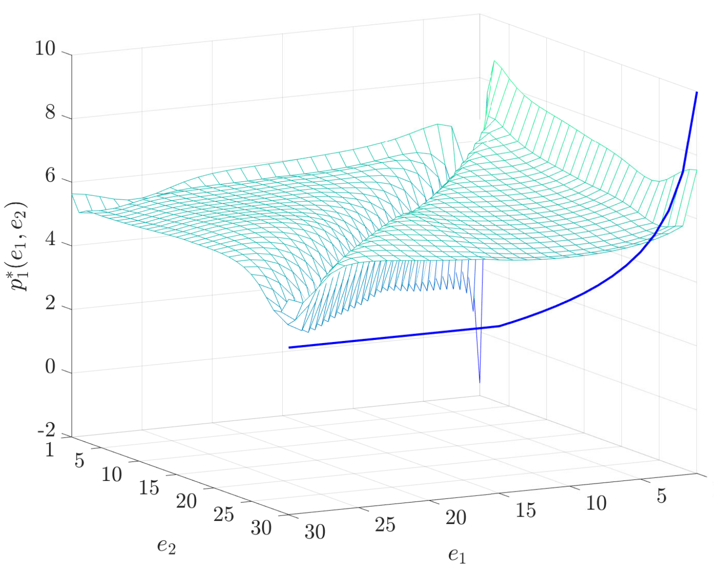
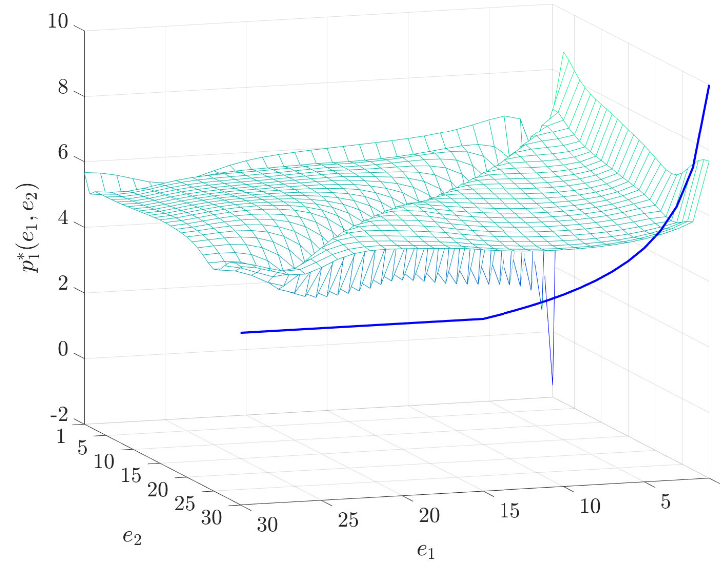


Figure 9: Equilibrium Prices when $\tau = 0$ for the Illustrative Parameters and HHI^{200} as a Function of τ. The blue curves on the price plots show firm 1 marginal costs.

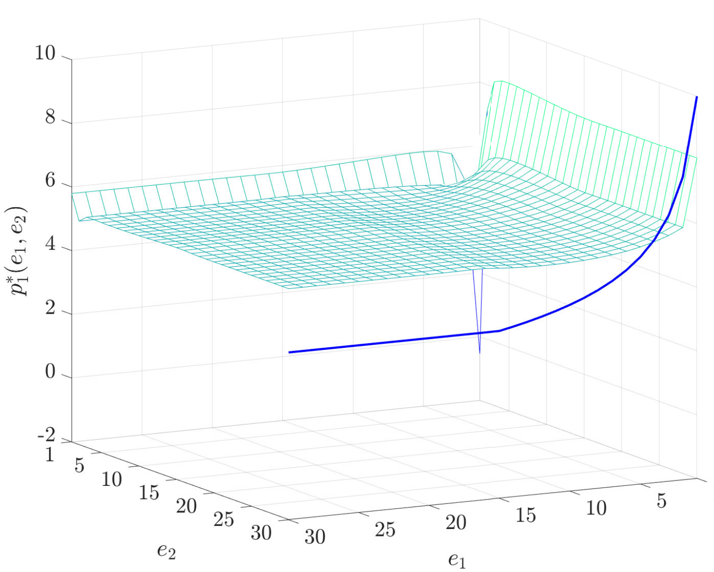
(a) $\tau = 0$ High-HHI: $HHI^{200} = 0.526$



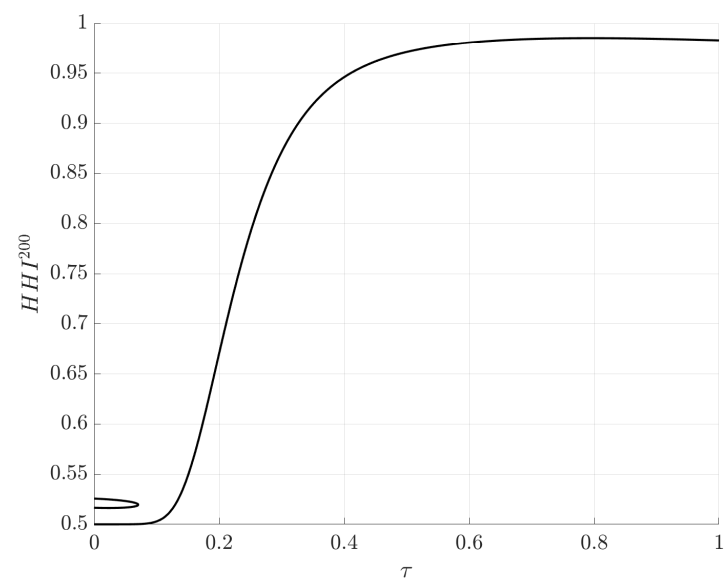
(b) $\tau = 0$ Mid-HHI: $HHI^{200} = 0.517$



(c) $\tau = 0$ Low-HHI: $HHI^{200} = 0.500$



(d) HHI^{200} Along τ-Homotopy Paths



However, further investigation has revealed that, while it is quite common for diagonal trench equilibria to be eliminated, there are also parameters where flat equilibria are eliminated and there are (τ, ρ, δ) combinations for which the only equilibria that we can identify have diagonal trenches (and possibly other features such as sideways trenches). It is therefore unclear that we can say anything about selection in general, and we do not try to pursue the issue further in thus paper.[19]

One should not interpret these results as implying that there are never scenarios with multiple equilibria for higher τ. For example, our policy analysis will consider a policy where the leader cannot consider its AD incentives. While eliminating consideration of all dynamic incentives by both firms would guarantee a unique outcome, we have identified more than 100 equilibria under our No Leader AD policy for some technology parameters when $\tau = 0.5$.

5.3 Effects of Increasing τ with No Policies

Our $M = 3$ example illustrated that reallocating bargaining power to buyers raised market concentration as a leader was more likely to sustain its lead, possibly taking concentration beyond the level that would be chosen by a social planner so that total surplus is non-monotonic in τ. The changes in market structure are driven both by how myopic buyers are more likely to buy from the leader when the leader's mark-up is restricted by the buyer's bargaining power, but also by how the leader's dynamic incentives to make sales/prevent a laggard from catching up can become much stronger if the leader expects that any lead will be sustained for longer.

Figure 10 shows that we observe similar effects in the $M = 30$ model, using the illustrative parameters as an example. The dashed lines in panel (a) show the paths of HHI^{32} (black) and HHI^{200} (red) in a model where both sellers and buyers act myopically but bargain over prices (given logit demand, the equilibrium will be unique). The dashed line in panel (b) shows TS^{PDV} under the same assumption.

The solid lines show the outcomes associated with equilibria in the dynamic model. When $\tau = 0$, concentration and the TS^{PDV} are fairly similar in the dynamic model and with static behavior. They are exactly identical when $\tau = 1$ (as prices equal production costs and

[19] Of course, it is possible that some other type of equilibrium classification would give a clear result.

Figure 10: The Effects of Changing the Allocation of Bargaining Power for the Illustrative Parameters With No Policies. In (c) only equilibria on the homotopy path from the Low-HHI equilibrium are shown.

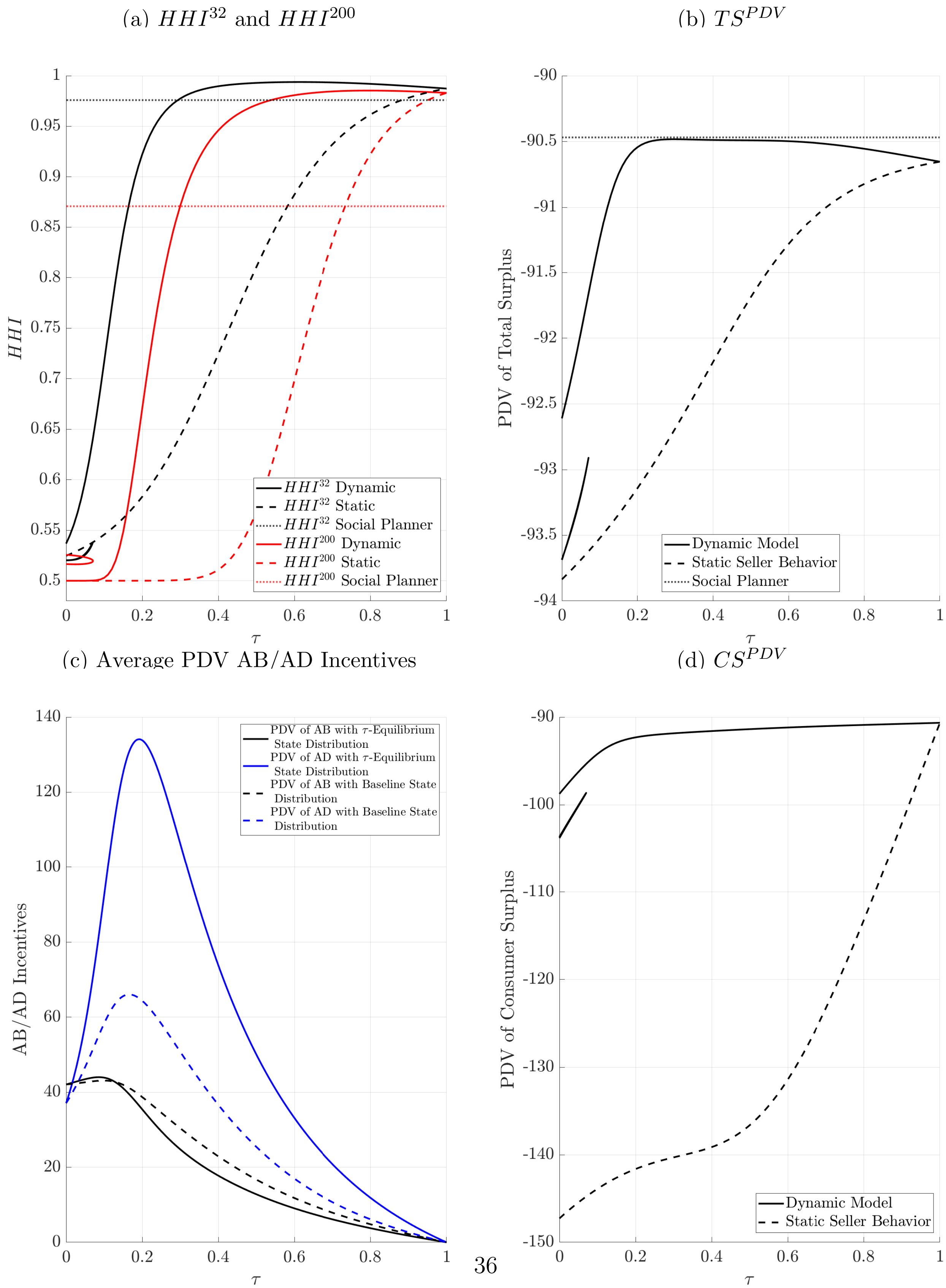
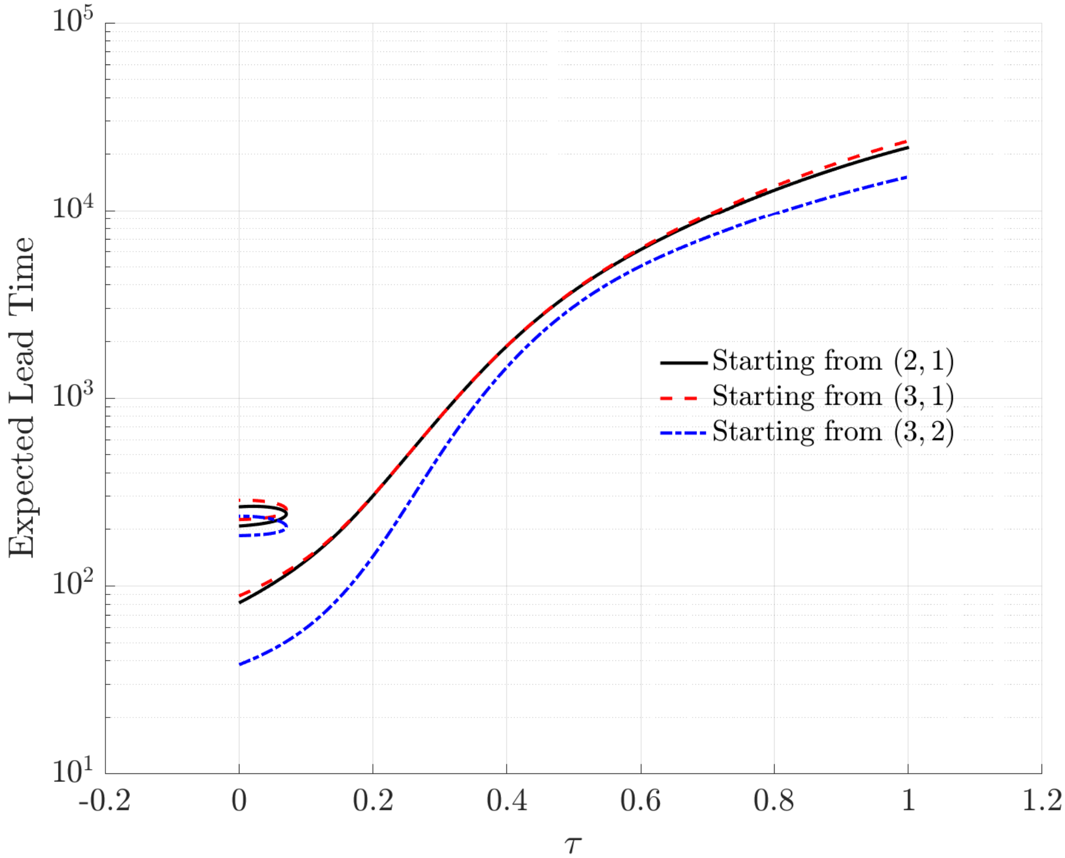


Figure 11: Expected Number of Periods A Leader Expects its Lead to Last in 3 Different States for the Illustrative Parameters in the $M = 30$ and $m = 15$ Model as a Function of τ. Notice that there is a log-scale on the y-axis.



buyers are myopic in both cases). However, when τ is between zero and one, differences can be substantial, and, in particular, concentration rises much more quickly with τ in the dynamic model, and TS^{PDV} is maximized when $\tau \approx 0.2$. Medium-run concentration declines in τ for $\tau > 0.6$.

Panel (c) shows that, as in our $M = 3$ example, the present value of AD incentives increases dramatically when τ increases from zero, partly because play shifts towards states where AD incentives are large (the difference between the solid blue and dashed blue lines). For $M = 3$, we noted that the increase in the relative value to having a lead as τ increases reflects how much longer the leader can expect its lead to last given equilibrium play. Figure 11 shows that, in the full model, the length of time that a leader in a low know-how state expects its lead to last increases exponentially for low τ.

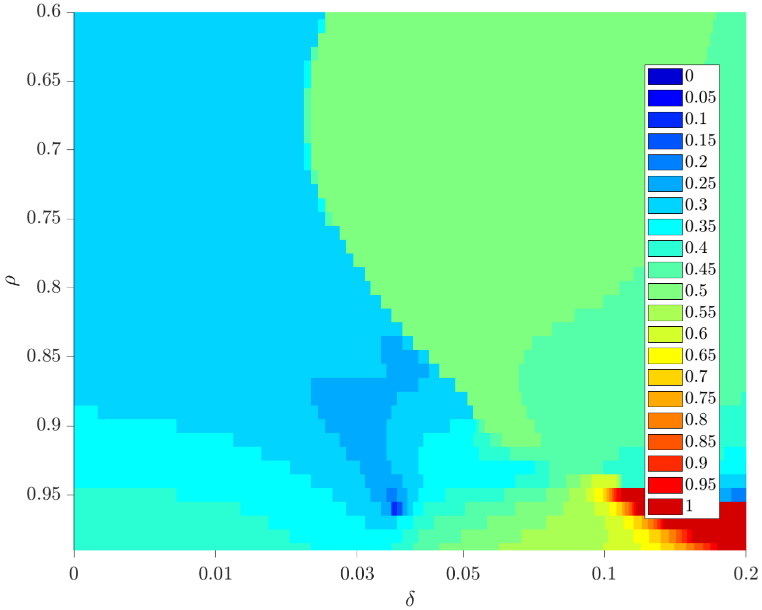
As AB and AD incentives tend to lower prices, including in the initial symmetric state, CS^{PDV} also increases sharply when τ increases from zero, although it continues to increases as τ goes to one, as buyers capture all of the available surplus. CS^{PDV} with static behavior is lower than with dynamic seller behavior for most τ, but it also increases much more dramatically from $\tau = 0.5$ to $\tau = 1$, than for $\tau = 0$ to $\tau = 0.5$.

These patterns hold fairly generally, i.e., for different technology parameters, when LBD effects on costs are strong. Figure 12 shows that for $\rho < 0.9$ and $\delta < 0.03$, the industry is most efficient for $\tau \approx 0.3$, even though efficiency is minimized by $\tau = 0$ for $\rho < 0.8$, and medium-run concentration is maximized for $0.5 \leq \tau \leq 0.7$. Similarly, seller dynamic incentives (i.e., the sum of AB and AD incentives) are maximized when buyers have some of the bargaining power for a large range of empirical relevant ρ parameters when forgetting effects are not too large.[20]

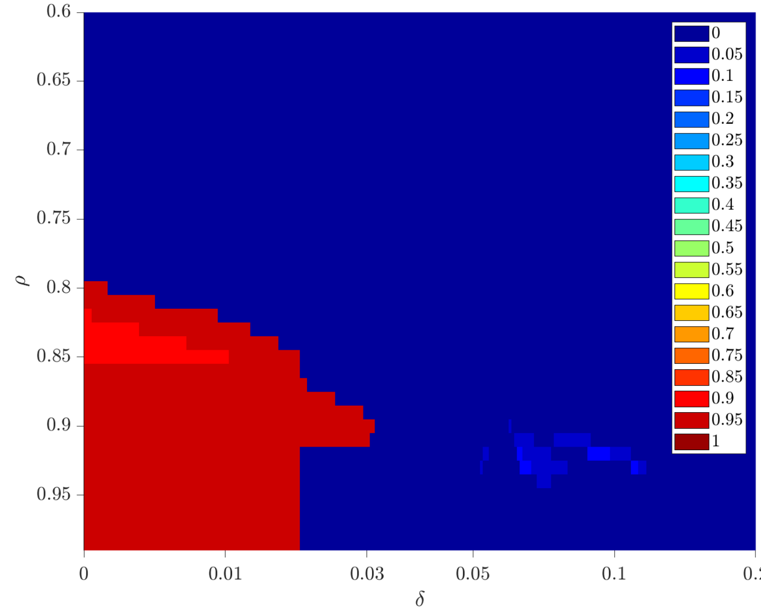
[20]If one restricts attention to seller AD incentives, then there are maximized for values $\tau \approx 0.6$ for $\rho = 0.9$ (and low δ), and for lower ρ they are also maximized for higher τs than is true in the figure.

Figure 12: Values of τ Maximizing TS^{PDV}, Concentration and the PDV of Seller Dynamic Incentives, and Minimizing TS^{PDV}. When there are multiple equilibria, we take the maximum value across equilibria for a given τ. In panel (d), AB and AD incentives for each seller are added together.

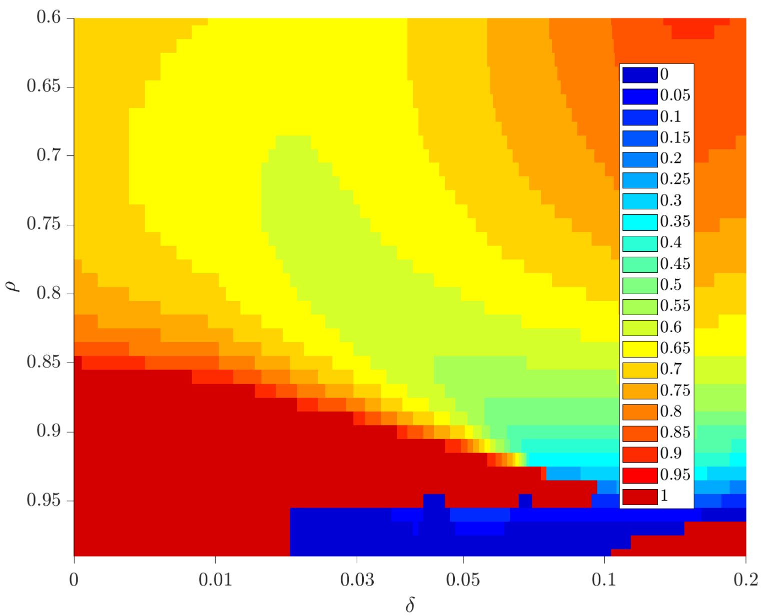
(a) τ Maximizing TS^{PDV}



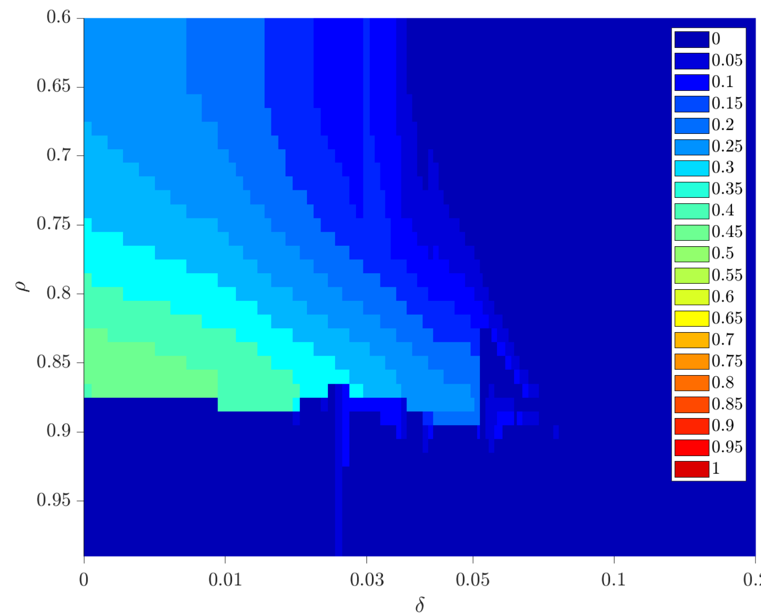
(b) τ Minimizing TS^{PDV}



(c) τ Maximizing HHI^{32}



(d) τ Maximizing PDV Seller Dynamic Incentives



6 Policies and the Allocation of Bargaining Power

We now turn to our analysis of policies in the full model. We particularly want to understand whether the optimal design of policies, and their costs/benefits, change quickly when we move away from $\tau = 0$. We begin by discussing optimal subsidy schemes which secure that same industry evolution that the social planner would choose. We then consider several policies that might be used to try to "increase competition". As these policies may tend to lower efficiency by slowing how quickly the first firm moves down its cost curve, we also consider policies that are introduced once one firm has made sufficient progress as well as policies that apply throughout the life the industry.

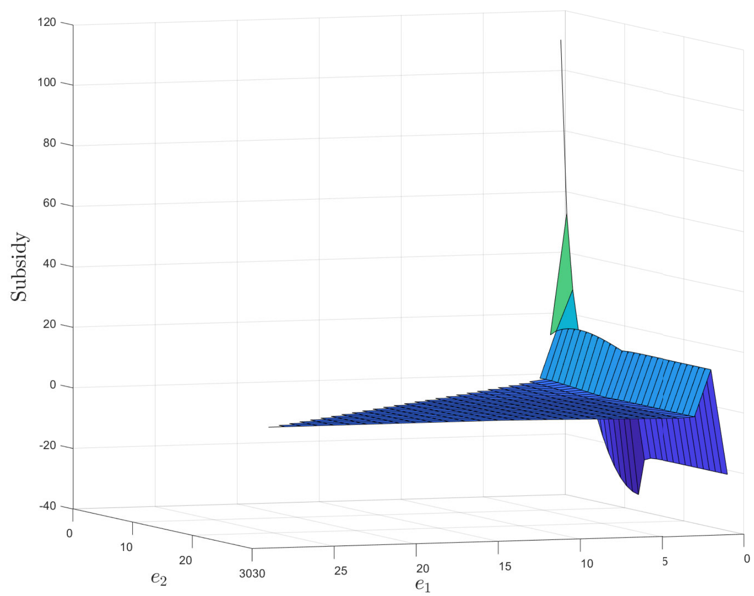
We note two limitations of our analysis. First, because our model is very stylized, we do not think it is appropriate to draw clear conclusions about which type of policy is better. We therefore focus more on how interactions between dynamic incentives, bargaining and market structure affect outcomes and why these interactions may mean that conclusions based on a $\tau = 0$ model are misleading. Second, our current discussion focuses on the illustrative parameters (with some results for one alternative set pf parameters in Appendix E). Future versions will try to understand which patterns appear to be general, although our current conclusion that policies *can* be sensitive to τ should not be read as implying that we believe that there are no parameters for which the level of τ is less important.

6.1 Optimal Subsidies

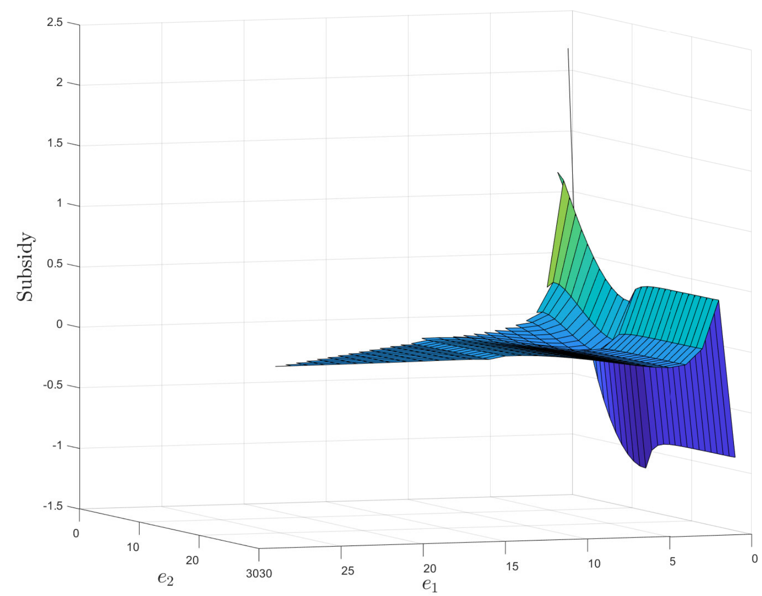
We can solve for the subsidies that implement the efficient evolution of the industry using the reformulated equations. Figure 13 shows the subsidies provided to the laggard, in the event that it sells, for $\tau = 0$, 0.25, 0.5 and 1 for the illustrative parameters. Figure 14(a) shows how optimal subsidies vary with τ continuously for a subset of states, and panel (b) shows the expected present discounted cost to the government. Increasing τ from zero has dramatic effects on optimal subsidies. Gor example, when $\tau = 0$ it is optimal to provide a very large subsidy in state (1,2), a large subsidy in states where the laggard has $e = 2$, but a tax when $e = 1$ and the leader has a large lead. When $\tau = 0.25$ optimal taxes and subsidies have much smaller scales. When $\tau \geq 0.5$, the pattern of which states have subsidies and

Figure 13: Optimal Subsidies to the Laggard for the Illustrative Parameters for $\tau = 0,\ 0.25,\ 0.5,\ 1$. Negative values indicate taxes. No value shown for leader states.

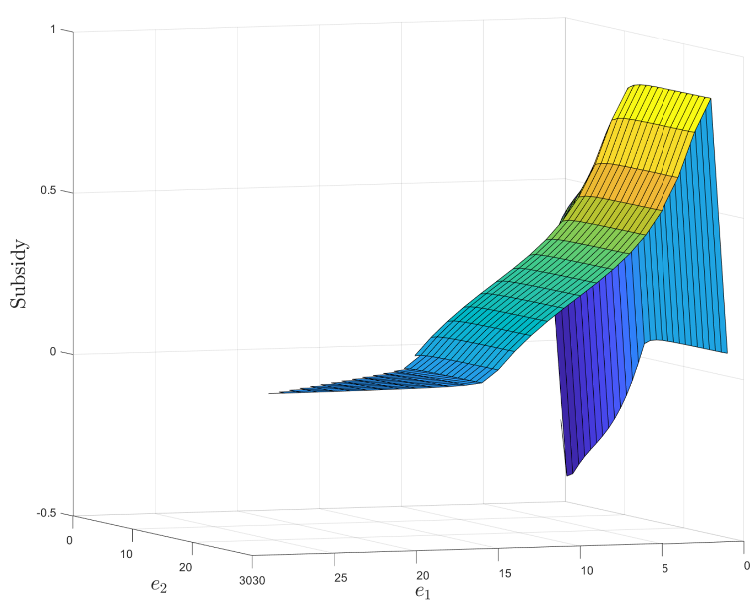
(a) $\tau = 0$



(b) $\tau = 0.25$



(c) $\tau = 0.5$



(d) $\tau = 1$

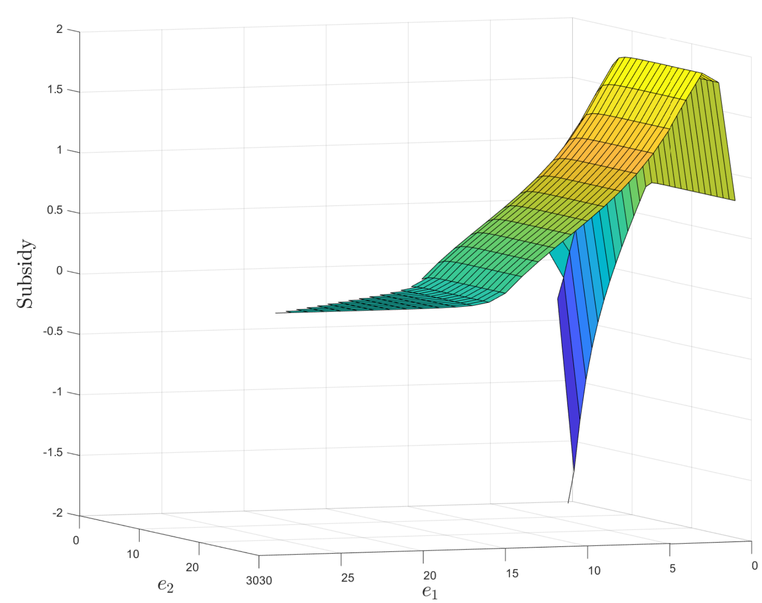
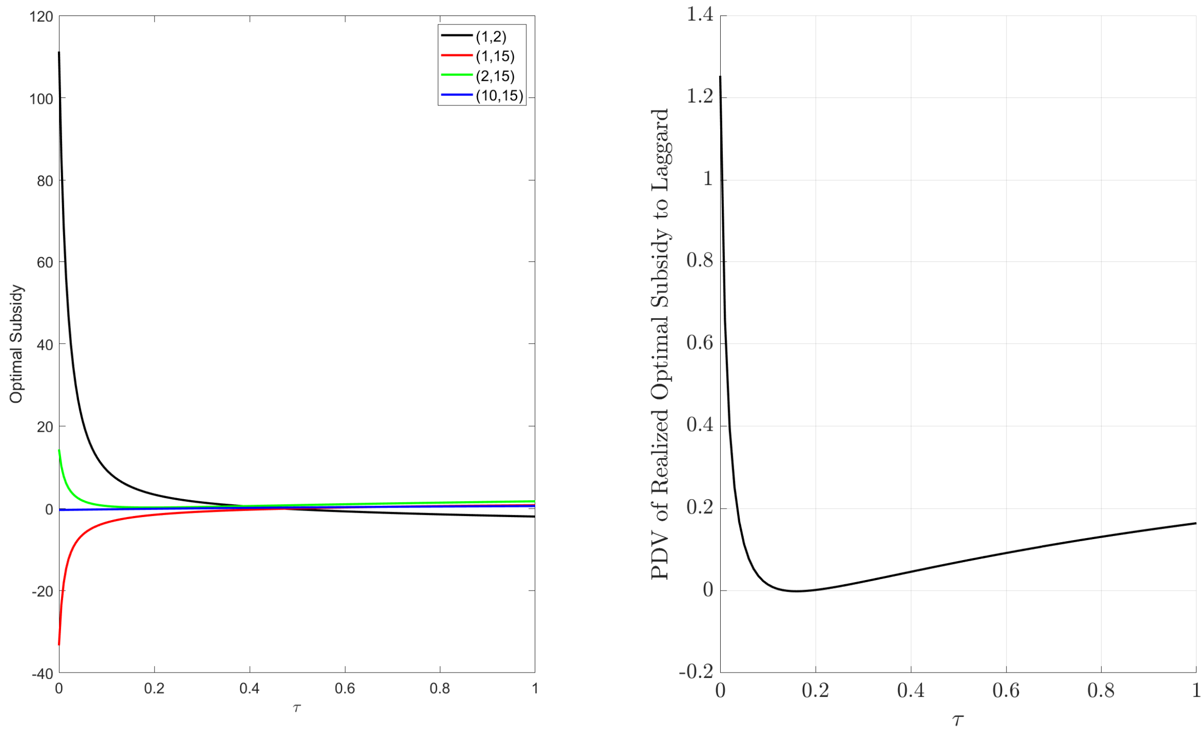


Figure 14: Optimal Subsidies and Expected Net Cost of Optimal Subsidies as a Function of τ for the Illustrative Parameters.

(a) Optimal Subsidies for Select States (b) Expected PDV Cost of Optimal Subsidy



which have taxes is different (e.g., the laggard is taxed if it makes a sale in (1,2)).

The structure of optimal subsidies may appear unintuitive. For example, when $\tau = 0$, the non-subsidy equilibrium probabilities that the laggard makes the sale in state (1,2) are *higher* than the social planner would choose but the optimal strategy is to *subsidize* a sale by the laggard. The fact that this can happen reflects the how incentives across states interact. As the leader is more likely to preserve its lead for longer in the social planner's solution (which is being implemented), a firm in state (2,1) has very strong dynamic incentives when $\tau = 0$ (its AB incentive is 127.3 and its AD incentive is 80.7). Therefore the leader would price as if it had a large, negative marginal cost, and the role of the subsidy to the laggard is to make sure that its probability of making a sale is at the socially optimal level rather than being vanishingly small. As the subsidy is very unlikely to be claimed, the expected net cost of the optimal subsidy is quite small for all τ.

We also find that the subsidies that are optimal when $\tau = 0$ can cause significant efficiency losses when $\tau > 0$ but small. For example, for the illustrative parameters, the optimal subsidies increase TS^{PDV} from -92.61 to -90.47 when $\tau = 0$, but if these subsidies were used when $\tau = 0.03$, TS would decrease from -92.24 to -95.34. Therefore, the sensitivity of the welfare effects of policies to whether sellers are assumed to have all of the bargaining power appears as pronounced in the $M = 30$ model as it was in our $M = 3$ example.[21]

6.2 Policies to Increase Competition

We now consider a set of policies that might be expected to "promote competition" by imposing some limits on the ability or the incentives of a market leader to get too far ahead.

We recognize that it might seem odd to consider this type of policy when, at least for low τ, the social planner would prefer more concentration in the early life of the industry than equilibrium will generate. However, we think it is interesting to consider these policies partly for this very reason as it seems plausible to us that there are policy-relevant sectors of the economy where commentators advocating for these policies (i) presuming that they will actually lead to more symmetric market structures and that (ii) presuming that efficiency losses are not too large. Without a model it is unclear whether either presumption is correct. Alternatively, one might motivate our analysis by how market structures may be too concentrated if bargaining power is roughly even split between buyers and sellers.

We proceed by describing the alternative policies that we consider before discussing how they affect outcomes (for different τ) for the illustrative parameters under the assumption that they are introduced in state (1,1) (i.e., when the industry begins). We will then discuss whether policies would be more effective if introduced only once the leader has achieved a particular level of know-how.

6.2.1 Policy Details

Restriction on Market Concentration. In his analysis of competition in the market for wide-bodied commercial aircraft, Benkard (2004) considers a counterfactual where limits

[21] The very large size of the subsidies when $\tau = 0$ leads to numerical problems when we try to calculate equilibria for alternative values of τ, as some of the choice probabilities are within numerical precision of zero. We will try to overcome this issue in future iterations.

are imposed on the market share of the largest firm in a given quarter. His estimates imply that market share limits of 60% and 51% slightly reduce consumer surplus and total surplus on average (changes are less than 1%), as prices not only tend to rise in states where the limits are binding, but also in states where firms would otherwise set low prices in order to try to gain a dominant position.[22]

In our duopoly single-buyer-per-period model, we implement the policy as a "soft" market share restriction assuming that a market leader has to pay a compliance cost of $\chi \times \max\{0, D_i - \psi\}^2$ whenever its sale probability is more than a threshold $\psi > 0.5$. As χ increases, it becomes more costly for a firm to have a high market share. The total surplus calculations that we present here assume that the compliance cost is not a real cost to society. If it was counted as a real cost then in the periods when the leader has a larger share, expected total surplus would be reduced and the policy would look worse (sometimes significantly so) in terms of efficiency.

Incorporating this penalty, the first-order condition for the negotiated price becomes

$$\begin{gathered}-\tau\left[D_1^*(\mathbf{e})(p_1^*(\mathbf{e})-\widehat{c}_1)-\chi\max\{0,D_1^*(\mathbf{e})-\psi\}^2\right]+(1-\tau)\log\left(\frac{1}{1-D_1^*(\mathbf{e})}\right)\times\\ \left[\sigma-(1-D_1^*(\mathbf{e}))(p_1^*(\mathbf{e})-\widehat{c}_1)+2\chi(1-D_1^*(\mathbf{e}))\max\{0,D_1^*(\mathbf{e})-\psi\}\right]=0,\end{gathered} \tag{15}$$

and the equation for the seller's value becomes

$$VS_1^*(\mathbf{e})-D_1^*(\mathbf{e})\left(p_1^*(\mathbf{e})-c(e_1)\right)-\sum_{k=1,2}D_k^*(\mathbf{e})\mu_{1,k}^S(\mathbf{e})-\chi\max\{0,D_1^*(\mathbf{e})-\psi\}^2=0. \tag{16}$$

The restriction can affect prices even when $\tau = 1$, because the seller can guarantee that it will not violate the constraint if it does not agree a price. In our analysis below we assume that $\chi = 50$ and $\psi = 0.75$. Appendix Figure C.2 compares several outcomes when $\chi = 10$

[22]While absolute restrictions on market shares are rare, market shares can play an important role in determining potential Sherman Act Section 2 liability for actions that agencies or rivals claim are anticompetitive (see discussion in the Department of Justice 2008 report "Competition and Monopoly: Single-Firm Conduct Under Section 2 of the Sherman Act", `https://www.justice.gov/sites/default/files/atr/legacy/2009/05/11/236681.pdf`, although the report was withdrawn as official policy in 2009). The European Union, the UK (e.g., `https://tinyurl.com/hx75cf44`) and legislation in the US Congress have proposed frameworks that impose potentially onerous restrictions or requirements on platforms identified as dominant, which is likely to, at least partially, reflect market shares.

and $\psi = 0.75$.[23]

Restrictions on Pricing Incentives. BDK and Besanko, Doraszelski, and Kryukov (2019b) consider the effects of alternative limitations on the dynamic incentives that firms are able to consider in the BDK model, motivated by standards that have been proposed in the antitrust literature for judging that a price is predatory.[24]

We consider how equilibrium outcomes change when the market leader (i.e., a firm i with $e_i > e_j$)[25] :

- is assumed to be unable to consider dynamic incentives at all. This is implemented by excluding the seller's continuation values (i.e., the μ^S terms) from the first-order conditions that determine prices, so that the perceived marginal cost is simply the current production cost.

- is assumed to be unable to consider AD incentives, but it is able to consider AB incentives (which, of course, may change in magnitude when AD incentives cannot be considered).

Restrictions on Pricing. In practice, restrictions on the incentives that firms can consider would be difficult to impose ex-ante. As a simple alternative, we therefore consider a restriction that the leader's price must be above its current cost of production, motivated by how below-cost pricing is often viewed as a necessary, but not sufficient, condition for a

[23] A notable feature is that a lower χ can increase long-run compliance costs because it is more likely to support market structures that violate the share threshold.

[24] We consider the effects of similar limitations in the BDKS setting where, instead of exit, there is some possibility that the rival forgets. While the lack of exit may appear to make the BDKS model less attractive for considering rules motivated by the legal literature on predation, it is not the case that the economic literature on investment motivated by anticompetitive intent (e.g., Caves and Porter (1977), Lieberman (1987)) necessarily assumes that exit has to happen. Further, as shown in SJHY, for many parameters in the BDK model it is certain that a firm will not exit once it has made a sale, so that firms that have made sales cannot be predated upon, whereas, of course, allegations of anticompetitive pricing often come from rivals that have had some initial success. In the BDKS model, where a firm's know-how can always decrease if $\delta > 0$, there is potential scope for a leader to act aggressively against a rival over a much wider range of states.

[25] We have also computed some results imposing these restrictions on both firms. However, allegations of anticompetitive conduct usually focus on the market leader.

court to conclude that pricing is anticompetitive.[26,27] The leader's price will be above cost when it cannot consider dynamic incentives, but considering some dynamic incentives may be consistent with below cost pricing in at least some states.

Implementing this restriction involves introducing an additional set of equations (to which homotopies can be applied) associated with the Lagrangian constraints on the leader's prices. To solve for an initial equilibrium from which to start homotopies, we use what can be thought of as an iterative guess-and-verify approach. We first solve the problem with no constraints, and then impose the constraints by setting prices equal to marginal cost in states where they are below marginal cost, before resolving for prices in all of the other states. We then check whether the constraints would still be binding, before re-solving appropriately.

Multiplicity of Equilibria. Multiple equilibria can complicate policy analysis by making comparisons between no policy and policy scenarios ambiguous. As noted previously, many equilibria can exist for some technology parameters under some of our policies even when $\tau = 0.5$. Our discussion will focus on the illustrative parameters. For these parameters we find some multiplicity for low τ under the Concentration Restriction and Leader $p \geq mc$ policies, but the predictions across these equilibria are sufficiently similar that they do not complicate our conclusions. Based on our searches, which do identify multiplicity for other technology parameters, equilibria under the incentive policies are unique.

6.3 Effects of Policies Introduced at the Start of the Industry's Life

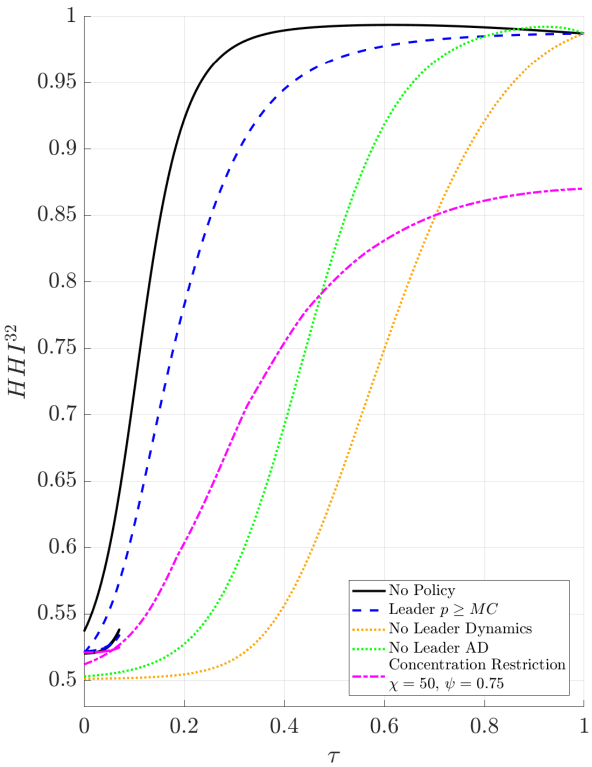
Figures 15 shows how policies change expected concentration and surplus. Focusing on $\tau < 0.8$, the directional effects are broadly as expected. In particular, the policies lower expected concentration. There is some cost to efficiency, but this cost is actually greater for τ values between 0.2 and 0.5 (when the no policy equilibrium produces too much concentration)

[26] See discussion in the Department of Justice 2008 report "Competition and Monopoly: Single-Firm Conduct Under Section 2 of the Sherman Act", `https://www.justice.gov/sites/default/files/atr/legacy/2009/05/11/236681.pdf`.

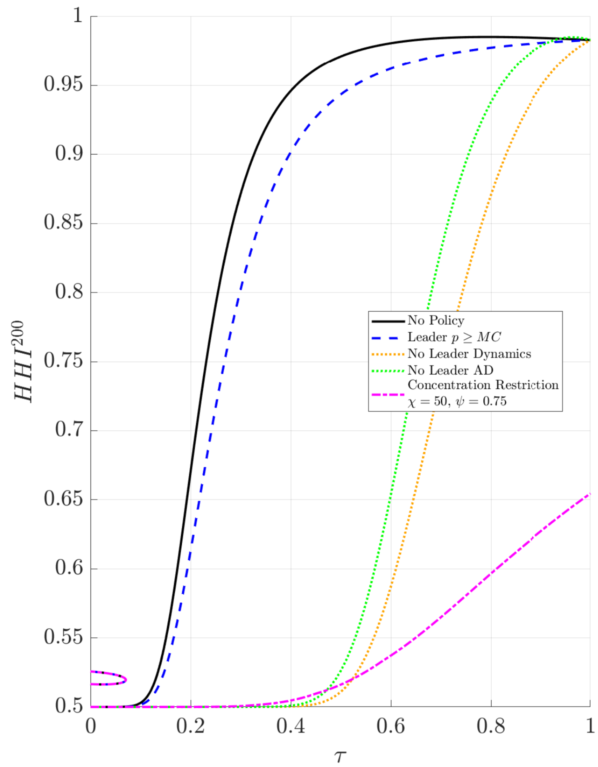
[27] In practice, there are many arguments about the appropriate measure of cost, and these would, of course, be significant in a setting where an incremental sale lowers expected future costs.

Figure 15: Effects of Policies Introduced at the Start of the Industry's Life on Concentration and Total and Producer Surplus, for the Illustrative Parameters. The compliance costs of the Concentration Restriction policy are not counted as costs to society in total surplus calculations, although they are costs to the sellers.

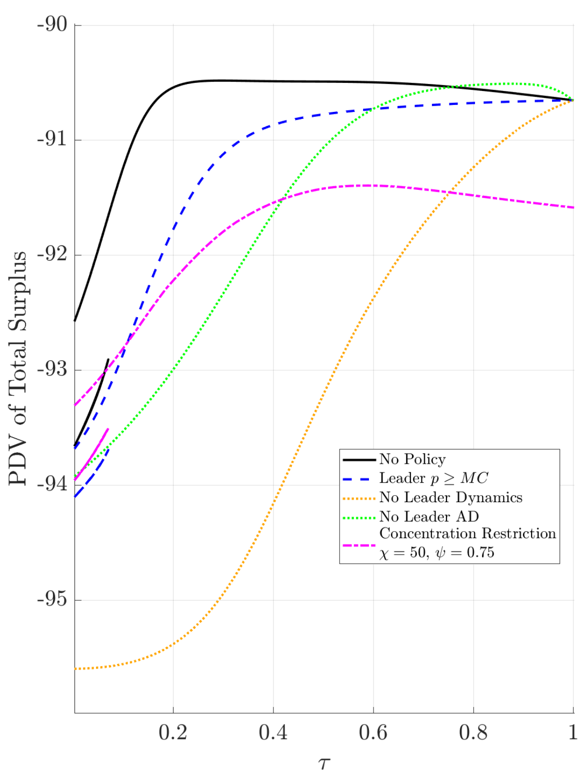
(a) HHI^{32}



(b) HHI^{200}



(c) TS^{PDV}



(d) PS^{PDV}

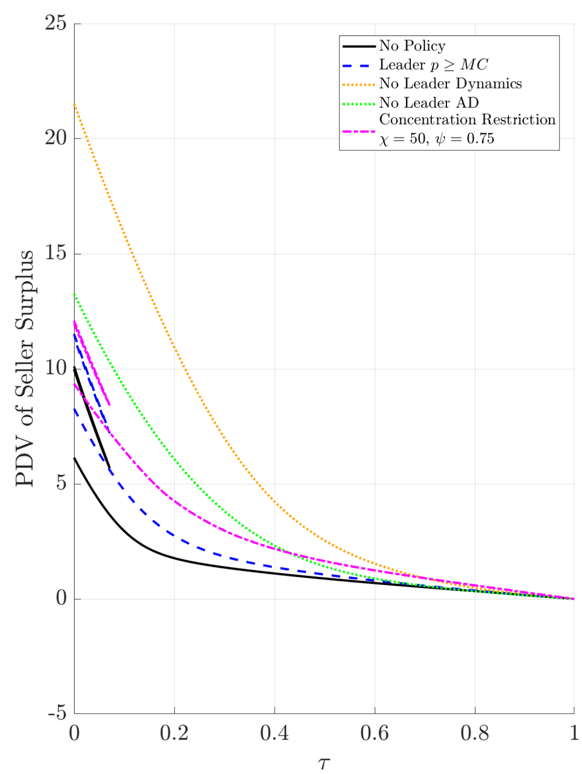
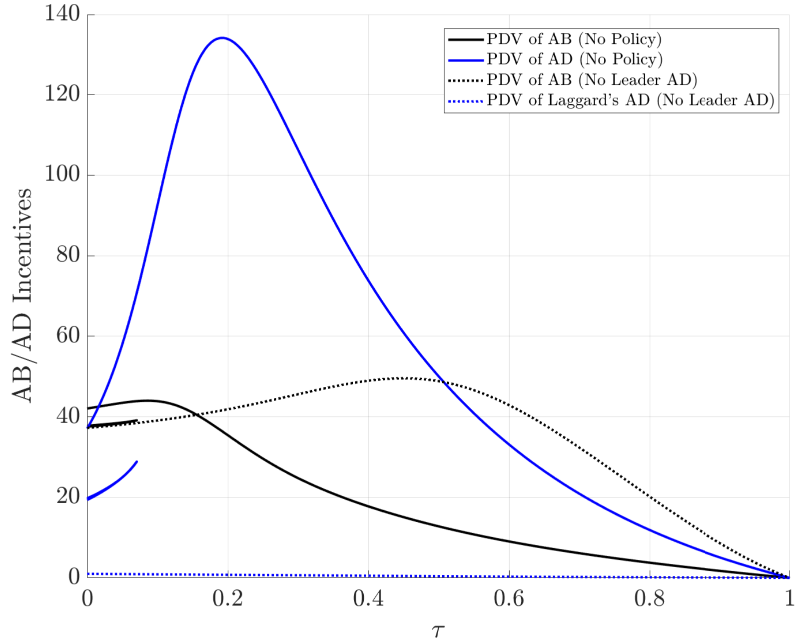


Figure 16: The Present Discounted Value of Sellers' AB and AD Incentives With and Without the No Leader AD Policy for the Illustrative Parameters.

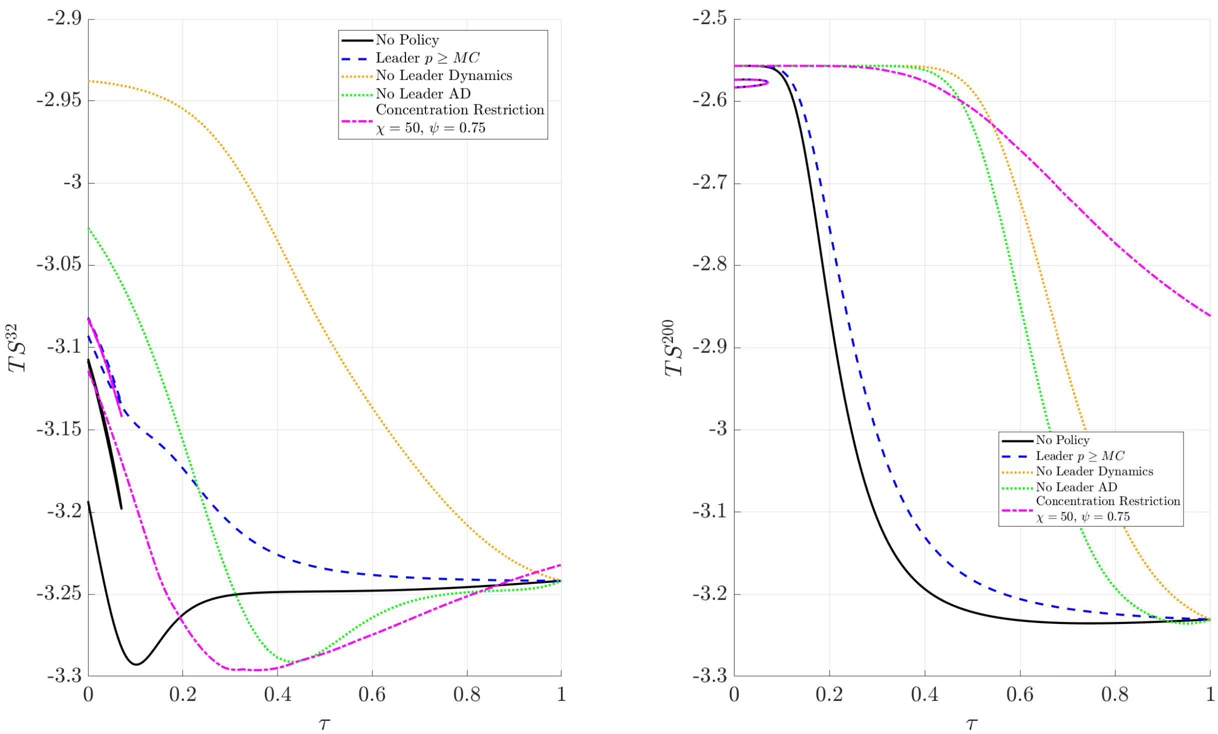


than it is for $\tau = 0$, reflecting how the effects on market structure can be larger when the market would be more concentrated with no policy. The policies soften competition so that even though the policies tend to promote symmetry, prices tend to increase, raising producer surplus, and lowering consumer surplus (not shown) and efficiency. The effects of the Leader $p \geq mc$ policy are quite modest, on both concentration and efficiency, for all τ, reflecting the fact that below marginal cost pricing only happens in a limited number of states even though dynamic incentives affect the evolution of market structure.

The effects are qualitatively different for the No Leader AD policy for $\tau \geq 0.8$. Medium-run concentration, TS^{PDV} and CS^{PDV} *increase* (i.e., concentration moves in the opposite of the expected direction). While we would not suggest that $\tau \geq 0.8$ is likely of particular relevance for empirical work, the logic is illustrative of interactions between dynamic incentives within the model. In particular, in the equilibrium with the policy, the discounted expected value of sellers' dynamic incentives (which lower prices) increases for high τ because AB incentives rise, even though the leader cannot consider AD incentives. This is illustrated in Figure 16, which shows the value of the AB and AD incentives with no policies, and the value of AB incentives and the laggard's AD incentives (which can affect prices) in the

Figure 17: Effects of Policies on Expected Total Surplus in 32^{nd} and 200^{th} Periods.

(a) TS^{32} (b) TS^{200}



equilibrium with the policy. When $\tau \approx 0$, the policy slightly lowers AB incentives, which strengthens how the policy tends to increase prices. For higher τ, the policy increases the value of AB incentives (compare the black solid and dotted lines), but the elimination of the AD incentive dominates until $\tau \approx 0.8$ when AB incentives with the policy exceed the sum of the AB and AD incentives without the policy. As discussed in Appendix D, the policy causes the leader's prices to fall in the early periods of the game which increases efficiency.[28]

6.4 Policies that are Introduced Only When the Leader Reaches a Specific State

In practice, there may only be political pressure for these policies once an industry has reached a sufficiently advanced state of development. Moreover, an analysis of the effects of policies introduced at the start of the industry shows that the effects of policies on the PDV

[28] The value of HHI^8 is actually lower in the $\tau = 1$ equilibrium than in the social planner solution (even though HHI^{32} and HHI^{200} are higher) implying that additional sales by the leader in low know-how states like (2,1) or (3,1) can increase efficiency.

of welfare measures and their effects on the expected values of those measures later in the game can be quite different. For example, Figure 17 show how the policies affect affect TS^{32} and TS^{200}. While the policies lower TS^{PDV} for almost all τ, efficiency may increase later in the game for at least some policy-τ combinations.[29]

We therefore also compute outcomes under what we call "trigger policies". These policies take the form that "the policy will be introduced as soon as one firm reaches know-how state e' and will then last forever". We assume that the government is committed to e', and that the sellers understand that the policy will be introduced as soon as e' is reached.[30]

Figure 18 shows how TS^{PDV} changes for four different τ, as a function of the trigger. As none of the policies constrains firms in state (1,1), the outcomes when the trigger is $e' = 2$ are identical to those when the policy is introduced at the start of the industry.[31] Trigger policies do raise the TS^{PDV} when $\tau = 0$ and the trigger is set so that the policy is introduced when one firm has made it (close to) state m (i.e., the lowest know-how state where costs are minimized).

However, this is not true if $\tau = 0.25$, with the policies lowering TS^{PDV} for all triggers (for the Leader $p \geq mc$ policy there is no effect for $e' \geq 6$ triggers). The changes in whether the policy increases TS^{PDV} happen for τs quite close to zero: for example, No Leader Dynamics and No Leader AD policies with triggers of $e' = 20$ lower TS^{PDV} if $\tau > 0.06$ (with negligible effect for $\tau \geq 0.8$) and the Concentration Restriction policy lowers TS^{PDV} if $\tau > 0.08$. This is another example of how allowing buyers to have even limited bargaining power change the qualitative conclusions that should be drawn.

The trigger policies raise TS^{PDV} for $\tau = 0$ not because they affect behavior only once they are enforced. Instead, it is because they actually increase concentration in the initial periods of the game. The intuition is that laggards know that they will benefit from the policy once the trigger is reached, so they compete less aggressively to try to catch-up before the leader reaches the trigger (so that the leader reaches the trigger more quickly). Figure

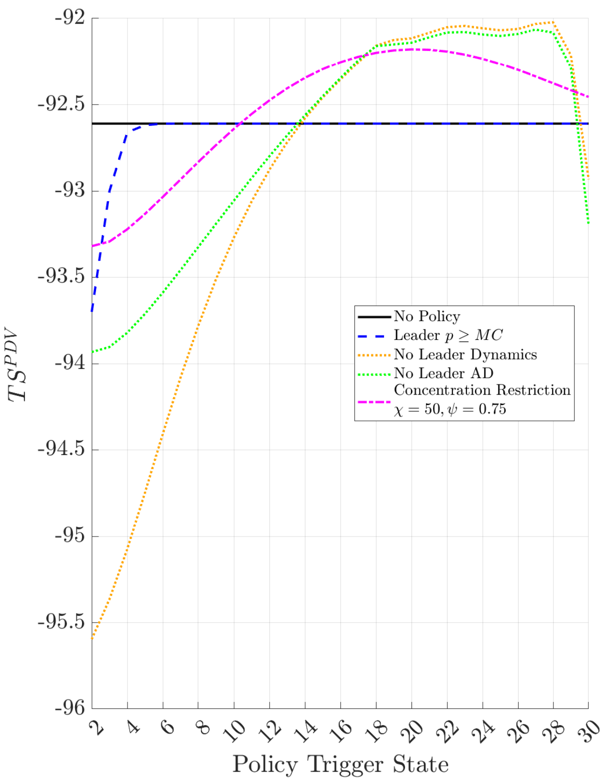
[29] The intuition is that policies that make it more likely that there will be a second low-cost producer, which raises efficiency once it has been achieved.

[30] Of course, outcomes might be different if the introduction of the policy is uncertain or is a probabilistic function of market structure or prices.

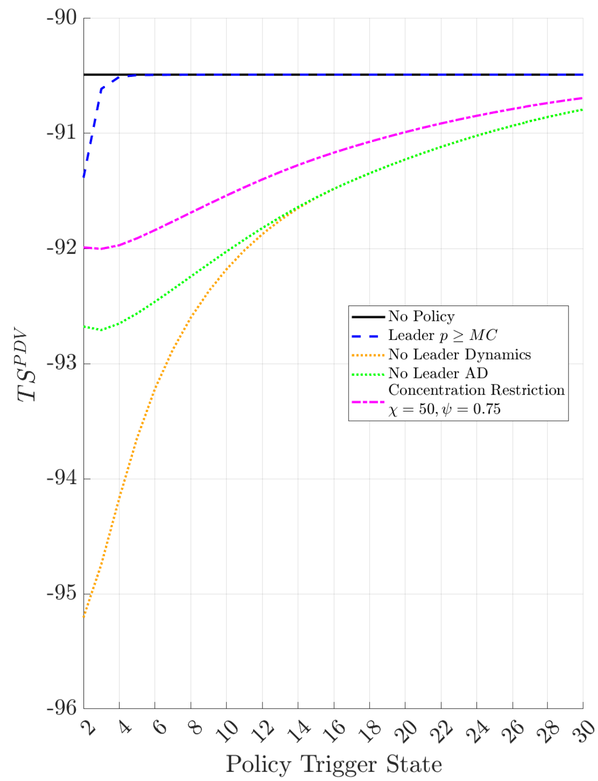
[31] A state 2 trigger implies that the policy is imposed as soon as there is a leader, which is the equivalent to imposing leader-only policies (like ours) at the start of the industry.

Figure 18: Effects of Policies on TS^{PDV} as a Function of the Trigger State for the Illustrative Parameters. The compliance costs of the Concentration Restriction policy are not counted as costs to society in total surplus calculations, although they are costs to sellers.

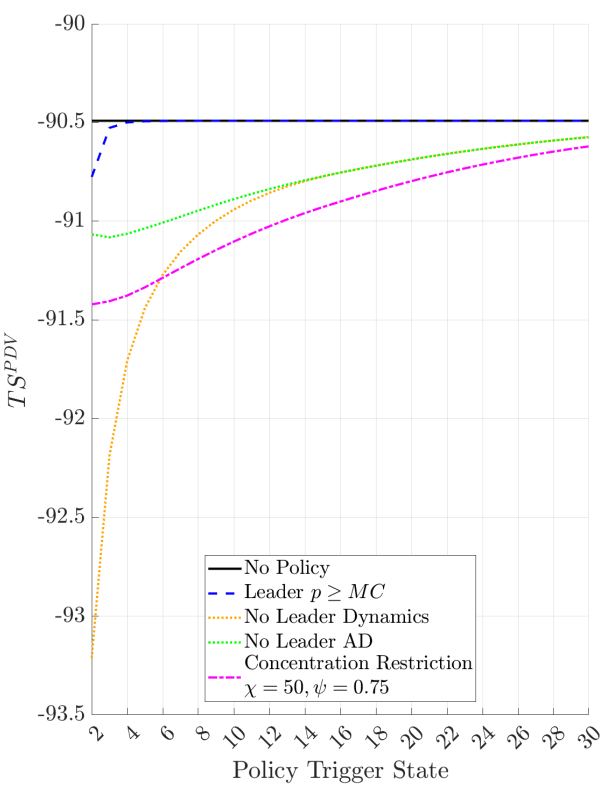
(a) $\tau = 0$ TS^{PDV}



(b) $\tau = 0.25$ TS^{PDV}



(c) $\tau = 0.5$ TS^{PDV}



(d) $\tau = 0.9$ TS^{PDV}

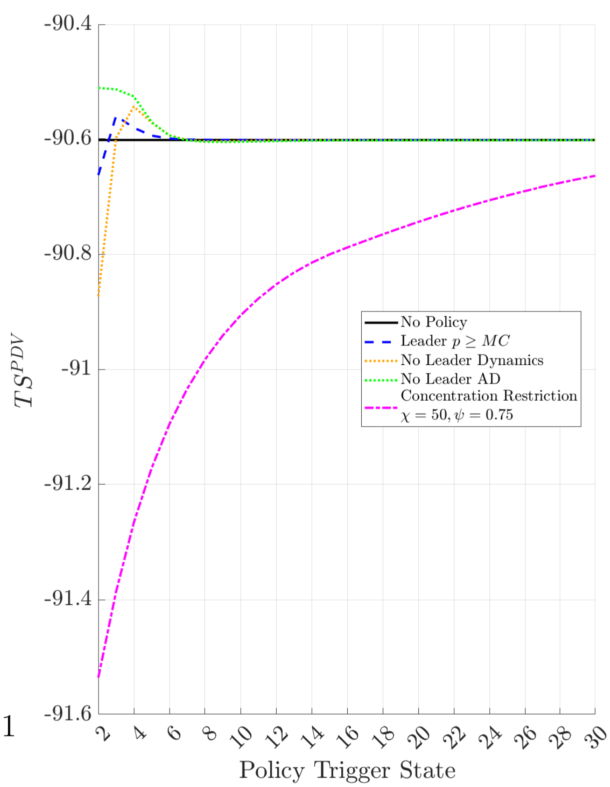
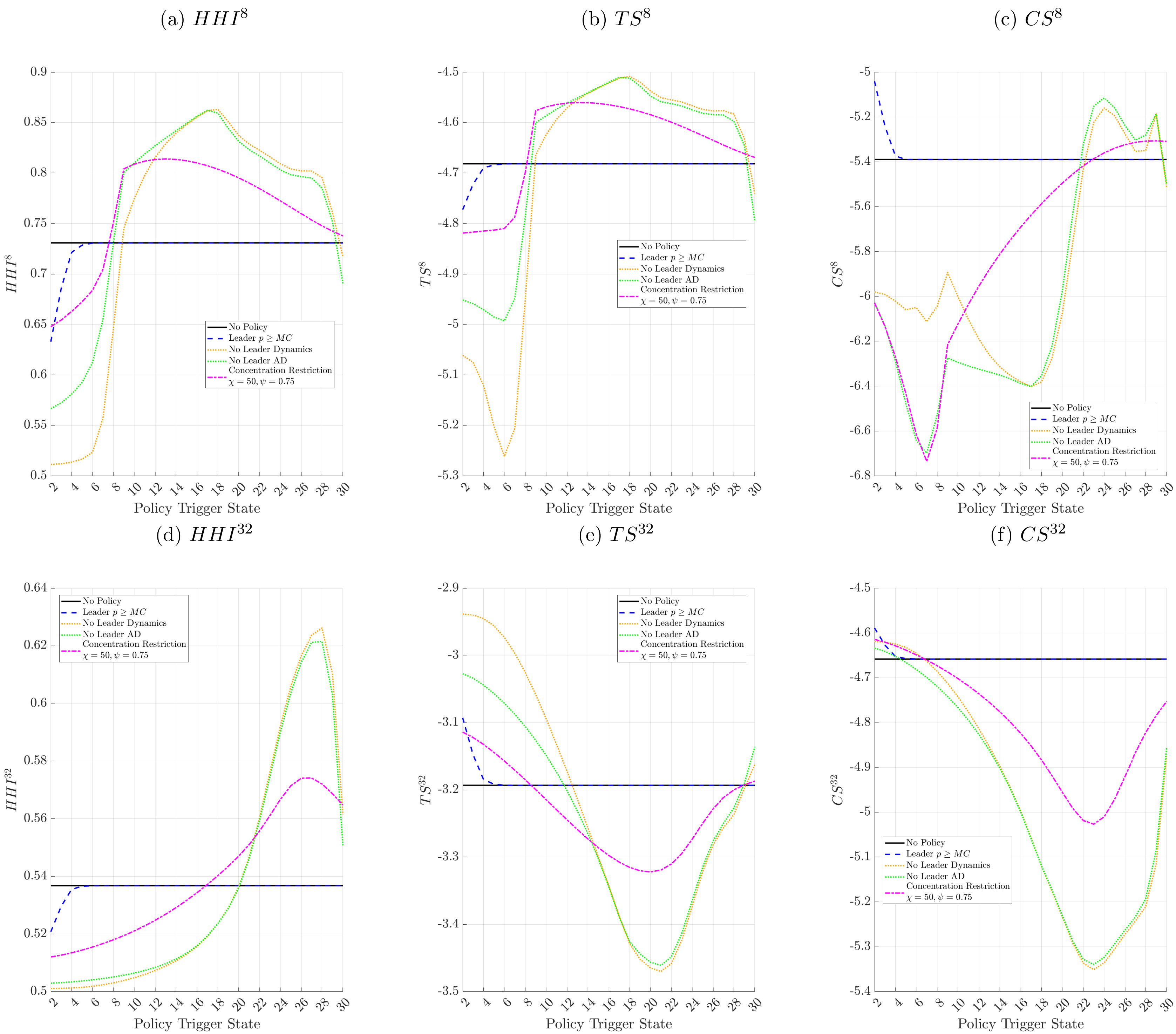


Figure 19: Effects of Policies on Concentration and Total Surplus After 8 and 32 Periods for $\tau = 0$. The compliance costs of the Concentration Restriction policy are not counted as costs to society in total surplus calculations, although they are costs to sellers.



19 shows that, for the triggers that increase total surplus, expected concentration and total surplus after 8 periods are higher than with no policy when $\tau = 0$, whereas expected total surplus and consumer surplus after 32 periods may be lower as it is more likely that only one firm will have a low cost, but the imposed policy will be reducing competitive pressure to lower prices.

7 Conclusion

Models in the dynamic competition literature have assumed that sellers compete by setting prices, even though these models are often motivated by issues arising in industries where firms are selling large capital goods and buyers are likely to engage in at least some negotiation over prices. The applied bargaining literature, which has found that bargaining affects mark-ups, pass-through and the effects of mergers, has focused on static settings where market structure is not itself affected by bargaining. This paper extends a well-known stylized model of dynamic competition, where sellers benefit from learning-by-doing, to ask whether allowing for bargaining, rather than assuming that sellers set prices, could significantly change the types of outcomes and policies that the literature has focused on. We are particularly interested in whether allowing for buyers to have limited bargaining power affects outcomes, as we believe that empirical researchers could often presume that, in these situations, assuming price-setting is a reasonable approximation. We analyze these issues using a model of bargaining that nests the standard price-setting assumption as a special case.

We find that outcomes, such as expected market concentration and efficiency, change quite dramatically as we move away from price-setting for many, although certainly not all, technology parameters, and that, absent policies, equilibria are unique once buyers have even quite moderate bargaining power. Our main intuition for this result comes from the fact that when buyers receive a small amount of bargaining power, the static effect of bargaining will have the largest (reducing) effect on the margins of sellers that have a large lead over their rivals (i.e., they are close to being monopolists). But, when buyers are myopic, this will tend to reduce the already small probability that laggard firms make sales, and therefore tend to increase how long a leader will expect its lead to last. The expected lifetime of a lead can increase exponentially, so that even as reallocating bargaining power lowers sellers' expected payoffs in any future state (holding continuation values fixed), the reallocation of bargaining power towards buyers raises the relative payoff to being a leader, rather than being a follower. As a result, firms compete more aggressively to become, and remain, leaders. This can raise efficiency when, because price-setting sellers do not capture all of the benefit from lowering their costs, the leader is likely to move too slowly down its cost curve in a price-setting equilibrium, but, even for these types of parameters, it can lead to concentration being excessive once buyers have even quite moderate bargaining power.

Our results illustrate how dynamic incentives, market structure and bargaining power interact in ways that we do not arise in the existing static literature on bargaining and have not been recognized in the literature on dynamics. Our results also show that these results matter for policy design, although we recognize that our model is more stylized than one would want to use to design policies for a specific industry. The effects are particularly clear when we consider optimal subsidies, with subsidies that would maximize efficiency when sellers have all of the bargaining power potentially lowering efficiency when sellers have the vast majority, but not quite all, of the bargaining power for at least some empirically relevant parameters. When analyzing policies that might seem designed to increase competition, we find that the efficiency costs and benefits of these policies (relative to having no policy) depend on the bargaining power parameter. In future work we will consider other policy-relevant variables, such as the size of magnitude of production cost efficiencies that required to make mergers procompetitive, are similarly sensitive.

The model could be enriched a number of ways. It would be interesting to consider multi-unit purchases and multiple purchases, and to understand how outcomes with bargaining over prices compare with outcomes when sellers choose quantities (as is sometimes assumed when researchers observe sales to a number of buyers in a given time period, such as a quarter, within the data). The framework in SJHY, where buyers can be moderately "forward-looking", can easily be incorporated into the current model. In SJHY we find that the primary effect of forward-looking buyers is to soften competition and these effects are significant when buyers only expect to capture a small share of future surplus. Therefore, bargaining power and forward-looking behavior can be viewed as two different forms of strategic buyer behavior that may have opposite effects.[32]

[32]However, we find that either type of buyer sophistication eliminates multiple equilibria, and, in initial work, we have found that this happens even more quickly when we allow for both aspects.

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ONLINE APPENDICES FOR "Bargaining and Dynamic Competition" by Deng, Jia, Leccese and Sweeting

A Sketch Proofs of Proposition 1

Recall Proposition 1.

Proposition 1 *1. If $\tau = 1$, prices will equal marginal production costs in all states, for all b^p, ρ and δ.*

2. when $b^p = 0$, there will be a unique symmetric MPNE when

(a) $\delta = 0$ for all ρ and τ (a result that also holds for intermediate values of b^p not considered in this paper).

(b) $\tau = 1$ for all ρ and δ.

Proof of Part 1. The structure of the proof is to show that VS must be zero in every state, which implies that price will equal marginal production costs. Since $\tau = 1$, the solution to the bargaining problem satisfies the following equation

$$(p_i(\mathbf{e}) - c_i(e_i))D_i(p(\mathbf{e}), \mathbf{e}) + \sum_{k=1,2} D_k(p(\mathbf{e}), \mathbf{e})\mu^S_{i,k}(\mathbf{e}) = \mu^S_{i,-i}(\mathbf{e}), \tag{17}$$

where $\mu^S_{i,k}(\mathbf{e}) = \beta \sum_{\mathbf{e}'} \Pr(\mathbf{e}'|\mathbf{e}, q_k) VS_i(\mathbf{e}')$ is firm i's continuation value after the buyer purchases from firm k, and the subscript $-i$ denotes the rival firm.

Recall that firm i's beginning of period value satisfies the following equation (18),

$$VS_i(\mathbf{e}) = (p_i(\mathbf{e}) - c_i(e_i))D_i(p(\mathbf{e}), \mathbf{e}) + \sum_{k=1,2} D_k(p(\mathbf{e}), \mathbf{e})\mu^S_{i,k}(\mathbf{e}). \tag{18}$$

Plugging (17) into (18) yields that

$$VS_i(\mathbf{e}) = \mu^S_{i,-i}(\mathbf{e}) = \beta \sum_{\mathbf{e}'} \Pr(\mathbf{e}'|\mathbf{e}, q_{-i}) VS_i(\mathbf{e}').$$

The above equation can be rewritten in a matrix form,

$$\mathbf{VS}_i = \beta \mathbf{Q} \mathbf{VS}_i,$$

where $\mathbf{VS}_i$ is an $M^2 \times 1$ vector, and $\mathbf{Q}$ is an $M^2 \times M^2$ Markov matrix. Therefore,

$$\mathbf{VS}_i = \left(\lim_{T \to \infty} \beta^T \mathbf{Q}^T \right) \mathbf{VS}_i = 0.$$

Plugging $VS_i(\mathbf{e}) = 0$ into (17) yields that $p_i(\mathbf{e}) = c_i(e_i)$.

Proof of Part 2(a). Follows from the recursive proof of BDKS, and the fact there can only be one (static) MPNE in the absorbing state (M, M).[33]

Proof of Part 2(b). The previous proof shows that , if $\tau = 1$, $VS_i(\mathbf{e}) = 0$ and $p_i(\mathbf{e}) = c_i(e_i)$ in all states. As a result, the MPNE is characterized by the equations concerning the buyer's value:

$$VB^*(\mathbf{e}) - b^p \sigma \log \left(\sum_{k=1,2} \exp \left(\frac{v_k - p_k^*(\mathbf{e}) + \mu_k^B(\mathbf{e})}{\sigma} \right) \right) - (1 - b^p) \sum_{k=1,2} D_k^*(\mathbf{e}) \mu_k^B(\mathbf{e}) = 0, \quad (19)$$

where

$$\mu_k^B(\mathbf{e}) = \beta \sum_{\mathbf{e}'} \Pr(\mathbf{e}' | \mathbf{e}, q_k) VB^*(\mathbf{e}').$$

If $b^p = 1$, (19) can be rewritten as

$$VB^*(\mathbf{e}) = \sigma \log \left(\sum_{k=1,2} \exp \left(\frac{v_k - p_k^*(\mathbf{e}) + \beta \sum_{\mathbf{e}'} \Pr(\mathbf{e}' | \mathbf{e}, q_k) VB^*(\mathbf{e}')}{\sigma} \right) \right).$$

One can view the right hand side as a functional of $VB^*(\cdot)$. We denote the functional by $\mathcal{T}$ and show next that $\mathcal{T}$ satisfies Blackwell's sufficient conditions for a contraction (see Theorem 3.3 in Stokey, Lucas and Prescott, 1989).

[33] Note that in state (M, M), the value of b^p has no effect on any buyer's strategy. However, this does not necessarily imply that there is uniqueness in earlier states even when movements through the state space are unidirectional. See Appendix D in SJHY for a discussion.

It is clear that $\mathcal{T}$ is monotone. That is, $[T(VB)](\mathbf{e}) \leq [T(\widehat{VB})](\mathbf{e})$ if $VB(\mathbf{e}) \leq \widehat{VB}(\mathbf{e})$. Note also that for any constant $a \geq 0$, $[T(VB + a)](\mathbf{e}) = \beta a + [T(VB)](\mathbf{e})$. Therefore, Blackwell's sufficient conditions are satisfied and $\mathcal{T}$ is a contraction mapping. By the Contraction Mapping Theorem, $\mathcal{T}$ has exactly one fixed point, which implies the existence and uniqueness of the MPNE.

If $b^p = 0$, the buyer's problem is effectively static and (19) can be rewritten in a matrix form,

$$\mathbf{VB} = \beta\widehat{\mathbf{Q}}\mathbf{VB},$$

where $\mathbf{VB}$ is an $M^2 \times 1$ vector, and $\widehat{\mathbf{Q}}$ is an $M^2 \times M^2$ Markov matrix. Therefore, $\mathbf{VB} = (\mathbf{I} - \beta\widehat{\mathbf{Q}})^{-1}\mathbf{0} = \mathbf{0}$ and $VB^*(\mathbf{e}) = 0$ uniquely solves (19).

B Methods for Finding Equilibria

B.1 Homotopies

This Appendix provides details of our implementation of the homotopy algorithm using the example of how we use a sequence of homotopies to try to enumerate the number of equilibria that exist for different values of (ρ, δ) for given values of τ. Our implementation of other homotopies, for exampl,e in τ, in the paper is similar to a single step in this sequence.

B.1.1 Preliminaries

We identify equilibria at particular gridpoints in (ρ, δ) space. We specify a 201-point evenly-spaced grid for the forgetting rate $\delta \in [0, 0.2]$ and a 41-point evenly-spaced grid for the learning progress ratio $\rho \in [0.6, 1]$. The state space of the game is defined by an (30×30) grid of values of the know-how of each firm.

B.1.2 System of Equations Defining Equilibrium

An MPNE is defined by a system of equations (one VS^* equation (text equation (16)) for each of 900 states and one p^* equation (text equation (4)) for each of 900 states. If $b^p > 0$, there are an additional 465 VB^* equations (text equation (3)), reflecting a buyer symmetry assumption. The grouping of all of the relevant equations is denoted F.

B.1.3 Homotopy Algorithm: Overview

The idea of the homotopy is to trace out an equilibrium correspondance as one of the parameters of interest is changed, holding the others fixed. Starting from any equilibrium, the numerical algorithm traces a path where a parameter (such as δ), and the vectors $V^B(\mathbf{e})$, $V^S(\mathbf{e})$ and $p(\mathbf{e})$ are changed together so that the equations F continue to hold, by solving a system of differential equations. The differential equation solver does not return equilibria exactly at the gridpoints so it is necessary to interpolate between the solutions returned by the solver. Homotopies can be run starting from different equilibria and varying different parameters. When these different homotopies return solutions at the same gridpoint it is necessary to define a numerical rule for when two different solutions should be counted as different equilibria.

B.1.4 Procedure Details

Step 1: Finding Equilibria for $\delta = 0$. The first step is to find an equilibrium (i.e., a solution to the 1,800 or 2,265 equations) for $\delta = 0$ for each value of ρ on the grid. There will be a unique MPNE for $\delta = 0$, as, in this case, movements through the state space are unidirectional, so that the state will eventually end up in the state (M, M) where no more learning is possible.[34]

We solve for an equilibrium using the Levenberg-Marquardt algorithm implemented using `fsolve` in MATLAB, where we supply analytic gradients for each equation. The solution for the previous value of ρ are used as starting values. To ensure that the solutions are precise we use a tolerance of 10^{-7} for the sum of squared values of each equation, and a relative tolerance of 10^{-14} for the price and value variables that we are solving for.

Step 2: δ-Homotopies. Using the notation of BDKS, we explore the correspondence

$$F^{-1}(\rho) = \{(\mathbf{V}^*, \mathbf{p}^*, \delta) | F(\mathbf{V}^*, \mathbf{p}^*; \rho, \delta) = \mathbf{0}, \quad \delta \in [0, 1]\},$$

The homotopy approach follows the correspondence as a parameter, s, changes (in our analysis, s could be δ, ρ or τ). Denoting $\mathbf{x} = (\mathbf{V}^*, \mathbf{p}^*)$, $F(\mathbf{x}(s), \delta(s), \rho) = \mathbf{0}$ can be implicitly differentiated to find how $\mathbf{x}$ and δ must change for the equations to continue to hold as s changes.

$$\frac{\partial F(\mathbf{x}(s), \delta(s), \rho)}{\partial \mathbf{x}} \mathbf{x}'(s) + \frac{\partial F(\mathbf{x}(s), \delta(s), \rho)}{\partial \delta} \delta'(s) = \mathbf{0}$$

where $\frac{\partial F(\mathbf{x}(s), \delta(s), \rho)}{\partial \mathbf{x}}$ is a (1,800 x 1,800, assuming $b^p = 0$) matrix, $\mathbf{x}'(s)$ and $\frac{\partial F(\mathbf{x}(s), \delta(s), \rho)}{\partial \delta}$ are both (1,800 x 1) vectors and $\delta'(s)$ is a scalar. The solution to these differential equations will

[34] BDKS discuss this result for $b^p = 0$. It will also hold for any higher value of b^p, as movements through the state space are unidirectional.

have the following form, where $y_i'(s)$ is the derivative of the ith element of $\mathbf{y}(s) = (\mathbf{x}(s), \delta(s))$,

$$y_i'(s) = (-1)^{i+1} \det\left(\left(\frac{\partial F(\mathbf{y}(s), \rho)}{\partial \mathbf{y}} \right)_{-i} \right)$$

where $_{-i}$ means that the ith column is removed from the (1,801 x 1,801) matrix $\frac{\partial F(\mathbf{y}(s),\rho)}{\partial \mathbf{y}}$.

To implement the path-following procedure, we use the routine FORTRAN routine FIXPNS from HOMPACK90, with the ADIFOR 2.0D automatic differentiation package used to evaluate the sparse Jacobian $\frac{\partial F(\mathbf{y}(s),\rho)}{\partial \mathbf{y}}$ and the STEPNS routine is used to find the next point on the path.[35,36]

The FIXPNS routine will return solutions at values of δ that are not equal to the gridpoints. Therefore we adjust the code so that after *each* step, the algorithm checks whether a gridpoint has been passed and, if so, the routine ROOTNX is used to calculate the equilibrium at the gridpoint, using information on the solutions at either side.[37]

The time taken to run a homotopy is usually between one hour and seven hours, when it is run on UMD's BSWIFT cluster (a moderately sized cluster for the School of Behavioral and Social Sciences).

Step 3: Enumerating Equilibria. Once we have collected the solutions at each of the (ρ, δ) gridpoints we need to identify which solutions represent distinct equilibria, taking into account that small differences may arise because of numerical differences that are within our tolerances. For this paper, we use the rule that solutions count as different equilibria if at least some elements of the price vector differ by more than 0.001.

[35] STEPNS is a predictor-corrector algorithm where hermetic cubic interpolation is used to guess the next point, and an iterative procedure is then used to return to the path.

[36] For details of the HOMPACK subroutines, please consult manual of the algorithm at `https://users.wpi.edu/~walker/Papers/hompack90,ACM-TOMS_23,1997,514-549.pdf`.

[37] It can happen that the ROOTNX routine stops prematurely so that the returned solution is not exactly at the gridpoint value of δ. We do not use the small proportion of solutions where the difference is more than 10^{-6}. Varying this threshold does not affect the reported results. We also need to decide whether the equations have been solved accurately enough so that the values and strategies can be treated as equilibria. The criteria that we use is that solutions where the value of each equation residual should be less than 10^{-10}. Otherwise, the solution is rejected. In practice, the rejected solutions typically have residuals that are much larger than 10^{-10}.

Step 4: ρ-Homotopies. With a set of equilibria from the δ-homotopies in hand, we can perform the next round of the criss-crossing procedure, using equilibria found in the last round as starting points.[38] From this round on, we run homotopies from starting points in both directions i.e., we follow paths where ρ is falling as well as paths where ρ is increasing. We have found that this is useful in identifying additional equilibria.

This second round of homotopies can also help us to deal with gridpoints where the first round δ-homotopies identify no equilibria because a homotopy run stops (or takes a long sequence of infinitesimally small steps). As noted by BDKS (p. 467), the homotopies may stop if they reach a point where the evaluated Jacobian $\frac{\partial F(\mathbf{y}(s),\rho)}{\partial \mathbf{y}}$ has less than full rank. Suppose, for example, that the δ-homotopy for $\rho = 0.8$ stops at $\delta = 0.1$, so we have no equilibria for δ values above 0.1. Homotopies that are run from gridpoints where we did find equilibria with higher values of δ and higher or lower values of ρ may fill in some of the missing equilibria.

Step 5: Repeat steps 3, 2 and 4 to Identify Additional Equilibria Using New Equilibria as Starting Points. We use the procedures described in Step 3 to identify new equilibria at the gridpoints. These new equilibria are used to start new sets of δ-homotopies, which in turn can identify equilibria that can be used for new sets of ρ-homotopies. This iterative process is continued until the number of additional equilibria that are identifed in a round has no noticeable effect on the heatmaps which show the number of equilibria. For the BDKS, $\tau = 0$ case, this happens after 8 rounds.

B.2 Method for Finding Equilibria Based on Three Reformulated Equations in the $M = 3$ Model

In this Appendix, we describe the novel method that we use to identify the number of equilibria that exist when $M = 3$.

As described in the text, the equilibrium conditions can be reformulated in terms of

[38] In practice, using all new equilibria could be computationally prohibitive. We therefore use an algorithm that continues to add new groups of 10,000 starting points when we find that using additional starting points yields a significant number of equilibria that have not been identified before. We have experimented with different rules, and have found that alternative algorithms do not find noticeably more equilibria, across the parameter space, than the algorithm that we use.

the probability that seller 1 is chosen in each state. If we restrict ourselves to symmetric equilibria then, together with the restriction that $D_1(e_1, e_2) = 1 - D_1(e_2, e_1)$, then there are just three unknown probabilities. We will use $D_1(1, 2)$, $D_1(1, 3)$ and $D_1(2, 3)$. The equilibrium equations for these three states are:

$$\sigma \log \left(\frac{1}{D_1(e_1, e_2)} - 1 \right) - p_1(e_1, e_2) - p_2(e_1, e_2) = 0 \tag{20}$$

and, from text Section 3.2,

$$\mathbf{p_1} = \mathbf{\Phi}(\mathbf{D_1}) + \mathbf{c_1} - \beta(\mathbf{Q_1} - \mathbf{Q_2})(\mathbf{I} - \beta\mathbf{Q_2})^{-1}[\mathbf{D_1} \circ \mathbf{\Phi}(\mathbf{D_1})]. \tag{21}$$

determines prices.

We proceed in the following steps for a given (ρ, δ, τ) combination.

Step 1. Define a grid of possible for $D_1(1, 2)$ and $D_1(1, 3)$. For each, we use a vector [1e-10, 1e-9, 1e-7, 1e-6, 1e-5, (0.0001:(0.9999-0.0001)/200:0.9999), 1-1e-5, 1-1e-6, 1-1e-7, 1-1e-8, 1-1e-9, 1-1e-10].

Step 2. For every combination on the grid, solve for the value of $D_1(2, 3)$ which solves the equilibrium equation for state (2,3), and record the values of the equations 20 for states (1,2) and (1,3), in matrices $M(1, 2)$ and $M(1, 3)$.[39]

Step 3. Use MATLAB `contour` command to define the shapes where the $M(1, 2)$ and $M(1, 3)$ surfaces are equal to zero.

Step 4. Count all of the intersections of these curves, using the user-defined MATLAB function `InterX` command.[40]

Of course, the contours are calculated using interpolation so the solutions are therefore not quite exact. Therefore,

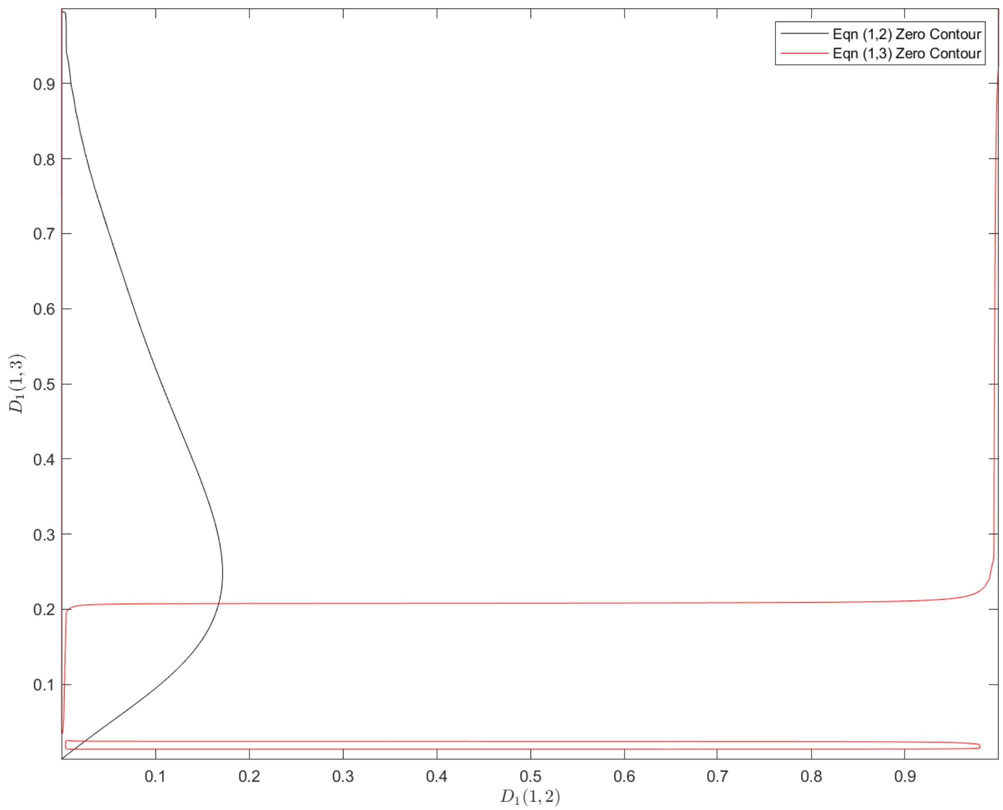
Step 5. Using the solutions based on the contours are starting points, solve the equilibrium equations using `fsolve`.

Step 6. Count the number of solutions where at least one choice probability is different

[39] We have not been able to prove uniqueness, but all of the the examples we have looked at there is a unique solution.

[40] `https://www.mathworks.com/matlabcentral/fileexchange/22441-curve-intersections`.

Figure B.2: Illustration of the Contour Plot for $\rho = 0.1$, $\delta = 0.05$ and $\tau = 0$.



from all of the other equilibria by at least 5e-4.

To give a sense of the procedure, consider the parameters $\rho = 0.1$, $\delta = 0.05$ and $\tau = 0.0$. Figure B.2 shows the contour plot, with the 3 intersections in the bottom left identifying equilibria. The intersection where $D_1(1,2) \approx 0$ and $D_1(1,3) \approx 1$ does not correspond to an equilibrium.

C Additional Tables and Figures

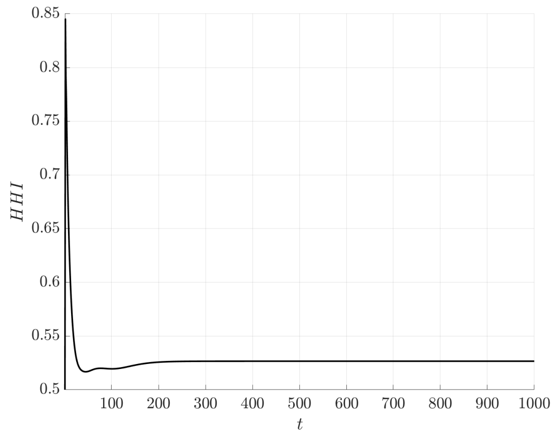
Table C.1: Equilibria in the BDKS Model for Illustrative Parameters ($\delta = 0.023$, $\rho = 0.75$) and $\tau = 0$ in the $M = 30$ and $m = 15$ Model.

						Low-HHI $HHI^{1000} = 0.500$		Mid-HHI $HHI^{1000} = 0.516$		High-HHI $HHI^{1000} = 0.527$	
e_1	e_2	c_1	c_2	Δ_1	Δ_2	p_1	p_2	p_1	p_2	p_1	p_2
1	1	10.00	10.00	0.0230	0.0230	-0.54	-0.54	-1.63	-1.63	-1.61	-1.61
2	1	7.50	10.00	0.0455	0.0230	4.91	7.21	5.16	7.60	5.15	7.60
2	2	7.50	7.50	0.0455	0.0455	4.22	4.22	0.77	0.77	0.77	0.77
3	1	6.34	10.00	0.0674	0.0230	5.82	8.18	6.56	8.71	6.55	8.70
3	2	6.34	7.50	0.0674	0.0455	4.65	5.46	4.06	5.97	4.05	5.97
3	3	6.34	6.34	0.0674	0.0674	5.11	5.11	1.49	1.49	1.47	1.47
4	1	5.62	10.00	0.0889	0.0230	5.95	8.29	6.67	8.55	6.67	8.54
4	2	5.62	7.50	0.0889	0.0455	4.86	5.85	5.46	7.08	5.46	7.08
4	3	5.62	6.34	0.0889	0.0674	5.08	5.42	3.81	5.53	3.80	5.54
4	4	5.62	5.62	0.0889	0.0889	5.22	5.22	1.72	1.72	1.70	1.70
10	1	3.85	10.00	0.2076	0.0230	5.89	8.16	6.14	7.71	6.13	7.69
10	2	3.85	7.50	0.2076	0.0455	5.05	6.06	5.75	6.43	5.75	6.42
10	3	3.85	6.34	0.2076	0.0674	5.20	5.81	5.80	6.31	5.80	6.31
10	8	3.85	4.22	0.2076	0.1699	5.10	5.20	4.49	5.85	4.49	5.86
10	9	3.85	4.02	0.2076	0.1889	5.11	5.15	3.26	4.56	3.25	4.55
10	10	3.85	3.85	0.2076	0.2076	5.12	5.12	2.47	2.47	2.43	2.43
15	1	3.25	10.00	0.2946	0.0230	5.79	8.05	5.98	7.36	5.97	7.38
15	2	3.25	7.5	0.2946	0.045	5.02	5.93	5.63	6.18	5.62	6.17
15	3	3.25	6.34	0.2946	0.0674	5.22	5.74	5.67	6.02	5.67	6.01
15	10	3.25	3.85	0.2946	0.2076	5.19	5.20	5.40	5.94	5.41	5.95
15	14	3.25	3.34	0.2946	0.2780	5.23	5.21	3.46	4.44	3.43	4.44
15	15	3.25	3.25	0.2946	0.2946	5.24	5.24	3.16	3.16	3.10	3.10
16	16	3.25	3.25	0.3109	0.3109	5.28	5.28	3.24	3.24	3.18	3.18
20	20	3.25	3.25	0.3721	0.3721	5.25	5.25	3.32	3.32	3.20	3.20
22	22	3.25	3.25	0.4007	0.4007	5.25	5.25	3.44	3.44	3.26	3.26
25	25	3.25	3.25	0.4411	0.4411	5.25	5.25	3.90	3.90	3.28	3.28
27	27	3.25	3.25	0.4665	0.4665	5.25	5.25	4.62	4.62	3.34	3.34
28	28	3.25	3.25	0.4787	0.4787	5.25	5.25	4.98	4.98	3.52	3.52
29	1	3.25	10.00	0.4907	0.0230	5.79	8.05	5.63	7.62	5.57	7.46
29	2	3.25	7.50	0.4907	0.0455	5.01	5.91	5.04	5.77	5.07	5.72
29	10	3.25	3.85	0.4907	0.2076	5.23	5.17	5.35	5.17	5.39	5.15
29	15	3.25	3.25	0.4907	0.2946	5.27	5.22	5.45	5.34	5.52	5.33
29	29	3.25	3.25	0.4907	0.4907	5.25	5.25	5.22	5.22	3.98	3.98
30	1	3.25	10.00	0.5024	0.0230	5.79	8.05	5.67	7.66	5.63	7.53
30	2	3.25	7.50	0.5024	0.0455	5.01	5.91	5.10	5.84	5.12	5.81
30	10	3.25	3.85	0.5024	0.2076	5.23	5.17	5.33	5.21	5.35	5.19
30	15	3.25	3.25	0.5024	0.2946	5.27	5.22	5.42	5.35	5.45	5.33
30	29	3.25	3.25	0.5024	0.4907	5.25	5.25	5.30	5.20	4.29	4.60
30	30	3.25	3.25	0.5024	0.5024	5.25	5.25	5.27	5.27	4.77	4.77

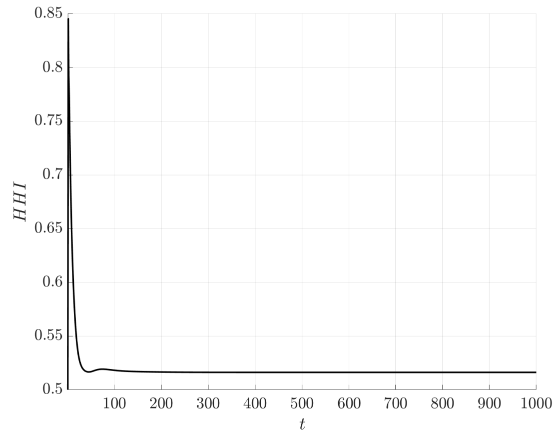
Notes: c_i, p_i, Δ_i are the marginal costs, equilibrium price and probability of forgetting for firm i. HHI^{∞} is the expected long-run value of the HHI.

Figure C.1: Expected Value of the HHI^t Measure for the Illustrative Parameters for Alternative Polar Values of b^p and τ. Note the different scales on the x-axis.

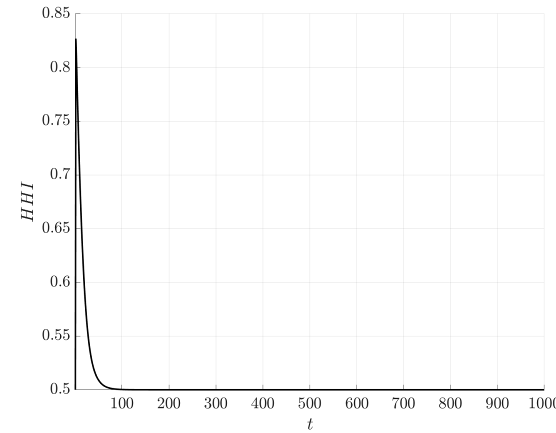
(a) $b^p = 0, \tau = 0$ (High-HHI)



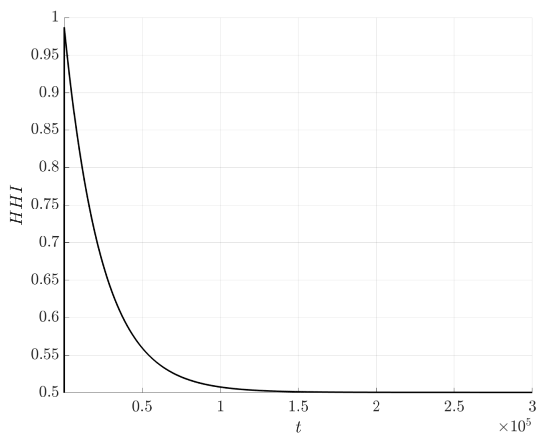
(b) $b^p = 0, \tau = 0$ (Mid-HHI)



(c) $b^p = 0, \tau = 0$ (Low-HHI)



(d) $b^p = 0, \tau = 1$



(e) $b^p = 1, \tau = 1$ (Social Planner)

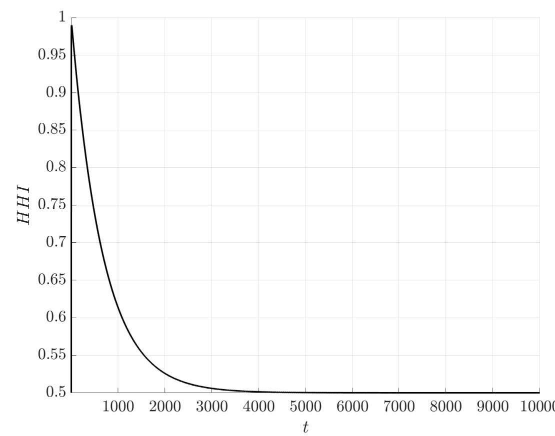
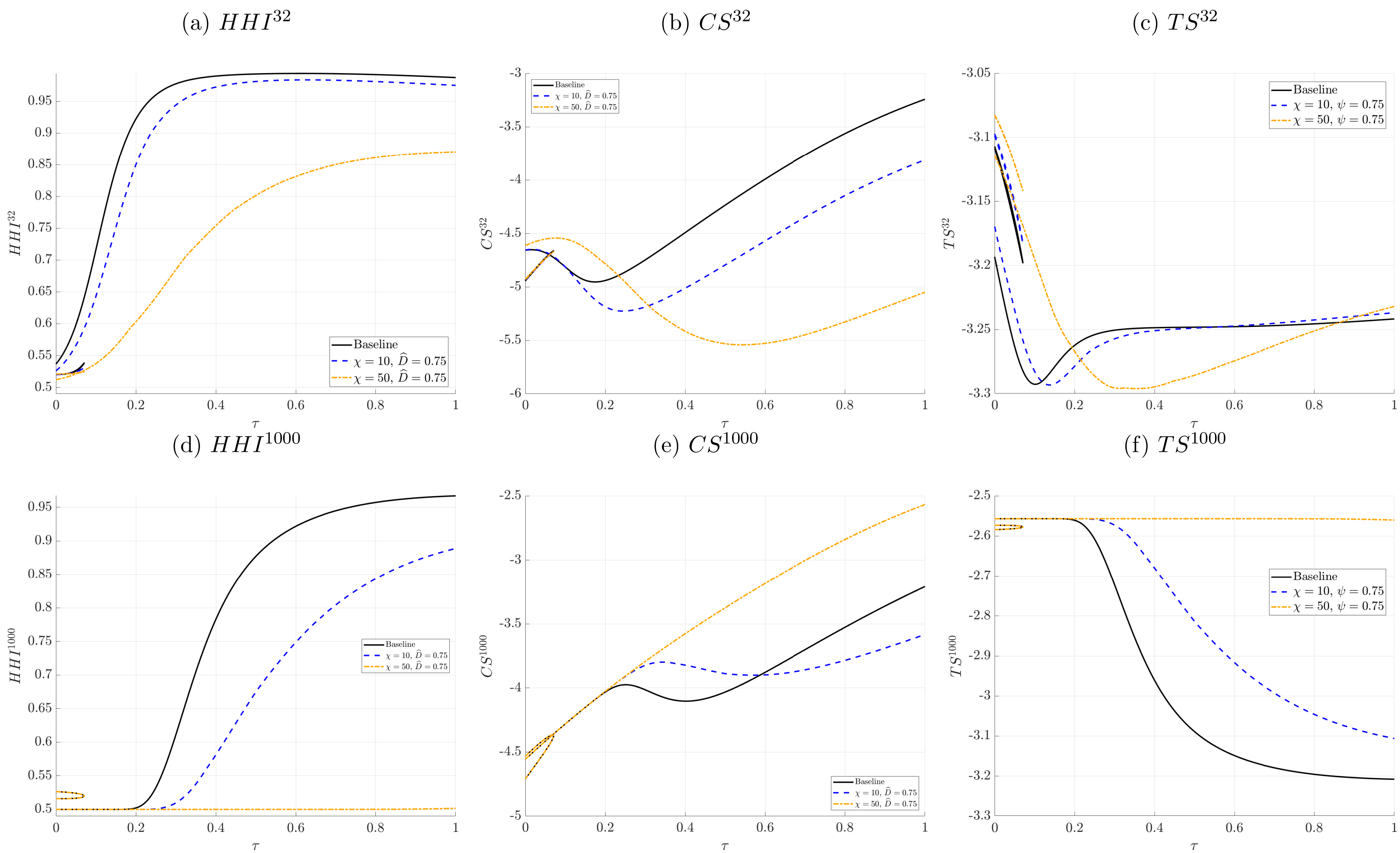


Figure C.2: Medium-Run and Long-Run Changes in Concentration and Expected Surplus as a Function of τ and Concentration Restriction Policies for the Illustrative Parameters. Total surplus calculations do not include the compliance costs as a social cost. Read $\widehat{D}$ as ψ.



D Why Does a Policy Where a Leader Cannot Consider Advantage Denying Incentives Raise Concentration, Total Surplus and Consumer Surplus When τ is High?

A surprising feature of the policy results is that, when buyers have almost all of the bargaining power, a policy that prevents the leader from considering advantage-denying incentives raises medium-run concentration and TS^{PDV} and CS^{PDV}, even though, when $\tau = 0$, large AD incentives have been viewed as being associated with a single firm establishing some dominance. In this Appendix, we provide some explanation for this result, focusing on how the policy endogenously changes AB incentives.

Figure D.1 shows the value of AB and AD incentives for a laggard firm 1 with $e_1 = 1$ for all possible values of e_2, for three alternative values of τ: 0 and 0.5 (for which the no leader AD policy reduces medium-run concentration and TS^{PDV}) and 0.9 (concentration and total surplus increase). While one could obviously look at alternative values of e_1 we will see that the key changes in prices occur when firms are at the top of their cost curves.

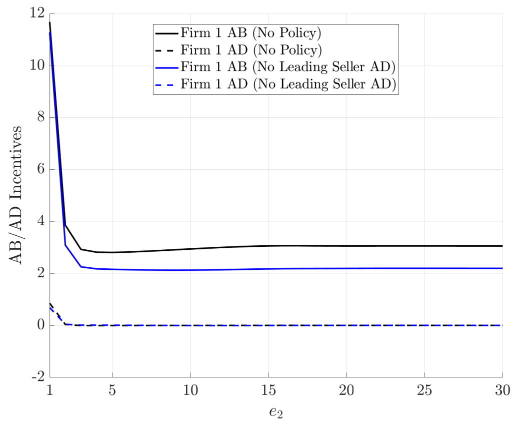
For $\tau = 0$, where we focus on the Low-HHI equilibrium with no policy, the policy slightly reduces the AB incentive of the laggard (firm 1), and has almost no effect on the AB incentive of the leader (firm 2). Therefore the only effect is the removal of the leader's large AD incentive. Equilibrium price changes are shown in the lower strip of Figure D.2 (prices without the policy are shown in the upper strip). Prices in symmetric states change little, but prices in non-symmetric states increase. Therefore the leader's progress down its cost curve is slowed (which is inefficient), and market structure tends to become more symmetric.

For $\tau = 0.5$, the policy leads to an increase in AB incentives in the initial (1,1) state, and prices in that state decrease. However, compared with the elimination of the leader's large AD incentives in states where there is a leader, the increase in AB incentives is smaller, so that prices tend to increase by small amounts.

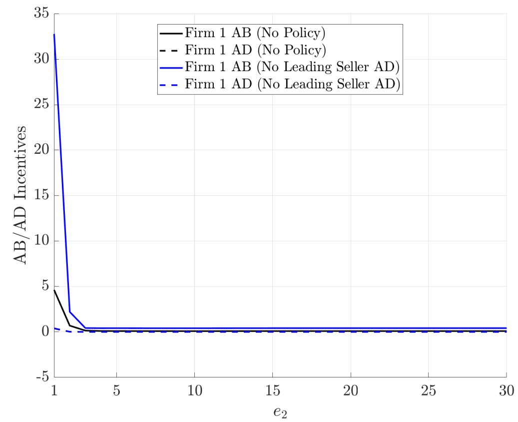
For $\tau = 0.9$, the leader's AD incentives are smaller (as buyers have most of the bargaining power), but now there is an increase in the leader's AB incentives over a significant number of states, and the effect of this (as shown in text Figure 16) is that the total PDV of dynamic

Figure D.1: Sellers' AB and AD Incentives when $e_1 = 1$ for the Illustrative Parameters.

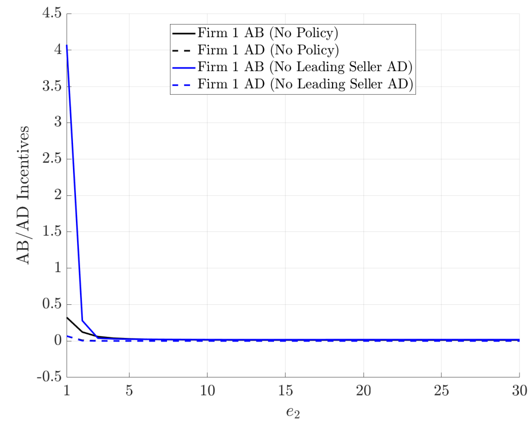
(a) $\tau = 0$ Firm 1



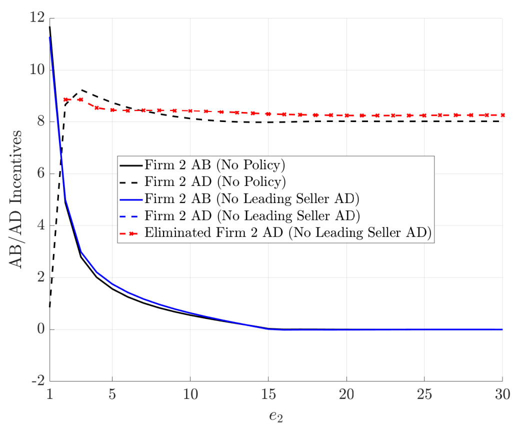
(b) $\tau = 0.5$ Firm 1



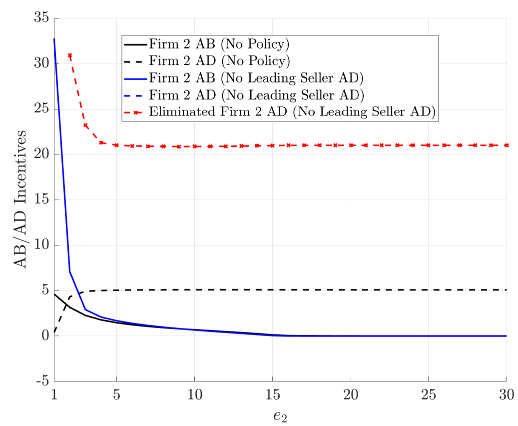
(c) $\tau = 0.9$ Firm 1



(d) $\tau = 0$ Firm 2



(e) $\tau = 0.5$ Firm 2



(f) $\tau = 0.9$ Firm 2

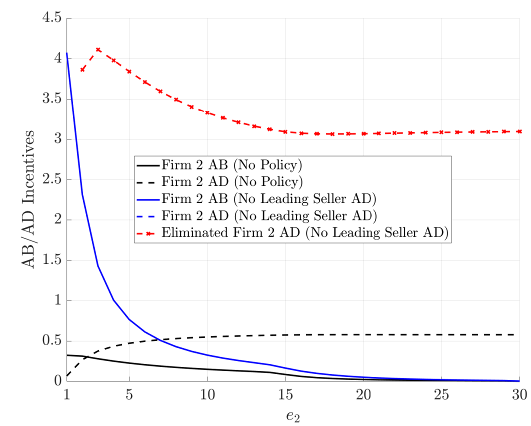
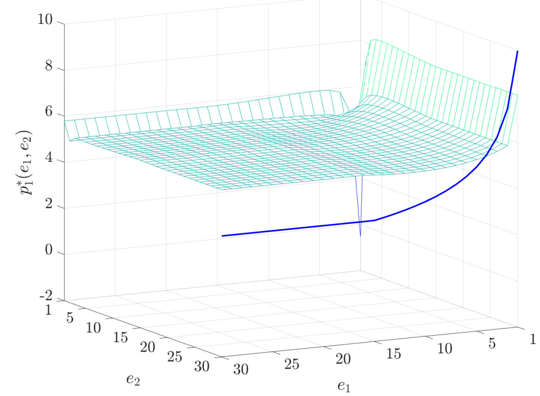
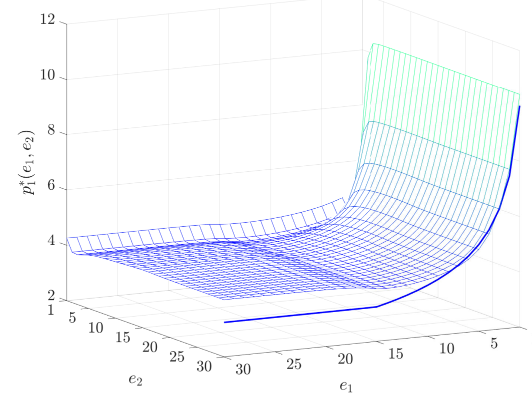


Figure D.2: No Policy Prices and Price Changes under a No Leader AD Policy for the Illustrative Parameters.

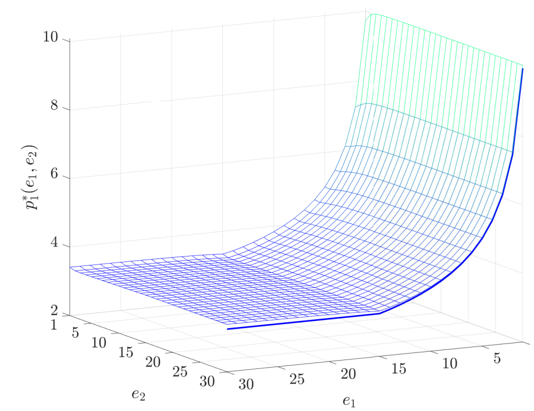
(a) $\tau = 0$ No Policy Prices: Low-HHI



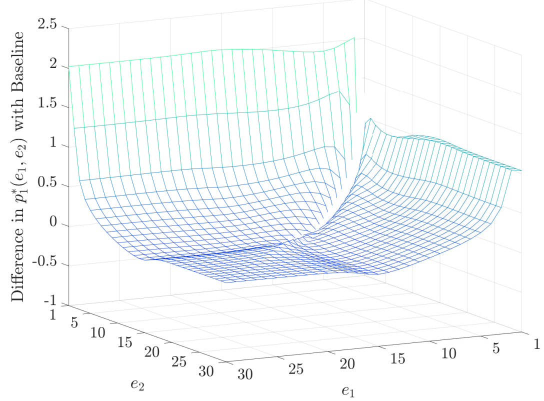
(b) $\tau = 0.5$ No Policy Prices



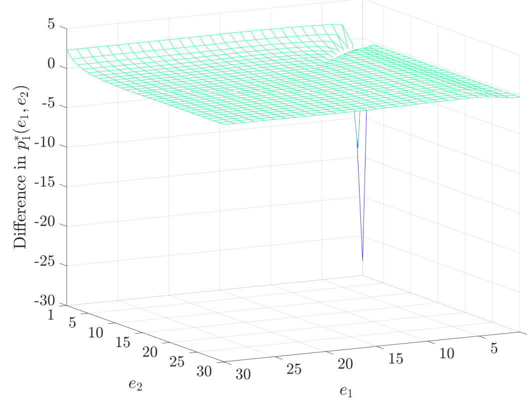
(c) $\tau = 0.9$ No Policy Prices



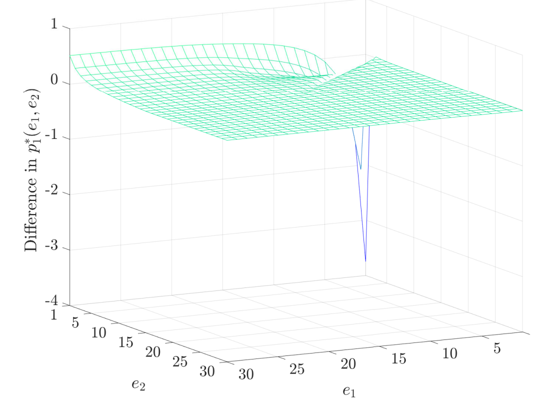
(d) $\tau = 0$ Changes in Price



(e) $\tau = 0.5$ Changes in Prices



(f) $\tau = 0.9$ Changes in Prices



incentives, given equilibrium play, actually increases. Even though prices are very close to production costs without the policy, the policy leads to lower prices at the start of the game, and one firm tends to move down its cost curve more quickly in the first few periods of the game, which is efficient.

E The Effects of Bargaining Power For Technology Parameters For Which Market Concentration is Too High with Price-Setting

In the text, we use $\rho = 0.75$ and $\delta = 0.023$ as our "illustrative" parameters. For these parameters, market concentration is significantly lower than the social planner would choose when firms set prices, and we find that allowing for bargaining can have significant effects. However, there are other technology parameters for which the social planner would prefer lower market concentration. In this Appendix, we consider $\rho = 0.95$ and $\delta = 0.03$ as an example. For these parameters LBD effects are small: even with the maximum possible know-how, costs are less than 20% lower than they are at the top of the cost curve.

Figure E.1 replicates text Figure 10 for our new parameters. For these parameters, there is a unique equilibrium for $\tau = 0$, but there is multiplicity for $0.07 \leq \tau \leq 0.175$, reflected by how the equilibrium path folds back on itself. As with all of the other parameters we consider, multiplicity is eliminated once buyers have moderate bargaining power. From an efficiency perspective, concentration is too high for $\tau = 0$ (although by a relatively small amount), but for $0.175 \leq \tau \leq 0.4$ the unique equilibrium is close to efficient, but, for higher τ, equilibrium concentration increases and efficiency declines.

For the text illustrative parameters, we observed that as τ increased from zero, the AB and AD incentives increased as the sellers expected that a lead would tend to last for longer and therefore potentially be more valuable (relative to being a laggard). However, as increasing τ from zero reduces concentration, that effect does not occur for these parameters and we observe AB and AD incentives declining in τ (ignoring the complication introduced by the multiplicity).

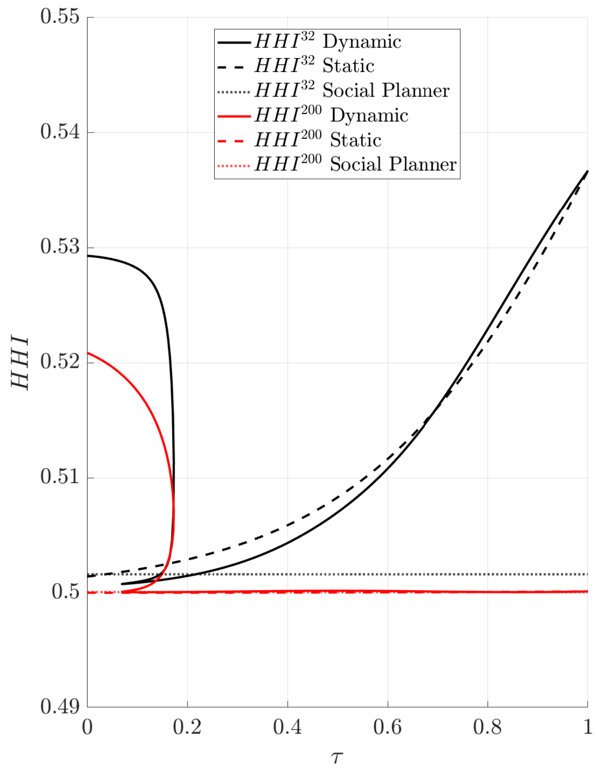
We now turn to the question of the effects of policies. We have not yet completed the analysis with policy triggers.

Subsidies. We can solve for the subsidies that implement the efficient evolution of the industry using the reformulated equations. Figure E.2 shows the subsidies provided to the laggard, in the event that it sells, for $\tau = 0$, 0.25, 0.5 and 1 for the illustrative parameters.

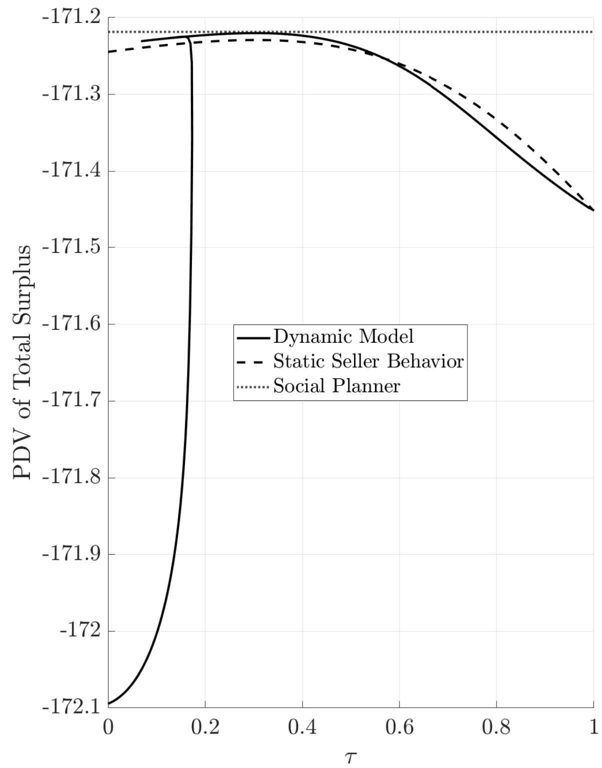
As with the text illustrative parameters, we can see that the optimal subsidy policy varies

Figure E.1: The Effects of Changing the Allocation of Bargaining Power for $\rho = 0.95$ and $\delta = 0.03$ With No Policies.

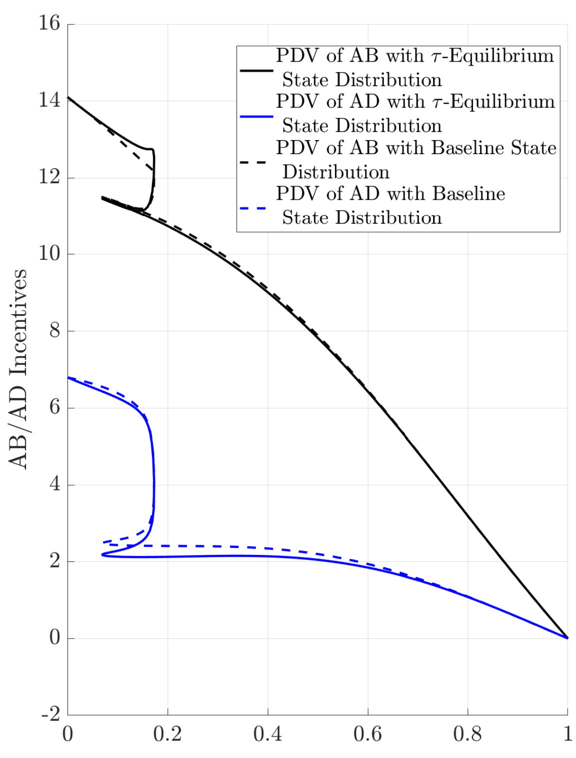
(a) HHI^{32} and HHI^{200}



(b) TS^{PDV}



(c) Average PDV AB/AD Incentives



(d) CS^{PDV}

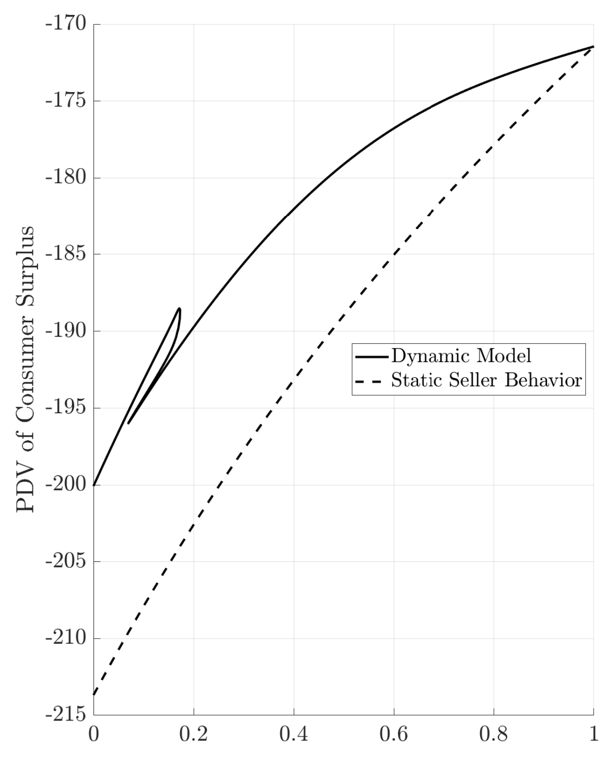
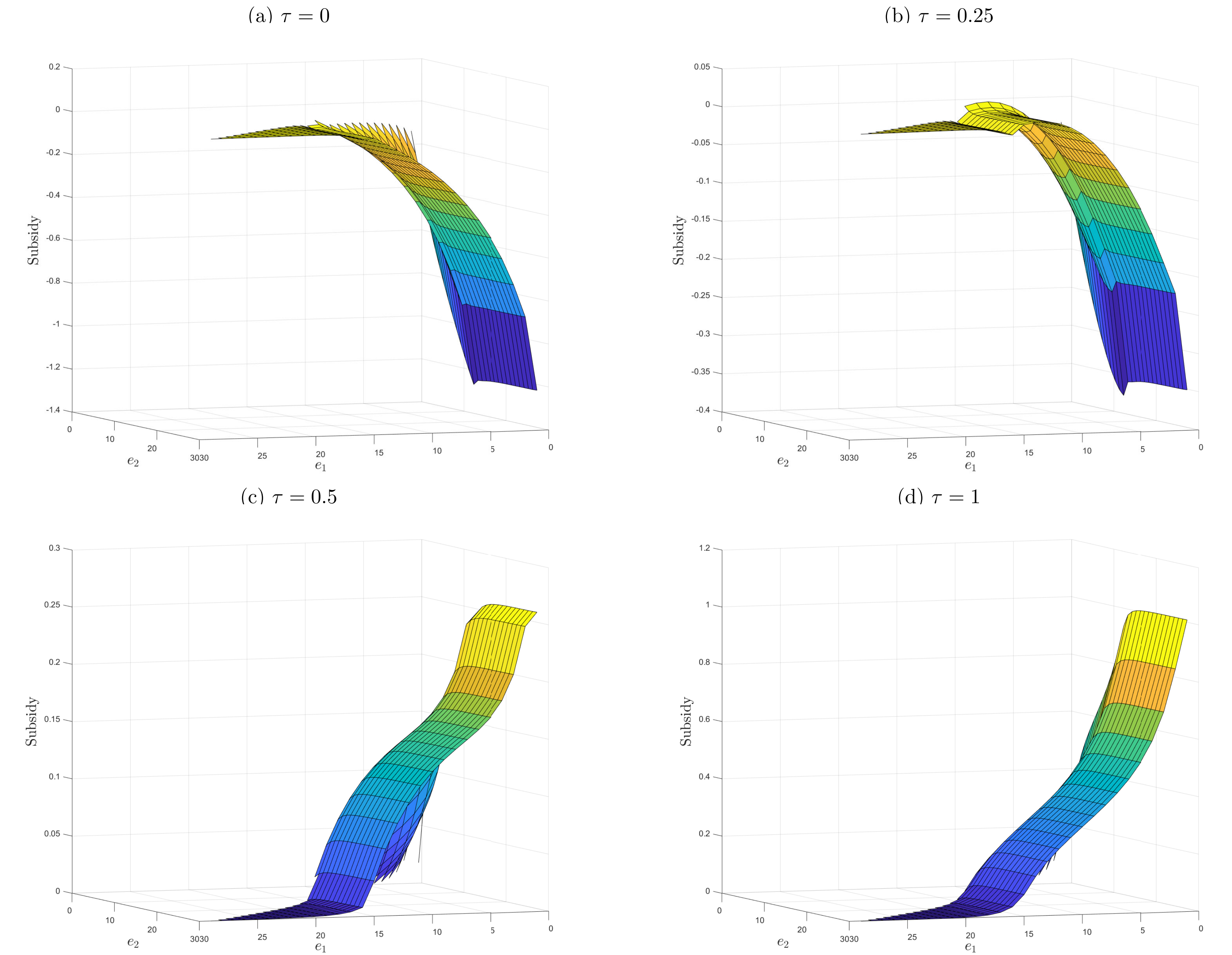


Figure E.2: Optimal Subsidies to the Laggard for $\rho = 0.95$ and $\delta = 0.03$ for $\tau = 0$, 0.25, 0.5, 1. Negative values indicate taxes. No value shown for leader states.



with the value of τ. In particular, the magnitude of subsidies/taxes is significantly smaller when $\tau = 0.25$ than when $\tau = 0$, and that the structure of subsidies (i.e., which states are subsidized and which are taxed) are different when $\tau = 0.5$ than when $\tau = 0$ or $\tau = 0.25$. This is slightly surprising as for both $\tau = 0$ and $\tau = 0.5$ the market equilibrium is more concentrated than the social planner would choose, but it confirms our conclusion that the interactions between bargaining power and incentives are both subtle and very relevant for the design of policies.

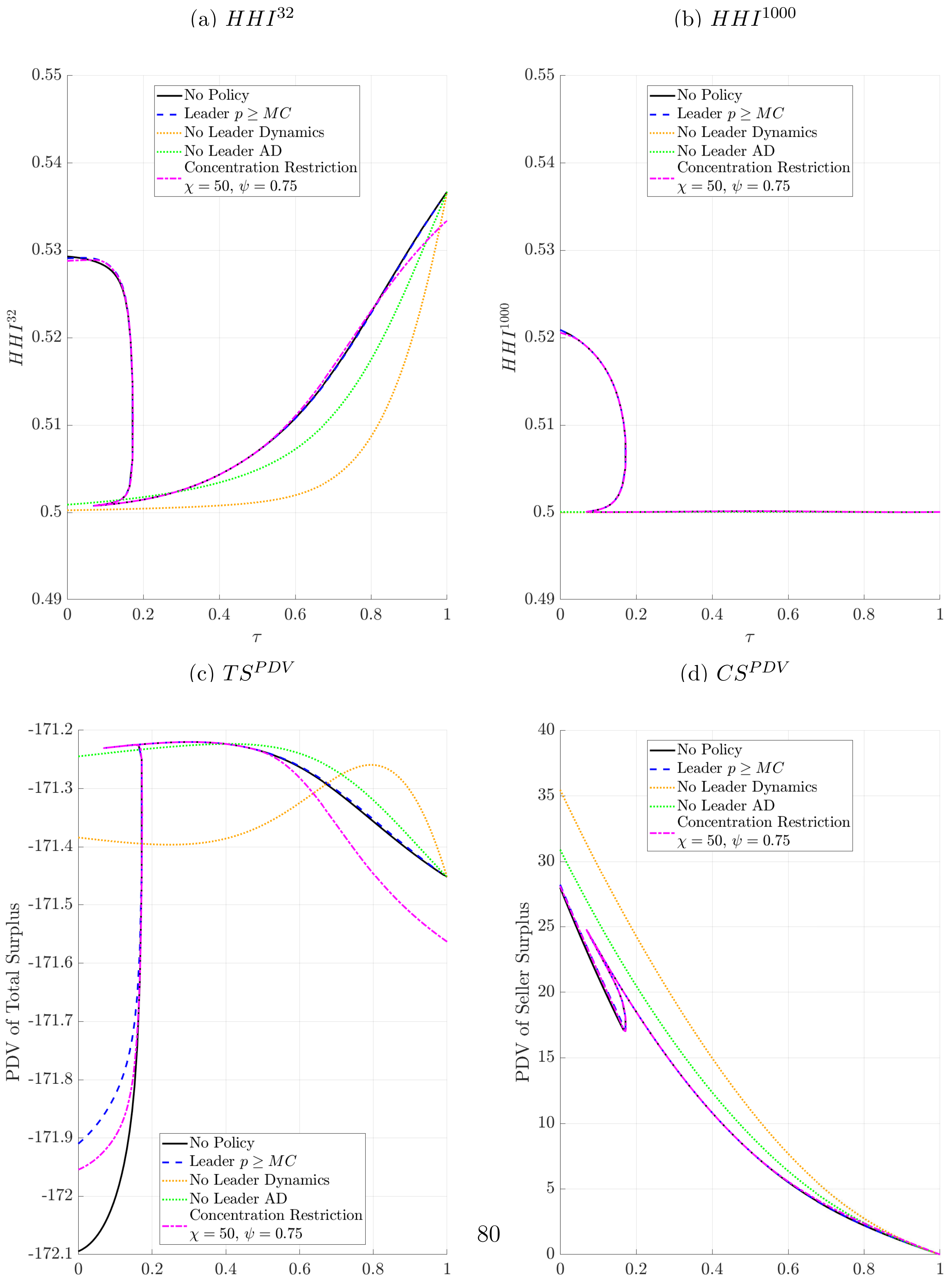
Policies Designed to Increase Competition. Figure E.3 replicates text Figure 15, by looking at how the policies designed to increase competition affect concentration and welfare, as a function of τ.

Similar to the text illustrative parameters, the policies all tend to soften competition and raise PS^{PDV}, and the Leader $p \geq mc$ policy has limited effects (although it does raise efficiency for low τ). For these parameters, the concentration restriction policy also has almost no effect, presumably because the level of concentration is typically much lower than the threshold we are using.[41] The incentive policies also raise efficiency for high τ (No Leader Dynamics for $\tau \geq 0.65$ and No Leader AD for $\tau \geq 0.4$), although, unlike for the text illustrative parameters, this is not associated with an increase in concentration.

Our broad conclusion is therefore that even for these quite different technology parameters, which limit the scope for these types of policies to raise efficiency, a cost-benefit analysis of policy will continue to depend on what is assumed about the allocation of bargaining power.

[41] With marginal cost pricing, the laggard would sell with probability 0.1824 even when the difference in costs is maximized, so the constraint will not bind once the leader charges even a relatively small markup in excess of the laggard.

Figure E.3: Effects of Policies Introduced at the Start of the Industry's Life on Concentration and total and Producer Surplus. The compliance costs of the Concentration Restriction policy are not counted as costs to society in total surplus calculations, although they are costs to the sellers.



F The Effects of Allowing an Outside Good or Variable Product Differentiation

Our text analysis assumes that there is no outside good, and that $\sigma = 1$. These assumptions mean that our results can be directly compared with those of BDKS, but it is natural to ask what might change if these assumptions were relaxed. We briefly address this issue here, assuming the illustrative parameters and that $\tau = 0$, by showing what happens to medium-run market concentration and the PDV of sellers' AD incentives.[42] We note, though, that there could be additional effects once one introduces policies.

Figures F.1(a) and (c) show equilibrium HHI^{32} and the PDV of AD incentives when $\sigma = 1$ and we allow for an outside good which gives the buyer an indirect utility of $v - P_0 + \varepsilon_0$ and P_0 has the fixed value that is shown on x-axis. The BDKS assumptions can be thought of as the limiting case where the x-axis is extended to the right.

$P_0 = 10$ corresponds to the assumption about the outside good in the BDK papers that allow for entry and exit (in this case, an outside good is required to define outcomes when there is monopoly). Allowing for an outside good with $P_0 = 10$ changes equilibrium outcomes very little compared to having no outside good. If we consider an outside good that is significantly more attractive, multiplicity of equilibria is eliminated, the industry tends towards medium-run monopoly, and the two sellers initially compete harder to become the single firm that will, in the medium-run, compete with the outside good. As the outside good becomes very attractive, the incentives to become the leading firm diminish as the ability to exercise market power will be limited.

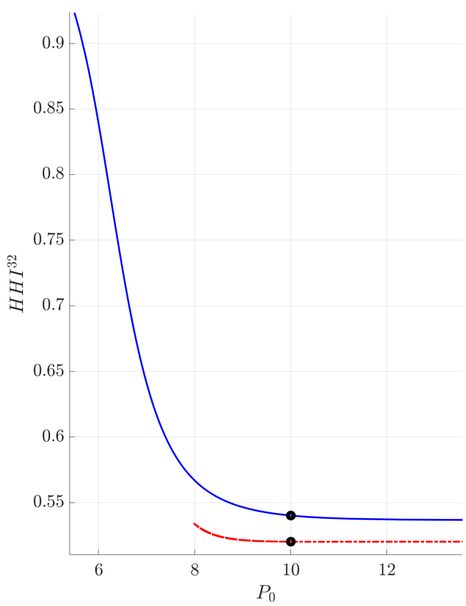
Figures F.1(b) and (d) show the same outcomes when there is no outside good but σ is varied. Lower values of σ correspond to reduced product differentiation between the sellers, which will tend to lower their equilibrium margins and reduce the probability that a higher priced firm makes a sale. Our base results correspond to $\sigma = 1$. Lowering σ does not eliminate multiplicity whereas we will see that raising τ does tend to eliminate multiplicity. However, lowering σ does tend to increase concentration and incentives to become a leader,

[42] We thank Jim Dana for prompting us to look more carefully at these changes, in order to understand the extent to which they lead to outcomes that are similar to, or different from, the effects of giving buyers more bargaining power.

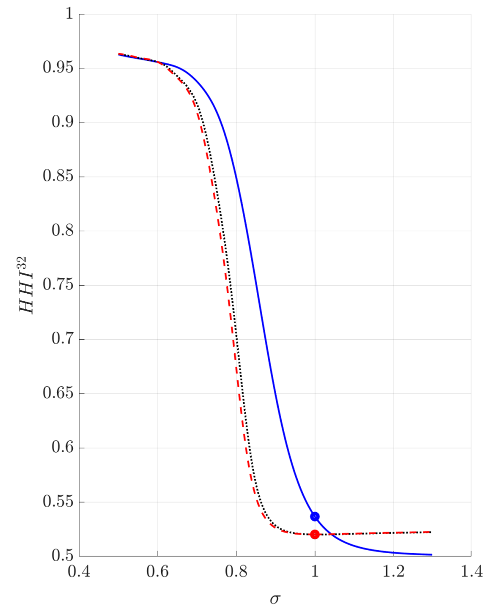
and moderate increases in buyer bargaining power will have similar effects.

Figure F.1: Effects of Allowing for an Outside Good and Variable Product Differentiation when $\tau = 0$ for the Illustrative Parameters.

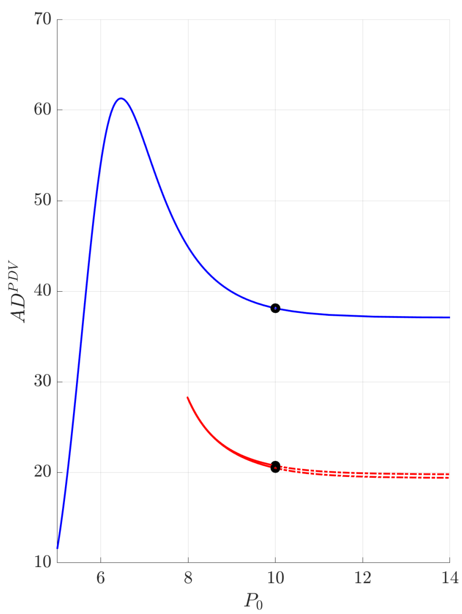
(a) Varying Outside Good Attractiveness: HHI^{32}



(b) Varying Product Differentiation: HHI^{32}.



(c) Varying Outside Good Attractiveness: PDV Seller AD Incentives.



(d) Varying Product Differentiation: PDV Seller AD Incentives.