The Value of A Statistical Judgment: A New Approach to the Insurer’s Duty to Settle*

Ezra Friedman

December 2014

Abstract

When liability insurance carries a limit that is smaller than the potential claim of a third party plaintiff, the insurer and the insured can have a conflict of interest as to settlement. The majority of jurisdictions in the United States impose upon insurers a duty to ignore liability limits when considering a settlement offer, and do not place limits on an insurer’s liability when the duty is violated. In this paper I take an incomplete contract approach and derive the optimal duty to settle. I propose limiting the insurer’s liability for an excess judgment to the Value of a Statistical Judgment (VSJ), which is derived from the amount a policy holder is willing to pay to avoid a risk of a large excess judgment. Setting damages for bad faith refusal to settle to the Value of Statistical Judgment causes the insurer to efficiently internalize the harm to the insured from an excess judgment. Although calculating the exact VSJ and corresponding duty to settle requires knowledge of the utility function of the insured, I show that courts can use a revealed preference approach to closely approximate the VSJ as a linear function of the liability limit and the vulnerable assets of the insured. The rule I propose applies excess liability which is a small multiple of the sum of the liability limit and the policy holder’s wealth. In addition to being nearly optimal rule from the ex-ante perspective for the insured, the proposed VSJ rule is attractive from a broader social welfare perspective; it discourages nuisance settlements, and directs more compensation towards victims with legitimate claims.

*I thank Ken Ayotte, Peter DiCola, Josh Fischman, Michael Frakes, Lance Liebman, Kate Litvak, Kyle Logue, Peter Molk, JJ Prescott, Matthew Spitzer, Abe Wickelgren, and participants at the Northwestern University Internal Workshop and the 2014 Midwestern Law and Economics Association Meetings. All errors are my own. Please send comments, questions, or suggestions to ezra-friedman@northwestern.edu.
1 Introduction

When liability insurance carries a limit that is smaller than the potential claim of a third party plaintiff, the insurer and the insured have a conflict of interest as to settlement. Because the liability for damages in excess of the liability limit falls on the insured, while the insurer bears the entire cost of a settlement up to the liability limit, the insurer has an incentive to refuse a settlement offer and ‘gamble with the insured’s money.’ We can see this by imagining that a pedestrian suffers $400,000 dollars worth of harm in an accident with a driver insured for $100,000. Let us suppose that the driver has $50,000 in a bank account, and that all of the driver’s other assets are out of reach of any tort claimant. Assume the pedestrian offers to settle with the driver’s insurer for $100,000, and there is some question about the driver’s liability. Absent a duty to settle, the insurer has nothing to lose by going to court, since it won’t be liable for more than $100,000 even if the plaintiff prevails. From an ex-post perspective, the insured driver clearly wants a settlement, since if the case goes to court and she loses she will be liable for the $300,000 excess judgment. Even though she will not be able to pay the entire $300,000 she may lose her $50,000 and be forced into bankruptcy. Courts have reacted to this conflict by the imposition of a duty to settle on the insurer, ruling that the explicit duty to provide a legal defense implies a duty to settle when reasonable. This duty is typically enforced by a (transferrable) claim for the insured against the insurer for unreasonable refusal to settle.\footnote{This paper examines the related questions of the appropriate extent of this duty to settle and the damages for violations.}

Although some commentators have questioned the necessity of a law imposing any duty to settle,\footnote{Some have noted that the policyholder and insured might be able specify their preferred duty to settle in the insurance contract. Others have taken note of the possibility that reputational concerns or interim bargaining between the insured and the policyholder could ameliorate the conflict. See Sykes[1994] for a thorough consideration of these arguments.} this paper operates from the premise that it is appropriate for courts to impose some duty. A court imposed duty can be justified by practical considerations such as the difficulty in drafting a complete contract or the limited legal sophistication of the policyholder precluding an optimal contract. Indeed, the majority of states impose a duty to settle, and require the insurer to place as much weight on the interests of the insured as on their own profits when considering a settlement offer.\footnote{See Syverud [1990] for a good review.} Most states interpret this as creating a duty to settle whenever “a prudent insurer without policy limits would have accepted the settlement offer.”\footnote{Crisci v. Sec Ins. Co. 426 P. 173,176 (Cal. 1967). This formulation is often attributed to Keeton[1954].} Likewise, the Tentative Draft of the ALI’s Principles of the Law of Liability Insurance defines a rea-
sonable settlement decision as “one that would be made by a reasonable person that bears the sole financial responsibility for the full amount of the potential judgment.”

This standard is often referred to as the disregard-the-limits rule (abbreviated here as DTL).

A simple mathematical interpretation of the DTL rule is that the insurer must accept any settlement demand that is less than the expected cost of paying any eventual judgment. In our example, since an insurer with no limit would be liable for $400,000 if the plaintiff prevailed, under DTL, the insurer would be required to accept a $100,000 settlement offer if there was more than a 1/4 chance the plaintiff would win in court. Insurers in violation of this rule are typically liable to the insured for at least the difference between the third party plaintiff’s judgment and the liability limit, and may be subject to consequential or punitive damages as well.

The disregard-the-limit rule has been criticized on the grounds that by making insurers consider liability above the limits of the policy, it gives policyholders a benefit they have not paid for. A problem with this argument is that if insurers are aware of the law, it seems implausible that they would not set premiums accordingly. Arguing from an incomplete contract perspective, Kyle Logue[1994] and Alan Sykes[1994a] make a more sophisticated argument: The disregard the limit rule could force policyholders to buy a benefit that costs more than its value to them. Logue suggests that the Michigan rule (described below) is preferable because it more closely represents the rule that the insured and insurer would find optimal in a complete contract. However, in a different paper, Sykes [1994b] shows that risk averse agents may prefer the DTL rule to no duty (or, implicitly, the Michigan rule), even when they are judgment proof, because, under certain circumstances, the DTL rule can protect the insured from an excess judgment while still allowing her to take some advantage of her judgment proofness.

From a more doctrinal perspective, it has been noted that the unlimited damages available under the DTL rule do not properly account for the fact that in many cases, because of the insured’s judgment proofness, the plaintiff would be unlikely to be able to collect the entire excess judgment. Defendants who have a plausible failure to settle claim against their insurer and who owe an excess judgment to a plaintiff will often transfer their claim to the plaintiff in exchange for a release from the excess judgment. Under these circumstances, awarding the full excess judgment to the insured results in a windfall to the original plaintiff, who is now able to collect the entire judgment, but would otherwise not have been able to collect much more than the insurance

---


6There is some question as how to treat the legal costs involved in defending the suit. The Draft PLLI concludes that it is appropriate to ignore them so as to not create an obligation to settle nuisance suits. See Chapter 2 §27 cmt.(h).

7See ibid Chapter 2 §30. See also Crisci, supra note 4, which upheld an award of $25,000 for mental anguish beyond the excess judgment
This was recognized by the supreme court of Michigan in *Frankenmuth Mutual v. Keeley*. The court was convinced by the dissent in an earlier hearing of the case that noted that the original defendant was unlikely to ever pay more than a small portion of the excess judgment. The dissent argued that awarding the full excess judgment is a departure “from the principle of confining the damages for breach of contract by an insurer to the economic loss suffered by the promisee.” The court instead adopted a rule that limits an insurer’s extra liability for failure to settle to what the plaintiff could have collected from the insured and any other actual damages to the insured.

This formulation of the Michigan rule only concerns damages. It does not explicitly state that at an insurer need not consider the portion of a judgment that could not be collected when considering a settlement offer, and is not an explicit rejection of the DTL rule. Nevertheless, by limiting the damages to what could be collected, the Michigan rule removes the incentives for the insurer to consider uncollectable damages. Returning to our example above, the Michigan rule would limit the damages for refusal to settle to $50,000, the amount of an excess judgment that could be collected. Since the insurer would only be liable for $150,000 (i.e. the $100,000 limit plus the $50,000 damages) if it refused to settle and the plaintiff prevailed, the insurer would only settle if it judged the chances the plaintiff would prevail was more than $66\%$. Thus, I interpret the Michigan rule as implying a duty to settle only when it provides an economic incentive for the insurer to actually settle, so that when the insured would not pay the entire excess judgment it implies less duty to settle than the DTL rule.

One might criticize the Michigan rule on the grounds that it provides a windfall to the insurer by allowing the insurer to take advantage of the judgment proofness of the insured. Furthermore, the fact that it implies more of a duty to an insured with more assets might be seen as inequitable. In this paper I develop what I think is a more powerful criticism, the Michigan rule does not take into account the risk aversion of the insured, which is likely to be the *raison d’etre* of the insurance in the first place. I show that for large claims, taking into account both the risk aversion and judgment proofness of the insured implies that a middle ground between the majority DTL rule and the Michigan rule is preferable.

Some courts have noted that a middle ground in which an insurance company valued the exposure of the insured at more than just the vulnerable assets, but less than the entire claim is possible. For example, in dicta, Judge Richard Posner notes that “damages need not be exactly equal to the amount of excess judgment, they could be more or less.” Posner may have had an intuition that risk aversion could push the appropriate damages for refusal to settle above the amount of the excess judgment.

---

judgment, whereas judgment proofness could push the appropriate damages below. Even the court that embraced the Michigan rule acknowledged that an insured could sue for financial ruin or damaged credit.\textsuperscript{11}

1.1 The Incomplete Contract Approach

I approach the question of the optimal duty to settle from an incomplete contract perspective, which explains the conflict between the interests of the insurer and insured by the incompleteness of the insurance contract. Economists refer to a contract as complete if it specifies what the promisor must do under any possible contingency. The typical liability insurance contract is incomplete because it does not specify exactly when an insurer must settle a case and when an insurer should litigate instead. The optimal complete contract (one that maximizes joint welfare between the insured and the insurer) would require settlement only in cases where the additional cost to the insurer of settling is less than the value of settling to the insured.

To see the tradeoff involved in requiring settlement, let us return to our example of a plaintiff who suffers a loss of $400,000, but has an uncertain case, and thus offers to settle with the defendant’s insurer for the policy limit of $100,000. Absent a duty to settle, the insurer would always wish to take the case to court, and would pay $100,000 only if the plaintiff prevailed. Note that if the insurer fights the case and wins, the insurer gains $100,000 relative to accepting the settlement, but if they lose they are liable for only $100,000 and lose nothing relative to settlement. Thus settling for the limit increases the insurer’s expected payout by the likelihood that the plaintiff loses at trial multiplied by the limit.

The insured, on the other hand, is put at risk of being liable for the difference between the claim and the liability limit when the insurer refuses to settle. Because the insurer would pay the entire settlement if it settled, from an \textit{ex-post} perspective the insured always wants the insurer to settle.\textsuperscript{12} That is to say, after the policyholder has paid for the insurance, when she is sued, she is always happier to have the insurer settle within the policy limits rather than exposing her to a chance of an excess judgment. However, an insurance policy in which the insurer always settled would cost more, so it is not immediately clear how much duty to settle the insure would prefer \textit{ex-ante}. Since the cost of a duty to settle is lower and the benefit to the insured is higher when the plaintiff is more likely to win, the optimal contract is more likely

\textsuperscript{11}“If Keeley could demonstrate that his credit had been damaged or he had suffered financial ruin, then he should no doubt recover for such economic loss caused by breach of contract”, \textit{Frankenmuth Mutual v. Keeley} 433 Mich. 525 at 559(1989),(Levin Dissenting)

\textsuperscript{12}Here I am assuming that the insured does not suffer a significant loss in reputation when the insurer settles. For physicians who are sued for malpractice, this may not be true, and this can create an opposite conflict of interest, where the insurer prefers to settle, while the insured prefers to fight, particularly if the claim does not exceed the policy limits. For this reason some malpractice policies may charge more for a clause that requires consent of the physician to settle.
to require settlement when the plaintiff is more likely to win.

I show that it is optimal to determine the duty to settle according to the Value of a Statistical Judgment (VSJ), which is derived from how much a policyholder would be willing to pay to reduce the risk of an excess judgment. This is analogous to the concept of the Value of a Statistical Life, derived from the willingness of people to accept small risks of death in exchange for increased wages or other benefits. Torts scholars have argued that setting damages for accidental death at the Value of a Statistical Life can give appropriate incentives to invest in care.\textsuperscript{13} Similarly, I show that the VSJ determines the appropriate weight for the insurer to place on the interests of the insured to produce an optimal duty to settle. Although the VSJ rule would replace the DTL rule’s specific interpretation of the duty to settle, it would not represent a fundamental change to the general principles behind the duty to settle. In fact, the VSJ rule is an economically sensible interpretation of the early statement of the duty to settle from \textit{Comunale v. Traders \\& General Ins. Co.}; that insurers considering settlement offers must “take into account the interest of the insured and give it at least as much consideration as it does to its own interests.”\textsuperscript{14}

To see how an ideal VSJ rule would work, let consider the policy holder mentioned above (with $50,000 in vulnerable assets), and suppose the maximum she was willing to pay to avoid a 1/10,000 chance of an excess judgment that would bankrupt her is $15, implying that the VSJ of this excess judgment is $150,000 ($15 × 10,000). The VSJ rule would impose a duty for the insurer to place full weight on the policyholder’s VSJ as the cost of an excess judgment. Therefore the insurer in our example should value the cost of losing in court as $250,000 (the $100,000 limit plus the $150,000 VSJ) and if the plaintiff has offered to settle for $100,000, the insurer should settle if the likelihood the plaintiff prevails is more than 40%. (\frac{100,000}{100,000+150,000}). To see why this emulates the optimal contract, imagine that the likelihood that the insured is faced with a claim for more than $150,000 with a $100,000 settlement demand and 40% chance of liability is exactly \frac{1}{10,000}. When the insured is faced with such a claim imposing the duty to settle increases the expected cost to the insurer by $60,000, so imposing the duty to settle in such cases would raise the premium by $15. Each time the insurer settles such a claim, it removes a 40% chance of an excess claim against the insured, so imposing this duty would reduce the total likelihood of an excess claim by \frac{0.4}{4000} = \frac{1}{10,000}. So, it is only be optimal to impose such a duty if the insured is willing to pay at least $15 to remove a 1 in 10,000 chance of excess judgment, that is to say if the VSJ is at least $150,000.

I show below that under the optimal VSJ rule, damages and the implied duty to settle are always greater than that of the Michigan rule. The intuition behind this is result is simple; the fact that the insured purchased insurance implies that she is willing to pay more to transfer risk from herself to the insurance company then

\textsuperscript{13}See Viscusi [2004]
\textsuperscript{14}50 Cal. 2d. 654, 659 (Cal. 1958)
it costs the company to bear it. Therefore, in cases where a settlement would only raise the cost of insurance by the expected amount that the insured would pay out of pocket, the insured would always prefer settlement. Returning to our example, if the VSJ of our insured was less than $50,000, she would never buy insurance in the first place. In cases where the excess judgment is less than the sum of the limit and the insured’s vulnerable assets, there is no difference between the Michigan rule and the DTL Rule, so the DTL Rule implies insufficient duty to settle as well. By continuity this also applies to cases where the potential claim is not much greater than the sum of the liability limit and the exposed assets of the insured.

In contrast, when the potential excess is substantially larger than the insured’s assets the DTL rule tends to lead to a greater duty to settle than the insurer and insured would prefer ex-ante. Under the DTL rule, an insurer’s potential liability is unlimited and it requires the insurer to act as if the entire judgment would be paid. However, once a judgment is big enough to bankrupt her, an insured does not care about further increases in the size of a judgment, so the VSJ does not keep increasing with the size of the claim. Furthermore, the fact that the insured chose a finite liability limit in the first place shows that the insured is willing to accept some risk of an excess judgment in exchange for lower premiums, implying there is a limit to the harm she would suffer from bankruptcy. The VSJ rule I propose would typically limit the insurer’s liability for breach of duty to a small multiple of the limit on the underlying insurance contract. This would imply no duty to settle cases that the plaintiff was relatively unlikely to win, regardless of the size of the potential judgment.

Exactly calculating the Value of a Statistical Judgment requires exact knowledge of the risk preferences and utility function of the insured, and would not be practical in a court of law. Simply asking an insured how much she is willing to pay to avoid a particular risk is unlikely to elicit an accurate answer, particularly if she knows how her answer will affect her lawsuit. The key insight of this paper is that the insured reveals a great deal about her risk preferences by her behavior in the insurance market. I show that a surprisingly accurate estimate can be constructed with knowledge of only three factors: 1) the liability limit chosen by the insured, 2) the relationship between liability limit and cost of insurance, and 3) the exposed assets of the insured. None of these factors are very difficult for the court to observe. Because the relationship between liability limits and cost of insurance is unlikely to vary substantially across cases, particularly within given lines of liability insurance, the court need not estimate the relationship between liability limit and cost of insurance for each case, but could use established values for each line of insurance. For example, the rule might impose maximum damages for breach equal to the sum of the liability limit plus 1.75 times the exposed wealth of the insured for automobile accidents, and 4/3 the liability limit plus double the exposed wealth for homeowner’s liability claims. Finally, I show that even if the court makes no attempt to estimate this relationship, and just uses a fixed
rule for VSJ for all lines of liability insurance such as the sum of the liability limit plus 1.75 times exposed wealth, it is likely to be superior to either the DTL or Michigan rule.

The insurer’s duty to settle is typically viewed as a duty to the insured, implying that the contractual principles I use are doctrinally appropriate to analyzing the duty. Nevertheless, the duty will have direct consequences for third party plaintiffs and will affect the incentives provided by the tort and insurance system as a whole. I argue below that in light of these considerations, the VSJ rule I propose remains attractive. The DTL rule may be superficially attractive to the plaintiffs, because there is no theoretical upper bound on the insurer’s liability for failure to settle. However, the DTL rule can encourage policy holders to choose lower limits, which would be harmful for plaintiffs with strong cases. In contrast, the proposed rule increases the incentive to choose higher insurance limits, while keeping insurance relatively affordable. Thus, the DTL rule leads to higher payouts to victims with ‘jackpot’ claims of low validity but high potential damages, while the proposed VSJ rule will tend to help victims with higher validity claims. Thus, I argue that implementing the VSJ rule could reduce the likelihood of nuisance settlements and is likely to have beneficial effects on the incentives provided by the tort system.

The bulk of the analysis in this paper is directed towards the case where the courts can accurately assess when refusal to settle constitutes a breach of the duty to settle. However, some have questioned the ability of the courts to accurately determine what beliefs would be reasonable for an insurer prior to trial, and have argued that the uncertainty and costs involved in determining whether there should be a duty makes a strict liability rule preferable. Under such a rule, the insurer would pay damages for failing to settle any claim that results in excess damage. I show that from an incomplete contract perspective, the insured would rather that damages for failure to settle are not imposed in cases where there would not be a duty to settle under the optimal rule, even if courts are imperfect. However, if the courts are constrained to use a strict liability rule, the incomplete contract approach would still imply damages that are related to the Value of a Statistical Judgment and are less than those available under the DTL rule.

In the following brief subsection, I present an example of how the estimated VSJ rule would work in practice. In the second section I present a formal model, derive the theoretically optimal VSJ rule and compare it to the disregard the limit rule and the Michigan rule. The next section describes a workable way of estimating the VSJ optimal duty and presents the results of simulations showing that the estimates are remarkably close to the optimal solution. Section 4 discusses the robustness of the results of the paper along with practical issues in implementing the rule including the question of whether to apply strict liability to failure to settle, or to apply a ‘negligence’ type standard. This is followed by some words regarding the policy implications and overall social welfare impact of the VSJ rule in Section 5. The final
1.2 An Example of the VSJ Rule

Consider two policy holders, Ms. H (Happy go lucky), and Mr. P (Play it safe). Both Ms. H and Mr. P have $2000 in assets that are vulnerable to an excess judgment. Let us assume that Ms. H is by disposition less risk averse than Mr. P, and also that Ms. H has $50 in judgment proof assets while Mr. P only has $30. We can interpret this in a number of ways, perhaps Ms. H has a hidden account in the Cayman Islands, or perhaps Ms. H just has more friends and family she can call on if she is down on her luck. In any case the consequences of bankruptcy would be less dire for Ms. H. Assume that they are both purchasing liability insurance where the probability of a loss decreases with the square of the loss, so that a loss that is between $10,000 and $10,001 is 1/4th as likely as a loss between $5,000 and $5,001. Although they both have the same vulnerable assets, and the same risk of being sued, Mr. P will choose a higher limit, both because he is more risk averse, and he is less judgement proof, so a very large loss will hurt him more. In particular, suppose that Mr. P chooses a liability limit of $50,000, while Ms. H chooses a limit of only $7,500.

In Section 3 we conduct simulations suggesting that for excess claims that exceed the vulnerable wealth of the policyholder, we can accurately estimate VSJ as $\hat{VSJ} = \frac{1}{\alpha - 1}L + (\frac{3}{4} + \frac{1}{\alpha - 1})V$, where $L$ is the limit chosen, $V$ is the vulnerable assets of the insured, and $\alpha$ is a parameter of the market indicating how quickly the probability of a loss decreases with the size of the loss. When the probability decreases with the square of the loss, this corresponds to $\alpha = 2$. Applying this estimated VSJ rule, we have $\hat{VSJ} = L + (1\frac{3}{4})V$. Since $L$ is 7500 and $V$ is 2000 for Ms. H, this implies that $\hat{VSJ}$ for Ms. H is $11000$. Similarly, for Mr. P, $L$ is 50,000 and $V$ is 2000, again. Estimating $\hat{VSJ}$, we obtain $53,500$. Suppose that both Ms. H and Mr. P were sued for $100,000 and the plaintiff offered to settle for $7500. Using the estimated VSJ rule, Ms. H’s insurer would face a duty to settle if the probability that Ms. H lost in court was greater than $\frac{7500}{7500+11000} \approx 40.5\%$, whereas Mr. P’s insurer would face a duty to settle if the probability that Mr P lost in court was greater than $\frac{7500}{50000+53500} \approx 7.2\%$. Because an excess judgment would hurt Mr. P much more, it is optimal for Mr. P’s insurer to have a greater duty to settle. In contrast, under the DTL rule, both insurers would face a duty only if the probability the plaintiff lost was greater than 7.5\%. Note that the DTL severely overstates the optimal duty to settle in the case of Ms. H, and the fact that it is approximately optimal with Mr. P results from the coincidence that the damages sought in the suit ($100,000) are very close to the

$\text{To be precise, we will assume that both of them have constant relative risk aversion, but the coefficient of risk aversion is } \gamma = 1.185055 \text{ for Ms. H, and is } \gamma = 1.707994 \text{ for Mr. P.}$

$\text{Using the results of Section 2 below, one can confirm that these limits are actually optimal given the parameters we have assumed.}$
sum of Mr. P's limit and VSJ($50,000+$53,000). Under the Michigan rule, Ms. H’s insurer would have an incentive to settle only if the likelihood the plaintiff prevailed was greater than $\frac{7500}{50000} \approx 78.9\%$ and Mr. P’s insurer would have an incentive to settle when the likelihood the plaintiff prevailed was greater than $\frac{7500}{52000} \approx 14.4\%$; in both cases the Michigan rule imposes too little duty to settle.

Since we created Ms. H and Mr. P, and know their utility functions, we can look ahead and use Proposition 1 to compute their actual VSJ’s. For Mr. P the actual VSJ is $53,841$, and for Ms. H it is $10,946$. Note that using only information likely to be observable to the court and our simple estimated VSJ rule, we were able to form estimates of VSJ’s within 1% of the true values. The following section develops a formal model of liability insurance, characterizes the optimal complete contract in that model, and shows what kind of duty to settle it would imply.

2 Model

For simplicity we imagine that all potential claims can be described by only three variables, the size of the potential judgment, which we call $x$, the plaintiff’s settlement demand, $s$, and the probability that the plaintiff would prevail in court $q$. We will assume that the insurer never has a duty to settle when the plaintiff demands a sum greater than the liability limit, so we only consider cases where $s \leq L$. If a claim goes to court, there are only two possibilities: With probability $q$ the defendant is liable for $x$, and with probability $1 - q$, the defendant is not liable. Prior to going to court, the plaintiff is able to make a take it or leave it settlement offer $s$. The insurer chooses to (R)eject or (A)cept. If the insurer chooses R and rejects the offer, the plaintiff sues and the court adjudicates the claim. If the insurer chooses A, it pays $s$ to the plaintiff, and the claim is settled.

Thus a complete insurance contract would specify whether or not the insurer must settle as a function of $s, x$ and $q$. This definition of a complete contract is suggestive of why it is unlikely to be observed in practice, the probability of winning a lawsuit is generally not observable to the insured. A complete contract would maximize the joint welfare of the risk averse insured and the risk neutral insurer. From a joint welfare perspective, the duty to settle should be increased until the costs to the insurer of any further increase in the duty exceeds the value, in terms of a lower likelihood of an excess judgment to the insured. We assume that insurers have no market power and face no administrative costs. Thus the premium for any policy will simply be the expected payouts from the insurer. Formally, the price of the policy is given by $p = E_\Omega y$, where $E_\Omega$ is the expectation operator over the set of all possible states of the world ($\Omega$) and $y$ represents the insurer’s liability to the plaintiff or policyholder. Let $L$ be the limit of the policy. If the insurer settles, $y = s$. If there is no settlement, and the policyholder is found liable, for any $x \leq L$, then $y = x$. If the insured is liable, and duty to settle does not apply then for any $x > L$, $y = L$. 

10
We assume that the policy holder is initially endowed with wealth $W_0$, and has a Von-Neumann Morgenstern utility function $u$. We assume that she is risk averse, so $u'(w)$ is strictly declining in $w$. We use the symbol $d$ to refer to any excess judgment paid by the policyholder, and assume that she has an amount of assets $Z$ that are judgment proof. Since she is left with $W_0 - p$ after she buys the insurance, the maximum excess judgment she pays will be $W_0 - p - Z$. For the sake of compactness we use $W = W_0 - p$ to refer to wealth after insurance is purchased and $V(Vulnerable assets)$ to represent the maximum excess judgment the insured ever pays, so $V = W - Z$. Thus if the insured is not liable for any excess judgment, her utility is $u(W)$, but when the insurer does not settle and the insured is found liable for a claim where $x > L$, her utility will be lower. If $x > L + V$, then $d = V$ and her utility is $u(Z)$, if $x < L + V$, but $x > L$ then $d = x - L$, and her utility is $u(W - x + L)$.

Our characterization of the optimal complete contract and duty to settle is is simplified by the following lemma:

**Lemma 1** An optimal duty can be expressed by a probability threshold that is a function of settlement offer ($s$), and claim ($x$) that denotes the minimum likelihood of plaintiff victory $q^*(s, x)$ where there is a duty to settle. Thus the insurer has a duty to settle whenever the probability the plaintiff wins is $q > q^*(s, x)$, and if $q \leq q^*(s, x)$, the insurer can go trial without liability for an excess verdict.

**Proof.** All proofs are in the appendix □

Intuitively, if it is worth it for the policyholder to pay the extra premium for a duty to pay $s$ to settle a claim with probability $q^*$ of an excess judgment, it is worth it for the policyholder to pay for a duty to pay the same $s$ to settle a stronger claim for the same amount. Settling the better claim is more likely to avoid an excess judgment, but it increases premiums less, because it would be more likely that the insurer would have to pay if it refused the settlement. Our first result characterizes the optimal duty to settle.

**Proposition 1**. Suppose that the policyholder has chosen the limit optimally. If $x > L + V$, then $q^*(s, x) = \frac{s}{L+ u(W) - u(Z)}$. If $x > L$, but $x < L + V$, then $q^*(s, x) = \frac{s}{L+ u(W) - u(W + L - x)} \cdot u'(W)$.

For a given utility function, if one knows the wealth of the insured, and how much of that wealth is judgment proof, it is relatively straightforward to calculate $q^*(s, x)$ for a given liability limit. The downside to the policyholder of imposing a duty is the utility loss from the increase in premium.17 This is just the product of $\left(1 - \frac{L}{W}ight)^{s}$.

---

17If the insurer suffers a loss greater than the limit, but not so great as to bankrupt him, he will be poorer, and he will have a greater marginal utility of money, so the loss from the increase
the marginal utility of money \((u'(W))\) times the probability that the duty is imposed (call it \(\pi(q,s,x)\)) times the cost to the insurer if it is imposed.

Since the insurer pays \(s\) if the duty is imposed, and expects to pay \(qL\) if it is not imposed, this expected cost is \(s - qL\). The benefit of imposing the duty is the likelihood it is imposed \((\pi(q,s,x))\) times the likelihood its imposition prevents an excess judgment \((q)\) times the utility benefit of preventing an excess judgment \((u(W) - u(W + L - x))\) or \((u(W) - u(Z))\) depending on whether the excess judgment is enough to bankrupt the policyholder. At the duty threshold, the policyholder is indifferent between imposing the duty or not. Solving \(u'(W) \times \pi(q^*, s, x) \times (s - q^*L) = \pi(q^*, s, x) \times q^* \times (u(W) - u(W + L - x))\) gets us the expression for \(q^*\) above.

We can interpret the expression above as determining how much weight to give to the possibility of an excess judgment. If an insurer needs to pay damages equal to the VSJ when it refuses to settle and the plaintiff prevails, then the insurer’s settlement decision will maximize the ex-ante joint payoff of the insurer and insured. We note that \(q^*(s,x) = \frac{s}{L + V}\), so according to proposition 1, if \(x < L + V\), then \(VSJ(x - L) = \frac{u(W) - u(W - x + L)}{u'(W)}\). We define VSJ \((d)\) as the Value of Statistical Judgment for a claim \(x\) that exceeds the insurance limit by \(d = x - L\), and define \(V\tilde{S}J = VSJ(V)\) for a judgment \(x > L + V\), that is to say a judgment that would bankrupt the insured and leave her with her judgment proof assets \((Z)\), so \(V\tilde{S}J = \frac{u(W) - u(Z)}{u'(W)}\).

Having defined the optimal extent to which an insurer should value an excess judgment and the optimal duty to settle, we compare these values to those implied by the majority duty-to-settle rule, and the minority Michigan rule. Let \(q^M\) and \(D^M\) be the duty to settle and damages for breach implied by the Michigan rule, and let \(q^{DTL}\) and \(D^{DTL}\) be the corresponding values implied by the disregard the limit rule. We note that the Michigan rule sets damages at the lesser of the excess judgment or vulnerable assets, so \(D^M = \min(x - L, V)\) and \(q^M = \max\left(\frac{s}{x}, \frac{s}{L + V}\right)\). The disregard-the-limits rule implies \(D^{DTL} = x - L\) and \(q^{DTL} = \frac{s}{x}\). The next result is that compared to the optimal rule, the Michigan rule always provides for an insufficient duty to settle, and that the majority disregard-the-limit rule can provide for an insufficient duty to settle as well. Note also that a higher threshold \(q\) implies that the insurer can refuse to settle cases that the plaintiff is more likely to win, so a higher threshold \(q\) implies less duty to settle.

**Proposition 2** For any \(x > L\), the Michigan rule implies too little duty to settle \((q^M(s,x) > q^*(s,x)\) and \(D^M < VSJ(d)\)). If the excess claim is not so large that it
could not be collected from the policyholder \((x \leq L + V)\), the duty to settle under the DTL rule is too little as well. \((q^{DTL}(s, x) > q^*(s, x) \text{ and } D^{DTL} = x - L < VSJ(x-L))\)

Suppose that the insured is risk averse, but has exposed assets greater than the excess of the claim over the liability limits, so the plaintiff would collect the entire claim if he prevails. In this case, both the Michigan and disregard-the-limits rule imply the same duty to settle, and provide the same damages. Both rules imply that the insurer should place the same value on any judgment that will actually be paid, regardless of who is responsible for paying it. Despite the intuitive appeal of this standard, there are good reasons why the insurer should place more weight on the portion of a judgment satisfied by the policy holder. This is the raison d’être of insurance. If the insured did not value getting rid of the risk of the excess judgment more than it costs to insure, she would not have bought insurance. So an optimal complete insurance contract would call for the insurer to settle to protect its policyholder in some circumstances where a risk neutral insurer would fight the case if the limit was higher and the only money at stake was its own.

To be fair, courts do not always ignore this risk aversion. Many courts that impose the DTL rule award consequential damages for loss of credit or mental anguish in addition to the excess judgment. However, a literal interpretation of the DTL rule implies too little duty to settle intermediate claims. We also note that by continuity, the fact that duty under the DTL rule is too small when \(x \leq L + v\) means that duty is also too small for some claims that would not be fully paid \((x > L + V)\). In fact we will see in a section below that duty under DTL is likely to be suboptimal until the claim \((x)\) is several times as great as the amount the plaintiff could collect \((L + V)\). However, as stated in the next proposition, if the claim is a large multiple of the amount the plaintiff could collect, duty to settle under the DTL rule will be too great.

**Proposition 3** For very large claims the disregard-the-limit rule provides too much duty to settle. For any policyholder and any finite limit \(L\), there is a \(\tilde{x}\) such that if \(x > \tilde{x}\) then \(q^{DTL}(s, x) < q^*(s, x) \text{ and } D^{DTL} > VSJ\)

Note that once the excess judgment exceeds the exposed wealth of the insured, the insured is no longer affected by the size of the claim. Returning to an example, suppose our insured has a $100,000 limit and exposed assets of $95,000. By continuity from proposition 2, this insured would be likely to prefer a rule that gave an insurer a duty to settle a claim for $200,000 with a 48% chance of liability if the plaintiff offers to do so for $100,000. Note that neither the DTL rule, nor the Michigan rule would impose a duty in these circumstances.

Now consider a proposed duty to pay $100,000 to settle a $2,000,000 claim with only a 6% chance of the plaintiff prevailing. First note that conditional on facing
the first claim imposing a duty raises the expected costs of the insurer by $52,000 ($100,000 − .48 × $100,000), but conditional on facing the second claim, imposing a duty raises the insurer’s expected costs by $94,000 ($100,000 − .06 × $100,000). The consequences to the policyholder of a refusal to settle followed by a plaintiff victory is the same in both cases, but these negative consequences are only 1/8th as likely in the second case. Compared to the first case, imposing a duty in the second case costs more, and provides less benefit to the insured, so it is quite possible that the insured would not be willing to pay ex-ante for a duty to settle in the latter case. Nonetheless, the DTL rule would impose a duty to settle only in the second case.

2.1 The Revealed Preference Approach: Towards an Accurate Estimate of VSJ

Having established that the Michigan rule always understates the optimal duty, and the DTL rule could understate or overstate the duty, one might wonder whether there is more practical significance to this exercise. In theory, proposition 1 could be used to compute the optimal duty, but any rule that requires the court determine the plaintiff’s Von Neumann-Morgenstern utility function in order to delimit a duty or compute damages would raise questions about implementation. Fortunately, I show that these results can be used to develop a practical way for courts to approximate the optimal rule. This subsection lays the groundwork for the approximation. We begin with the following comparative statics regarding VSJ and optimal duty to settle.

**Proposition 4** Suppose the policy holder has decreasing absolute risk aversion. 19 Consider a claim so large that the policy holder will not pay the excess judgment ($x > L + V$). Holding all else equal, $\bar{V}_S J$ and duty to settle will be greater (so $q^\ast$ will be lower) when:

A: The policy holder is more risk averse
B: The policy holder has more vulnerable assets ($V$ is increased)
C: The policy holder has fewer judgment proof assets ($Z$ is lower)

These relationships are all intuitive: When the policy holder is more risk averse, she is willing to pay more to insure against the same risk of a monetarily loss, implying higher VSJ. When the policy holder is richer, so she has more vulnerable assets, a total

\[18\] In fact, my simulations suggest that if this insured would prefer a duty to settle in the latter case, she would have probably chosen a higher policy limit.

\[19\] Decreasing absolute risk aversion implies that as an individual’s wealth increases he or she is willing to pay less to avoid a risk of constant absolute magnitude. Constant relative risk aversion implies decreasing absolute risk aversion. Formally it implies that if $x$ is positive, then $\frac{u(y+x) - u(y)}{u'(y)}$ is increasing in $y$. 

14
loss of those assets will hurt her more.\textsuperscript{20} Finally, when $Z$ is lower, the policyholder is left worse off after paying the excess judgment, and is willing to pay more to avoid that outcome.

However, aside from part $B$ of the proposition, it is not clear how these relationships can be used to inform a decision of what the duty or damages should be in a particular case. Even in cases where we believe that an individual might have good knowledge of his relative degree of risk aversion or his value of a statistical judgment, direct testimony about his subjective values is unreliable at best, particularly if he knows how such testimony is likely to affect his award. Furthermore, determining the value of judgment proof assets ($Z$) can be difficult. Some assets, such as pensions or home equity protected by a homestead exemption, are judgment proof because they are protected by law, and might be observable to a court. Other assets are judgment proof precisely because they are hidden (such as oversees bank accounts or cash in a mattress), or because they are uncertain and not susceptible to easy quantification (future earning opportunities, goodwill of friends and family).

One could argue that having a formulation of the ideal rule is helpful even if the inputs are relatively speculative, and that using imprecise inputs into the correct theory is better than precise inputs into an incorrect theory. With a revealed preference approach, we can do much better than this, we can use the policyholder’s choice of policy to guide our estimate of the VSJ and optimal duty to settle. As I show in the next section, this approach is capable of getting very close to the optimal result.

The key insight of this approach is that although the utility function of the insured might not be directly observed, the behavior of the insured provides evidence of her preferences. One learns something about the risk aversion of the insured, particularly about her disutility from an excess judgment, by her choice of a liability limit. As the next proposition shows, when the policyholder is deciding upon a liability limit, she will be guided by many of the same factors that determine her individual VSJ, and all else equal will choose a higher limit if she is more risk averse or less judgment proof.

**Proposition 5** Suppose the policyholder has decreasing absolute risk aversion, and that actuarially fair insurance is available at the policyholder’s choice of limit $L$. Holding all else equal, the policyholder will choose a higher limit when:

A: The policyholder is more risk averse

B: The policyholder has fewer judgment proof assets ($Z$ is lower)

C: The likelihood of a claim is decreasing more quickly with the size of the claim (Formally: if $f_1(x)$ is the probability density of claim size at $x$ in market 1, and if $f_2(x)$ is the probability density in market 2, then if $f_1(x)/f_2(x)$ is decreasing in $x$, the insured will demand

\textsuperscript{20}Note that if $x < L + V$, so the policy holder would pay, VSJ would decrease as wealth $W$ and assets $V$ increased, since increasing $V$ would imply more wealth left over, so the policy holder would be less risk averse over the judgment
A quick comparison of proposition 5 and proposition 4 reveals that the unobservable factors (i.e., higher risk aversion and less judgment proofness) which lead to a higher VSJ lead to demand for a high limit. One factor that affects liability limit demanded but not VSJ, is the rate at which claims decrease in likelihood as their size increases.\textsuperscript{21} We can think of this last factor as a proxy for the price of a high limit, if the likelihood decreases quickly, it is relatively cheap to choose a high limit and protect oneself against a large excess judgment.

A policyholder chooses an optimal liability limit so that the marginal benefit to her of increasing the limit is balanced by the marginal cost. Abstracting away from moral hazard and adverse selection, the monetary cost of increasing the liability limit is the increase in the insurer’s expected payout. If we assume that the limit does not affect the disposition of claims below the limit, increasing the limit by $1 will lead to an increased payout of $1 when there is a claim at or above the limit, and will have no effect otherwise. Thus the marginal price increase should simply be the probability that the insurer will pay a claim at or above the limit. Formally, let us assume that the cumulative probability distribution over total payouts under the policy is given by $F(x)$, with density $f(x)$. If $p$ is the price of the policy, and the policy is actuarially fair $p = \int_0^L xf(x)dx + \int_L^\infty Lf(x)dx$. The marginal cost of increasing the limit is given by $\frac{dp}{dL} = \int_L^\infty f(x)dx = 1 - F(L)$.

Turning to the benefit, increasing the limit only helps the policyholder with regard to claims above the limit; specifically claims between $L$ and $L+V$. If the claim is smaller than $L$ the policyholder faces no liability in any case, and if the claim is above $L+V$, the extra dollar of insurance just goes to the plaintiff, and the policyholder is still left with only her judgment proof assets. A more technical approach is to note that benefit of increasing the limit is that it decreases the probability of an excess judgment by $f(L)$. In particular the marginal decrease in the likelihood of a judgment that bankrupts the policyholder is $f(L+V)$. It also decreases\textsuperscript{22} the likelihood of an excess judgment that does not bankrupt the policyholder by $f(L) - f(L+V)$.

The marginal benefit of increasing the limit is directly related to the benefit of avoiding an excess judgment. Increasing the policyholder’s disutility from excess judgments increases the benefit of an increase in the liability limit, leading to an increase in the limit demanded. The demand for a liability limit also depends on the price of decreasing the likelihood of an excess judgment; that is to say the ratio of the price of increasing the limit to the likelihood that the increase prevents an excess judgment. This ratio will be a function of how quickly the likelihood of a claim decreases as the size of claim decreases. If the likelihood of the claim decreases quickly, any claim above the limit is likely to be near the limit, and it is more likely...

\textsuperscript{21}This factor was parameterized by $\alpha$ in the examples and simulations.

\textsuperscript{22}If $f(x)$ is increasing, this will be an increase.
that the policyholder will benefit from the increase in the limit. On the other hand, if the likelihood decreases relatively slowly, claims above the limit are more likely to be well above the limit, and the benefit of the increase in the limit is more likely to go to the plaintiff. Mathematically, when \( f(x) \) is decreasing slowly \( \frac{1-F(x)}{f(x)} \) will be high, and it will be more expensive to decrease the likelihood of an excess judgment.

The final part of our analysis is to note that not all excess judgments are the same; an excess judgment that takes all of one’s vulnerable assets is worse than an excess judgment which only requires part of them to satisfy. If \( f(x) \) is decreasing, as \( V \) increases and we hold all else constant, the ratio \( \frac{f(L+V)}{f(L)} \) will be decreasing. This means that of the marginal decrease in excess judgments \( f(L) \), the proportion that consists of excess judgments that bankrupt the policyholder will be lower. If two individuals with different amounts of vulnerable assets \( (V) \) demand the same limit \( (L) \), they must each value the average decrease in expected excess judgment the same. Since these avoided excess judgments are less likely to be full excess judgments for the policyholder with higher \( V \), it must be the case that her VSJ is higher.

To illustrate how all these factors affect demand for liability limits, let’s return to our example of the two policyholders, Mr. P and Ms. H. Recall that they both have vulnerable assets of $2000, but Mr. P is more risk averse and less judgment proof, so he chooses an insurance limit of $50,000 while Ms. H chooses a limit of only $7500.

Let us take a look at the consequences of increasing the limit for each policyholder. We see that Mr. P will be bankrupted only by a judgment of over $52,000, while Ms. H will be bankrupted by any judgment above $9500. We assumed the likelihood of loss decreases with the square of the size of the loss (\( \alpha = 2 \)), this implies that the likelihood of any loss over \( x \) decreases proportionately with \( x \). Since $50000 is \( \frac{2}{3} \times $7500, the likelihood a judgment will be above $50000 is only \( \frac{1}{\frac{2}{3}} \) or 15% of the likelihood a judgment will be greater than $7500. Thus increasing the limit by $1 will cost about 15% as much for Mr. P. Next, note that for Ms. H, A $1 increase in the limit will increase the amount she has left after judgment for any claim between 7500 and 9501, while for Mr. P, a $1 increase in the limit will increase the amount he has left for any claim between 50000 and 52001. The mid point between 50000 and 52001 is about 6 times as large the midpoint between 7500 and 9501, so an increase in the limit of $1 is about \( 6 \times 6 \) or 36 times more likely to benefit Ms. H. Our very rough estimate would be that because Mr P is willing to pay 15% as much for an increase that will only benefit him with 1/36th the likelihood, his value of a statistical judgment is \( .15 \times 36 \), or 5.4 times as large as Ms. H’s.

The last thing we note is that because $9500/7500 > 52000/50000, the relative decrease in probability of loss over the interval where increasing the limit benefits the policyholder will be greater for Ms. H. This implies that of the cases where the

\[^{23}\text{To see this note that a loss between }$10,000 \text{ and } $10,001 \text{ is } 1/4 \text{ as likely as a loss between }$5000 \text{ and } $5001, \text{ but a loss between }$10,000 \text{ and } $20,000 \text{ is } 1/2 \text{ as likely as a loss between }$5000 \text{ and } $10000. Formally, } \int_{-\infty}^{\infty} \frac{1}{x^2} dx = \frac{1}{x} \]
policy holder is benefited, they are more likely to be small losses for Ms. H, and relatively more likely to be greater losses for Mr. P. This suggests our estimate that Mr. P’s VSJ is 5.4 times as much is likely to be an overestimate, as is confirmed when we look back at Section 1.2, and see that our estimated VSJ rule provides a more accurate estimate of $\frac{53.5}{11}$, and that our calculation using utility functions gives a ratio of 4.92.

In summary, our analysis suggests that we will be able to estimate the value of an excess judgment based on factors which are likely to be observed with relative accuracy by the courts. This paper proposes that rather than the disregard-the-limit rule, courts apply a duty to settle according to the VSJ rule, based on an estimate of the policyholder’s value of statistical judgment (VSJ). To use the language of the DTL rule, the VSJ rule would impose a duty to accept a settlement offer whenever a prudent insurer who faced a claim for the sum of the policy limit $L$ and the value of a statistical judgment $\hat{VSJ}$ would accept the settlement offer. More formally, the rule would impose a duty to accept a settlement $s$ if the probability of the plaintiff prevailing is greater than $\hat{q} = \frac{s}{L + \hat{VSJ}}$. The next section uses a simulation analysis to test how closely courts can estimate the optimal duty to settle.

3 Simulations

In order to identify an implementable approximation of the ideal duty to settle rule and to test its accuracy, I conducted a simulation. I randomly assigned wealth, judgment proofness, and risk aversion to 599 simulated insurance buyers. The buyers were then each placed in an individual, actuarially fair insurance market, where they ‘chose’ an optimal insurance limit. I then computed the buyers’ actual $\bar{VSJ}$’s and used these to develop a simple linear approximation rule to estimate $\bar{VSJ}$ as a function of three factors that are likely to be available to the court: Insurance limit, vulnerable assets, and the rate at which claims decrease in probability as they grow larger. I then estimated $\bar{VSJ}$ and the associated duty to settle for each agent and compared these to the actual values, as well as to the values that would be implied by the DTL and Michigan rules. The next subsections describe the methods of the simulation in more detail.

---

24Since the ratio of the likelihood that an increase prevents a total loss is $\left( \frac{9500}{5200} \right)^2 \approx \frac{1}{30}$, and this is an upper bound on the ratio of the likelihood that the increase prevents a total loss equivalent, we know that a lower bound on the ratio of values of statistical judgment between Mr. P and Ms. H is $0.15 \times 30$, or 4.5
3.1 Agents

The agents (indexed by \( i \)) were all assumed to have constant relative risk aversion utility functions,\(^{25}\) with a coefficient of risk aversion \( \gamma_i \), chosen from a uniform distribution between \( .7 \) and \( 2.5 \). They were each assigned initial wealth \( W_i \) from a uniform distribution between 500 and 2000. Some of this wealth, \( Z_i \) was assumed to be judgment proof, with \( Z_i \) randomly assigned between 5 and 20.

Each agent was then placed in an insurance market characterized by a claim distribution parameter \( \alpha_i \), with \( \alpha_i \) randomly chosen between 1.5 and 3. The parameter \( \alpha \) is a measure of how quickly the probability of a claim decreases as the size of the claim increases. Specifically we assumed that the probability density of a claim of size \( x \) is given by \( f(x) = k_0 x^{-\alpha} \). We assumed a competitive market with no administrative costs, so insurance was priced at exactly the expected payout.\(^{26}\) The constant factor \( (k_0) \) was chosen to be small enough so that the price of insurance did not represent a significant portion of the agents’ wealth. The agents then ‘chose’ the liability limit \( L_i \) that would maximize their expected utility. That is to say, for every agent, the simulation identified the optimal liability limit: the point where agent’s marginal disutility from the price increase associated with increasing the limit was exactly balanced by the increase in expected utility from a decreased likelihood of an excess judgment. The simulation then computed the agent’s true VSJ\(_i \), the value of a statistical judgment for a claim that would leave them with only their judgment proof wealth. This was used to compute the optimal duty to settle \( (q^*_i) \) when the insurer was faced with a very large claim \( (x > L_i + V_i) \) and an offer to settle at the policy limit.

The variation in wealth, risk aversion, and judgment proofness was sufficient so that the agents differed widely in their demand for insurance and their values of a statistical judgment. The optimal liability limits ranged from under 500 to over 150,000, and the VSJ’s ranged from about 1500 to over 200,000.

3.2 Constructing a Rule for Estimates

The next step was developing a simple rule to approximate the optimal duty to settle. I looked for a rule that for any insurance market could determine the penalty for a breach of the duty to settle as a linear function of \( L \) and \( V \) of the type \( \hat{\text{VSJ}} = k_1(\alpha)L + k_2(\alpha)V \). Such a rule implies a duty to settle when \( q > \hat{q}^* \), where \( \hat{q}^* = \frac{L}{L + \text{VSJ}} \).

I was guided by the following analytical result.

\(^{25}\)Constant relative risk aversion means that as wealth varies, an agent would be willing to risk a constant share of wealth in a gamble with the same return. Formally \( u(w) = \frac{w^{1-\gamma}}{1-\gamma} \) where \( \gamma \) is the coefficient of risk aversion. See Chiaporri and Paiella[2011] for evidence supporting constant relative risk aversion from panel data of individual’s investment portfolio’s.

\(^{26}\)We assumed that claims where \( x < 1 \) were not worth filing, so price is given by \( p(L) = \int_1^L xf(x)dx + \int_L^\infty Lf(x)dx \).
Proposition 6 Suppose that insurance is actuarially fair and that \( f(x) = k_0 x^{-\alpha} \),
\[
\lim_{V \to 0} \frac{\bar{V}_S J}{L} = \frac{1}{\alpha - 1}
\]
This suggests that setting \( k_1(\alpha) = \frac{1}{\alpha - 1} \) is a good starting point.\(^{27}\) Having set \( k_1 \), we estimated \( k_2(\alpha) = 1 + \frac{3}{4(\alpha - 1)} \). This second factor was arrived at primarily through trial and error.

3.3 Results
The performance of the VSJ rule is demonstrated in Figures 1 and 2. Figure 1 compares the logs of \( \hat{V}_S J \) (the estimates of VSJ) on the y axis with the logs of the true values (on the x axis). The diagonal line in the figure is the 45° line, representing perfect accuracy. As illustrated, the estimates are remarkably close to the true values. In fact the maximum error was 8%, and 90% of the time, the error was less than 4.2%. It should be noted that because the figures displays log values, the figure shows that the rule produces accurate estimates over a very wide range of true values.

Figure 2 compares the duty to settle implied by the estimated VSJ rule to the optimal duty to settle. Again, the estimates are very close to the true values. Note that despite the great variance in the VSJ’s, there is much less variation in the optimal duty to settle threshold. The probability threshold at which the duty applies (\( q^* \)) generally falls between .2 and .65. This implies that (at least with these parameters), insurers should rarely have a duty to settle any case where the defendant has less than a 20% likelihood of prevailing.

At this point, it is worthwhile to compare the performance of the VSJ rule with the majority DTL rule and the minority Michigan rule, in doing so, I note that the damages provided can be seen as the dollar value that a particular rule requires the insurer to consider as the costs of an excess judgment to the insured, so I refer to these damages as implied VSJ. For both the VSJ rule, and the Michigan rule, once an excess claim is large enough that it will not be satisfied, duty to settle (and implied VSJ) do not depend on the size of the claim, but for the DTL rule, the size of the claim is important. Therefore, for each individual \( i \), the simulation randomly assigns a claim size \( x_i > L_i + V_i \) to form a basis for the disregard-the-limits duty to settle and implied VSJ .\(^{28}\) A comparison between actual VSJ and \( D^M \), the implied value under the Michigan rule is presented in Figure 3. The duty to settle, as expressed by the threshold probability \( q \) of plaintiff success under the Michigan rule is compared to the optimal duty in Figure 4. The corresponding comparisons with implied values (\( D^{DTL} \)) and duty to settle under DTL are shown in Figures 5 and 6. As explained in

\(^{27}\) The proof of proposition 6 suggests that even if the claims distribution is not geometric, when \( V \) is small compared to \( L \), VSJ \( \approx \frac{1 - F(L)}{f(L)} \)

\(^{28}\) The claim size was chosen randomly according to the distribution of claims in the individual’s insurance market.
proposition 2, the Michigan rule always underestimates VSJ, and in the simulation, it was occasionally off by a factor of 100. The disregard-the-limits rule sometimes grossly overestimated the VSJ (by a factor of up to 1000), and sometimes severely underestimated the VSJ as well. Translating these results into duty to settle, as shown in Figure 4, Michigan rule often imposed a duty only when the plaintiff’s likelihood of victory was above 90% or 95%. Similarly, Figure 6 shows that the DTL rule sometimes imposed a duty when the likelihood of plaintiff victory was only 5%, and sometimes would not impose a duty unless the likelihood was well above 90%. In contrast to the DTL rule and the Michigan rule, which often result in large errors, the VSJ rule produces estimates that closely track the theoretical optimal values for damages and duty to settle.

4 Robustness and Implementation

As shown above, if courts can observe $\alpha$, the rate at which the likelihood of claims decreases as they grow large, then they can use a simple linear rule to accurately estimate what value an insurer should place on an excess judgment. Having observed $\alpha$ for the relevant insurance market, the courts can generate an accurate estimate using only the limit chosen and the policyholder’s vulnerable assets, both of which
should be relatively easy to ascertain.

It is true that exactly how much a plaintiff would be able to collect $(V)$ might not be observed accurately by the court. Given the typical history of a bad faith refusal to settle claim, it is likely that the policyholder transfers the claim against the insurer to the plaintiff in the underlying suit in exchange for an agreement not to collect from the policyholder. Unlike a case where the plaintiff is trying to collect from the policyholder, in this case the policyholder has little incentive to hide assets, so the amount of vulnerable assets might be overstated. \(^{29}\) One might also worry about the court’s ability to calculate $\alpha$, the rate at which large claims become unlikely. Although this information might not be simple to observe, it is readily estimated from actuarial table or simply from the relationship between insurance prices and insurance limits.\(^{30}\) Furthermore, the court need not estimate $\alpha$ individually for each case, for example, courts might use a particular $\alpha$ for all automobile liability insurance, and a different $\alpha$ for all medical malpractice.

Even if one is convinced that engaging in any estimates $\alpha$ is beyond the scope of what courts can do, a simplified version of the proposed rule could be implemented. Figure 7 shows the simulated results of a very simple rule in which the court did not adjust damages or duty for the characteristics of the insurance market ($\alpha$), and simply estimate damages (call them $\text{VSJ}^P$, where P stands for ‘punt’) at the sum of the limit plus 1.75 times collectable assets. By comparing to figures 3 and 5, one can see that this very simple rule vastly outperforms the DTL and Michigan rule.

One might question the premise that policyholders choose their liability limits rationally. Some liability insurance is legally mandated, or is purchased primarily to demonstrate financial responsibility to customers or suppliers. Undoubtedly, many policyholders use rules of thumb or simply choose the limit that their insurance agent suggests. Zeiler et al. [2007] suggest that in Texas, physicians rarely changed their nominal limits, implying that real limits declined over time, which is difficult to explain if physicians are optimizing in their choice of limits. Nevertheless, the fact that most purchasers give little thought to an insurance limit does not mean that the limit chosen is completely uninformative. Insurance agents will often suggest higher liability limits for those with more assets, and an individual’s likelihood of accepting a higher limit would naturally reflect a subjective value of protecting those assets. In any case, there is still value in having an insurance contract be coherent. That is to say that an insurance contract with a given limit should provide a duty to settle that is compatible with that limit.

In order to generate estimates of the relationship between liability limit and VSJ, it was assumed that insurance was priced at an actuarially fair rate. Of course, insurers

\(^{29}\)One could argue that the Michigan rule would also be vulnerable to this criticism.

\(^{30}\)If insurance is actuarially fair, and there is no selection bias, then $\alpha$ can be estimated by looking at three prices $p(L_1), p(L_2)$, and $p(L_3)$, where $L_3 = \frac{L_2}{L_1} L_2$. Then $\alpha \approx (2 - \log\left(\frac{p(L_3) - p(L_2)}{p(L_2) - p(L_1)}\right)) / \log(L_2/L_1)$. 

23
must on average charge more than their expected liability that to cover administrative costs and the costs of maintaining reserves. Nevertheless, it is not clear that insurers must charge more than marginal cost for an increase in liability limit; they might just charge a fixed administrative cost per policy. However, if insurers do charge a proportional markup, this would imply that the policyholders are paying more for the same protection, so VSJ would be underestimated by the same degree.

So as to keep the simulation tractable, I assumed that both the utility function of the insured and the distribution of claims followed simple parameterized functions. Specifically, I assumed constant relative risk aversion (CRRA), and a geometric decay in claim probability. I do not believe that relaxing any of these assumptions would result in substantially different results. Varying the judgment proof wealth ($Z$) in the simulations has a similar effect as departing from CRRA, and did not significantly degrade the accuracy of the estimates. Regarding the distribution in claim density, in proposition 6, I showed that in the limit VSJ is approximately determined by the inverse hazard ratio ($\text{VSJ} \approx \frac{(1-F(x))}{f(x)}$). This is a non-parametric result, and does not depend on the shape of the claims distribution (or utility function). Assuming a geometric decay simply allows us to know the relationship between $\alpha$, $L$, and the hazard ratio. However, when $L$ is not large compared to $V$, so intermediate size excess claims that do not bankrupt the policyholder become more important, the shape of the utility function and claims distribution might have more effect. That is
to say the estimate of the parameter $k_1$ is more robust than the estimate of $k_2$. I also assumed that only the defendant’s liability was in doubt, and there was no uncertainty about the damages that would be awarded if liability was found. It would be easy to generalize the rule and account for uncertainty about the damage award by replacing the VSJ from a certain damage award with the expected VSJ, and derive the optimal duty to settle with uncertain damages.\(^{31}\)

Another theoretical difficulty is that insurers may base their estimate of the risk posed by an individual policyholder partially on the limit chosen. In that case, the relationship between price and limit reflects perceived risk, rather than real risk. If insurers judged policyholders who choose high limits to be better risks, this would cause rational policyholders to buy more insurance than they would if insurance were priced according to real risk, so VSJ would be overestimated. The reverse would occur in a market with adverse selection, where choosing a high limit is indicative risk, so VSJ would be underestimated.\(^{32}\) However, if $\alpha$ was estimated according to the price of insurance, rather than according to an actuarial table, the presence of selection would affect the estimate of $\alpha$ in an ambiguous manner, so it would be difficult to generalize regarding the bias.

For the sake of computational ease, the estimates above of the relationship between VSJ and liability limit were generated under an assumption that in most cases liability is clear, so I could ignore the feedback effect between duty to settle and the price and demand for insurance. Of course if duty to settle had no effect whatsoever, the whole question would be purely academic, therefore it is worth asking whether these results are robust to a situation where duty to settle is more important. To determine the impact of the duty to settle on insurance prices and demand, it is necessary to model plaintiff settlement behavior. I ran simulations where liability was sometimes uncertain and assumed that for any claim above the limit, when liability was uncertain the plaintiff made a take it or leave it offer at the liability limit.\(^{33}\) This behavior is consistent with the ‘spike’ of settlements at or near the limit observed by Zeiler, Silver et al [2007] in their study of Texas physician malpractice insurance and claims payouts. The insurer would then observe the settlement offer and settle if there was a duty to settle or if it minimized its expected payout.

With these assumptions about plaintiff behavior, increasing the limit has two effects on the disposition of uncertain claims. First, it increases the settlement demand, and secondly for a given settlement demand, it will increase the likelihood that there

\(^{31}\)Let $\phi(\xi)$ be the probability density of damages, $\xi$, conditional on liability, and let $\Phi(\xi)$, be the cumulative probability. The insurer would have a duty to settle if $q >$

$$
\int_0^s \phi(\xi) \xi d\xi + \int_s^{L+V} \phi(\xi) \left( L + \frac{1}{\sqrt{\omega}} \left( 1 + 1 - \Phi(L+V) \right) \right) \xi + \left( 1 - \Phi(L+V) \right) \left( L + \overline{\text{VSJ}} \right)
$$

\(^{32}\)See Siegelman [2004] for an argument that both scenarios are possible

\(^{33}\)Such a strategy might be rational for a plaintiff if he did not know the probability of victory, and collecting any excess judgment from the policyholder defendant would be difficult, but going to trial did not significantly increase his costs of collecting from the insurer.
will be a duty to settle for large claims under the VSJ or Michigan rule. From an ex-ante perspective, the policyholder is always hurt when the plaintiff demands a greater settlement, so the first effect always makes increasing the limit less attractive. The ex-ante welfare effects from the change in the duty to settle are more ambiguous, when the duty to settle is optimal, the policyholder is neither helped nor hurt by a marginal change. When the duty to settle is not optimal, welfare is increased (or decreased) if changing the limit moves the duty towards (or away from) the optimal level. Overall, taking note of the fact that many plaintiffs strategically make demands exactly at the settlement offer will reduce limit demanded under all duty to settle rules. This implies that the estimated VSJ rule will tend to underestimate VSJ as more plaintiffs make demands exactly at the liability limit.

The simulations shed some light on the interesting question of the interaction between duty to settle and liability limit desired. The general result is that policyholders will demand the highest limits under rules where there is the least duty to settle, so the limit demanded will be greatest under the Michigan rule. Comparing between the Michigan rule the DTL rule, increasing the limit is less attractive with the DTL rule, because doing so increases the costs of insurance more under DTL, and provides a similar benefit, thus policyholders will demand a lower limit under the DTL rule. Recall that there is some ambiguity as to whether the DTL rule or the VSJ rule gives more duty to settle. If the DTL duty to settle is interpreted narrowly according to a strict interpretation of the Prosser formula, so that the insured’s risk aversion is never taken into account, then the VSJ rule implies more duty to settle for intermediate claims. For large claims with $x > \hat{\text{VSJ}}$ the VSJ rule implies less duty to settle than DTL. Thus, whether or not changing from DTL to the VSJ rule would decrease or increase rates depends on the relative likelihood of intermediate high validity claims, and large low validity claims.

In practice, it is not clear that courts really do not consider the risk aversion of the client when considering duty to settle. If courts do fully consider risk aversion from intermediate sized claims under the DTL rule, the DTL rule would tend to always imply more duty to settle than VSJ, and would tend to lead to policyholders to chose lower limits. Finally, it should be noted that even if the insured chooses the same limit under both the DTL and the VSJ rule, the insured gets more surplus from insurance under the VSJ rule. To the degree that insurance is optional and not universally consumed, consumers should be more willing to purchase it under the VSJ rule, if we believe insurance is socially desirable, this is an argument in favor of VSJ.

4.1 Negligence and Strict Liability

This paper has focused on a presumption that courts will use a ‘negligence’ rule; that is to say that courts will only award damages when an insurer actually violates its duty to settle. In order for this to be workable, it must be the case that the court can
accurately estimate $q$, the likelihood that the plaintiff would win, properly calculated from the information available to the insurer prior to trial. Out of fear that courts may be unable to accurately assess what was actually reasonable to believe at the time of settlement, some have suggested strict liability for failure to settle. This subsection examines the implications of a strict liability rule as well as an imperfectly administered negligence rule on our analysis.

It should be noted that for any duty standard, insurers will make the same decisions under a perfectly executed ‘negligence’ rule, and a strict liability rule. Holding limits constant, a plaintiff would prefer a strict liability rule, since then a plaintiff would be able to collect the full claim, or at least the limit plus damages, whenever he made a settlement offer that was rejected, regardless of whether the insurer was reasonable in rejecting the offer. Whether the insured would ex-ante prefer strict liability is a more subtle question, and is closely related to the question of whether or not the duty is set optimally. Imposing strict liability would increase the cost of insurance, but could protect the insured from excess claims whenever the plaintiff made a settlement offer. Note that this is the same tradeoff that is considered in the duty to settle analysis. This leads to the conclusion that a plaintiff would ex-ante prefer strict liability only in cases where it would be optimal to impose a duty to settle. So even if a rule, like the Michigan rule, imposes a suboptimal duty to settle, a policyholder would be weighing the benefit of strict liability in the cases she (ex-ante) wished the insurer would settle against the losses in the cases she (ex-ante) did not want the insured to have a duty to settle. From the point of view of the optimal (or nearly optimal) VSJ rule, it is clear she would not prefer strict liability to a perfectly executed negligence rule. However, if a court sometimes underestimates $q$, an insurer who is faced with a settlement offer where there just should be a duty to settle (i.e. $q$ is just barely greater than $q^*$) might find it profitable to reject the settlement offer because even when the third party plaintiff wins in court, the insurer might avoid responsibility for the excess judgment if the court finds there was no duty to settle. A way to ameliorate is to enhance the damages for failure to settle to restore incentives, that is to say the court could use the estimated VSJ rule to establish the duty to settle, but impose damages of some multiple of the VSJ. Another partial solution is to apply a low burden of persuasion to a bad faith failure-to-settle claim.\footnote{For example, in Shamblin v. Nationwide Mutual Insurance Co. 396 S.E. 2d 766(W. Va. 1990) the court placed a burden on the insurer to show by clear and convincing evidence that refusal to settle was reasonable.}

There is some tension between the argument that the parties are unable to write a contract that explicitly delimits duty to settle as a function of the quality of the defendant’s case and the notion that courts could make an ex-post judgment of whether an insurer violated its duty to settle under any of the standards discussed in the paper. If it is impossible for courts to estimate how a reasonable insurer would judge $q$ at the settlement stage, the only practical rule for the court to apply is the strict
liability rule, which would hold the insurer liable for damages whenever the insurer rejected an offer to settle within the policy limits, and the insured was subsequently held liable for excess damages. Since damages under the DTL rule are set equal to the excess judgment, the effect of a DTL strict liability rule is to remove any liability limit once the plaintiff offers to settle within the original policy limit. Under the strict liability VSJ rule, which would require the insurer to pay $\text{VSJ}(x - L)$ when it refuses a settlement and the plaintiff prevails, an insurer would prefer to simply indemnify the policyholder if the penalty under the VSJ rule is greater than the excess judgment.\(^{35}\) For large claims (above $L + \text{VSJ}$), the insurer would not be obliged to pay the entire judgment under the VSJ rule, (but would under DTL), so the insurer would not indemnify. Thus the VSJ rule has the effect of increasing the limit to $L + \text{VSJ}$, rather than to infinity as would occur under DTL, and results in less liability for the insurer and less settlement of very large claims. Under strict liability, the VSJ rule provides for damages less than or equal to damages under the DTL rule.

From the incomplete contracting point of view, applying strict liability changes the desirability of increasing or decreasing the damages for failure to settle in several ways. Under strict liability, increasing the damages increases the amount the insurer will pay when it reasonably fails to settle but the plaintiff prevails in court. Assuming that the plaintiff has the first claim on these damages, increasing damages is essentially equivalent to increasing the liability limit when the insurer rejects the settlement offer. Since imposing a penalty $D_{SL}$ has the effect of increasing the limit to $L + D_{SL}$, and the policyholder implicitly prefers a policy limit of $L$, increasing the limit is presumably undesirable. On the other hand, applying strict liability changes the policyholder’s preferences over the settlement threshold. Let us define $q_{SL} = \frac{s}{L + D_{SL}}$ as the settlement threshold, so if the insurer is minimizing its expected liability, the insurer will settle if $q(L + D_{SL}) > s$, that is if $q > q_{SL}$. Thus settling more cases (decreasing $q_{SL}$), has no first order effect on the insurer’s expected payment (since $q_{SL}(L + D_{SL}) = s$). However, settling more cases does have the effect of reducing the likelihood of an excess judgment, so the fact that increasing damages makes the insurer settle more cases is always beneficial from the point of view of the policyholder.

At this point one might wonder what the optimal damage rule would be under the strict liability rule. Unfortunately it is difficult to do so without making some additional assumptions regarding the distribution of settlement offers over case quality.\(^{36}\) However, with some assumptions we are able to obtain an analytical result.

**Proposition 7** Suppose that the likelihood of a claim decreases geometrically with the size of the claim so $f'(x) = k_0x^{-\alpha}$, and suppose further that the proportion of plaintiffs who make a settlement offer $L$ is $\Phi$, and is independent of $x$. Finally,\(^{35}\)

\(^{35}\)If the claim $(x)$ is less than $L + \text{VSJ}$ then $\text{VSJ}(x - L) > x - L$.

\(^{36}\)Because the net benefit of increasing the penalty is negative for the infra-marginal cases (where $q < q_{SL}$, but it is positive for the marginal cases (when $q = q_{SL}$) we need to make assumptions about how those densities compare.
suppose that for plaintiffs who offer to settle claim quality $q$ is distributed uniformly and independently of $x$. If $V$ is small relative to $L$, the optimal strict liability damages for failure to settle is given by $D_{SL} \approx 2VSJ$.

The above result implies that it might be desirable to increase the penalty for refusal to settle when courts are constrained to use strict liability. This implies that all else equal, imposing strict liability decreases the benefits of not settling more than it increases the cost of imposing the duty to settle. Nevertheless, the optimal damages (and the optimal implied duty to settle) are still closely related to the VSJ, and this suggest it is still appropriate to use a VSJ type rule, but with different multipliers. Most importantly, optimal damages would still be a linear function of liability limit and vulnerable wealth, and unlike under DTL there would be a finite limit to damages available.

5 External Implications for Third Party Plaintiffs and the Tort System

The analysis so far has focused on the implicit contract approach and has shown that the VSJ rule would be likely to increase the efficiency of the insurance contract and the joint welfare of the policyholder and the insured. However, the extent of the duty to settle also affects other people. It directly affects the plaintiff in the underlying tort case, and it can indirectly affect all of society by altering the functioning of the tort system. In this section, I argue that the VSJ rule can improve the deterrent function of the tort system. I do not argue that the VSJ rule can solve the problem that judgment proof individuals may have insufficient incentive to purchase insurance or otherwise take into account the risks they impose on others.\footnote{See Shavell[1986]} The decision not to mandate purchase of insurance without limits reflects a social policy judgment not to force individuals to insure against all risk. Within these constraints, I argue that the external social welfare effects of moving towards the VSJ rule is likely to be beneficial on average.

Because an insured with a plausible claim for breach of duty to settle typically transfers his claim to the third party plaintiff in exchange for an agreement not to pursue the assets of the insured, the direct beneficiary of the duty is typically the plaintiff. Therefore, the rule I propose, which decreases the insurer’s maximum liability relative to the DTL rule, might be seen as hurting plaintiffs with large claims. However, holding the limit constant, the VSJ rule is likely to result in an increase in the duty to settle intermediate sized claims and implies more payments to plaintiffs with intermediate size claims of moderate validity. Moreover, making insurance more efficient from the perspective of the insured should tend to increase the discretionary

\footnote{See Shavell[1986]}
purchase of insurance. So while it is not clear whether moving from DTL to the VSJ rule is likely to help or hurt plaintiffs in general, it is likely to hurt some plaintiffs, and help others. The results of the simulations implied that the insurer would be unlikely to have a duty to settle when the plaintiff’s likelihood of victory was less than 20% unless the insured had assets well in excess of the policy limit. Thus the VSJ rule would increase recovery from claims that are moderately above the insurance limits that are likely to be valid, while decreasing the recovery from large claims that are unlikely to be valid. Some of these large claims may involve very sympathetic plaintiffs who have suffered greatly, but where the contribution or liability of the insured to the harm is doubtful.

If we assume that the tort system is sensibly designed, it should apply liability in cases where doing so is societally efficient, either because applying liability provides socially valuable deterrence, or provides a valuable compensation function. On the other hand, in cases where applying liability does not provide good deterrence, or the deterrence is outweighed by a possible chilling effect, liability should not be applied. Presumably, in cases where liability is unlikely, imposing liability on the defendant is less likely to lead to socially efficient incentives.

This is borne out by the research of Studdert, Mello et al [2006], which looked at hospital records of inpatient stays that led to claims of malpractice. They then had physicians with training in the relevant specialty independently review the records and make judgments regarding the likelihood that there had been an injury caused by medical error. The reviewers graded the claims according to the actual medical validity (as opposed to legal validity) of the malpractice claim. They found that in cases where the reviewing physicians found that when there was little to no evidence that the injury was caused by medical error, only 19% of claims led to payments, while when the evidence of error increased from “close call, but less than 50-50” to “virtually certain”, the likelihood of payment increased from 52% to 84%. Of the cases where that went to trial there was a payment in only about 9% of the cases that the reviewers adjudged were unlikely to involve medical error, whereas whereas of the cases adjudged to be involve medical error, the plaintiffs prevailed in 43% of the cases.38 These results are all consistent with a hypothesis that success at trial is a good indicator of the validity of the claim, and that many cases with an intermediate likelihood of success at trial might involve medical error.

Claims where the plaintiff has a good probability of prevailing may involve some likelihood that the plaintiff could prove that the defendant did something clearly blameworthy, or may involve actions by the defendant that are close to the edge of blameworthy or not. In either case, increasing payments in these cases is likely to increase deterrence. On the other hand, large claims with a low probability of success may be more likely to be attempts to capitalize on the jury’s sympathy for an unlucky plaintiff, or to involve behavior by the defendant that involves complicated issues, but

---

38Table 2 of Studdert, Mello et al [2006]
would not be seen as blameworthy by most experts. Thus, although I formally justify
the VSJ rule on the basis of private efficiency between the insurer and policyholder,
I believe it is likely to have beneficial external effects.

This body of this paper has not examined how the strategic behavior of the plain-
tiff could be affected by the choice of duty to settle rule. If the plaintiff has good
information about the likelihood he prevails in court, he is likely to adjust his settle-
ment demand according to the insurer’s duty to settle. In particular, the plaintiff is
likely to demand more under rules which give the insurer a stronger duty to settle. As
pointed out by Meurer[1992], this might imply that the policyholder would prefer an
insurance contract that imposed a weaker duty to settle. This would give the insurer
a better bargaining position vis-a-vis the plaintiff, allowing the insurer to capture
more of the surplus from settlement and pass this on to policyholder by lowering the
premium. However, as Meurer points out this gain comes solely from allowing the
insurer to capture more of the surplus of settlement from the plaintiff.

Given that plaintiffs are generally presumed to be risk averse, and often have
high discount rates or liquidity constraints due to having suffered a severe injury, one
might presume that the insurer is already able to capture a good deal of surplus from
settlement. This is to say the insurer facing a claim under the limit, who would pay
the whole claim, but faces no other duty to settle, would often be able to settle for less
than the expected payout in trial. Eliminating or curtailing the duty to settle would
have the effect of allowing the insurer to take advantage of the defendant’s judgment
proofness in addition to the risk aversion of the plaintiff. It is not clear why there
would be a strong societal interest in improving the ability of an insurer to leverage
these factors for a lower settlement, so from a policy perspective, ignoring the effect
of the duty to settle on the strategic behavior of the plaintiff may be justified.

As mentioned above, the VSJ rule will sometimes lead to a stronger duty to settle
than the DTL rule. This implies that an insurer would sometimes have a duty to
settle claims above the liability limit, when it would not want to settle those claims
if the insurer had a limit above the claim. Recall the example in section 2, where the
plaintiff had a $200,000 claim and demanded $100,000
to settle. If the defendant had a $100,000 limit, and $100,000 of vulnerable assets, the
VSJ rule would almost certainly give the insurer a duty to settle. However, if the limit
was $200,000, so there was no danger of an excess claim, the insurer would rather face
an expected payment of $96,000 than settle for $100,000. Thus, the insurer could save
money by indemnifying the policyholder for any excess judgment and avoiding the
duty to settle. One might ask if the courts should intervene in such a strategy. From
the point of view of the policyholder, she receives the same protection from excess
claims, and allowing the insurer to gamble with its own money when it is profitable to
do so lowers premiums. The plaintiff is hurt somewhat, because recovery goes down
(here his expected recovery drops from $100,000 to $96,000), although it should be
noted that allowing the insurer to indemnify is equivalent to allowing the insurer to
increase the limit ad-hoc, and would only have the effect of denying the plaintiff a
settlement that is better than he could expect from a defendant who was fully insured.
This would lead to less settlement of uncertain claims, and could increase the ability
of the legal process to maintain deterrence without chilling pro-social actions.39

6 Conclusion

This paper tackles the question of an insurer’s duty to settle claims that could lead
to an excess judgment. Using an incomplete contract approach, I have shown that it
is optimal for the insured to internalize the risk of an excess judgment at the Value
of a Statistical Judgment to the insured. Depending on the size of the claim, and
the risk aversion and judgment proofness of the insured, this value can be orders of
magnitude higher or lower than the value implied by the disregard-the-limits rule used
by the majority of states. Through a revealed preference approach, a policyholder’s
choice of liability limit provides very accurate information about risk attitudes. So,
although the Value of a Statistical Judgment cannot be directly observed, Once the
characteristics of the insurance market are known, the it can be accurately estimated
as a simple linear function of the liability limit and the collectable assets of the
insured. Even when the characteristics of the insurance market cannot be observed,
the courts could use a very simple rule of thumb which results in a duty to settle that
is likely to be much closer to optimal than the alternative rules in common use.

In addition to providing a duty to settle that is much closer to the optimal duty
for the insured, the approach I propose could improve the functioning of the tort
system. In contrast to the disregard-the-limit rule, the VSJ rule which I propose
rarely imposes a duty to settle claims that have a very low probability of success.
This should decrease nuisance litigation, and could lead to more recovery by deserving
plaintiffs. The rule that I propose has the attractive feature of tying the duty to settle
to the limit chosen, thus making the limit a good proxy for the overall quality of the
insurance contract. It should be attractive to insurers because it limits their liability
for failure to settle to a small multiple of the policy limit, and thus reduces the long
tail of underwriting risk. Meanwhile it should be attractive to policyholders because
it keeps premiums down while ensuring that insurers have incentive to take the risk
aversion of the policyholder into account when making settlement decisions.

39See Friedman and Wickelgren [2010]
7 References


**Chiappori, Pierre-Andre, and Monica Paiella.** 2011, “Relative Risk Aversion is Constant: Evidence from Panel Data,” 9 *J of European Econ Assoc* 1021-1052

**Friedman, Ezra and Abraham Wickelgren.** 2010, “Chilling, Settlement, and the Accuracy of the Legal Process,” 26 *J of Law, Econ and Org* 144-157


**Sykes, Alan.** 1994a, “Judicial Limitations on the Discretion of Liability Insurers to Settle or Litigate: An Economic Critique”, 72 *Tex. L. Rev.* 1345

**Sykes, Alan.** 1994b, “Bad Faith Refusal to Settle, Some Implications of the Judgment Proof-Problem” 23 *J of Legal Stud.* 77-110


8 Appendix

Proof of lemma 1:

Let $H$ be a partition of $\Omega$ into information sets $h$ observable to the insurer and verifiable to the court ex-post. Without loss of generality we assume no atoms in the distribution of $h$. (We can always add an observable randomization device). A duty to settle rule is a map $D : H \to \{0, 1\}$ where $D(h) = 1$ implies there is duty to settle when the court observers $h$. The optimal duty to settle maximizes $E_{\Omega} u(W_0 - p - d)$ where $E_{\Omega}$ is once the expectation operator over $\Omega$. Every $h$ is associated with an $s$, $x$ and $q$. We will abuse notation and denote these as $s(h)$, $x(h)$ and $q(h)$. Suppose that the optimal duty cannot be expressed as a threshold rule, and $D$ represents an optimal duty. Then there are two sets of positive probability measure $H^+ \subset H$ and $H^- \subset H$ such that for any $h^+ \in H^+$ there is an $h^- \in H^-$ of equal measure such that $q(h^+) > q(h^-)$, $s(h^+) = s(h^-)$, and $x(h^+) = x(h^-)$ and that $D(h^-) = 1$, while $D(h^+) = 0$ and where the insurer does not settle cases in $h^+$. Define $h^-$ as the sibling of $h^+$. Let $G^+$ be a subset of $H^+$ such that every $h^+ \in H^+$ has a sibling in $H^-$. Let $G^- \subset H^-$ be the set of siblings. Consider the effect on the policyholder’s expected utility is of imposing a duty to settle on $G^+$, while removing a duty to settle on cases in $G^-$. Let $\rho$ be the probability density function. By construction, $\rho(h^+) = \rho(h^-)$ and $\int_{G^-} \rho(h^-)dh^- = \int_{G^+} \rho(h^+)dh^+$. Holding price constant, the change in expected utility is given by: $\Delta u = \int_{G^+} \rho(h^+) (q(h^+) - q(h^-)) (u(W_0 - p) - u(\max(W_0 - p - x(h^+)+L, z)))dh^+$. This is positive since $q(h^+) - q(h^-) > 0$ and $x(h^+) > L$. Note also that since the duty is binding at $h^-$, $q(h^-) L < s(h^+)$, so $\Delta p = \int_{G^+} \rho(h^+) (q(h^-) L - s(h^+) - \min(q(h^-) L - s(h^+), 0))dh^+$ so $\Delta p$ is negative, and the policyholder’s utility is improved, so $D$ was not optimal. Q.E.D.

Proof of proposition 1:

Let us use the function $\rho : \mathbb{R}^3 \to \mathbb{R}^+$ to represent the probability density of the information set $h = \{s, x, q\}$ so $\rho(s, x, q)$ is the likelihood of demand $s$, loss $x$ and probability of plaintiff prevailing $q$. Let us consider simultaneously imposing a duty to settle at an information set and increasing the limit $L$ by the (infinitesimally) increased cost of this duty to settle. This implies that the entire cost of the increased duty is borne when there is no loss above $L$. The cost density to the insurer of imposing a duty to settle is $\rho(s, x, q) \times (s - qL)$. Let us use $F(L)$ to represent the likelihood that there is no valid claim with a loss above $L$. Since $L$ was originally chosen optimally, there is no utility effect from the infinitesimal change in $L$.$^{40}$ Thus

$^{40}$This adjustment of the limit is an implicit proof of the claim in footnote 17 of the text.
the price increase is $\frac{\rho(s,x,q) - (s-q)L}{F(L)}$. The probability that the insured actually pays this price increase is $F(L)$, so the loss in utility to the insured of paying for the increased cost is simply $F(L)u'(W)\frac{\rho(s,x,q) - (s-q)L}{F(L)} = u'(W)\rho(s,x,q) - (s-q)L$. The benefit to the insured of increasing the duty to settle is that with probability $q \times \rho(s,x,q)$ it prevents an excess judgment. If $x > L + V$, the expected utility benefit is thus $q \times \rho(s,x,q) \times (u(W) - u(Z))$. Thus imposing a duty will be optimal if $q \times \rho(s,x,q) \times (u(W) - u(Z)) > u'(W)\rho(s,x,q) - (s-q)L$, and the marginal case occurs when $q^*(u(W) - u(Z)) = u'(W)\rho(s,x,q) \times (s-q)L$ or $q^* = \frac{s}{L + u(W) - u(x + L)}$. If $x < L + V$ the expected utility benefit will be $q \times \rho(s,x,q) \times (u(W) - u(W - x + L))$, and the marginal case occurs when $q^* = \frac{s}{L + u(W) - u(W - x + L)}$.

**Proof of proposition 4:**

Under DTL, for any $x > L$, there is a duty to settle only when $q > q^D_{TL} = \frac{s}{x}$. Again risk aversion implies $u(W) - u(W - x + L) > u'(W)\rho(s,x,q) - (s-q)L$, and thus $D_{TL} > V_{SJ}$, and since $L + V_{SJ} < x$ then $q^* = \frac{s}{L + V_{SJ}} > q^D_{TL} = \frac{s}{x}$. Q.E.D.

**Proof of proposition 5:**

Under DTL, for any $x > L$, there is a duty to settle only when $q > q^D_{TL} = \frac{s}{x}$. Damages will be given by $D_{TL} = x - L$. Note that for any $x > L + V$, $V_{SJ} = V_{SJ} = \frac{u(w) - u(x)}{u'(W)}$. Note that $V_{SJ}$ does not depend on $x$ if $x > x > L + V_{SJ}$, then $D_{TL} > V_{SJ}$, and since $L + V_{SJ} < x$ then $q^* = \frac{s}{L + V_{SJ}} > q^D_{TL} = \frac{s}{x}$. Q.E.D.
so VSJ$_1 >$ VSJ$_2$.

C: Suppose $Z_1 < Z_2$, but they share the same vulnerable assets. Note that this implies $W_1 < W_2$. By decreasing absolute risk aversion, if $y_1 > y_2$, $-\frac{u''(y_1)}{u'(y_2)} < -\frac{u''(y_2)}{u'(y_2)}$. Recall $\frac{u'(W_1)-u(Z_1)}{u'(W_1)} = \int_0^V u'(W_1-t)dt$ and $\frac{u'(W_2)-u(Z_2)}{u'(W_2)} = \int_0^V u'(W_2-t)dt$. Since $W_1 < W_2$, by decreasing risk aversion, $-\frac{u''(W_1)-y}{u'(W_1-y)} > -\frac{u''(W_2)-y}{u'(W_2-y)}$. So for any $t > 0$, $\frac{u'(W_1-t)}{u'(W_1)} = e^{-(\int_0^t-u''(W_1-y)/u'(W_1-y)dy)} > \frac{u'(W_2-t)}{u'(W_2)} = e^{-(\int_0^t-u''(W_2-y)/u'(W_2-y)dy)}$. Thus $\int_0^V \frac{u'(W_1-t)}{u'(W_1)}dt > \int_0^V \frac{u'(W_2-t)}{u'(W_2)}dt$ and VSJ$_1 >$ VSJ$_2$. Q.E.D.

Proof of proposition 5.

For actuarially fair insurance, the derivative of price with respect to limit is given by $\frac{dP}{dL} = 1 - F(L)$. Thus increasing the limit decreases utility by $u'(W)(1 - F(L))$ if there is no loss above $L$. This occurs with probability $F(L)$. If there is a claim above $L + V$, the policyholder is bankrupt anyway, so a marginal increase in the limit has no effect. If there is a claim between $L + V$, the policyholder gets $dL$ more insurance, but needs to pay $dP$ more for the insurance. Thus the policyholder gets to keep $(1 - \frac{dP}{dL})$ or $F(L)dL$ of the increase. The likelihood of a claim is $x = f(x)$ so the expected benefit from increasing the limit is an increase of utility of $\int_L^{L+V} f(x)u'(W + L - x)F(L)dx$. If the limit is chosen optimally $F(L)u'(W)(1 - F(L)) = F(L)\int_L^{L+V} f(x)u(W + L - x)dx$ or $u'(W)(1 - F(L)) = \int_L^{L+V} f(x)u'(W + L - x)dx$. Compare two policyholders indexed by 1 and 2.

A: Suppose that 1 is more risk averse, but $V_1 = V_2 = V$, $Z_1 = Z_2 = Z$, so $W_1 = W_2 = W$. As established in the proof of proposition 4, since 1 is more risk averse, $\frac{u_1'(W-t)}{u_1'(W)} > \frac{u_2'(W-t)}{u_2'(W)}$ for any $t > 0$. $\int_L^{L+V} f(x)u_1'(W + L - x)dx = \int_0^V f(L + V - t)u_1'(W-t)dt$, so if $L_1$ is optimal for 1, then $u_1'(W)(1 - F(L)) = \int_0^V f(L_1 + V - t)u_1'(W-t)dt$, Since $\frac{u_1'(W-t)}{u_1'(W)} > \frac{u_2'(W-t)}{u_2'(W)} \int_0^V f(L_1 + V - t)u_1'(W-t)dt > \int_0^V f(L_1 + V - t)u_2'(W-t)dt$ so $u_2'(W)(1 - F(L_1)) > \int_0^V f(L_1 + V - t)u_2'(W-t)dt$. Thus, at $L_1$ the cost of increasing the limit is outweighs the benefit, so agent 2 prefers a lower limit.

B: Suppose $Z_1 < Z_2$, but they share the same utility function and judgment proof wealth. Note that since $W = V + Z$ this implies that $W_1 < W_2$. By decreasing absolute risk aversion, since $W_1 < W_2$, $-\frac{u''(W_1-y)}{u'(W_2-y)} > -\frac{u''(W_2-y)}{u'(W_2-y)}$ for any positive $y$. Since $\frac{u'(W-t)}{u'(W)} = e^{-\int_0^t-u''(W-y)/u'(W-y)dy}$, this implies $\frac{u'(W_1-t)}{u'(W_1)} > \frac{u'(W_2-t)}{u'(W_2)}$, so if $u'(W_1)(1 - F(L)) = \int_0^V f(L_1 + V - t)u'(W_1-t)dt$, then $u'(W_2)(1 - F(L_1)) > \int_0^V f(L_1 + V - t)u'(W_2-t)dt$ and at $L_1$ the cost of increasing the limit is outweighs the benefit, so agent 2 prefers a lower limit.

C: Suppose $\frac{f_1(x)}{f_2(x)}$ is decreasing in $x$. Note that $1 - F(L) = \int_L^\infty f(x)dx$. Let $L$ be the limit that is optimal for distribution 2, so $u'(W)(1 - F_2(L)) = \int_L^{L+V} f_2(x)u'(W + L - x)dx$ and $u'(W)(1 - F_2(L))f_2(L+V) = f_2(L+V)\int_L^{L+V} f_2(x)u'(W + L - x)dx$. Note

36
also that

\[ (1-F_1(L)) = \int_L^{L+V} f_1(x) dx + \int_{L+V}^{\infty} f_1(x) dx < \int_L^{L+V} f_1(x) dx + \frac{f_1(L+V)}{f_2(L+V)} (1-F_2(L+V)) \]

and

\[ \int_{L_2}^{L+V} f_1(x) dx = \int_L^{L+V} f_2(x) \frac{f_1(L+V)}{f_2(L+V)} dx + \int_L^{L+V} f_1(x) - f_2(x) \frac{f_1(L+V)}{f_2(L+V)} dx \]

So

\[ (1-F_1(L)) < \int_L^{L+V} f_1(x) - f_2(x) \frac{f_1(L+V)}{f_2(L+V)} dx + (1-F_2(L)) \frac{f_1(L+V)}{f_2(L+V)} \quad (1) \]

and

\[ (1-F_1(L)) < \int_L^{L+V} f_1(x) - f_2(x) \frac{f_1(L+V)}{f_2(L+V)} \frac{u'(W+L-x)}{u'(W)} dx + \int_L^{L+V} f_2(x) \frac{f_1(L+V)}{f_2(L+V)} \frac{u'(W+L-x)}{u'(W)} dx \]

Note that

\[ \int_L^{L+V} f_1(x) \frac{u'(W+L-x)}{u'(W)} = \int_L^{L+V} f_2(x) \frac{f_1(L+V)}{f_2(L+V)} \frac{u'(W+L-x)}{u'(W)} dx \]

So by 2

\[ (1-F_1(L)) < \int_L^{L+V} f_1(x) - f_2(x) \frac{f_1(L+V)}{f_2(L+V)} dx + \int_L^{L+V} f_2(x) \frac{f_1(L+V)}{f_2(L+V)} \frac{u'(W+L-x)}{u'(W)} dx \quad (3) \]

and

\[ (1-F_1(L)) < \int_L^{L+V} f_1(x) \frac{u'(W+L-x)}{u'(W)} dx - \int_L^{L+V} (f_1(x) - f_2(x) \frac{f_1(L+V)}{f_2(L+V)} \frac{u'(W+L-x)}{u'(W)} dx \quad (4) \]

Since \( \frac{f_1(x)}{f_2(x)} \) is decreasing in \( x \), \( f_1(x) > f_2(x) \frac{f_1(L+V)}{f_2(L+V)} \) for any \( x < L + V \). So since \( u'(W+L-x) > u'(W) \), \( \int_L^{L+V} (f_1(x) - f_2(x) \frac{f_1(L+V)}{f_2(L+V)} \frac{u'(W+L-x)}{u'(W)} dx \) is positive.

Thus

\[ (1-F_1(L)) < \int_L^{L+V} f_1(x) \frac{u'(W+L-x)}{u'(W)} dx \iff (1-F_1(L)) \frac{u'(W)}{u'(W+L-x)} < \int_L^{L+V} f_1(x) \frac{u'(W+L-x)}{u'(W)} dx \]

37
and the policyholder would prefer a higher limit under \( f_1 \). Q.E.D.

**Proof** of proposition 6

Start by noting that when \( L \) is optimal, \( u'(W - p(L))p'(L) = f(L)(u(W - p(L)) - u(Z)) + \int_L^{L+V} f'(x)u(W - p(L) - x + L) - u(Z)dx \). As \( V \to 0 \), \( \int_L^{L+V} f'(x)u(W - p(L) - x + L) - u(Z)dx \to 0 \) as well, and \( u'(W - p(L))p'(L) = f(L)(u(W - p(L)) - u(Z)) \).

By definition \( \text{VSJ} = \frac{u(W - p(L)) - u(Z)}{u'(W - p(L))} \), so we have \( p'(L) = f(L)\text{VSJ} \). But \( p'(L) = \int_L^\infty f(x)dx = k_0 \frac{L^{-(\alpha-1)}}{\alpha-1} \). So we have

\[
\frac{k_0}{\alpha-1} = L^{-\alpha}\text{VSJ} \Leftrightarrow \frac{\text{VSJ}}{L} = \frac{1}{\alpha-1}
\]

Q.E.D.

**Proof** of proposition 7

Formally, we can describe a strict liability rule as a map \( f : \mathbb{R}^+ \to \mathbb{R}^+ \) that defines damages for any excess judgment. Note also that if \( D(x) < x - L \) it is cheaper to indemnify and pay \( x \) than to pay \( L + D \), so an insurer will indemnify. Thus we can define \( \tilde{D} \) as effective damages, where for any \( x > L \), \( \tilde{D}(x, L) = \min(x - L, D(x)) \).

For simplicity, we will only consider rules of the type \( \tilde{D}(x, L) = \min(x - L, D_{SL}) \), where \( D_{SL} \) is the maximum penalty for the insurer. Define \( \Phi(x, s) \) as the cumulative distribution function of \( s \), conditional on \( x \), so that if harm is \( x \), the likelihood that the plaintiff makes a settlement offer of \( s \) or lower is \( \Phi(x, s) \). Furthermore, let us define \( \rho(x, s, q) \) as the probability density of case quality \( q \), conditional on harm \( x \) and plaintiff’s offer \( s \), so that \( \int_0^\lambda f(x) \int_0^s f'_L \int_0^q \rho(x, s, \psi)d\psi d\Phi(x, s)dx \) is the probability that the damages are less than \( \lambda \), the plaintiff makes an offer below or at \( L \), and the quality of the plaintiff’s case is below \( q \).

Let us consider the impact of increasing \( D_{SL} \). This increases the price of insurance by the probability that the insurer will pay these damages, so the price increase directly decreases the utility of the insured by

\[
\int_{L + D_{SL}}^\infty f(x) \int_0^L \int_0^{L + D_{SL}} q\rho(x, s, q)dq d\Phi(x, s)dx
\]

For the proposition we have assumed that the plaintiff has the same likelihood (\( \Phi \)) of offering a settlement \( L \) for any \( x > (L + D_{SL}) \). We might justify this by noting that once the claim exceeds the insurer’s maximum liability, the payout to the plaintiff from winning is unlikely to further increase in the plaintiff’s claim. We also assume that the distribution of claim quality \( q \) is uniform over the relevant region \( (0, \frac{L}{L + D_{SL}}) \), and is independent of the size of the claim. So we can write the utility loss from the increase in price as \( \int_{L + D_{SL}}^\infty f(x) \int_0^{L + D_{SL}} q\rho(x, L, q)dq d\Phi dx \)

We can decompose the benefit from increasing the penalty \( D_{SL} \) into two components. First, an increase in the penalty has the same direct effect as increase in
the limit for claims the insurer does not settle. That is to say, it directly decreases the excess liability, and increases the residual wealth when the plaintiff wins a claim between \( L + D_{SL} \) and \( L + D_{SL} + V \). Second, increasing the penalty increases the probability that the insurer settles, so it decreases the probability that there is an excess judgment. The insurer will be willing to settle for \( L \) when \( q > \frac{L}{L + D_{SL}} \). So if \( q_{SL} = \frac{L}{L + D_{SL}} \) is the settlement threshold, \( \frac{dq_{SL}}{dD_{SL}} = \frac{-L}{(L + D_{SL})^2} \).

The increase in utility from the direct increase in residual wealth is:

\[
\int_{L+D_{SL}}^{L+D_{SL}+V} f(x) \left( L \right) \frac{L}{(L+D_{SL})^2} \left( \frac{L}{L+D_{SL}} - u(W-x+L+D_{SL}) \right) \rho(x,L,L+D_{SL}) \Phi dx
\]

and the increase from the change in settlement threshold is:

\[
\int_{L+D_{SL}}^{L+D_{SL}+V} f(x) \left( L \right) \frac{L}{(L+D_{SL})^2} \left( \frac{L}{L+D_{SL}} - u(Z) \right) \rho(x,L,L+D_{SL}) \Phi dx
\]

Since we assume \( \rho(x,L,q) \) does not vary over the relevant region, we replace it with the constant \( \rho \). We can now write the utility loss from increased price as

\[
\rho \Phi u'(W) \frac{L^2}{2(L+D_{SL})^2} (1 - F(L + D_{SL}))
\]

Since \( V \) is small compared to \( L \), we can now approximate the benefit from the direct increase in residual wealth as:

\[
\rho \Phi \frac{L^2}{2(L+D_{SL})^2} f(L + D_{SL}) (u(W) - u(Z))
\]

Recall that by definition \( V_{SJ} = \frac{u(W) - u(Z)}{u'(W)} \), so the benefit from the direct increase in residual wealth is approximately:

\[
\rho \Phi u'(W) \frac{L^2}{2(L+D_{SL})^2} f(L + D_{SL}) V_{SJ}
\]

If \( V \) is small compared to \( L \), we can approximate the benefit from the change in settlement threshold as:

\[
\rho \Phi \frac{L^2}{(L+D_{SL})^2} \left( 1 - F(L + D_{SL}) \right) \frac{u(W) - u(Z)}{L+D_{SL}}
\]

Note that if \( x < L + V + D_{SL} \) then \( u(W) - u(W-x+L+D_{SL}) < u(W) - u(Z) \), so the benefit from the change in settlement threshold is actually less.

Substituting in \( V_{SJ} = \frac{u(W) - u(Z)}{u'(W)} \), we have the benefit from the change in settlement threshold as approximately:

\[
\rho \Phi u'(W) \frac{L^2}{(L+D_{SL})^2} \left( 1 - F(L + D_{SL}) \right) \frac{V_{SJ}}{L+D_{SL}}
\]

If \( D_{SL} \) is optimal, then
\[ \rho \Phi u'(W) \frac{L^2}{2(L + D_{SL})^2} \left( f(L + D_{SL})V\bar{S}J - (1 - F(L + D_{SL})) + 2(1 - F(L + D_{SL})) \frac{V\bar{S}J}{L + D_{SL}} \right) \approx 0 \]

So \( (1 - F(L + D_{SL})) \frac{L + D_{SL} - 2V\bar{S}J}{L + D_{SL}} \approx f(L + D_{SL})V\bar{S}J \) and

\[ \frac{L + D_{SL} - 2V\bar{S}J}{L + D_{SL}} = \frac{f'(L + D_{SL})}{(1 - F(L + D_{SL}))} V\bar{S}J \]

But when limit is chosen optimally, if \( V \) is small compared to \( L \) then \( (1 - F(L)) \approx f(L)\bar{V}\bar{S}J \), So if \( D_{SL} \) is optimal, \( \frac{L + D_{SL} - 2V\bar{S}J}{L + D_{SL}} \approx \frac{f'(L + D_{SL})}{(1 - F(L + D_{SL}))} V\bar{S}J \). Recall that if \( f(x) = kx^{-\alpha} \), then \( 1 - F(x) \approx k^{1/\alpha}x^{-(\alpha - 1)} \) so \( \frac{f'(L + D_{SL})}{(1 - F(L + D_{SL}))} \frac{1 - F(L)}{f(L)} \approx \frac{L}{L + D_{SL}} \). Thus

\[ \frac{L + D_{SL} - 2V\bar{S}J}{L + D_{SL}} \approx \frac{L}{L + D_{SL}} \iff D_{SL} \approx 2V\bar{S}J \]

Q.E.D.