Coordinated Effects∗

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Abstract

While concerns of coordinated effects—that is, the possibility that a merger facilitates collusion—play a prominent role in merger review and antitrust economics, they have proved challenging to define and quantify. We provide a setup that allows us to model coordination as distinct from perfect collusion and to provide a definition of coordinated effects as well as a powerful test of whether (and if so, how much) a market is at risk for coordination. Coordination among a subset of suppliers in a procurement setting is modelled as a bidder selection scheme that selects each supplier as the designated bidder with some probability, while all nondesignated suppliers refrain from bidding. Whether a market is more or less at risk for coordination is quantified using the sum of the critical probabilities required for each supplier to be willing to coordinate. When this sum is less than (exceeds) 1, the market is (not) at risk for coordination. We show that a merger can, but need not, cause a market not at risk for coordination to become at risk and that coordinated effects are a greater concern when coordinating suppliers are stronger and more symmetric and when buyers are not powerful. Our framework allows a definition of a “maverick” firm and provides support for some aspects of maverick-based merger evaluation while challenging others.

Keywords: merger review, buyer power, maverick

JEL Classification: D44, D82, L41

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1 Introduction

Competition authorities regularly review proposed mergers and oppose those that they
determine will have sufficiently detrimental effects for competition.\textsuperscript{1} They recognize that
one source of detrimental effects is that a merger can change “the nature of competition in
such a way that firms that previously were not coordinating their behavior, are now signif-
ically more likely to coordinate and raise prices or otherwise harm effective competition”
(\textit{EC Guidelines}, para. 22(b)).\textsuperscript{2}

Adverse competitive effects that arise in this way as a result of a merger are referred
to as “coordinated effects.” These play a prominent role in current antitrust thinking
and practice. For example, Kolasky (2002) observes: “Concern over what we now call
coordinated effects has long been at the core of U.S. merger policy.” Similarly, in evaluating
a hospital merger, Judge Richard Posner wrote that “the ultimate issue is whether the
challenged acquisition is likely to facilitate collusion” and, in particular, “whether the
challenged acquisition is likely to hurt consumers, as by making it easier for the firms in a
market to collude, expressly or tacitly, and thereby force price above or farther above the
competitive level.”\textsuperscript{3} However, thus far, coordinated effects of mergers have proved elusive
to define, let alone analyze or test for.\textsuperscript{4} In light of the far reaching implications of decisions
that are based on coordinated effects, this state of affairs is, obviously, problematic.

A big part of the challenge comes from difficulties modelling coordination. On the
one hand, if one models coordination as perfect collusion, then coordination always pays
off, and so, by that measure, any market in which a merger might occur is at risk.\textsuperscript{5} On

\textsuperscript{1}The U.S. \textit{Horizontal Merger Guidelines} (\textit{U.S. Guidelines}) guide courts in the United States on how
to evaluate the potential anticompetitive effects of a merger. A similar role is played in Europe by the
European Commission’s \textit{Guidelines on the Assessment of Horizontal Mergers} (\textit{EC Guidelines}) and in
Australia by the Australian Competition and Consumer Commission’s \textit{Merger Guidelines}.

\textsuperscript{2}Similarly, U.S. competition authorities recognize that a merger “can enhance market power by in-
creasing the risk of coordinated, accommodating, or interdependent behavior among rivals” (\textit{U.S. Guide-
lines}, p. 2). See Aigner et al. (2006) on the evolution of coordinated effects’ assessment in the EU.

\textsuperscript{3}Hospital Corp. Of America v. FTC, 807 F.2d 1381, 1386 (7th Cir. 1986), paras. 1 and 7.

\textsuperscript{4}As stated by a former U.S. DOJ official, “One particular problem is that neither the theoretical
nor empirical literature tells us much at all about whether the disappearance of a single firm through
merger will increase the likelihood of coordination, other than, perhaps, in the extreme case where a
merger reduces the number of firms in a market from three to two” (Kolasky, 2002). In contrast to
the traditional approach based on tacit collusion being more likely in a market with fewer firms, Baker
(2002) argues for a “maverick-centered approach” that focuses on the role of so-called maverick firms in
part because “support for the traditional analysis in both the case law and the economics literature has
been eroding” (Baker, 2002, pp. 136–137).

\textsuperscript{5}Of course, in a repeated games setup one may be able to obtain different thresholds for the common
discount factor necessary to sustain perfect collusion, and one may interpret differences in these thresholds
as differences in the likelihood that coordination will occur. However, because discount factors are not
observable, evidence-based tests prove difficult in repeated-games settings. Moreover, the approach based
on critical discount factors does not quantify how much the participants gain from successful coordination,
which is one of the core questions according to the \textit{U.S. Guidelines}. 
the other hand, lacking a well-established economic model of suboptimal collusion, any alternative modelling assumption may appear, to a larger or lesser extent, ad hoc. Yet, a model of suboptimal coordination seems necessary to shed light on the matter. As a case in point, the _U.S. Guidelines_ (p. 26) say:

> The Agencies regard coordinated interaction as more likely, the more the participants stand to gain from successful coordination.

Motivated by the above, this paper provides a model of imperfect coordination that allows us to define what it means for a market to be at risk for coordination, to quantify this risk and, thereby, the gains from successful coordination, and to analyze the effects of mergers on the risk of coordination. Our approach is novel in two fundamental ways. We model coordination by suppliers in a procurement setting as a bidder selection scheme, whereby each coordinating supplier is chosen to be the designated bidder with some prespecified probability. With private information about stochastic costs, bidder selection schemes are suboptimal coordination devices, but are appealing because they do not require the communication of any private, production-relevant information, which would be likely deemed per se illegal. This allows us to ask whether, for a given set of candidate coordinators, there are probabilities that sum up to less than one such that each supplier is better off participating in the bidder selection scheme than not. This also means that we focus on whether coordination is individual rational, which contrasts with the previous literature that has emphasized incentive compatibility of (typically optimal) coordination.

Our approach is based on the idea that individual rationality is a necessary condition for coordination. Moreover, individual rationality seems to be the question of interest and relevance for competition authorities. For example, the _U.S. Guidelines_ explicitly refer to how much “the participants stand to gain from successful coordination,” and the threshold probabilities we derive provide a way of quantifying these gains. That being said, we do not, of course, dispute the relevance of incentive compatibility, but merely suggest that it may be useful to look at individual rationality separately, and possibly first.

We show that a merger can, but need not cause a market not at risk for coordination to become at risk. We also show that coordinated effects are a greater concern when, in senses that we make precise, coordinating suppliers are stronger and more symmetric and buyers are not powerful. We quantify the extent to which a market is at risk for coordination in a _coordinated effects index (CEI)_). Because we focus on individual rationality and disregard incentive compatibility, the test we develop will be biased in that it may overestimate the gains from coordination. Importantly, under appropriate parametric assumptions, data on pre-merger market shares, to which agencies often have access, will typically be sufficient to construct the _CEI_, making it operational for practical purposes.
Our clear-cut and operational definition of coordinated effects also allows us to give precise meaning to another term that features prominently in antitrust thinking and practice, which is that of a *maverick* firm. For example, Baker (2002, pp. 140–141) argues that “the identification of a maverick that constrains more effective coordination is the key to explaining ... which particular changes in market structure from merger or exclusion are troublesome, and why.” Commenting on the recently proposed merger between AT&T and T-Mobile, the American Antitrust Institute states that the “merger failed in large part because it eliminated T-Mobile as a disruptive competitor, or a ‘maverick’.” Baker (2002) describes Northwest Airlines as being a maverick with respect to coordination by the other major hub-and-spoke carriers (leaving aside Southwest Airlines) and Liggett as a maverick among the major cigarette manufacturers. Related to the market for baby food, the U.S. Federal Trade Commission raised the concern that Heinz and Gerber would coordinate following Heinz’s acquisition of the maverick Beachnut.\(^6\)

Notwithstanding its prominence, the definition of a maverick has remained vague. Antitrust officials have described a maverick as “a firm that declines to follow the industry consensus and thereby constrains effective coordination” (Kolasky, 2002), while the *U.S. Guidelines* (p. 4) describe a maverick as “a firm that has often resisted otherwise prevailing industry norms to cooperate on price setting or other terms of competition.” Kwoka (1989) identifies a maverick as the relatively “more rivalrous” firm, and Ivaldi and Lagos (2017) take the view that a maverick is a small firm. Current thinking is that an acquisition that eliminates a maverick is likely to cause adverse coordinated effects if the market is vulnerable to coordinated conduct (*U.S. Guidelines*, p. 25), with some arguing that the elimination of a maverick may be necessary for a coordinated effects argument because “loss of a firm that does not behave as a maverick is unlikely to lead to increased coordination” (Kolasky, 2002).

Undeterred by the difficulties arising from such vagueness, Baker (2002, p. 197) argues for a “maverick-centered approach” to coordinated effects, saying: “In many settings, regulators reliably can identify an industry maverick that prevents or limits coordination” by either observing that the firm constrains industry pricing or identifying factors that would cause the firm to prefer low prices (e.g., excess capacity). This presumably leaves merging parties guessing as to what evidence and arguments they could possibly put forth to counter agencies’ maverick-based concerns.

An operational definition of a maverick requires that a maverick firm be identifiable based only on pre-merger observables, which we are able to provide. For a fixed set of firms that contemplate coordinating, we say that some other firm is a maverick if

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\(^6\)American Antitrust Institute, “AAI Says the Proposed Sprint-T-Mobile Merger Should be ‘DOA at the DOJ,’” June 5, 2018).

coordination does not pay off for the potentially coordinating firms when the maverick is present in the market, but does pay off when the maverick is not present. We show that whether the acquisition of a maverick puts a market at risk for coordination may depend on the capacity, or strength, of the acquiring firm. Our framework and analysis thus provide support for some aspects of maverick-based merger evaluation while challenging others. A key insight here is that, although the elimination of a maverick would, by definition, put the market at risk for coordination, the acquisition of a supplier is not the elimination of the production capacity of that supplier: The newly merged entity’s production capacity will be stronger after the merger than the acquirer’s capacity before the merger. In general, this affects every firm’s critical share.

We are also able to make precise and corroborate, for example, notions that a merger that increases the symmetry of a group of firms increases the risk of coordination by those firms, and that powerful buyers reduce concerns of coordinated effects. At the same time, we show that other ideas, such as the notion that a merger to duopoly necessarily increases the risk of coordination or that the elimination of a maverick is necessary for coordinated effects, are not supported. Further, from our framework arise new results, such as coordinated effects being more of a concern when a merger generates cost synergies, and new questions, such as whether incentives for coordination induce mergers rather than, as in current thinking, mergers increase incentives for coordination.

Formally, we study a procurement model with independent private types to define and analyze coordinated effects. This setup is tractable, explicitly models price formation in the presence of cost uncertainty, and permits incorporating buyers that are “powerful” along the lines of Loertscher and Marx (forth.), who build on Myerson (1981) and Bulow and Klemperer (1996). In addition, the setup neither presumes nor precludes efficiency, and in general involves a tradeoff between profit and social surplus that is at the heart of industrial organization and antitrust economics.

8"An acquisition eliminating a maverick firm ... in a market vulnerable to coordinated conduct is likely to cause adverse coordinated effects" (U.S. Guidelines, para. 25).

9Amelio et al. (2009) explain that for the ABF/GBI Business case, the European Commission identified the increase in competitors’ symmetry as one of the critical factors that would make tacit collusion easier to implement, monitor, and sustain.

10The U.S. Guidelines suggest a possible relation between buyer power and coordinated effects, saying that buyer power can mitigate merger effects if, for example, “the conduct or presence of large buyers undermines coordinated effects” (U.S. Guidelines, p. 27) The EC Guidelines discuss the possibility that “buyer power would act as a countervailing factor to an increase in market power resulting from the merger” (para. 11). The Australian Merger Guidelines view “countervailing power” as a competitive constraint that can limit merger harms (paras. 1.4, 5.3, 7.48). See Grout and Sonderegger (2005) on the implications of buyer power for the ability of suppliers to sustain collusion. See Green et al. (2015) illustrating how explicit collusion can defeat the buyer power of even a small number of large strategic buyers.

11For more comprehensive arguments as to why the independent private values models is, in a sense, the right setup, see Loertscher and Marx (forth.).
Importantly, this procurement model is not only well-accepted in the theoretical literature, but it also enables us to capture the key ideas of George Stigler, whom antitrust officials have referred to as “the true father of modern oligopoly theory,” regarding the importance of information costs (and incomplete information) in determining firm behavior (Kolasky, 2002). As noted by Stigler (1961, p. 213), “some important aspects of economic organization take on a new meaning when they are considered from the viewpoint of the search for information” and, in particular, “one important problem of information—the ascertainment of market price.” Stigler (1964, p. 44) emphasizes the particular role of information in the problem on firms coordination, which he says “proves to be a problem in the theory of information.”

We model a merger by assuming that the two merging suppliers form a merged entity whose cost is the minimum of the merging suppliers’ costs. This implies that the merged entity draws its type from a “better” distribution than the pre-merger suppliers. Suppliers that coordinate by suppressing all but one of their bids may lose sales to outside suppliers and may reduce total surplus even when one of the coordinating suppliers wins if the “wrong” (not lowest cost) supplier wins. But the coordinating suppliers stand to gain higher payments by suppressing all but one of the coordinating suppliers’ bids. Thus, on net, the change in expected surplus from coordination can be positive or negative for the coordinating suppliers.

There is related legal and economics literature on coordinated effects. Representative of the economics literature, Ivaldi and Lagos (2017) consider a repeated differentiated products Bertrand model and focus on the critical discount factor required to support all-inclusive perfect collusion based on grim trigger strategies and assuming any deviations are perfectly observed. A decrease in a firm’s critical discount factor is interpreted as an increase in its incentive to collude. Ivaldi and Lagos (2017) define the overall effect of a merger between two suppliers as the difference between the critical discount factor for

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12Baker (2002, pp. 137–138) describes the related history of thought as initially having the dominant view (prior to the rise of Chicago School ideas) that “when only a few firms competed in an industry, they readily would find a way to reduce rivalry, collude tacitly, and raise prices above the competitive level.” Later, George Stigler explained that “the success of collusion, whether tacit or express, could not be assumed,” which led to the likelihood of collusion becoming a matter of analysis rather than one of assumption.

13This approach is also taken by, e.g., Farrell and Shapiro (1990), Salant et al. (1983), and Perry and Porter (1985).

14In a procurement-based setup, in the absence of buyer power, the incentives of the noncoordinating suppliers are not affected by the presence of coordination by rivals, which is in contrast to the effects in Cournot and Bertrand setups, where outsiders incentives are affected by coordination.

15For auction-based model of unilateral effects, see Waehrer (1999), Waehrer and Perry (2003), Miller (2014), and Froeb et al. (2017).

16Davis (2006), Davis and Huse (2010), and Brito et al. (2014) provide simulations of coordinated effects in similar setups. Moresi et al. (2011, 2015) provide indices for scoring incentives for coordination in certain contexts.
the merged entity and the maximum of the critical discount factors for the two merging suppliers in the pre-merger market. Their model assumes complete information and does not admit non-all-inclusive collusion. Further, Ivaldi and Lagos (2017) focus on perfect (joint surplus maximizing) collusion—in their setup there is no disciplined way to model less-than-perfect collusion. Simulations in Ivaldi and Lagos (2017) suggest that mergers strengthen the incentives for the merged entity to collude but weaken incentives for the nonmerging firms, with the effect on the merged firm being stronger.

Our results are strikingly different. While the literature described above suggests that mergers strengthen the incentives to collude for the merged firm, we show that a merger can either increase or decrease incentives to coordinate. Results in the literature suggest that mergers weaken the incentives to collude of nonmerging parties. In contrast, in our model, a merger with no cost synergies has no effect on the incentives for nonmerging parties to coordinate. Furthermore, in our model with cost synergies, a merger increases incentives for nonmerging suppliers to coordinate. Finally, results in the literature suggest that the overall effect of a merger is to increase incentives for collusion. In contrast, we show that the overall effect can be to increase or decrease incentives for coordination.

In other coordinated effects literature, Kovacic et al. (2007, 2009) and Gayle et al. (2011) view coordination as analogous to incremental mergers among post-merger firms and propose quantifying coordinated effects by using existing merger simulation tools to model these incremental mergers. For an empirical analysis of coordinated effects following the MillerCoors joint venture, see Miller and Weinberg (2017). Case studies involving coordinated effects include Aigner et al. (2006) for Sony/BMG and Impala, Amelio et al. (2009) for ABF/GBI Business, and Motta (2000) for Airtours/First Choice. For an overview of the related legal literature, see Baker (2002, 2010).

As mentioned, to the extent that it has studied the question, the previous literature has focused on incentive compatibility—when is coordination a Nash equilibrium—while modelling coordination as perfect collusion. In contrast, we study individual rationality (and mutual consistency of the individual rationality constraints) of coordination, modelled deliberately and necessarily as suboptimal collusion, paying at this stage no attention to incentive compatibility. Satisfying individual rationality is necessary (but of course not sufficient) for coordination. So whenever our index is negative, coordination (as a bidder selection scheme) cannot happen, but if the index is positive, it may.

The paper is structured as follows. Section 2 defines the setup and describes our model of coordination and our approach to quantification of coordinated effects. It also establishes that coordination harms the buyer and reduces social surplus. Section 3 analyzes

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17Ivaldi et al. (2003) identify market characteristics that may affect the sustainability of collusion, including in bidding markets.
which markets are (not) at risk for coordination. Section 4 consider extensions. Section 5 concludes.

2 Framework

In this section, we introduce the procurement setup, our model of coordination within this setting, and the critical shares that we use to measure the risk of coordination.

2.1 Setup

We assume a procurement setup with one product and one representative buyer. In the pre-merger market, there are \( n \geq 2 \) suppliers, indexed by the elements of \( \{1, \ldots, n\} \equiv N \). In the post-merger market following the merger of suppliers \( k \) and \( \ell \), there are \( n - 1 \) suppliers, including the suppliers in \( N \) other than \( k \) and \( \ell \) plus the merged entity, which is denoted by \( \mu_{k,\ell} \). Thus, we index the post-merger set of suppliers by elements of \( N \setminus \{k, \ell\} \cup \mu_{k,\ell} \equiv \hat{N}_{k,\ell} \). We refer to suppliers in \( N \) as the active suppliers in the pre-merger market and suppliers in \( \hat{N}_{k,\ell} \) as the active suppliers in the post-merger market following the merger of suppliers \( k \) and \( \ell \).

In either the pre-merger or post-merger market, each active supplier \( i \) draws a cost \( c_i \) independently from continuously differentiable distribution \( G_i \) with support \([c, \bar{c}]\) and density \( g_i \) that is positive on the interior of the support. We model a merger as allowing the merging suppliers to rationalize production by producing using the lower of their two costs, which implies that the merged entity \( \mu_{k,\ell} \) draws its cost from the distribution of the minimum of the pre-merger costs of suppliers \( k \) and \( \ell \), i.e., for \( k, \ell \in N \), \( G_{\mu_{k,\ell}}(c) \equiv 1 - (1 - G_k(c))(1 - G_\ell(c)) \) with density \( g_{\mu_{k,\ell}} \). Each supplier is privately informed about its type, and so the suppliers’ types are unknown to the buyer. The buyer has value \( v > c \) for one unit of the product. All of this is common knowledge.

The buyer and suppliers are risk neutral. The buyer’s payoff is zero if it does not trade and equal to its value minus the price it pays if it does trade. Similarly, a supplier’s payoff is zero if it does not trade and equal to the payment that it receives minus its cost if it does trade.

To model buyer power, we assume that a buyer with buyer power uses an optimal procurement. In contrast, a buyer without power is assumed to use an efficient procurement. This approach and these definitions are consistent with Bulow and Klemperer (1996) and Loertscher and Marx (forth.). It means that a buyer without buyer power uses a second-lowest-price auction (or, equivalently, a descending clock auction) with reserve equal to the minimum of the buyer’s value and the upper support of the suppliers’ cost distribution, \( r \equiv \min \{v, \bar{c}\} \). We assume that suppliers follow their non-weakly dominated
strategies of reporting truthfully. As a result, the lowest cost supplier wins whenever its cost is less than or equal to $r$ and is paid the minimum of the second-lowest cost and $r$. Otherwise, there is no trade.

We assume that a powerful buyer uses the dominant strategy implementation of the optimal mechanism that can be derived using the methods developed by Myerson (1981). In the optimal procurement, the buyer ranks suppliers according to their virtual costs, defined below, and applies a supplier-specific reserve price to the supplier with the lowest virtual cost. Given any pre-merger or post-merger supplier $i$, we denote supplier $i$’s virtual cost function by

$$\Gamma_i(c) \equiv c + \frac{G_i(c)}{g_i(c)}.$$

We impose the standard regularity assumption that $\Gamma_i$ is increasing. Because we allow the possibility that the densities are zero at $c$ (and also possibly at $\bar{c}$), define $\Gamma_i(c) = \lim_{c \to \bar{c}} \Gamma_i(c) = \bar{c}$. For $x > \Gamma_i(\bar{c})$, we define $\Gamma_i^{-1}(x) \equiv \bar{c}$. Notice that $\Gamma_i(c) > c$ for $c > \bar{c}$, which implies that $\Gamma_i^{-1}(v) < v$ for any $v > \bar{c}$.

Loertscher and Marx (forth.) show that it is useful to think of buyer power as consisting of two components: (i) the ability to exert *monopsony power*, that is, to reduce the quantity traded below the efficient level by committing to a binding reserve below $\min\{\bar{c}, v\}$, and (ii) the ability to *discriminate* among suppliers. While these abilities or powers do not vary with a merger or the environment, their optimal use does. For example, setting a binding reserve to supplier $i$ is not optimal if $v$ is so large that $\Gamma_i^{-1}(v) = \bar{c}$. Likewise, discrimination among suppliers (based on the virtual cost functions) is only optimal if the suppliers’ virtual cost functions differ. Thus, with symmetric suppliers the

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18We assume that buyer power itself is not affected by a merger among suppliers, which seems natural if buyer power derives from the size and/or sophistication of the buyer, as suggested by the *EC Guidelines* (para. 65), or from the ability to vertically integrate upstream or sponsor entry, as suggested by the *U.S. Guidelines* (p. 27). However, the *EC Guidelines* also raise the possibility that a merger could reduce buyer power “because a merger of two suppliers may reduce buyer power if it thereby removes a credible alternative” (*EC Guidelines*, para. 67). A nuance on the view that mergers decrease buyer power is provided by Loertscher and Marx (forth.), who observe that, with symmetric suppliers, a merger increases the buyer’s incentive to become powerful.

19An intuitive interpretation of the virtual cost function and an understanding of the role of its monotonicity can be developed using standard monopsony pricing (Bulow and Roberts, 1989). Consider a buyer with value $v \leq \bar{c}$ who faces a single supplier $i$ who draws his cost from the distribution $G_i$. The buyer’s pricing problem is $\max_p (v - p) G_i(p)$, the first-order condition for which is $0 = g_i(p)(v - \Gamma_i(p))$. If $\Gamma_i$ is increasing, the second-order condition is satisfied if the first-order condition is, that is, the problem is quasi-concave.

20In Loertscher and Marx (forth.), we called the ability to set a binding reserve price—that is, a reserve price that sometimes prevents efficient trade from occurring—*monopsony power* and the ability to discriminate among ex ante heterogeneous suppliers *bargaining power*. In light of the fact that bargaining power has other connotations and, in particular, is meaningful even in bilateral bargaining (see, e.g., Loertscher and Marx, 2018a), calling the latter *discrimination power* seems preferable.
buyer optimally chooses not to exert its power to discriminate; see Loertscher and Marx (forth.) for more detailed discussions and derivations.

The assumption that suppliers follow their non-weakly dominated strategies of reporting truthfully implies that if supplier $i$ has the lowest virtual cost, then supplier $i$ wins if $c_i \leq \Gamma_i^{-1}(v)$ and is paid $\min_{j\neq i}\{\Gamma_i^{-1}(v), \Gamma_i^{-1}(\Gamma_j(c_j))\}$. Otherwise, there is no trade.

In order to investigate the effects of changes in the “strength” and “weakness” of suppliers on incentives for coordination, it is useful for some results to consider a parameterized class of cost distributions. In the “capacity-based parameterization,” we assume that $v > \bar{c}$ and that for all $i \in N$, $G_i(x) = 1 - (1 - x)^{\alpha_i}$ on $[0, 1]$, where we refer to $\alpha_i > 0$ as supplier $i$’s capacity.21 It follows that the merged entity that combines suppliers $k$ and $\ell$ has capacity $\alpha_k + \alpha_\ell$. In this parameterization, a supplier with a larger capacity has a cost distribution that is first order stochastically dominated by the cost distribution of a supplier with a smaller capacity.

As shown by Loertscher and Marx (forth.), a merger decreases expected buyer surplus. Thus, even in the absence of coordinated effects, a merger among suppliers is harmful for the buyer—termed “unilateral effects” in the argot of antitrust. Here, we study the possibility and quantification of incremental harm related to coordinated effects.

### 2.2 Coordination

We focus on coordination that does not involve the communication of firms’ private information and does not involve transfers. Without such communication, the coordinating suppliers are not able to identify which of them is the lowest cost supplier, and, consequently, efficient coordination is not possible. Further, without transfers, for coordination to be individually rational, it must increase the individual expected surplus of each coordinating supplier rather than merely increase the joint expected surplus of the coordinating suppliers.22

To model coordination among a set $K \subseteq N$ of pre-merger suppliers or among a set $K \subseteq \hat{N}_{\ell,k}$ of post-merger suppliers, we assume the existence of a bidder selection scheme that randomly designates one supplier in $K$ to be the only supplier from $K$ to bid in the procurement.

When the subset $K$ of suppliers coordinate, the bidder selection scheme is defined by a vector $s = (s_i)_{i \in K}$ of selection probabilities, where for all $i \in K$, $s_i \in [0, 1]$ and $\sum_{i \in K} s_i = 1$. With probability $s_i$, supplier $i$ is selected to participate in the procurement.

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21 See Froeb et al. (2017) on the usefulness of this class of distributions in merger evaluation.

22 Baker (2002, p. 164) notes that: “coordinating firms may not be able to allocate the monopoly rents they achieve in a manner satisfactory to all the participants, because they may be unable to compensate each other directly.”
and the bids of suppliers in $K$ other than $i$ are suppressed. We assume that a coordinating supplier that is not designated by the bidder selection scheme has no ability to participate in the procurement, other than perhaps submitting a deliberately losing bid.\textsuperscript{23}

Given a set of suppliers $K$ and coordination probabilities $s$, which are common knowledge, the timeline is as follows:

**Stage 1: Coordination stage.** All suppliers in $K$ simultaneously and independently decide whether to coordinate by participating in the bidder selection scheme. If all suppliers in $K$ choose to coordinate, then the bidder selection scheme operates, designating one and only one of the coordinating suppliers, according to probability vector $s$, to participate in the procurement. If one or more supplier in $K$ chooses not to coordinate, then the bidder selection scheme does not operate.\textsuperscript{24}

**Stage 2: Procurement stage.** All suppliers’ costs are realized, and the procurement is held: if the bidder selection scheme does not operate, then all active suppliers participate in the procurement; otherwise, active suppliers not in $K$ plus the supplier in $K$ designated by the bidder selection scheme participate in the procurement.

Regardless of whether the bidder selection scheme operates, by the usual second-price auction logic, truthful bidding is a non-weakly dominated strategy for all suppliers participating in the procurement. In particular, the incentives of the noncoordinating suppliers are not affected by the presence of coordination by rivals, and the incentives of the designated coordinating bidder are not affected by the coordination. We focus on the equilibrium with truthful bidding in the procurement stage.

Suppliers contemplating coordination face tradeoffs. Coordinating suppliers may lose surplus in two ways. They may lose profitable sales to outside suppliers by suppressing winning bids, and they may make less profitable sales even when winning because the bidder selection scheme does not select the lowest-cost supplier as the designated bidder. However, the coordinating suppliers stand to gain higher payments by suppressing all but one of the coordinating suppliers’ bids. Thus, on net, coordination can be positive or negative for the coordinating suppliers.

\textsuperscript{23}If $v < \bar{c}$, deliberately losing bids (those with probability zero of being winning bids) include any bid in $[v, \bar{c}]$. If $v \geq \bar{c}$, then nondesignated coordinating suppliers must submit a bid of $\bar{c}$ or decline to bid in order to have zero probability of winning. In practice, there are examples where a bidding ring’s designated bidder ensures that another ring member does not submit a competitive bid by turning in the bid form for that other ring member (\textit{U.S. v. W.F. Brinkley & Son Construction Company, Inc.}, 783 F.2d 1157, 4th Cir. 1986).

\textsuperscript{24}We leave for future research the consideration of other possibilities for what might happen when one or more supplier in $K$ chooses not to coordinate. For example, in that event, some strict subset of the suppliers in $K$ might pursue coordination.
When a bidder decides whether to coordinate, the bidder contrasts its expected surplus from coordination with its expected surplus from no coordination, given truthful bidding in the procurement stage. We characterize when it is a weakly dominant strategy for all of the coordinating suppliers to coordinate.\textsuperscript{25}

Throughout the paper, we assume that the buyer’s procurement mechanism is the same with and without coordination.

### 2.3 Critical shares and the Coordinated effects index

Given events $x$ and $y$, it is useful to define $\hat{E}[x \mid y] \equiv E[c \mid y] \Pr_c(y)$. We let $c_X$ denote the vector of costs for suppliers in set $X$, i.e., $c_X \equiv \times_{i \in X} c_i$.

Given a market with a set of active suppliers $M$, where $M = N$ for the pre-merger market and $M = \tilde{N}_{k,\ell}$ for the post-merger market following the merger of suppliers $k$ and $\ell$, for all $i \in M$, let $\pi_i$ denote supplier $i$’s expected surplus in the absence of coordination. Thus, when the buyer is not powerful,

$$\pi_i(M) = \hat{E} \left[ \min \{r, c_{M\setminus\{i\}} \} - c_i \mid c_i < \min \{r, c_{M\setminus\{i\}} \} \right].$$

and when the buyer is powerful,

$$\pi_i(M) = \hat{E} \left[ \min_{j \in M\setminus\{i\}} \{\Gamma_i^{-1}(v), \Gamma_i^{-1}(\Gamma_j(c_j))\} - c_i \mid c_i < \min_{j \in M\setminus\{i\}} \{\Gamma_i^{-1}(v), \Gamma_i^{-1}(\Gamma_j(c_j))\} \right].$$

For $K \subseteq M$ and $i \in K$, we let $\hat{\pi}_i^K$ denote supplier $i$’s expected surplus when it is the bidder in $K$ designated by the bidder selection scheme to participate in the auction. When the buyer is not powerful,

$$\hat{\pi}_i^K(M) = \hat{E} \left[ \min \{r, c_{M\setminus K} \} - c_i \mid c_i < \min \{r, c_{M\setminus K} \} \right],$$

and when the buyer is powerful,

$$\hat{\pi}_i^K(M) = \hat{E} \left[ \min_{j \in M\setminus K} \{\Gamma_i^{-1}(v), \Gamma_i^{-1}(\Gamma_j(c_j))\} - c_i \mid c_i < \min_{j \in M\setminus K} \{\Gamma_i^{-1}(v), \Gamma_i^{-1}(\Gamma_j(c_j))\} \right].$$

Thus, given $M$ and a set of coordinating suppliers $K \subseteq M$, the expected payoff of supplier $i \in K$ is $s_i \hat{\pi}_i^K(M)$ if $i$ and all other suppliers in $K$ coordinate and if $i$ is the designated bidder with probability $s_i$, and $\pi_i(M)$ otherwise. It follows that choosing to coordinate

\textsuperscript{25}Given that suppliers do not lose anything if they indicate that they want to coordinate but then coordination fails, in stage 1 the set of Nash equilibria is equal to the set of dominant strategy equilibria.
weakly dominates not doing so for supplier $i$ if and only if

$$s_i \hat{\pi}_i^K(M) > \pi_i(M).$$

Letting

$$s_i^K(M) \equiv \frac{\pi_i(M)}{\hat{\pi}_i^K(M)},$$

choosing to coordinate is weakly dominant for supplier $i \in K$ if and only if

$$s_i > s_i^K(M).$$

We refer to $s_i^K(M)$ as the critical share for supplier $i \in K$ when subset $K$ of suppliers in market $M$ coordinate. Given a set $M$ of suppliers in the market and a subset $K \subseteq M$ who may coordinate, we refer to one minus the sum of the critical shares for suppliers in $K$ as the coordinated effects index (CEI) for suppliers in $K$, or $CEI_K$. That is,

$$CEI_K(M) \equiv 1 - \sum_{i \in K} s_i^K(M).$$

We drop the argument $M$ when the market at issue is clear. Because the critical shares are elements of $[0,1]$, the CEI is a number less than or equal to 1 and possibly negative.

We use the $CEI_K$ to measure the extent to which a market is at risk for coordination by suppliers in $K$, where a larger $CEI_K$ (closer to 100%) means that market is more at risk. Further, using the CEI, we can define what it means for a market to be at risk for coordination.

**Definition 1.** The market is “at risk for coordination by suppliers in $K$” if $CEI_K > 0$. The market is “not at risk for coordination by suppliers in $K$” if $CEI_K \leq 0$.

When a market is at risk for coordination by suppliers in $K$, by definition there exists a vector of coordination probabilities $\times_{i \in K} s_i^*$ such that for all $i \in K$, $s_i^* \in (s_i^K, 1)$ and $\sum_{i \in K} s_i^* = 1$. This implies that a bidder selection scheme based on coordination probabilities $(s_i^*)_{i \in K}$ is feasible and makes coordination a weakly dominant strategy for each supplier in $K$. However, if $CEI_K$ is less than or equal to zero, then $\sum_{i \in K} s_i^K \geq 1$ and there is no feasible vector of coordination probabilities such that coordination increases the expected payoff of all suppliers in $K$, relative to not coordinating. For any feasible vector of coordination probabilities, at least one supplier would have weakly greater expected surplus from no coordination than from coordination.\(^{26}\)

\(^{26}\)One could potentially incorporate a “cost of coordination” by requiring that the $CEI_K$ be greater than some positive threshold in order to view a market as at risk.
Let $L_X$ be the distribution of the minimum cost among suppliers in $X$, i.e., $L_X(c) = 1 - \Pi_{i \in X} (1 - G_i(c))$. For a given market with a set of active suppliers $M$ and without buyer power, $\pi_i(M)$ is given as

$$\pi_i(M) = \int_{\xi}^{r} (1 - L_{M \backslash \{i\}}(x)) G_i(x) dx,$$

which follows from the payoff (or revenue) equivalence theorem; see, e.g., Myerson (1981), Krishna (2002) or Börgers (2015). Similarly, given $K \subseteq M$ and $i \in K$, without buyer power, one can write $\hat{\pi}_i^K(M)$ as

$$\hat{\pi}_i^K(M) = \int_{\xi}^{r} (1 - L_{M \backslash K}(x)) G_i(x) dx.$$

Thus, in the absence of buyer power, given a market with active suppliers $M$ and $K \subseteq M$, the critical share for $i \in K$ is

$$s_i^K(M) = \frac{\pi_i(M)}{\hat{\pi}_i^K(M)} = \frac{\int_{\xi}^{r} (1 - L_{M \backslash \{i\}}(x)) G_i(x) dx}{\int_{\xi}^{r} (1 - L_{M \backslash K}(x)) G_i(x) dx}.$$

Similarly, with buyer power, letting $\hat{L}_X^i$ be the distribution of $\min_{j \in X} \Gamma_i^{-1}(\Gamma_j(c_j))$, then the expressions for $\pi_i$, $\hat{\pi}_i^K$, and $s_i^K$ are the same as above, but replacing $r$ with $\Gamma_i^{-1}(v)$ and $L_X$ with $\hat{L}_X^i$.

In the capacity-based parameterization without buyer power, letting $A_X \equiv \sum_{i \in X} \alpha_i$, we can write the critical share for supplier $i$ as a function of the suppliers’ capacities given $K \subseteq M$ and $i \in K$ as follows:

$$s_i^K(M) = \frac{(1 + \alpha_i + A_{M \backslash K})(1 + A_{M \backslash i})}{(1 + A_{M \backslash \{i\}})(1 + A_{M})}.$$

### 2.4 Effects on the buyer

It is straightforward to show that a buyer is harmed by coordination among suppliers. Letting the operator $2^{nd}$ select the second lowest element of a set, in the absence of buyer power, the change in the buyer’s expected payoff as a result of coordination by suppliers in $K \subseteq M$ using designation probabilities $s$ is

$$\begin{align*}
- \sum_{i \in K} s_i \hat{E}_c \left[ 2^{nd} \{ r, c_i, c_{M \backslash K} \} - 2^{nd} \{ r, c \} \mid \min \{ c_i, c_{M \backslash K} \} < r \right] \\
- \sum_{i \in K} s_i \hat{E}_c \left[ v - 2^{nd} \{ r, c \} \mid \min \{ c_{K \backslash \{i\}} \} < r < \min \{ c_i, c_{M \backslash K} \} \right],
\end{align*}$$

13
where the first summation corresponds to the buyer having to pay more in expectation as a result of the suppression of some bids, and the second summation corresponds to the buyer no longer receiving any bids below the reserve as a result of the suppression of some bids. Because this expression is negative, it follows that a buyer without power is harmed by coordination. Further, because coordination decreases the expected quantity traded when \( r < \bar{c} \), coordination decreases expected social surplus in that case. As we show in the proof of the following proposition, the result that coordination decreases expected buyer and social surplus continues to hold when the buyer is powerful. Thus, regardless of buyer power, the buyer and society are harmed by coordination.

**Proposition 1.** Regardless of buyer power, expected buyer surplus is reduced by coordination. Without buyer power, expected social surplus is reduced by coordination.

*Proof.* See the appendix.

Proposition 1 underscores the relevance of understanding when a merger increases the risk of coordination by transforming a market from one that is not at risk for coordination into one that is. Interestingly, with buyer power, coordination can increase social surplus for some type realizations. This occurs when, because of discrimination, the buyer purchases from a non-lowest-cost supplier when there is no coordination, but when there is coordination, the bid of that non-lowest-cost supplier is suppressed and the buyer instead purchases from the lowest-cost supplier.

### 3 Which markets are (not) at risk for coordination?

We now analyze which markets are at risk for coordination (at risk, for short).

#### 3.1 No merger

We begin with a characterization of when a market with symmetric suppliers is at risk for coordination. Under the assumption of symmetric suppliers, we let \( c_{(i:j)} \) denote the \( i \)-th lowest order statistic out of \( j \) draws from the common cost distribution. Also, for the case of symmetric suppliers, we drop the supplier subscripts on the virtual type function.

**Proposition 2.** Without buyer power, a market with \( m \) symmetric suppliers is at risk for coordination among \( k \in \{2,\ldots,m\} \) suppliers if and only if

\[
\frac{\mathbb{E} \left[ \min \{r, c_{(2:m-k+1)} \} - c_{(1:m-k+1)} \mid c_{(1:m-k+1)} < r \} \right]}{\mathbb{E} \left[ \min \{r, c_{(2:m)} \} - c_{(1:m)} \mid c_{(1:m)} < r \} \right]} > \frac{k}{m} \frac{m - k}{m} ,
\]

(3)
and with buyer power if and only if (3) holds with \( r \) replaced by \( \Gamma^{-1}(v) \).

Proof. See the appendix.

Proposition 2 implies that without buyer power and \( r = \overline{r} \) (or with buyer power and \( \Gamma^{-1}(v) = \overline{r} \)), a market with \( m \) symmetric suppliers is at risk for coordination among all \( m \) suppliers if and only if

\[ \overline{r} - E_c[c_i] > E_c[c_{(2:m)} - c_{(1:m)}]. \]  

(4)

Notice that with \( \Gamma^{-1}(v) = \overline{r} \) and symmetric suppliers, buyer power is effectively moot because it is optimal for the buyer to exert neither its monopsony nor its discrimination power. Inequality (4) says that the expected cost and the upper support of the cost distribution is greater than the expected distance between the first and second order statistics. Inequality (4) is satisfied, for example, for the uniform distribution on \([0, 1]\) and \( n = 2 \), but is not satisfied in other cases, such as when \( n = 2 \) and suppliers draw their types from a distribution with density \( g(x) = 0.05 \) for \( x \in [0, 0.9] \) and \( g(x) = 9.55 \) for \( x \in (.9, 1] \), which has a long left tail and high probability close to the upper support.

In this case, even with competition, the price paid to the winning supplier is likely to be close to the reserve, so the incremental expected payment under cooperation is small and outweighed by the incremental cost associated with possibly having the “wrong” supplier trade.

To explore this further, note that when there are two suppliers drawing their costs from the uniform distribution on \([0, 1]\), coordination increases the expected payment to the winner from \( 2/3 \) to \( 1 \), which is an increase of \( 1/3 \). Coordination also increases the expected cost of the winner, from \( 1/3 \) to \( 1/2 \), that is, by \( 1/6 \). Because the increase in expected payment exceeds the increase in expected cost, coordination is profitable.

However, if the suppliers draw their costs from the long left tail distribution described above, coordination only increases the expected payment to the winner from 0.965 to 1, that is, by 0.035, while it increases the expected cost of the winner from 0.891 to 0.928, which is an increase of 0.037. Thus, for the long left tail distribution, coordination increases the expected cost to the winner by more than it increases the expected payment to the winner, making coordination unprofitable.

This establishes the following result.

**Corollary 1.** Some, but not all, markets are at risk for coordination, including coordination by all suppliers in the market.

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27 In this case, \( \overline{r} - E_c[c_i] = 1/2 \) and \( E_c[c_{(2:m)} - c_{(1:m)}] = 1/3 \).

28 The corresponding cdf has \( G(x) = 0.05x \) for \( x \in [0, 0.9] \) and \( G(x) = 9.55x - 8.55 \) for \( x \in (0.9, 1] \).

29 In this case, \( 1 - E[c] = 0.0725 \) and \( E[c_{(2:n)} - c_{(1:n)}] = 0.0740 \).
Corollary 1 shows that our test for markets at risk is powerful in that some, but not all, markets are at risk. This is in contrast to notions of a market being at risk for perfect explicit collusion in which only the lowest cost supplier bids and transfers are made among the colluding suppliers. Coordination of the type we consider introduces an inefficiency because it is not necessarily the lowest cost agent who trades, and if the buyer’s reserve is binding, coordination can cause there to be no trade when there would be trade in the absence of coordination.

Under symmetry, we can also show that the risk for coordination eventually vanishes as the number of suppliers $m$ increases while the number $k$ of coordinators is kept fixed.

**Proposition 3.** With or without buyer power, assuming symmetric suppliers, and fixing $k \in \{2, 3, \ldots\}$, a market with a sufficiently large number $m$ of total suppliers is not at risk for coordination among $k$ suppliers, i.e., $\lim_{m \to \infty} CEI_{\{1, \ldots, k\}}(\{1, \ldots, m\}) \leq 0$.

*Proof.* See the appendix.

Thus, with symmetric suppliers, markets with a sufficiently large number of noncoordinating suppliers are not at risk for coordination. Relatedly, with symmetric suppliers, holding fixed the total number of suppliers, as the number of coordinating suppliers increases, outside competition diminishes, which increases incentives for coordination.

**Proposition 4.** With or without buyer power, assuming $m$ symmetric suppliers, a market is at risk for coordination among $k \in \{2, \ldots, m - 1\}$ suppliers only if it is also at risk for coordination among $j \geq k + 1$ suppliers.

*Proof.* See the appendix.

Proposition 4 provides a monotonicity result: Once a market with symmetric suppliers is at risk for coordination, it continues to be so when more of the suppliers participate in that coordination, keeping fixed the total number of suppliers.

### 3.2 Mergers

In order to consider the effects of a merger among suppliers in $K$ on incentives for coordination, we take as given a set $K$ of three or more suppliers. Recall from the discussion above that without buyer power $s^K_i$ depends only on the distribution of the minimum cost of suppliers other than $i$ and on the distribution of the minimum cost of suppliers outside $K$. Because a merger of two suppliers does not affect the distribution of the minimum cost of the two merging suppliers (in the absence of cost synergies, which we consider in Section 4), it follows that in the absence of buyer power, a merger of two suppliers in $K$
does not affect the critical shares of the nonmerging firms in $K$. With buyer power, a merger of suppliers in $K$ increases the critical shares of the nonmerging suppliers in $K$ because it increases their expected payoffs without coordination without affecting their expected payoffs if they are designated by the coordination mechanism. Thus, the change in $CEI_K$ as a result of a merger depends on how the critical share of the merged entity compares to the sum of the critical shares of two merging suppliers in the pre-merger market. We state this formally in the following lemma:

**Lemma 1.** Given $K \subseteq N$ and merging suppliers $k, \ell \in K$ and letting $\hat{K} \equiv K \setminus \{k, \ell\} \cup \mu_{k,\ell}$, 

$$CEI_K(\hat{N}_{k,\ell}) - CEI_K(N) \geq s^K_k(N) + s^K_\ell(N) - s^K_{\mu_{k,\ell}}(\hat{N}_{k,\ell}),$$

with equality in the absence of buyer power.

The following proposition follows immediately, where we say that a merger of suppliers $k$ and $\ell$ in $K$ “puts the market at greater risk” if $CEI_K(N)$ is greater than $CEI_{\hat{K}}(\hat{N}_{k,\ell})$ for $\hat{K} \equiv K \setminus \{k, \ell\} \cup \mu_{k,\ell}$.

**Proposition 5.** With (resp. without) buyer power, a merger of suppliers in $K$ puts a market at greater risk if (resp. if and only if) the critical share of the merged entity is less than the sum of the pre-merger critical shares of the merging suppliers.

In the absence of buyer power, a merger of suppliers $i$ and $j$ is always profitable for those suppliers, i.e., $\pi_i + \pi_j < \pi_{\mu_{i,j}}$ (Loertscher and Marx, forth., Proposition 6). At the same time, a merger increases the merging suppliers’ payoff from coordination by improving the efficiency of coordination. To see this, note that the merged entity has cost $\min \{c_i, c_j\}$, so the merger eliminates the possibility that a supplier with the higher cost $\max \{c_i, c_j\}$ is designated to be the sole bidder to represent the coordinating bidders, i.e., $\max \{\hat{s}^K_i, \hat{s}^K_j\} < \hat{s}^K_{\mu_{i,j}}$, where $\hat{K} \equiv K \setminus \{i, j\} \cup \{\mu_{i,j}\}$. Because either effect can dominate, the sum of the pre-merger critical shares of merging suppliers can be greater than or less than the critical share of the merged entity, implying that the CEI can increase or decrease. We illustrate the forces at work in Figure 1, which provides an example of a merger that decreases the CEI and one that increases the CEI.

In the example of Figure 1(a), the pre-merger market, which has $CEI_{\{1,2,3\}} = 0$, is not at risk for coordination by suppliers in the set $\{1,2,3\}$. In contrast, the post-merger market is at risk for coordination by supplier 1 and the merged entity $\mu_{2,3}$, with $CEI_{\{1,\mu_{2,3}\}} > 0$. Figure 1(b) shows a related example in which a merger causes the CEI to decrease.

In contrast to the case of a merger of suppliers in $K$, a merger of suppliers outside $K$ not affect the $CEI_K$ in the absence of buyer power. Without buyer power, the $CEI_K$
(a) Merger of weak suppliers

<table>
<thead>
<tr>
<th>Pre-merger market</th>
<th>Post-merger market</th>
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<tr>
<td>( i )</td>
<td>( s_i^{(1,2,3)} )</td>
</tr>
<tr>
<td>1 (strong)</td>
<td>3/7</td>
</tr>
<tr>
<td>2 (weak)</td>
<td>2/7</td>
</tr>
<tr>
<td>3 (weak)</td>
<td>2/7</td>
</tr>
<tr>
<td>4 (weak)</td>
<td></td>
</tr>
<tr>
<td>total</td>
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</tr>
<tr>
<td>CEI</td>
<td>0</td>
</tr>
</tbody>
</table>

(b) Merger of strong suppliers

<table>
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<th>Pre-merger market</th>
<th>Post-merger market</th>
</tr>
</thead>
<tbody>
<tr>
<td>( i )</td>
<td>( s_i^{(1,2,3)} )</td>
</tr>
<tr>
<td>1 (strong)</td>
<td>5/16</td>
</tr>
<tr>
<td>2 (strong)</td>
<td>5/16</td>
</tr>
<tr>
<td>3 (weak)</td>
<td>3/14</td>
</tr>
<tr>
<td>4 (weak)</td>
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</tr>
<tr>
<td>total</td>
<td>0.84</td>
</tr>
<tr>
<td>CEI</td>
<td>0.16</td>
</tr>
</tbody>
</table>

Figure 1: Effects of merger on the CEI. Each supplier \( i \) draws its cost from distribution \( G_i(c) = 1 - (1 - c)^{\alpha_i} \), where a higher value for \( \alpha_i \) indicates a “stronger” distribution (greater weight on lower costs). We label suppliers with \( \alpha_i = 1 \) as “weak,” with \( \alpha_i = 2 \) as “strong,” and with \( \alpha_i = 4 \) as “2x strong.” The pre-merger market has 5 suppliers, where the set \( K \) includes suppliers 1, 2, and 3. We consider the mergers indicated. Assumes no buyer power and \( r = \bar{c} \).

depends only on suppliers outside of \( K \) through the distribution of the minimum of their costs, which is not affected by a merger (in the absence of cost synergies). With buyer power, a merger of suppliers outside \( K \) results in increased discrimination by the buyer against the merged entity and so increases both \( \pi_i \) and \( \pi^K_i \) for each supplier \( i \in K \). As a result, the effect on the \( CEI_K \) is ambiguous.

**Proposition 6.** A merger of suppliers in \( K \) can, but need not, cause a market not at risk for coordination among suppliers in \( K \) to become at risk for coordination among the corresponding post-merger suppliers. In the absence of buyer power, a merger of suppliers not in \( K \) does not affect the risk for coordination among suppliers in \( K \).

### 3.3 Coordinating suppliers with larger and outside suppliers with smaller capacities increases risk

We can obtain more detailed results on what makes a market more or less at risk for coordination by considering a parameterization of type distributions in which suppliers are ranked according to their capacities, so we consider the capacity-based parameterization. In that setup, an increase in the total capacity of the outsiders decreases both the noncoordinated and coordinated payoffs of suppliers in \( K \). However, as we show in Proposition 7(i), the effect of increased outside capacity on coordinated payoffs outweighs its effect on competitive payoffs, and so the market becomes less at risk for coordination.

Intuitively, greater outside capacity reduces coordinated payoffs in two ways by (i) decreasing the benefit to the coordinating suppliers from suppressing their bids because
the outsiders’ bids are more likely to determine the price and (ii) by increasing the expected inefficiency associated with not selecting the lowest cost coordinating supplier as the designated bidder because that supplier is more likely to lose to one of the outsiders.

Further, Proposition 7(ii) shows that under certain conditions, the relation in 7(i) is monotonic—an increase in the capacity of a noncoordinating supplier increases the critical shares of all of the coordinating suppliers. Thus, Proposition 7(i)–(ii) extends Propositions 3 and 4 to a setup with asymmetric suppliers. Recall that for a market with a set $M$ of suppliers, $A_{M\setminus K}$ denotes the sum of the capacity parameters for suppliers not in $K$.

**Proposition 7.** In the capacity-based parameterization without buyer power, given a set $M$ of suppliers and $K \subset M$:

(i) a market with sufficiently large total outside capacity is not at risk, i.e., for $A_{M\setminus K}$ sufficiently large, $CEI_K \leq 0$;

(ii) the critical shares of suppliers in $K$ are increasing in the total capacity of the outside suppliers if $1 + A_{M\setminus K} > A_K$.

As an illustration of Proposition 7(i), when $m$ symmetric suppliers draw their costs from the uniform distribution on $[0, 1]$, $CEI_{\{1, 2\}}(\{1, \ldots, m\}) = (3 - m)/(m + 1)$, which is decreasing in $m$ and is less than or equal to 0 for $m \geq 3$.

Thus, increasing the total capacity of the outside suppliers decreases the CEI and so reduces the risk of coordination.

We can also consider for which set of suppliers the market is most at risk for coordination by those suppliers. Proposition 8(i) shows that, conditional on $k$ suppliers coordinating, the CEI is greatest if the coordinating suppliers are the $k$ highest capacity suppliers in the market. Thus, the market is more at risk for coordination among a group of the highest capacity suppliers in the market, relative to an equal number of other suppliers. Further, Proposition 8(iii) shows that ordering suppliers by their capacities and including suppliers in the set of coordinating suppliers in order starting with the highest capacity supplier, the critical share of the marginal (lowest capacity) member is lower when more suppliers are included in the set. In addition, a coordinating supplier’s critical share decreases when an additional coordinator is added. Thus, increasing the number of coordinating suppliers to include the next highest capacity supplier makes coordination easier “on the margin,” and makes each inframarginal critical share smaller, which is (also) good for coordination.

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$^{30}$In the case of costs drawn from the uniform distribution on $[0, 1]$, $\pi_i = \frac{1}{m(m+1)}$ and for $i \in \{1, \ldots, k\}$, $\hat{\pi}_i^{\{1, \ldots, k\}} = \frac{1}{m-k+1} \frac{1}{m-k+2}$, so $s_i^{\{1, \ldots, k\}} = (m - k + 1)(m - k + 2)/(m(1 + m))$ and $CEI_{\{1, \ldots, k\}}(\{1, \ldots, m\}) = 1 - ks_i^{\{1, \ldots, k\}}$. Given $k$, as $m$ increases, $CEI_{\{1, \ldots, k\}}(\{1, \ldots, m\})$ is eventually decreasing in $m$ and less than or equal to 0 for $m$ sufficiently large.
Proposition 8. In the capacity-based parameterization without buyer power, given a set of $m$ suppliers and $k \in \{2, \ldots, m\}$:

(i) the market is most at risk for coordination by $k$ suppliers if the $k$ coordinating suppliers are the $k$ highest capacity suppliers in the market;

(ii) the critical shares of any $k$ coordinating suppliers are increasing in their capacities, i.e., if $\alpha_1 \geq \ldots \geq \alpha_k$, then $s_{\{1,\ldots,k\}}^{1,\ldots,k} \geq \ldots \geq s_{\{1,\ldots,k\}}^{1,\ldots,k}$;

(iii) as successively lower capacity suppliers are added to the set of coordinating suppliers, the critical shares of the marginal and all inframarginal coordinating suppliers decrease, i.e., if $\alpha_1 > \ldots > \alpha_m$ and $k,k' \in \{2,\ldots,m\}$ with $k < k'$, then $s_{\{1,\ldots,k'\}}^{1,\ldots,k} < s_{\{1,\ldots,k\}}^{1,\ldots,k}$ and $s_{\{1,\ldots,k'\}}^{1,\ldots,k'} < s_{\{1,\ldots,k\}}^{1,\ldots,k}$.

Proof. See the appendix.

These results suggest that a competition authority should be most concerned about coordination among relatively high capacity suppliers.

In many settings, a competition authority evaluating a merger may have concerns about coordination among a specific subset of suppliers in the industry, perhaps because of a history of coordination or channels of communication available to the suppliers. But in other settings there may not be a specific subset of suppliers that is uniquely a concern for coordinated effects. In that case, one can consider calculating the CEI for all possible subsets of two or more suppliers. If the CEI for any of those subsets is greater than zero, then more attention can be targeted towards an evaluation of possible coordination by those suppliers.

Definition 2. A market with suppliers $M$ is universally not at risk for coordination if there is no subset $K \subseteq M$ of two or more suppliers such that $CEI_K > 0$.

For example, the market described at the beginning of Section 3 in which suppliers draw their types from a distribution with a long left tail is universally not at risk for coordination for any number of suppliers below some bound.\textsuperscript{32}

\textsuperscript{31}The U.S. Guidelines, p. 25 state: “The Agencies presume that market conditions are conducive to coordinated interaction if firms representing a substantial share in the relevant market appear to have previously engaged in express collusion affecting the relevant market, unless competitive conditions in the market have since changed significantly.” Priest (2003) discusses how patent cross-licensing agreements can facilitate anticompetitive exchange of information.

\textsuperscript{32}Numerical calculations show that the result holds at least for $n \in \{2,\ldots,10\}$. 

20
3.4 More symmetry among coordinators increases risk

Antitrust authorities have expressed the view that a merger that increases symmetry among potentially coordinating suppliers is more likely to raise concerns of coordinated effects. As we show, our framework provides some support for this view.

Let $K$ be a set of three or more suppliers, including suppliers 1 and 2, and suppose that suppliers 1 and 2 are symmetric with one another (although not necessarily with other suppliers) with cost distribution $G$. From this scenario, we construct a second scenario by keeping the distributions for suppliers other than 1 and 2 the same and by making supplier 1 stronger and supplier 2 weaker, while preserving the distribution of the minimum cost of suppliers 1 and 2. That is, letting $\hat{G}_1$ and $\hat{G}_2$ be the distributions of suppliers 1 and 2 in the second scenario, for all $x \in [c, \bar{c}]$,

$$\hat{G}_1(x) \geq G(x) \quad \text{and} \quad \hat{G}_2(x) \leq G(x), \quad (5)$$

with a strict inequality for a positive measure subset of $[c, \bar{c}]$, and for all $x \in [c, \bar{c}]$,

$$(1 - \hat{G}_1(x))(1 - \hat{G}_2(x)) = (1 - G(x))^2. \quad (6)$$

We refer to a shift in the distributions of two symmetric suppliers that satisfies (5) and (6) as a *competitively neutral spread*. It follows from (6) that a competitively neutral spread applied to two symmetric suppliers in $K$ does not affect the critical shares of any other suppliers in $K$. And, as we show in the proof of Proposition 9, it causes the sum of the critical shares of supplies 1 and 2 to increase, thereby decreasing the CEI.

**Proposition 9.** *Without buyer power, a competitively neutral spread applied to two symmetric suppliers in $K$ decreases the $CEI_K$.*

**Proof.** See the appendix.

An implication of Proposition 9 is that, given two otherwise identical markets that differ by a competitively neutral spread for two suppliers in $K$, the market in which the suppliers in $K$ are more symmetric with one another is at greater risk for coordination among those suppliers. This has implications for merger effects as we now describe.

Consider two markets $A$ and $B$ that differ only with respect to suppliers 1, 2, and 3. Suppose that suppliers 1 and 2 merge and that the merged entity and supplier 3 are part of the set $K$ of coordinating suppliers in the post-merger market. We can contrast two cases. In case one, the merged entity and supplier 3 are symmetric, both with cost distribution $G$, and in case two, the merged entity and supplier 3 have distributions that differ by a competitively neutral spread with respect to $G$. Referring to case one as a
“symmetry-inducing merger at $G$” and case two as a “merger that induces a competitively neutral spread with respect to $G$,” we have the following result:

**Corollary 2.** Without buyer power, a symmetry-inducing merger at $G$ results in a higher post-merger CEI than a merger that induces a competitively neutral spread with respect to $G$.

Corollary 2 formalizes and, in a certain sense, corroborates the popular view that a merger that increases symmetry among firms facilitates coordination. Note, however, that the corollary compares the effects of two different mergers on the CEI. It is silent about whether the CEI increases or decreases with the merger. In contrast, if the merged entity is not part of the coordinating set $K$, then we know from Proposition 6 that the CEI$_K$ is the same before and after the merger.

Intuition suggests that the merger of two relatively weak suppliers in $K$ will increase the symmetry of suppliers in $K$, whereas the merger of two relatively strong suppliers in $K$ will decrease the symmetry of those suppliers. We next conceptualize this within the capacity-based parameterization, by using capacities as a measure of strength and the range of the capacities of the suppliers in $K$ as a measure of the symmetry of suppliers in $K$. Then in the absence of buyer power, a merger of sufficiently low capacity suppliers increases symmetry and increases the CEI, whereas a merger of sufficiently high capacity suppliers decreases symmetry and decreases the CEI. Further, any merger that increases the CEI associated with all-inclusive coordination necessarily increases symmetry, assuming that the total capacity of the nonmerging suppliers is sufficiently large.

**Proposition 10.** In the capacity-based parameterization without buyer power, a merger of two suppliers in $K$:

(i) increases (resp. decreases) symmetry and the CEI$_K$ if the merging suppliers have sufficiently low (resp. high) capacities;

(ii) causes the CEI$_N$ to increase only if it increases symmetry, assuming that the total capacity of the nonmerging suppliers is sufficiently large.

**Proof.** See the appendix.

### 3.5 Buyer power reduces risk

There is a notion in the *U.S. Guidelines*, p. 27 that “the conduct or presence of large buyers” could undermine coordinated effects.\(^{33}\) To examine how coordinated effects are

\(^{33}\)The *U.S. Guidelines* (p. 27) also state: “In some cases, a large buyer may be able to strategically undermine coordinated conduct, at least as it pertains to that buyer’s needs, by choosing to put up for bid a few large contracts rather than many smaller ones, and by making its procurement decisions opaque to suppliers.”
affected by buyer power, we first abstract from mergers and focus on ex ante symmetric suppliers. In this case, the powerful buyer never uses its power to discriminate, so the sole effect of buyer power is to reduce the reserve price from $r$ to $\Gamma^{-1}(v)$. In what follows, this allows us to focus on the effects of a change in the reserve. Using (1), with symmetric suppliers $\{1, \ldots, m\} \equiv M$ and $\{1, \ldots, k\} \equiv K \subseteq M$, where $k \geq 2$, for $i \in K$, in the absence of buyer power

$$s^K_i(M) = \frac{\int_r^\infty (1 - L_{M\setminus\{i\}}(x)) G(x) dx}{\int_r^\infty (1 - L_{M\setminus K}(x)) G(x) dx},$$

(7)

and with buyer power the expression is the same but with $r$ replaced by $\Gamma^{-1}(v)$. If $v \geq \Gamma(\bar{\tau})$, then $r = \Gamma^{-1}(v) = \bar{\tau}$, and so the critical shares are not affected by buyer power. Focusing on the case with $r < \bar{\tau}$ and differentiating the expression in (7) with respect to $r$, we get an expression with sign equal to the sign of

$$(1 - L_{M\setminus\{i\}}(r)) \int_r^\infty (1 - L_{M\setminus K}(x)) G(x) dx - (1 - L_{M\setminus K}(r)) \int_r^\infty (1 - L_{M\setminus\{i\}}(x)) G(x) dx$$

$$= (1 - G(r))^{m-1} \int_r^\infty (1 - G(x))^{m-k} G(x) dx - (1 - G(r))^{m-k} \int_r^\infty (1 - G(x))^{m-1} G(x) dx$$

$$< 0,$$

where the final inequality is established in the proof of the proposition below. Thus, critical shares are weakly decreasing in buyer power, and strictly so for $v < \Gamma(\bar{\tau})$.

**Proposition 11.** Assuming symmetric suppliers, $CEI_K$ is increasing in the buyer’s reserve price, and thus decreasing with buyer power.

**Proof.** See the appendix.

Let us now turn to a merger. With buyer power, a merger of two suppliers in $K$ increases the noncoordinated payoff of the other suppliers in $K$—with buyer power, a merger shifts market share away from the merged entity. However, a merger does not affect the payoff of a nonmerging supplier under coordination. Conditional on being selected as the designated bidder, its payoff does not depend on the merger and, trivially, its payoff is always 0 if it is not selected as the designated bidder. Thus, a merger of two suppliers in $K$ increases the critical share of the other suppliers in $K$, giving the following result:

**Proposition 12.** With buyer power, a merger of two suppliers in $K$ increases the critical shares of the other suppliers in $K$. 

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Proposition 12 implies that with buyer power, a merger of two suppliers in \( K \) reduces the incentive for the other suppliers in \( K \) to participate in coordination.

Propositions 11 and 12 provide a foundation for the view that coordinated effects from a merger are less likely in the face of powerful buyers. In particular, buyer power increases the critical designation probabilities for coordinated suppliers, which is to say that in the face of buyer power, coordinating suppliers are more constrained in their ability to establish a bidder selection scheme that is profitable for all of the coordinating suppliers.

4 Extensions

In this section, we consider a number of extensions.

4.1 Merger-related cost synergies

In order to consider cost synergies, consider the capacity-based parameterization. If suppliers \( k \) and \( \ell \) merge, then without cost synergies the cost distribution of the merged entity \( \mu_{k,\ell} \) is \( G_{\mu_{k,\ell}}(c) = 1 - (1 - c)^{\alpha_k + \alpha_\ell} \). We model cost synergies as an increase in the value of the capacity parameter for the merged entity. Specifically, for \( s \geq 1 \), let the merged entity’s cost distribution be

\[
G_{\mu_{k,\ell}}(c) \equiv 1 - (1 - c)^{s(\alpha_k + \alpha_\ell)},
\]

where \( s = 1 \) corresponds to no cost synergies (that is, \( G_{\mu_{k,\ell};1}(c) = G_{\mu_{k,\ell}}(c) \)) and \( s > 1 \) corresponds to merger-related cost synergies. It then follows from Proposition 8(ii) that cost synergies increase the critical share of the merged entity for any group of coordinating suppliers that it might be a part of. But cost synergies for the merged entity decrease the noncoordinated payoffs for other suppliers, without affecting their coordinated payoffs, and so decrease the critical shares of those suppliers. As we show, the decrease in critical shares for the nonmerging suppliers dominates for a range of cost synergies. Specifically, when the post-merger set of coordinating suppliers \( \hat{K} \) includes the merged entity and another supplier with capacity larger than or equal to that of the individual merging suppliers, then cost synergies increase the \( CEI_{\hat{K}} \) as long as they are not too large.

**Proposition 13.** In the capacity-based parameterization without buyer power, if \( \hat{K} \) consists of the merged entity and one other supplier with capacity greater than or equal to the average of the merging suppliers’ pre-merger capacities, then cost synergies increase the \( CEI_{\hat{K}} \) as long as they are not too large.
Proof. See the appendix.

Proposition 13 shows that, for capacity-based parameterization, coordinated effects between two post-merger suppliers are more of a concern when the merger generates (moderate) cost synergies than when it does not. To our knowledge, this relation between cost synergies and coordinated effects is novel. It raises the interesting possibility that two merging firms might choose not to fully implement available cost synergies in order to preserve the possibility of post-merger coordination.

4.2 Mergers that eliminate a maverick

As discussed in the introduction, the term “maverick” fares prominently in current antitrust economics and practice, but lacks a clear-cut and operational definition. If, as has been argued, regulators can reliably identify an industry maverick that prevents or limits coordination, but are unable to define what a maverick is, then it is difficult if not impossible for merging parties to provide evidence and arguments that could invalidate the regulators’ concerns. The vagueness surrounding the term maverick also begs the question whether being a maverick is a hard-wired property of a firm that is, essentially, written in its DNA, or whether it is an endogenous characteristic that is subject to change with changes in circumstances. If it is the latter, but treated as the former, then maverick-based analysis and decisions would be subject to what has become known as the Lucas critique (Lucas, 1976) and, as such, of debatable value for predictive purposes.

An operational definition of a maverick requires that a maverick firm be identifiable based only on pre-merger observables. In what follows, we provide such a definition and then analyze mergers involving a maverick in our setup.

Definition 3. Given a pre-merger market with set of suppliers $N$ and subset $K \subset N$ containing two or more suppliers, supplier $m \in N \setminus K$ is a maverick with respect to $K$ if $CEI_K(N) \leq 0$ and $CEI_K(N \setminus \{m\}) > 0$.

According to this definition, a maverick with respect to a set of suppliers $K$ is a supplier whose presence prevents a pre-merger market from being at risk for coordination among suppliers in $K$, i.e., the market is not at risk for coordination by suppliers in $K$ when the maverick is in the market, but is at risk when the maverick is not in the market.$^{34}$

This definition allows the possibility that more than one supplier in a market could

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$^{34}$For example, when the market contains three firms drawing their cost types from the uniform distribution on $[0, 1]$, then supplier 3 is a maverick with respect to coordination by suppliers 1 and 2, i.e., $CEI_{\{1,2\}}(\{1,2,3\}) \leq 0$ and $CEI_{\{1,2\}}(\{1,2\}) > 0$. 

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be a maverick with respect to a particular set $K$ of suppliers. However, because merger approval is typically required for each pairwise merger, we focus on transactions that involve the acquisition of only one maverick. If two mavericks with respect to $K$ are themselves the merging parties, then Proposition 6 implies that in the absence of buyer power, the merger does not affect the $CEI_K$, so the post-merger market is not at risk for coordination. With buyer power, a merger of two mavericks increases the payoffs of suppliers in $K$ both with coordination and without, with an ambiguous effect on the critical shares and hence on the $CEI_K$.

Following the merger between a supplier in $K$ and a maverick, the critical share of a nonmerging supplier $i \in K$ decreases relative to the case in which the maverick is not present, i.e., relative to $s^K_i(N\{m\})$, because their coordinated expected payoff remains the same, but their noncoordinated payoff decreases. For the merged entity, its noncoordinated and coordinated expected payoffs increase, and in the capacity-based parameterization, the critical share of the merged entity increases relative to the critical share of the acquiring supplier in the market with suppliers $N\{m\}$. As we show in the following proposition, under certain conditions, the decrease in the critical shares of the nonmerging suppliers dominates, so that an acquisition of a maverick by a supplier in $K$ puts the market at risk for coordination, but under other conditions, it does not.

**Proposition 14.** In the capacity-based parameterization without buyer power, if the market contains a maverick with respect to $K = \{1, 2\}$, then the acquisition of the maverick by the lower capacity supplier in $K$ puts the market at risk for coordination, but the acquisition of the maverick by the higher capacity suppliers in $K$ does not put the market at risk for coordination if the capacity of the non-acquiring supplier in $K$ is sufficiently small.

**Proof.** See the appendix.

As Proposition 14 shows, under certain circumstances, the acquisition of the maverick by the lower capacity of two suppliers in $K$ puts the post-merger market at risk for coordination. This is consistent with the view that a merger that eliminates a maverick raises concerns of coordinated effects. However, Proposition 14 also shows that the acquisition of a maverick by the higher capacity of two suppliers in $K$ does not put the market at risk when the capacity of the other supplier in $K$ is sufficiently small. This is inconsistent with the view that any merger that eliminates a maverick raises coordinated effects concerns.

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35Indeed, in the U.S. mobile communications market, at one point both T-Mobile and MetroPCS were considered “‘mavericks’ with a history of disrupting the industry” (“Sprint CEO Sees ‘Enormous’ Synergies in T-Mobile Merger,” Kansas City Star, June 12, 2017). T-Mobile was permitted to acquire MetroPCS.
Whether the elimination of a maverick puts a market at risk for coordination can depend on which supplier acquires the maverick.

Turning our attention to some other aspects of the perceived wisdom regarding mavericks, we find that they are not supported in our framework. In particular, one can easily construct examples in which the acquisition of a supplier that is not a maverick leaves a market at risk for coordination. Thus, it is not the case that the elimination of a maverick is necessary for coordinated effects. It is also not the case, as some have suggested (Baker, 2002, p. 180) that the presence of a maverick prevents coordinated effects from mergers not involving the maverick.

### 4.3 Coordination-induced mergers

In the absence of buyer power, a merger is always profitable for the merging suppliers. That profitability is potentially enhanced by post-merger coordination if the merger results in a market that is at risk for coordination.

With buyer power, a merger is not necessarily profitable for the merging suppliers because after the merger, a powerful buyer more aggressively asserts its monopsony power and discrimination power against the merged entity. Thus, in an environment with buyer power, the prospect of post-merger coordination can make the difference between a merger being profitable and not. In that sense, the possibility of post-merger coordination can make a merger profitable that would not otherwise be so. Thus, coordinated effects can induce a merger.

More formally, the merger of suppliers $k$ and $\ell$ is not profitable in the absence of coordinated effects if $\pi_k(N) + \pi_\ell(N) \geq \pi_{\mu_k,\ell}(\hat{N}_{k,\ell})$, but it is profitable in the presence of post-merger coordinated effects if there exists a set of post-merger suppliers $\hat{K}$ including the merged entity such that $C E I_{\hat{K}}(\hat{N}_{k,\ell}) > 0$ and $\pi_k(N) + \pi_\ell(N) < \hat{\pi}_{\mu_k,\ell}(\hat{N}_{k,\ell})$. In this case, the presence of post-merger coordinated effects induces a merger.

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36 For example, Figure 1(a) shows that the merger of suppliers 2 and 3 puts the post-merger market at risk for coordination, but supplier 3 is not a maverick with respect to coordination by suppliers 1 and 2. (The pre-merger market is not at risk for coordination by suppliers 1 and 2 and also not at risk for coordination by suppliers 1 and 2 if supplier 3 is eliminated from the market.)

37 For example, when the market contains five firms drawing their cost types from the uniform distribution on $[0,1]$, then supplier 5 is a maverick with respect to coordination by suppliers in $\{1,2,3\}$, i.e., $C E I_{\{1,2,3\}}(\{1,2,3,4,5\}) \leq 0$ and $C E I_{\{1,2,3\}}(\{1,2,3,4\}) > 0$, and following the merger of suppliers 1 and 2, the market is at risk for coordination by the merged entity and supplier 3, i.e., $C E I_{\{\mu_1,2,3\}}(\{\mu_1,2,3,4,5\}) > 0$. 

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5 Conclusion

We provide a framework for analyzing mergers in procurement-based markets that permits a quantification of coordinated effects. Our model of coordination is distinctly not one of explicit collusion because we assume that coordinating suppliers do not make transfers and do not share their private cost information. Instead, coordination is modelled as a bidder selection scheme among coordinating suppliers, where only one of the coordinating suppliers is designated as the bidder in the procurement. This approach allows the possibility of identifying markets that are not at risk for coordination prior to a merger but that are at risk for coordination after a merger and for quantifying the extent of that risk.

Our analysis suggests that coordinated effects of a merger are more of a concern when: (i) the coordinating suppliers are stronger and the noncoordinating suppliers are weaker and/or smaller in number; (ii) the merger increases the symmetry of the coordinating suppliers; (iii) the merger combines suppliers that are weak relative to other coordinating suppliers; (iv) buyers are not powerful; and (v) the merger generates moderate cost synergies. We provide a definition of a maverick firm and analyze the effects of mergers that eliminate a maverick or that only involve nonmavericks. We find some support for the view that a merger that eliminates a maverick puts a market at risk for coordination, but only in some settings, such as when the acquiring supplier has low capacity relative to the supplier with which it would coordinate.

Much remains for future research. For example, in a dynamic setup, the type of bidder selection scheme with critical shares that we analyze may form a basis for the division of surplus in subsequent explicit collusion, which may suitably be analyzed along the lines of Cramton et al. (1987).

Likewise, the question remains to what extent the ideas put forth in this paper extend to standard oligopoly models. Our procurement setting, and for that matter, any auction of a single unit that endows agents with dominant strategies, has the convenient feature that a bidding ring does not change the optimal strategies of the ring’s non-collusive rivals, regardless of the ring’s collusive arrangement. This allows us to focus on the individual rationality constraints of the insiders without departing from equilibrium for outsiders, and to define a bidder selection scheme, which is a suboptimal coordination device because suppliers’ costs are random variables. We can then use this bidder selection scheme to derive the critical shares that serve as the inputs into our coordinated effects index and test.

In an oligopoly setting, say a Cournot model, keeping fixed a suboptimal coordination scheme, one could analogously derive critical shares, which may be interpreted as market shares. The challenges are that one needs to take a stand on what kind of coordination
scheme is suboptimal and whether the outsiders best-respond to coordination—if they do not, then they will find out ex post that they were “duped,” and if they do, then the merger paradox may be back quickly. For example, fixing market shares may not do the trick because, with known and identical marginal costs, it may be equivalent to optimal collusion. While addressing these two questions—what is suboptimal coordination and how do outsiders react—is best left for future research, our analysis and approach suggest that progress along these lines would be valuable for further improving our understanding of coordinated effects and related concepts.
A Appendix: Proofs

Proof of Proposition 1. The proof that expected buyer surplus is reduced by coordination in the absence of buyer power follows from the text. For a powerful buyer, the change in the buyer’s expected payoff as a result of coordination by suppliers in $K$ using designation probabilities $s$ is

$$
- \sum_{i \in K} s_i E_c \left[ \Gamma_i(c_i) \cdot 1_{\Gamma_i(c_i) < \min_j \{ \Gamma_j(c_j), v \}} - \Gamma_i(c_i) \cdot 1_{\Gamma_i(c_i) < \min_j \{ \Gamma_j(c_j), v \}} \right]
- \sum_{i \in K} s_i E_c \left[ (v - \min_{j \in K \setminus \{i\}} \{ \Gamma_j(c_j) \}) \cdot 1_{\min_j \{ \Gamma_j(c_j) \} < v < \min_j \{ \Gamma_j(c_j) \}} \right].
$$

Because this expression is negative, a powerful buyer is harmed by coordination. Turning to the effect of coordination on social surplus, without buyer power, trade is efficient without coordination but not with coordination because with positive probability the cost of the supplier whose bid is suppressed is the lowest. ■

Proof of Proposition 2. Under symmetry and no buyer power, for all $i \in K = \{1, \ldots, k\}$, the definition of $\hat{\pi}_i^K$ implies that

$$
\hat{\pi}_i^K = \hat{E} \left[ \min\{r, c_{M \setminus K}\} - c_i \mid c_i < \min\{r, c_{M \setminus K}\} \right]
= \frac{1}{m - k + 1} \hat{E} \left[ \min\{r, c_{(2:m-k+1)}\} - c_{(1:m-k+1)} \mid c_{(1:m-k+1)} < r \right]
$$

The definition of $\pi_i$ implies that

$$
\pi_i = \frac{1}{m} \hat{E} \left[ \min\{r, c_{(2:m)}\} - c_{(1:m)} \mid c_{(1:m)} < r \right].
$$

Using symmetry, the market is at risk for coordination among $k$ suppliers if and only if coordination based on symmetric coordination probabilities increases the expected surplus for all suppliers, i.e., for all $i \in K$, $\frac{k}{k} \hat{\pi}_i^K > \pi_i$, which holds if and only if

$$
\frac{1}{m - k + 1} \hat{E} \left[ \min\{r, c_{(2:m-k+1)}\} - c_{(1:m-k+1)} \mid c_{(1:m-k+1)} < r \right] > \frac{k}{m} \hat{E} \left[ \min\{r, c_{(2:m)}\} - c_{(1:m)} \mid c_{(1:m)} < r \right],
$$

completing the proof for the case without buyer power.

With buyer power, for all $i \in K$,

$$
\hat{\pi}_i^K = \frac{1}{m - k + 1} \hat{E} \left[ \min\{\Gamma^{-1}(v), c_{(2:m-k+1)}\} - c_{(1:m-k+1)} \mid c_{(1:m-k+1)} < \Gamma^{-1}(v) \right]
$$

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and
\[ \pi_i = \frac{1}{n} \hat{E} \left[ \min \{ \Gamma^{-1}(v), c_{(2:n)} \} - c_{(1:n)} \mid c_{(1:m)} < \Gamma^{-1}(v) \} \right], \]
so \( \frac{1}{k} \hat{\pi}_i^K > \pi_i \), which holds if and only if
\[
\frac{1}{m - k + 1} \hat{E} \left[ \min \{ \Gamma^{-1}(v), c_{(2:m-k+1)} \} - c_{(1:m-k+1)} \mid c_{(1:m-k+1)} < \Gamma^{-1}(v) \} \right]
> \frac{k}{m} \hat{E} \left[ \min \{ \Gamma^{-1}(v), c_{(2:m)} \} - c_{(1:m)} \mid c_{(1:m)} < \Gamma^{-1}(v) \} \right],
\]
which completes the proof for the case with buyer power. ■

**Proof of Proposition 3.** In the limit as \( m \) grows large, the conditioning events in (3) are irrelevant because with probability 1 both the lowest and second-lowest order statistics are less than the reserve \( r \) for the case without buyer power or \( \Gamma^{-1}(v) \) for the case with buyer power. Thus, it is sufficient to show that
\[
\lim_{m \to \infty} \left( E \left[ c_{(2:m-k+1)} - c_{(1:m-k+1)} \right] - \frac{k}{m} (m - k + 1) E \left[ c_{(2:m)} - c_{(1:m)} \right] \right) \leq 0. \quad (8)
\]
Using Loertscher and Marx (2018b, Lemma 2), we can write the left side of (8) as
\[
E \left[ \frac{G(c_{(1:m-k+1)})}{g(c_{(1:m-k+1)})} \right] - \frac{k}{m} (m - k + 1) E \left[ \frac{G(c_{(1:m)})}{g(c_{(1:m)})} \right],
\]
where \( E \left[ \frac{G(c_{(1:j)})}{g(c_{(1:j)})} \right] = \int_{\xi}^\pi jG(x)(1 - G(x))^{j-1}dx \). Thus, we can write the left side of (8) as
\[
\int_{\xi}^\pi (m - k + 1)G(x)(1 - G(x))^{m-k}dx - \frac{k(m - k + 1)}{m} \int_{\xi}^\pi mG(x)(1 - G(x))^{m-1}dx
= (m - k + 1) \int_{\xi}^\pi G(x)(1 - G(x))^{m-k} \left( 1 - k(1 - G(x))^{k-1} \right)dx,
\]
which has the sign of
\[
\int_{\xi}^\pi G(x)(1 - G(x))^{m-k} \left( 1 - k(1 - G(x))^{k-1} \right)dx.
\]
Letting \(c^* \equiv G^{-1}(1 - 1/k)\),

\[
\lim_{m \to \infty} \int_{c^*}^{ \pi} G(x)(1 - G(x))^{m-k} \left(1 - k(1 - G(x))^{k-1}\right) dx
\]

\[
= \lim_{m \to \infty} \int_{c^*}^{ \pi} G(x)(1 - G(x))^{m-k} \left(1 - k(1 - G(x))^{k-1}\right) dx
\]

\[
+ \lim_{m \to \infty} \int_{c^*}^{ \pi} G(x)(1 - G(x))^{m-k} \left(1 - k(1 - G(x))^{k-1}\right) dx
\]

\[
= \lim_{m \to \infty} \int_{c^*}^{ \pi} G(x)(1 - G(x))^{m-k} \left(1 - k(1 - G(x))^{k-1}\right) dx
\]

\[
\leq 0,
\]

which completes the proof. ■

**Proof of Proposition 4.** Without buyer power, if \(k \geq 1\) out of \(m\) symmetric suppliers coordinate, then for all \(i \in K \equiv \{1, ..., k\},

\[
\hat{\pi}_i^K = \int_r^c (1 - G(c))^{m-k} G(c) dc
\]

and

\[
\pi_i = \frac{1}{m} E[\min\{c_{(2:m)}, r\} - c_{(1:m)} \mid c_{(1:m)} < r] \Pr \left(c_{(1:m)} < r \right).
\]

Thus, \(s_i^K < \frac{1}{k}\) if and only if

\[
\int_r^c (1 - G(c))^{m-k} G(c) dc > \frac{k}{m} E[\min\{c_{(2:m)}, r\} - c_{(1:m)} \mid c_{(1:m)} < r] \Pr \left(c_{(1:m)} < r \right).
\] (9)

At \(k = 1\), both sides in (9) are the same. To see this, note that by Lemma 2 of Loertscher and Marx (2018b), \(E[c_{(2:m)} - c_{(1:m)}] = E \left[G(c_{(1:m)})/g(c_{(1:m)})\right]\), so

\[
\frac{1}{m} E[c_{(2:m)} - c_{(1:m)} \mid c_{(1:m)} < r] \Pr \left(c_{(1:m)} < r \right)
\]

\[
= \frac{1}{m} E \left[G(c_{(1:m)})/g(c_{(1:m)}) \mid c_{(1:m)} < r\right] \Pr \left(c_{(1:m)} < r \right)
\]

\[
= \frac{1}{m} \int_r^c \frac{G(x)}{g(x)} m(1 - G(x))^{m-1} g(x) dx
\]

\[
= \int_r^c (1 - G(x))^{m-1} G(x) dx.
\]

The right side of (9) increases linearly in \(k\). The sign of the derivative of the left side with
respect to $k$ is the same as the sign of

$$-\ln(1 - G(c))(1 - G(c))^{m-k} > 0,$$

and the sign of the second derivative of the left side is the same as the sign of

$$[\ln(1 - G(c))]^2(1 - G(c))^{m-k} > 0.$$

Thus, the left side of (9) is an increasing and convex in $k$. It follows then that if (9) holds for $k \in \{1, ..., m - 1\}$, then it holds for any $k \in \{k + 1, ..., m\}$.

With buyer power, the same argument applies, replacing $r$ with $\Gamma^{-1}(v)$. ■

Proof of Proposition 7. Part (i). Given the assumed parameterization of costs

$$s^K_i = \frac{\pi_i}{\pi^K} = \frac{(1 + \alpha_i + A_{M\setminus K})(1 + A_{M\setminus K})}{(1 + A_{M\setminus K} + A_K - \alpha_i)(1 + A_{M\setminus K} + A_K)}.$$

The limit of $s^K_i$ as $A_{M\setminus K}$ goes to infinity is

$$\lim_{A_{M\setminus K} \to \infty} \frac{2 + \alpha_i + 2A_{M\setminus K}}{2 + 2A_{M\setminus K} + 2A_K - \alpha_i} = 1.$$

Part (ii). Differentiating $s^K_i$ with respect to $A_{M/K}$, we get a numerator of

$$A_K(2(1 + A_{M\setminus K})(1 + A_K + A_{M\setminus K}) - \alpha_i(2 + A_K + 2A_{M\setminus K}) + \alpha_i^2),$$

which has sign equal to the sign of

$$2(1 + A_{M\setminus K})(1 + A_K + A_{M\setminus K}) - \alpha_i(2 + A_K + 2A_{M\setminus K}) + \alpha_i^2,$$

which is convex in $\alpha_i$ and has minimum at $\alpha_i = 1 + A_K/2 + A_{M\setminus K}$. Thus, it is positive for all $\alpha_i$ if it is positive at $\alpha_i = 1 + A_K/2 + A_{M\setminus K}$. So, it is positive for all $\alpha_i$ if

$$2(1 + A_{M\setminus K})(1 + A_K + A_{M\setminus K}) > (1 + A_K/2 + A_{M\setminus K})^2.$$

It is sufficient that $1 + A_{M\setminus K} > A_K$. ■

Proof of Proposition 8. Part (i). Let $K \equiv \{1, ..., k\}$ and $M \equiv \{1, ..., m\}$. Then

$$\sum_{i \in K} s^K_i = \sum_{i \in K} \frac{(1 + \alpha_i + A_{M\setminus K})(1 + A_{M\setminus K})}{(1 + A - \alpha_i)(1 + A)}.$$
Thus, the CEI decreases when a member of a lower capacity. It follows that the CEI is maximized when its members are the set \( \alpha \). Part (ii) of suppliers with the largest capacities.

Differentiating with respect to \( \varepsilon > 0 \) to the capacity \( \alpha_j \) for some \( j \in M \setminus K \) and deduct \( \varepsilon \) from the capacity \( \alpha_1 \). Then the CEI is

\[
\sum_{i \in (K \setminus \{1\})} \frac{(1 + \alpha_i + A_{M \setminus K} + \varepsilon)(1 + A_{M \setminus K} + \varepsilon)}{(1 + A - \alpha_i)(1 + A)} + \frac{(1 + \alpha_1 + A_{M \setminus K})(1 + A_{M \setminus K} + \varepsilon)}{(1 + A - \alpha_1 + \varepsilon)(1 + A)}.
\]

Suppose we add \( \varepsilon > 0 \) to the capacity \( \alpha_j \) for some \( j \in M \setminus K \) and deduct \( \varepsilon \) from the capacity \( \alpha_1 \). Then the CEI is

\[
\sum_{i \in (K \setminus \{1\})} \frac{(1 + A_{M \setminus K} + \varepsilon)(1 + A_{M \setminus K} + \varepsilon)}{(1 + A - \alpha_i)(1 + A)} + \frac{(1 + \alpha_1 + A_{M \setminus K})(1 + A_{M \setminus K} + \varepsilon)}{(1 + A - \alpha_1 + \varepsilon)(1 + A)}.
\]

Differentiating with respect to \( \varepsilon \), we have

\[
\sum_{i \in (K \setminus \{1\})} \frac{(1 + A_{M \setminus K} + \varepsilon)(1 + A_{M \setminus K} + \varepsilon)}{(1 + A - \alpha_i)(1 + A)} + \frac{(1 + \alpha_1 + A_{M \setminus K})(1 + A_{M \setminus K} + \varepsilon)}{(1 + A - \alpha_1 + \varepsilon)(1 + A)} > 0.
\]

Thus, the CEI decreases when a member of \( K \) is replaced with a member of \( M \setminus K \) with a lower capacity. It follows that the CEI is maximized when its members are the set \( K \) of suppliers with the largest capacities.

\textit{Part (ii).} In the capacity-based parameterization,

\[
\hat{\pi}_i^K = \int_{\varepsilon}^{r} (1 - x)^{\sum_{j \in M \setminus K} \alpha_j} (1 - (1 - x)^{\alpha_i}) dx,
\]

\[
= \frac{1 - (1 - r)^{1+\sum_{j \in M \setminus K} \alpha_j} - 1 + \alpha_i + \sum_{j \in M \setminus K} \alpha_j}{1 + \sum_{j \in M \setminus K} \alpha_j} + \frac{1 - (1 - r)^{1+\alpha_i + \sum_{j \in M \setminus K} \alpha_j}}{1 + \alpha_i + \sum_{j \in M \setminus K} \alpha_j},
\]

and

\[
\pi_i = \int_{\varepsilon}^{r} (1 - x)^{\sum_{j \in M \setminus \{i\}} \alpha_j} (1 - (1 - x)^{\alpha_i}) dx.
\]

\[
= \frac{1 - (1 - r)^{1+\alpha_i + \sum_{j \in M \setminus \{i\}} \alpha_j}}{1 + \sum_{j \in M \setminus \{i\}} \alpha_j} + \frac{1 - (1 - r)^{1+\sum_{j \in M \setminus \{i\}} \alpha_j}}{1 + \alpha_i + \sum_{j \in M \setminus \{i\}} \alpha_j},
\]

\[
= \frac{1 - (1 - r)^{1+A-\alpha_i}}{1 + A - \alpha_i} + \frac{1 - (1 - r)^{1+A}}{1 + A}.
\]

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If \( r = \bar{r} \), then \( \hat{s}_i^K = \frac{\alpha_i}{(1 + \alpha_i + A_{M\setminus K})(1 + A_{M\setminus K})} \) and \( \pi_i = \frac{\alpha_i}{(1 + A - \alpha_i)(1 + A)} \), so

\[
s_i^K = \frac{\pi_i}{\hat{s}_i^K} = \frac{(1 + \alpha_i + A_{M\setminus K})(1 + A_{M\setminus K})}{(1 + A - \alpha_i)(1 + A)}.
\]

It follows that for \( i \in \{1, ..., k - 1\} \),

\[
s^K_i \frac{s^K_i}{s^K_{i+1}} = \frac{(1 + \alpha_i + A_{M\setminus K})(1 + A - \alpha_{i+1})}{(1 + \alpha_{i+1} + A_{M\setminus K})(1 + A - \alpha_i)} > 1,
\]

implying that \( s^K_1 \geq ... \geq s^K_k \).

**Part (iii).** Suppose that \( \alpha_1 > ... > \alpha_m \). We show that \( s^{\{1,...,i+1\}}_i < s^{\{1,...,i\}}_i \). To see this, note that

\[
s^{\{1,...,i\}}_i = \frac{(1 + \alpha_i + \alpha_{i+1} + A_{M\setminus\{1,...,i+1\}})(1 + A_{i+1} + A_{M\setminus\{1,...,i+1\}})}{(1 + A_i - \alpha_i)(1 + A_i)}
\]

and

\[
s^{\{1,...,i+1\}}_{i+1} = \frac{(1 + \alpha_{i+1} + A_{M\setminus\{1,...,i+1\}})(1 + A_{M\setminus\{1,...,i+1\}})}{(1 + A_i - \alpha_{i+1})(1 + A_i)}.
\]

Thus, \( s^{\{1,...,i\}}_i - s^{\{1,...,i+1\}}_{i+1} \) has sign equal to the sign of

\[
\frac{1 + \alpha_i + \alpha_{i+1} + A_{M\setminus\{1,...,i+1\}}}{1 + A_i - \alpha_i} - \frac{1 + A_{M\setminus\{1,...,i+1\}}}{1 + A_i - \alpha_{i+1}} > 0.
\]

\[\blacksquare\]

**Proof of Proposition 9.** For suppliers 1 and 2, we have critical shares in the first scenario of (dropping the argument \( x \) to conserve of notation and integrating from \( c \) to \( r \))

\[
s_1 = s_2 = \frac{\int (1 - L_{M\setminus\{1,2\}})(1 - G)G \, dx}{\int (1 - L_{M\setminus K})G \, dx} \quad (10)
\]

and critical shares in the second scenario of

\[
\hat{s}_1 = \frac{\hat{s}_2}{\int (1 - L_{M\setminus\{1,2\}})(1 - G_2)G_1 \, dx} \quad \text{and} \quad \hat{s}_2 = \frac{\int (1 - L_{M\setminus\{1,2\}})(1 - G_1)G_2 \, dx}{\int (1 - L_{M\setminus K})G_2 \, dx} \quad (11)
\]

The competitively neutral spread causes the CEI to decrease if and only if \( \hat{s}_1 + \hat{s}_2 - s_1 - s_2 > \)

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0, which holds as we now show.

Using the definitions of \( s_1, s_2, \hat{s}_1, \) and \( \hat{s}_2 \) in (10) and (11) we have

\[
\hat{s}_1 + \hat{s}_2 - s_1 - s_2 = \frac{\int (1 - L_{M\backslash\{1,2\}})(1 - \hat{G}_2)\hat{G}_1 dx}{\int (1 - L_{M\backslash\{1,2\}})G_1 dx} + \frac{\int (1 - L_{M\backslash\{1,2\}})(1 - \hat{G}_1)\hat{G}_2 dx}{\int (1 - L_{M\backslash\{1,2\}})G_2 dx} - 2\frac{\int (1 - L_{M\backslash\{1,2\}})(1 - G)G dx}{\int (1 - L_{M\backslash\{1,2\}})G dx}
\]

\[
> \frac{\int (1 - L_{M\backslash\{1,2\}})(1 - \hat{G}_2)\hat{G}_1 dx}{\int (1 - L_{M\backslash\{1,2\}})G_1 dx} + \frac{\int (1 - L_{M\backslash\{1,2\}})(1 - \hat{G}_1)\hat{G}_2 dx}{\int (1 - L_{M\backslash\{1,2\}})G_2 dx} - 2\frac{\int (1 - L_{M\backslash\{1,2\}})(1 - G)G dx}{\int (1 - L_{M\backslash\{1,2\}})G dx}
\]

\[
= \frac{\int (1 - L_{M\backslash\{1,2\}})(1 - \hat{G}_2)\hat{G}_1 dx}{\int (1 - L_{M\backslash\{1,2\}})G_1 dx} + \frac{\int (1 - L_{M\backslash\{1,2\}})(G^2 - \hat{G}_1)\hat{G}_1 dx}{\int (1 - L_{M\backslash\{1,2\}})G dx}
\]

\[
> \frac{\int (1 - L_{M\backslash\{1,2\}})(1 - \hat{G}_2)\hat{G}_1 dx}{\int (1 - L_{M\backslash\{1,2\}})G_1 dx} + \frac{\int (1 - L_{M\backslash\{1,2\}})(G^2 - \hat{G}_1)\hat{G}_1 dx}{\int (1 - L_{M\backslash\{1,2\}})G_1 dx}
\]

\[
= \frac{\int (1 - L_{M\backslash\{1,2\}})\frac{(G - \hat{G}_1)^2}{1 - \hat{G}_1} dx}{\int (1 - L_{M\backslash\{1,2\}})G_1 dx}
\]

\[
> 0,
\]

where the first inequality uses \( G \geq \hat{G}_2 \) (with a strict inequality for a positive measure subset of \([c, \bar{c}]\)), the second equality collects terms and uses the implication of (6) that \((1 - \hat{G}_1)\hat{G}_2 - 2(1 - G)G = G^2 - \hat{G}_1\), the second inequality uses \( \hat{G}_1 \geq G \) (with a strict inequality for a positive measure subset of \([c, \bar{c}]\)), the third equality collects terms and uses the implication of (6) that \((1 - \hat{G}_2)\hat{G}_1 + G^2 - \hat{G}_1 = \frac{(G - \hat{G}_1)^2}{1 - \hat{G}_1}\) (and also equal to \(\frac{(G - \hat{G}_2)^2}{1 - \hat{G}_1}\)), and the final inequality rearranges. ■

**Proof of Proposition 10.** To prove the first part of the proposition, it is straightforward to show that in the capacity-based parameterization without buyer power, for \( \alpha_1 \) and \( \alpha_2 \) sufficiently large, \( s_{1+2}^K > s_1^K + s_2^K \) and for \( \alpha_1 \) and \( \alpha_2 \) sufficiently close to zero, \( s_{1+2}^K < s_1^K + s_2^K \).

Turning to the second part of the proposition, given Proposition 5, the CEI increases with the merger of suppliers 1 and 2 (assumed in \( K \)) if and only if \( s_1^K + s_2^K > s_{1+2}^K \), which
we can write as
\[
\frac{1 + \alpha_1 + A_{N\setminus K}}{1 + A_N - \alpha_1} + \frac{1 + \alpha_2 + A_{N\setminus K}}{1 + A_N - \alpha_2} > \frac{1 + \alpha_1 + \alpha_2 + A_{N\setminus K}}{1 + A_N - \alpha_1 - \alpha_2}.
\]
Assume that \(\alpha_1 = \alpha_2 = \alpha\). Then the condition is
\[
2 \frac{1 + \alpha + A_{N\setminus K}}{1 + A_{N\setminus K} + A_{K\setminus\{1,2\}} + \alpha} > \frac{1 + 2\alpha + A_{N\setminus K}}{1 + A_{N\setminus K} + A_{K\setminus\{1,2\}}},
\]
which we can rewrite as
\[
(1 + A_{N\setminus K}) (1 + A_{N\setminus K} + A_{K\setminus\{1,2\}}) > \alpha (1 + 2\alpha + A_{N\setminus K}).
\]
This is satisfied at \(\alpha = 0\), and the right side is increasing without bound in \(\alpha\). Thus, given \(K\), there exists a unique positive \(\alpha^*_K\) such that it is satisfied if and only if \(\alpha < \alpha^*_K\). Solving for \(\alpha^*_K\), we have:
\[
\alpha^*_K \equiv \frac{1}{4} \left(1 + A_{N\setminus K}\right) \left(\sqrt{9 + 8 \frac{A_{K\setminus\{1,2\}}}{1 + A_{N\setminus K}}} - 1\right),
\]
which is increasing in \(A_{K\setminus\{1,2\}}\) and in \(A_{N\setminus K}\).

It follows that in the capacity-based parameterization, a merger of two symmetric suppliers, labelled 1 and 2, in \(K\) increases the CEI if and only if each merging supplier has capacity \(\alpha < \alpha^*_K\). Further, if \(N = K\) and \(A_{K\setminus\{1,2\}} > 2\), then \(\alpha^*_K < A_{K\setminus\{1,2\}}\). To see this, note that if \(N = K\) and \(A_{K\setminus\{1,2\}} > 2\), then
\[
\alpha^*_K = \frac{1}{4} \left(\sqrt{9 + 8A_{K\setminus\{1,2\}}} - 1\right)
< \frac{1}{4} \left(3\sqrt{1 + A_{K\setminus\{1,2\}}} - 1\right)
< \frac{1}{4} \left(3(1 + A_{K\setminus\{1,2\}}) - 1\right)
= \frac{1}{2} + 3/4A_{K\setminus\{1,2\}}
< A_{K\setminus\{1,2\}},
\]
where the final inequality uses \(A_{K\setminus\{1,2\}} > 2\).

Thus, if \(N = K\), we have \(\alpha^*_N = 1/4 \left(\sqrt{9 + 8A_{N\setminus\{1,2\}}} - 1\right)\). Further, if \(A_{N\setminus\{1,2\}} = 2\), then \(\alpha^*_N = 1\) and that if \(A_{N\setminus\{1,2\}} > 2\), then \(2\alpha^*_N < A_{N\setminus\{1,2\}}\). If, in addition, \(|N| = |K| = 3\) and \(\alpha_3 > 2\), then \(2\alpha^*_N < \alpha_3\), which implies that whenever a merger of suppliers 1 and 2 increases the CEI (i.e., whenever \(\alpha < \alpha^*_N < \alpha_3/2\)), that merger increases the symmetry of the suppliers because the merger increases the capacity of the merged entity to \(2\alpha\), where \(\alpha < 2\alpha < \alpha_3\).
More generally, if \( \alpha_k = \min_{i \in K \setminus \{1, 2\}} \alpha_i \) and \( \alpha_k > 1/2(1 + \sqrt{9 + 8A_{K \setminus \{1, 2, k\}}} \), then \( 2\alpha_k < \alpha_k \) and again whenever the merger of suppliers 1 and 2 increases the CEI, it also increases the symmetry of the suppliers. ■

**Proof of Proposition 11.** To complete the proof, it is sufficient to show that the following expression is negative for \( r \in (c, \bar{c}) \):

\[
(1 - G(r))^{m-1} \int_{\xi}^{r} (1 - G(x))^{m-k} G(x)dx - (1 - G(r))^{m-k} \int_{\xi}^{\bar{r}} (1 - G(x))^{m-1} G(x)dx. \tag{12}
\]

If \( m = k \), we can write (12) as

\[
(1 - G(r))^{m-1} \int_{\xi}^{r} G(x)dx - \int_{\xi}^{\bar{r}} (1 - G(x))^{m-1} G(x)dx,
\]

which is zero at \( r = c \) and has first derivative with respect to \( r \) of

\[
-(m - 1)(1 - G(r))^{m-2} g(r) \int_{\xi}^{r} G(x)dx < 0.
\]

Thus, if \( m = k \), then (12) is negative for all \( r \in (c, \bar{c}] \), which completes the proof.

If \( m - k \geq 1 \), which implies that \( m \geq 3 \), then (12) is equal to zero at \( r = c, r = \bar{c} \), and at a unique interior point \( r^* \in (c, \bar{c}) \) defined by

\[
(1 - G(r^*))^{k-} = \frac{m - k}{m - 1} \frac{\int_{\xi}^{r^*} (1 - G(x))^{m-1} G(x)dx}{\int_{\xi}^{r^*} (1 - G(x))^{m-k} G(x)dx}.
\]

Thus, it is sufficient to show that (12) is decreasing in \( r \) at \( r = c \) or that (12) is increasing in \( r \) at \( r = \bar{c} \).

Differentiating (12) with respect to \( r \), we get

\[
-(m - 1)(1 - G(r))^{m-2} g(r) \int_{\xi}^{r} (1 - G(x))^{m-k} G(x)dx
\]

\[
+(m - k)(1 - G(r))^{m-k-1} g(r) \int_{\xi}^{r} (1 - G(x))^{m-1} G(x)dx.
\]

If \( m - k = 1 \), this is

\[
-(m - 1)(1 - G(r))^{m-2} g(r) \int_{\xi}^{r} (1 - G(x)) G(x)dx + g(r) \int_{\xi}^{r} (1 - G(x))^{m-1} G(x)dx,
\]

which is positive at \( r = \bar{c} \), completing the proof for the case of \( m - k = 1 \).
If \( m - k \geq 2 \), then (13) is zero at both \( r = x \) and \( r = \bar{c} \), so we need to differentiate again. Taking the second derivative of (12) with respect to \( r \), we get

\[
(m - 1)(m - 2)(1 - G(r))^{m-3} (g(r))^2 \int_x^r (1 - G(x))^{m-k} G(x) dx
\]

\[
-(m - 1)(1 - G(r))^{m-2}g'(r) \int_x^r (1 - G(x))^{m-k} G(x) dx
\]

\[
-(m - 1)(1 - G(r))^{m-2}g(r) (1 - G(r))^{m-k} G(r)
\]

\[
-(m - k)(m - k - 1) (1 - G(r))^{m-k-2} g(r) \int_x^r (1 - G(x))^{m-1}G(x) dx
\]

\[
+(m - k) (1 - G(r))^{m-k-1} g'(r) \int_x^r (1 - G(x))^{m-1}G(x) dx
\]

\[
+(m - k) (1 - G(r))^{m-k-1} g(r)(1 - G(r))^{m-1}G(r).
\]

This is zero at \( r = c \) and \( r = \bar{c} \). Taking the third derivative with respect to \( r \), we get the following two terms that do not involve the integral, which is zero at \( r = c \) and that do not involve \( G(r) \), which is also zero at \( r = c \):

\[
(m - k) (1 - G(r))^{m-k-1} g(r)(1 - G(r))^{m-1}g(r)
\]

\[
-(m - 1)(1 - G(r))^{m-2}g(r) (1 - G(r))^{m-k} g(r)
\]

\[
= -(k - 1)(1 - G(r))^{2m-k-2}g^2(r),
\]

which is negative at \( r = c \) if \( g(c) > 0 \). If \( g(c) = 0 \), then our assumptions guarantee that there exists a \( j \)-th derivative of \( g \) such that \( g^{(0)}(c) = ... = g^{(j-1)}(c) \) and \( g^{(j)}(c) > 0 \). Then continuing to differentiate (12), the “first time” the derivative is nonzero at \( r = c \), it is negative, completing the proof. □

**Proof of Proposition 13.** Let \( \mu \) be the index of the merged entity. In the capacity-based parameterization with \( \alpha_\mu \) equal to the sum of the merging suppliers’ pre-merger capacities and \( \varepsilon \equiv s\alpha_\mu - \alpha_\mu \), we have

\[
\sum_{i \in \hat{K}} s_i^\hat{K} = \frac{(1 + \alpha_\mu + \varepsilon + A_{M\setminus\hat{K}})(1 + A_{M\setminus\hat{K}})}{(1 + A_{M\setminus\set{\mu}})(1 + A_M + \varepsilon)} + \sum_{i \in K \setminus \set{\mu}} \frac{(1 + \alpha_i + A_{M\setminus\set{i}})(1 + A_{M\setminus\set{i}})}{(1 + A_{M\setminus\set{i}} + \varepsilon)(1 + A_M + \varepsilon)},
\]

where first term increases in \( \varepsilon \) and each term in the sum decreases in \( \varepsilon \). Taking the
derivative with respect to \( \varepsilon \), we have

\[
\frac{(1 + A_{M \setminus \hat{K}})}{(1 + A_{M} + \varepsilon)^2} \left[ \frac{A_{\hat{K} \setminus \{\mu\}}}{1 + A_{M \setminus \{\mu\}}} - \sum_{i \in \hat{K} \setminus \{\mu\}} \frac{(1 + \alpha + A_{M \setminus \{i\}})(2 + A_{M} + 2\varepsilon + A_{M \setminus \{i\}})}{(1 + A_{M \setminus \{i\}} + \varepsilon)^2} \right]
\]

\[
\leq \frac{(1 + A_{M \setminus \hat{K}})}{(1 + A_{M} + \varepsilon)^2} \left[ \frac{A_{\hat{K} \setminus \{\mu\}}}{1 + A_{M \setminus \{\mu\}}} - 2 \sum_{i \in \hat{K} \setminus \{\mu\}} \frac{(1 + \alpha + A_{M \setminus \{i\}})(1 + \varepsilon + A_{M \setminus \{i\}})}{(1 + A_{M \setminus \{i\}} + \varepsilon)^2} \right]
\]

\[
= \frac{(1 + A_{M \setminus \hat{K}})}{(1 + A_{M} + \varepsilon)^2} \left[ \frac{A_{\hat{K} \setminus \{\mu\}}}{1 + A_{M \setminus \{\mu\}}} - 2 \sum_{i \in \hat{K} \setminus \{\mu\}} \frac{1 + \alpha + A_{M \setminus \{i\}}}{1 + A_{M \setminus \{i\}} + \varepsilon} \right].
\]

If there is just one other supplier, denoted by index \( j \), in \( \hat{K} \) besides the merged entity, then we can rewrite the above expression as

\[
\frac{(1 + A_{M \setminus \hat{K}})}{(1 + A_{M} + \varepsilon)^2} \left[ \frac{\alpha_j}{1 + A_{M \setminus \hat{K}} + \alpha_j} - 2 \frac{1 + \alpha_j + A_{M \setminus \hat{K}}}{1 + A_{M \setminus \hat{K}} + \alpha_{\mu} + \varepsilon} \right],
\]

which has sign equal to the sign of

\[
\alpha_j \left( 1 + A_{M \setminus \hat{K}} + \alpha_{\mu} + \varepsilon \right) - 2 \left( 1 + \alpha_j + A_{M \setminus \hat{K}} \right)^2,
\]

which is negative if and only if

\[
\alpha_{\mu} < \frac{1}{\alpha_j} \left[ 2 \left( 1 + A_{M \setminus \hat{K}} + \alpha_j \right)^2 - \alpha_j \left( 1 + A_{M \setminus \hat{K}} + \varepsilon \right) \right],
\]

which holds if \( \varepsilon = 0 \) and \( \alpha_j \geq \alpha_{\mu}/2 \). To see this, let \( \kappa \) be such that \( \alpha_j = \alpha_{\mu}/\kappa \). Then the right side of (14) can be written as

\[
\frac{\kappa}{\alpha_{\mu}} \left[ 2 \left( 1 + A_{M \setminus \hat{K}} + \alpha_{\mu}/\kappa \right)^2 - \alpha_{\mu}/\kappa \left( 1 + A_{M \setminus \hat{K}} \right) \right]
\]

\[
= \frac{\kappa}{\alpha_{\mu}} \left[ 2(1 + A_{M \setminus \hat{K}})^2 + 2(1 + A_{M \setminus \hat{K}})\alpha_{\mu}/\kappa + 2\alpha_{\mu}^2/\kappa^2 - \alpha_{\mu}/\kappa \left( 1 + A_{M \setminus \hat{K}} \right) \right]
\]

\[
= \frac{\kappa}{\alpha_{\mu}} \left[ 2(1 + A_{M \setminus \hat{K}})^2 + (1 + A_{M \setminus \hat{K}})\alpha_{\mu}/\kappa + 2\alpha_{\mu}^2/\kappa^2 \right]
\]

\[
= \alpha_{\mu} \frac{2}{\kappa} + \frac{\kappa}{\alpha_{\mu}} 2(1 + A_{M \setminus \hat{K}})^2 + (1 + A_{M \setminus \hat{K}}),
\]

which is greater than \( \alpha_{\mu} \) for \( \kappa \leq 2 \). ■
**Proof of Proposition 14.** As we show above, in the capacity-based parameterization,

\[ s_i^K(N) = \frac{(1 + \alpha_i + A_{N\setminus K})(1 + A_{N\setminus K})}{(1 + A_{N\setminus\{i\}})(1 + A_N)}. \]

Let \( N = \{1, \ldots, n\} \) with \( n \in \{3, 4, \ldots\} \) and \( K = \{1, 2\} \), and assume that supplier \( n \) is a maverick with respect to \( K \). In what follows, to conserve on notation, we use \( A \) in place of \( A_N \). It will also be useful to define \( X = \alpha_{k+1} + \ldots + \alpha_{n+1} \). Then by the definition of \( n \) being a maverick,

\[
\sum_{i \in K} \frac{(1 + \alpha_i + X)(1 + X)}{(1 + \alpha_1 + \alpha_2 - \alpha_i + X)(1 + \alpha_1 + \alpha_2 + X)} < 1. \tag{15}
\]

Following the merger of suppliers 1 and \( n \), the sum of the critical shares for suppliers in \( \hat{K} \equiv \{\mu_{1,n}, 2\} \) is

\[ CEI_{\hat{K}} = \frac{(1 + \alpha_1 + X + \alpha_n)(1 + X)}{(1 + \alpha_2 + X)(1 + \alpha_1 + \alpha_2 + X + \alpha_n)} + \frac{(1 + \alpha_2 + X)(1 + X)}{(1 + \alpha_1 + X + \alpha_n)(1 + \alpha_1 + \alpha_2 + X + \alpha_n)}. \]

It follows that

\[
\lim_{\alpha_2 \to 0} CEI_{\hat{K}} = 1 + \frac{(1 + X)^2}{(1 + \alpha_1 + X + \alpha_n)^2} > 1,
\]

which proves the second part of the proposition.

Using (15),

\[
CEI_{\hat{K}} < \frac{(1 + \alpha_1 + X + \alpha_n)(1 + X)}{(1 + \alpha_2 + X)(1 + \alpha_1 + \alpha_2 + X + \alpha_n)} + \frac{(1 + \alpha_2 + X)(1 + X)}{(1 + \alpha_1 + X + \alpha_n)(1 + \alpha_1 + \alpha_2 + X + \alpha_n)} - \frac{(1 + \alpha_1 + X)(1 + \alpha_1 + \alpha_2 + X)}{(1 + \alpha_1 + X)(1 + \alpha_1 + \alpha_2 + X)} + 1
\]

\[ \equiv (1 + X)f(\alpha_1, \alpha_2, X) + 1. \]

Thus, \( CEI_{\hat{K}} < 1 \) if \( f(\alpha_1, \alpha_2, X) < 0 \). Collecting the terms in \( f(\alpha_1, \alpha_2, X) \) over the common denominator of

\[
(1 + \alpha_1 + X)(1 + \alpha_2 + X)(1 + \alpha_1 + \alpha_2 + X + \alpha_n)(1 + \alpha_1 + X + \alpha_n)(1 + \alpha_1 + \alpha_2 + X),
\]

it follows that \( CEI_{\hat{K}} < 1 \) if the associated numerator, denoted by \( \hat{f}(\alpha_1, \alpha_2, X) \), is negative,
implies that for all $i$, i.e., if $\hat{f}(\alpha_1, \alpha_2, X) < 0$, where

$$\hat{f}(\alpha_1, \alpha_2, X) = (1 + \alpha_1 + X)(1 + \alpha_1 + X + \alpha_n)(1 + \alpha_1 + \alpha_2 + X)$$

$$-(1 + \alpha_1 + X + \alpha_n)(1 + \alpha_1 + \alpha_2 + X + \alpha_n)(1 + \alpha_1 + X)^2$$

$$+(1 + \alpha_2 + X)^2(1 + \alpha_1 + X)(1 + \alpha_1 + \alpha_2 + X)$$

$$-(1 + \alpha_1 + \alpha_2 + X + \alpha_n)(1 + \alpha_2 + X)^2(1 + \alpha_1 + X + \alpha_n).$$

Rearranging the first two terms and the second two terms, we have

$$\hat{f}(\alpha_1, \alpha_2, X) = (1 + \alpha_1 + X + \alpha_n)(1 + \alpha_1 + X)\alpha_2\alpha_n$$

$$+\alpha_n(1 + \alpha_2 + X)^2[(\alpha_2 + \alpha_n) - 2(1 + \alpha_1 + \alpha_2 + X + \alpha_n)]$$

$$\equiv \alpha_n\hat{f}(\alpha_1, \alpha_2, X).$$

So, $CEI_K < 1$ if $\hat{f}(\alpha_1, \alpha_2, X) < 0$. Rewriting $\hat{f}(\alpha_1, \alpha_2, X)$, we have

$$\tilde{f}(\alpha_1, \alpha_2, X) = (1 + X + \alpha_1)^2\alpha_2 + (1 + X + \alpha_2)^2(\alpha_2 + \alpha_n) + \alpha_n\alpha_2(1 + X + \alpha_1)$$

$$-2(1 + X + \alpha_2)^2(1 + X + \alpha_1 + \alpha_2 + \alpha_n).$$

Differentiating with respect to $X$, we have

$$\frac{\partial \hat{f}(\alpha_1, \alpha_2, X)}{\partial X} = 2(1 + X + \alpha_1)\alpha_2 + 2(1 + X + \alpha_2)(\alpha_2 + \alpha_n) + \alpha_n\alpha_2$$

$$-4(1 + X + \alpha_2)(1 + X + \alpha_1 + \alpha_2 + \alpha_n) - 2(1 + X + \alpha_2)^2,$$

and, evaluating at $X = 0$,

$$\frac{\partial \tilde{f}(\alpha_1, \alpha_2, 0)}{\partial X} = 2(1 + \alpha_1)\alpha_2 + 2(1 + \alpha_2)(\alpha_2 + \alpha_n) + \alpha_n\alpha_2 - 4(1 + \alpha_2)(1 + \alpha_1 + \alpha_2 + \alpha_n) - 2(1 + \alpha_2)^2$$

$$= -2(1 + \alpha_1)\alpha_2 - \alpha_2(2\alpha_2 + \alpha_n) - 2(\alpha_2 + \alpha_n) - 4(1 + \alpha_1) - 2(1 + \alpha_2)^2$$

$$< 0.$$

Further, $\frac{\partial^2 \tilde{f}(\alpha_1, \alpha_2, 0)}{\partial X^2} < 0$, so $\tilde{f}(\alpha_1, \alpha_2, X)$ is concave in $X$ and decreasing at $X = 0$, which implies that for all $X \geq 0$, $\tilde{f}(\alpha_1, \alpha_2, X) \leq \tilde{f}(\alpha_1, \alpha_2, 0)$. Thus, $CEI_K < 1$ if $\tilde{f}(\alpha_1, \alpha_2, 0) <$
where
\[ \tilde{f}(\alpha_1, \alpha_2, 0) \]
\[ = (1 + \alpha_1)2\alpha_2 + (1 + \alpha_2)^2(\alpha_2 + \alpha_n) + \alpha_n\alpha_2(1 + \alpha_1) - 2(1 + \alpha_2)^2(1 + \alpha_1 + \alpha_2 + \alpha_n) \]
\[ = (1 + \alpha_1)^2\alpha_2 + \alpha_n\alpha_2(1 + \alpha_1) - (1 + \alpha_2)^2[(\alpha_2 + \alpha_n) + 2(1 + \alpha_1)]. \]

It is straightforward to show that \( \tilde{f}(\alpha_1, \alpha_1, 0) < 0 \). Differentiating with respect to \( \alpha_2 \), we get
\[ \frac{\partial \tilde{f}(\alpha_1, \alpha_2, 0)}{\partial \alpha_2} = (1 + \alpha_1)^2 + \alpha_n(1 + \alpha_1) - 2(1 + \alpha_2)[(\alpha_2 + \alpha_n) + 2(1 + \alpha_1)] - (1 + \alpha_2)^2, \]
which at \( \alpha_2 = \alpha_1 \) is
\[ \frac{\partial \tilde{f}(\alpha_1, \alpha_1, 0)}{\partial \alpha_2} = -2(1 + \alpha_1)\alpha_1 - (1 + \alpha_1)\alpha_n - 4(1 + \alpha_1)^2 < 0. \]

The second derivative with respect to \( \alpha_2 \) is
\[ \frac{\partial^2 \tilde{f}(\alpha_1, \alpha_2, 0)}{\partial \alpha_2^2} = -2[(\alpha_2 + \alpha_n) + 2(1 + \alpha_1)] - 2(1 + \alpha_2) - 2(1 + \alpha_2) < 0. \]

Thus, \( \tilde{f}(\alpha_1, \alpha_2, 0) \) is concave in \( \alpha_2 \) and negative and decreasing in \( \alpha_2 \) at \( \alpha_2 = \alpha_1 \), which implies that \( \tilde{f}(\alpha_1, \alpha_2, 0) < 0 \) for all \( \alpha_2 \geq \alpha_1 \), i.e., as long as the acquiring supplier has the weakly smaller capacity, which completes the proposition. ■
References


