Price Competition between Platforms: Equilibrium Coexistence on Competing Online Auction Sites revisited

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May 15, 2013

Abstract

We investigate the equilibrium market structure on competing online auction sites such as those of eBay or Yahoo!. Building on the model of Ellison, Fudenberg, & Möbius (2004) we take full account of the complexity of network effects on such platforms. We extend the model by looking at the implication of exogenous and endogenous buyer and seller charges making use of contingent tariffs. This extension brings in line the theory with the empirical findings of Brown & Morgan (2006). Eventually we investigate welfare effects, look at the viability of duopoly with size differentials, and the implications for large markets and policy.

1 Introduction

Virtual market platforms such as auctions often reveal very different price strategies despite the fact that such intermediaries offer homogenous products. Competition between eBay and Yahoo! auctions are a case in point with Yahoo! having substantially lower fees and commissions than eBay both in the US and in Japan. Despite these similarities markets were eventually dominated by eBay in the US and by Yahoo! in Japan (see Yin (2004)). One explanation for this observation is the presence of network externalities.

Intermediation between heterogenous agents such as bargaining buyers and sellers generates direct, congestion externalities (from agents of their own type) and indirect network externalities (from agents of the same type). This complex interaction of network externalities often remains unmodelled an exception being the work of Ellison, Fudenberg, and Möbius (EFM, 2004). The analysis in EFM shows that stable equilibria in such duopoly markets exist and that they may be asymmetric. The consequences of asymmetry for optimal platform pricing strategies are however not pursued.

The economic literature on *two-sided markets* in particular on platform competition has been blossoming recently. Auctions being special kinds of platforms with a clear and well understood bargaining structure are thus amenable to this analysis. Based on the pioneering work of Caillaud and Jullien (2003) and Armstrong (2006) one focus has been to try to deal with the inherent *multiplicity of equilibria.* These may arise for example from coordination failures, congestion externalities as in EFM, the possibility of multi-homing, i.e. joining multiple platforms, and/or even the simplest competitive process.

In order to tackle the issue, early research has resorted to *belief restrictions* such as "bad-expectation beliefs", (originating in Caillaud and Jullien (2003), and refined in Armstrong and Wright (2007)) such that agents are assumed to join one particular platform unless it is a dominant strategy for them not to do so. The consequences of such "responsive" as compared to "passive" expectations and hybrid forms of monopoly and duopoly pricing have been investigated recently in Hagiu and Halaburda (2013).

An alternative approach is taken by Ambrus and Argenziano (2009) who make use of an *equilibrium refinement* that is strictly weaker than Coalition-Proof Nash Equilibrium (CPNE) originating in Bernheim, Peleg, and Whinston (1987). Lee (2013) makes use of CPNE in a non-atomistic one-sided market setting to mitigate the problem of coordination failures allowing for participation contingent contracts.

Viewing the problem of platform competition as a (multi)principal-agent problem (albeit with important externalities) with platforms trying to induce players to take certain actions using take-it-or-leave-it offers, the issue has parallels with the common agency literature originating in Bernheim and Whinston (1986). In an incomplete information setting, Halaburda and Yehezkel (2013) are employing these parallels in a *sequential* game under "ex-post asymmetric information", i.e. the same informational structure as EFM. Argenziano (2008) and Jullien and Pavan (2012) are investigating these issues within a global-game framework.

A paper that is close to our motivation to explain the coexistence of asymmetric outcomes in two-sided markets is Ambrus and Argenziano (2006). While

[0]I am grateful for comments by Matthias Blonski, Björn Brand, Federico Boffa, Thomas Gall, Germain Gaudin, Dominik Grafenhofer, Michael Katz, Lucia Visconti Parisio, Alex White, Julian Wright, and Pai-Ling Yin as well as participants of the "Two-sided Markets" Seminar at Universität Frankfurt, INTERTIC 2010 Milan, the 10th ICT conference at ZEW, CRESSE 2012, RSA Matera 2012, and Oligo2013 Budapest. Financial support of the PREMIUM project of the BMBF is gratefully acknowledged. Author's homepage www.stefanbehringer.com and e-mail: s.behringer@gmx.de.

this paper also starts with heterogenous consumers, their informational structure differs strongly as agents know their types when choosing the platform. Due to their assumption that agents are atomistic the reasons for finding equilibrium coexistence are fundamentally different.

The implications of the EFM framework for online trading platform competition have been investigated in Brown and Morgan (2009).[1] In their extension of the EFM model they look at *exogenous* vertical platform differentiation. One of their findings (see Proposition 4 in their paper), is that given eBay is the dominant platform and provides an exogenous vertical differentiation advantage to sellers, 1. more buyers are attracted to a given Yahoo! auction than an eBay auction, and 2. prices for the traded goods are higher on Yahoo! than on eBay. The authors note that both predictions are *exactly contradicted* by their evidence from field experiments. As an alternative they offer a dynamic disequilibrium model with boundedly rational players that will eventually lead to 'tipping'.

In this paper we are offering a more parsimonious extension of EFM that is in accordance with equilibrium coexistence taking differences in seller charges into account. This extension is empirically warranted as eBay has almost always been the more expensive platform for sellers in practice, charging listing fees *and* commissions. However treating pricing/vertical differentiation as exogenous is clearly not fully satisfactory in the context of competing platforms either.

We thus investigate the effects of *endogenous* seller charges on the equilibrium market structure. Following the analogy with the common agency literature it seems straightforward that platforms as principals should be able to make their actions (the payments required from agents) *contingent* on the choice of the agent, i.e. the observable allocation decision of buyers and sellers. This is strongly related to the idea of "insulating tariffs" on two-sided platforms (put forward in Weyl (2010) for monopoly and White and Weyl (2012) for competition) where platform pricing may be made conditional on this eventual allocation mitigating the coordination problems.

A major advantage of such contingent tariffs over previous refinements is that they are flexible enough to accommodate the potentially *asymmetric* market shares of buyers and sellers on each platform and endogenously reflect these asymmetries. While we are aware of the fact that such tariffs are only sufficient to bring about the observed changes in charges as eBay gains market dominance in many empirical markets (see Yin (2004)) we think that the investigation of price menus is warranted in its own right and may open new perspectives for platform operators. For example it would be technically feasible for eBay to make seller charges contingent on the number of competing products and/or the number of buyers who decide to monitor a given auction on their watchlist.

[1] Alternative applications include the competition of exchanges, see Cantillon & Yin (2010).

In this paper we are also able to tighten the characterization of the set of equilibria compared to EFM, show that our extensions can explain the empirical evidence in Brown and Morgan (2009), and allow for equilibrium coexistence with asymmetric charges under ex-post asymmetric information.

2 The Model

We model the duopolistic platform competition departing from a simple two-stage game presented in EFM (2004).

The timing of the game is as follows: In the first stage B risk-neutral buyers ($B \in \mathcal{N}_0$) with unit demand and S risk-neutral sellers ($S \in \mathcal{N}_0$) with a single unit of the good to sell and no reservation value simultaneously decide whether to attend platform 1 or platform 2. In the second stage buyers learn their valuations that are uniformly i.i.d. distributed and bargaining for the object takes place. We model this bargain as a uniform price (multiobject if $S > 1$) auction on each platform. By the revenue equivalence theorem this choice of the bargaining process is quite general. Each buyer only demands one homogeneous good. In order to guarantee strictly positive prices we make the 'non-triviality assumption' that

$$B > S + 1 \tag{1}$$

for both being positive integers. Risk neutral sellers have zero reservation value and their expected utility is given by the expected price on their chosen platform. A buyer's utility on a platform with B buyers and S sellers is given by his expected net utility conditional on winning the good i.e.

$$u_B = E\left\{ v - v^{S+1,B} \middle| \, v \geq v^{S,B} \right\} \Pr\left\{ v \geq v^{S,B} \right\} \tag{2}$$

where $v^{k,n}$ gives the k highest order statistic of a draw of n values and thus in this auction format the uniform price is simply the $S+1$ highest of the buyers valuations $v^{S+1,B}$ (i.e. the highest losing bid). This is the typical mathematical convention as long as we deal with a discrete model.

Larger markets are more efficient than smaller ones as they come closer to the ex-post efficient outcome to allocate a good to a buyer iff his valuation is high. The ex-post efficient outcome implies that the buyers with the S highest values obtain the good, so that the expectation of the maximum total ex-ante surplus (welfare) is

$$\begin{aligned} &B \Pr\left\{ v \geq v^{S,B} \right\} E\left\{ v \middle| \, v \geq v^{S,B} \right\} = SE\left\{ v \middle| \, v \geq v^{S,B} \right\} = \\ &SE\left\{ v \middle| \, v > v^{S+1,B} \right\} = S \int_0^1 \left(\int_x^1 v f(v \,| v > x) dv \right) f^{S+1,B}(x) dx \end{aligned} \tag{3}$$

where $f^{S+1,B}$ is the density function of $v^{S+1,B}$, the $S+1$ highest order statistic of a draw of B values under the uniform distribution.

Lemma (EFM)

Under the uniform distribution total welfare on one platform can be written as the sum of buyer and seller utilities

$$w(B,S) = S(1 - \frac{1}{2}\frac{1+S}{B+1}) = S\left(\frac{B-S}{B+1}\right) + B\left(\frac{S(1+S)}{2B(B+1)}\right).$$

Proof:
See Appendix.■

The result is intuitive: The total value of a sale is $E\left\{v|\, v > v^{S+1,B}\right\}$, i.e. expected value of v *given* $v > p$. Under the uniform distribution this is $1 - \frac{1}{2}\frac{1+S}{B+1} = p + \frac{1-p}{2}$. Clearly the second term is the value for one buyer $E\left\{v - v^{S+1,B}\middle|\, v > v^{S+1,B}\right\} = \frac{1-p}{2}$ with the remaining p (as calculated above) going to the seller and to obtain total welfare we multiply with the number of sales.

Note that

$$\frac{\partial w(1, \frac{S}{B} = \bar{x} < 1)}{\partial B} = \frac{1}{2}\bar{x}\frac{(2-\bar{x})(B+2)B+1}{(B+1)^2} > 0 \tag{4}$$

showing that for constant shares of sellers to buyers larger markets are more efficient than smaller ones. The efficiency deficit makes it more difficult for small markets to survive but the sequential structure of the game allows for equilibria with two active platforms whenever the impact of switching of buyer and/or seller on his expected surplus more than outweighs the efficiency advantage.

The game is solved by backward induction and the solution concept is Subgame Perfect Nash equilibrium (SPNE). The transaction of the good in stage two yields ex-ante utility in stage one for a seller of

$$u_S(B,S) = p = \frac{B-S}{B+1} \tag{5}$$

and for a potential buyer of

$$u_B(B,S) = \frac{1-p}{2}\frac{S}{B} = \frac{S(1+S)}{2B(1+B)}. \tag{6}$$

Note that holding S/B (the relative advantages of buyers and sellers) constant, sellers prefer larger, *more liquid* markets (where the expected equilibrium price is higher) and buyers prefer small, less efficient markets as

$$\frac{\partial u_S(1, \frac{S}{B} = \bar{x} < 1)}{\partial B} = \frac{\partial p(1, \frac{S}{B} = \bar{x} < 1)}{\partial B} > 0 \tag{7}$$

and

$$\frac{\partial u_B(1, \frac{S}{B} = \bar{x} < 1)}{\partial B} < 0 \tag{8}$$

Extending the setting of EFM we assume that platforms can charge buyers and/or sellers some fee for participating that need not be homogenous. Without loss of generality we assume that such a fee takes a non-negative value.

As buyers and sellers simultaneously decide which platform to join in stage one, we can set up the relevant constraints that determine the set of *all* possible SPNE of the game subject to the qualification that the integer constraint holds. Otherwise we will speak of a *quasi-equilibrium*. This restriction is investigated in detail in Anderson, Ellison and Fudenberg (2010). The constraints to keep buyers in place in stage one given buyer charge difference $p_{2B} - p_{1B} \equiv \Delta_B \geq 0$ are (B1)

$$u_B(B_1, S_1) \geq u_B(B_2 + 1, S_2) - \Delta_B \tag{9}$$

and (B2)

$$u_B(B_2, S_2) - \Delta_B \geq u_B(B_1 + 1, S_1) \tag{10}$$

In words: A buyer on platform 1 needs to have an expected utility from the bargaining stage correcting for charges paid to the platform owner such that a change to the other platform and the implied effect on the equilibrium bargaining outcome there deters him from doing so.

To keep sellers in place in stage one given seller charge difference $\Delta_S \geq 0$ we need (S1)

$$u_S(B_1, S_1) \geq u_S(B_2, S_2 + 1) - \Delta_S \tag{11}$$

and (S2)

$$u_S(B_2, S_2) - \Delta_S \geq u_B(B_1, S_1 + 1) \tag{12}$$

to hold. The motivation for the constraints is analogous. Clearly these constraints matter only for interior equilibria.

2.1 Exogenous buyer charges

We now look explicitly at the form of the constraints and thus at the set of possible SPNE with some exogenous charge differences $\Delta_B > 0$ to (winning) buyers in auction two. Note that this does not imply that charges are made only by one of the platforms but only that it is the difference between such charges that influence location incentives.

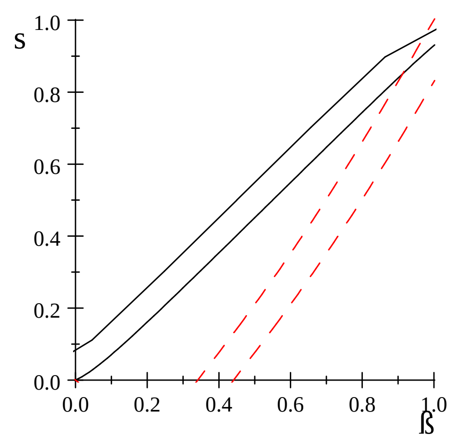
Denoting s as the *share* of sellers on platform one and β as the *share* of buyers at platform one the buyer constraint (9) becomes

$$\frac{sS(1+sS)}{2\beta B(1+\beta B)} \geq \frac{(1-s)S(1+(1-s)S)}{2((1-\beta)B+1)(1+(1-\beta)B+1)} - \Delta_B \tag{13}$$

and (B2) is

$$\frac{(1-s)S(1+(1-s)S)}{2(1-\beta)B(1+(1-\beta)B)} - \Delta_B \geq \frac{sS(1+sS)}{2(\beta B+1)(1+\beta B+1)} \tag{14}$$

A numerical example (with $B = 10, S = 5$) illustrates how the buyer constraints change. The two buyer constraints with $\Delta_B = 0$ (*solid lines*) and $\Delta_B = 0.3$ (*dashed red lines*) are:



where the share of sellers on platform one (s) is on the ordinate and the share of buyers on platform one (β) is on the abscissa.

The interpretation of this finding is as follows: The lower *solid line* is the (B1) constraint gives the condition that buyers stay on platform one if the fraction of sellers s is large enough or, alternatively if β is low enough. The higher solid line is the (B2) constraint gives the condition under which buyers stay on platform 2, i.e. if s is small (and thus $(1-s)$ the fraction of seller on his own platform is large enough). Between the two curves is the candidate set of SPNE (we still need to check if the seller constraints hold).

Now with a charge of $\Delta_B > 0$ to buyers on the second platform both the (B1) and the (B2) constraint shift downwards to the *dashed red lines*, i.e. the set of SPNE allows for equilibria with a lower share of sellers on platform one for a given share of buyers. The (B2) constraint also shifts downwards. i.e. buyers move from the second platform at higher levels of s already, (and thus for a lower fraction of seller $(1-s)$ on his own platform) than before given the new charge.

2.2 Exogenous seller charges

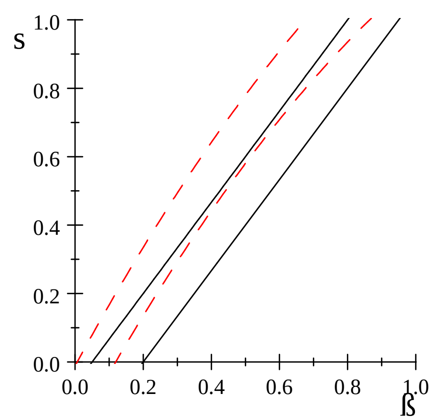
We now introduce an exogenous charge difference Δ_S for sellers of platform 2. Seller constraints are (S1)

$$\frac{\beta B - sS}{\beta B + 1} \geq \frac{(1-\beta)B - ((1-s)S + 1)}{(1-\beta)B + 1} - \Delta_S \tag{15}$$

and (S2)

$$\frac{(1-\beta)B - (1-s)S}{(1-\beta)B + 1} - \Delta_S \geq \frac{\beta B - (sS + 1)}{\beta B + 1} \tag{16}$$

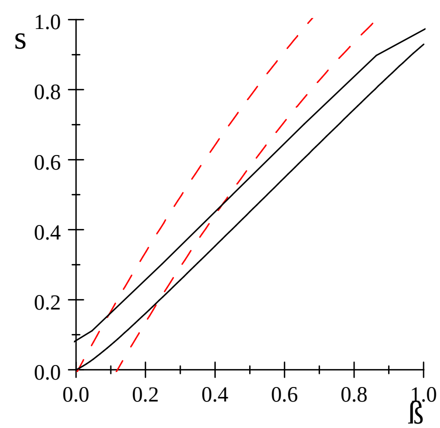
With $\Delta_S = 0$ (*solid lines*) and $\Delta_S = 0.3$ (*dashed red lines*) we find the picture with the seller constraints becomes:



The interpretation of this finding is as follows: For the upper *solid line* is the linear (S1) constraint, a seller stays on platform 1 if s is not too high for a given share of β, otherwise he will go to platform 2. For the lower solid line, the linear (S2) constraint, a seller stays at platform 2 if s is high (i.e. his own seller share $1-s$ is low) otherwise he will go to platform one. Between the two curves is the candidate set of SPNE (we need to check if the buyer constraint holds simultaneously).

Now that there is a charge of $\Delta_S > 0$ to the sellers on the second platform, the (S1) constraint is no longer linear and shifts upwards to the upper *dashed red line*: Sellers stay on platform 1 even if s is much higher than before for given β. Similarly the (S2) constraint is no longer linear and also shifts upwards to the lower dashed red line: Sellers will move from platform 2 even if s is much higher (hence their own seller share $1-s$ much lower) than before.

The numerical example with $\Delta_S = 0.3$ (*dashed red lines*) and the original buyer constraints for $\Delta_B = 0$, *(solid lines)* yields both seller and buyer constraints as



Only $\beta = 0.2, s = 0.2$ is a viable equilibrium here and the previous candidate $\beta = 0.4, s = 0.4$ is no longer viable.

The result reveals that charging sellers on platform 2 allows for higher s tolerance for given β on platform 1. Also, equally sized platforms are no longer viable. As sellers like larger, more liquid platforms where the uncertainty about the resulting final price is lower we find that a positive and exogenous relative seller charge difference of platform 2 can only be an equilibrium if platform 2 also has the larger share of sellers.

3 Endogenous seller price competition

In order to investigate the issue we extend setup and timing of the game:

In the *first stage* platforms $j = 1, 2$ simultaneously choose seller charges that may be *contingent* on realized allocations of buyers and sellers, i.e. a *function* that specifies price as a function of buyer and seller allocations, $p^{S,j}, p^{S,-j} : (\beta, B, s, S) \to \mathbb{R}^{+}$.

In the *second stage* buyers and sellers simultaneously decide which platform to join, learn their valuations that are uniformly i.i.d. distributed and bargaining for the object takes place. Payoffs to buyers, sellers, and platforms are realized (this was described as a separate stage above but can be collapsed into a single stage as we have shown for the EFM model above).

We make use of a very weak equilibrium refinement for the second stage : Instead of implementing belief/expectation restrictions or a new solution concept we define:

Definition 1 *An* efficient *equilibrium for the EFM model is the welfare maximizing equilibrium taken from the set of stable candidate equilibria.*

Lemma 2 *If* $\Delta_B = 0$ *the efficient equilibrium is unique.*

Proof:
See Appendix.■

Hence the efficient equilibrium refinement selects a *unique* equilibrium outcome for any subgame following the first stage.

In order to discuss price formation in the above platform game we now introduce platforms' *pricing strategies* to the first stage of the model that have the spirit of the "*insulating tariff*" used in Weyl (2010) and White & Weyl (2012) (i.e. a mapping or menu from buyer and seller allocations into charges) to select their "*target allocations*" to mitigate the coordination problems. Lee (2013) also investigates contingent transfers in a one-sided setting. In the continuation equilibrium the buyers and sellers then choose the platforms contingent on these price mappings. Platform profits are simply

$$\Pi_j = S_j p^{S,j} \text{ for } j = 1, 2 \tag{17}$$

where where $p^{\cdot}$ can be interpreted to be a markup net of seller unit costs.

Definition 3 *A seller pricing strategy is a map that specifies the price charged to sellers as a function of the buyer and seller platform choices:*

$$p^{S,1}(B_1, B_2, S_1, S_2) \text{ and } p^{S,2}(B_2, B_1, S_2, S_1)$$

or

$$\Delta^S(\beta, B, s, S).$$

This allows prices to be used as a non-cooperative tool to induce coordination on a desired outcome. The solution concept for the first stage is then *pure strategy Nash equilibrium* so that

$$p^{S,j*} = p^{S,j}(B_j^*, B_{-j}^*, S_j^*, S_{-j}^*) \text{ for } j = 1, 2 \text{ when } -j = 2, 1.$$

denotes the subgame perfect Nash Equilibrium of the two-stage game where platforms maximize profits holding the other platforms price map constant and anticipate correctly that buyers and sellers will play the unique *efficient* equilibrium in stage two.

In most analyses of platform competition it is assumed that *participation constraints* of the buyers are always met. For example in a Hotelling where the fixed benefit always outweighs possibly high charges by platforms. An explicit treatment of participation constraints often obscures results substantially (e.g. Lee, 2013, fn.11) or renders solutions implausible. On the contrary we are able to investigate participation constraints and profit conditions (feasibility) in detail below.

Furthermore demand also should depend on price: If platform j charges a seller charge high enough, then it should loose all its sellers. In order to meet these intuitive requirements we will assume (*Assumption A*) that agents do react to price changes of platforms in a way that their incentive (switching) constraints remain binding.[2] Hence a bounded and non-trivial demand elasticity can be calculated.

Proposition 4 *The equilibrium seller charge difference (of the price maps) resulting under competition and Assumption A are given by*

$$\Delta^{S*R} = -3\frac{B_1 - B_2 - S_1 + S_2 + B_1S_2 - B_2S_1}{(B_2 + 1)(B_1 + 1)}$$

if prices are rigid (R) and by

$$\Delta^{S*F} = -3\frac{2B_1 - B_2 - S_1 + S_2 + B_1S_2 - B_2S_1 + 1}{(B_2 + 1)(B_1 + 1)}$$

if prices are fully flexible (F).

[2] I owe this suggestion to a referee.

Proof:

The game is solved using backward induction. W.l.o.g. we will focus on platform 1 with profits $\Pi_1 = S_1 p^{S_1,1}$. Using A we can assume that (S1) is strictly binding thus

$$\frac{\beta B - sS}{\beta B + 1} - p^{S_1,1} \geq \frac{(1-\beta)B - ((1-s)S+1)}{(1-\beta)B+1} - p^{S_2+1,2} \tag{18}$$

can be written as

$$p^{S_1,1} = p^{S_2+1,2} + \frac{B_1 - S_1}{B_1 + 1} + \frac{S_2 - B_2 + 1}{B_2 + 1} \tag{19}$$

We investigate two cases:

a) Prices are *rigid* (R) so that platforms cannot change their prices quickly and hence

$$p^{S_2+1,2} = p^{S_2,2}$$

Substituting into the profit condition the concave programme for the optimal target allocation is

$$\underset{S_1}{Max}\Pi_1 = S_1(p^{S_2+1,2} + \frac{B_1 - S_1}{B_1 + 1} + \frac{S_2 - B_2 + 1}{B_2 + 1} - c^{S,1})$$

with the necessary and sufficient first order conditions yielding best responses

$$S_1^{BR} = \frac{1}{2}\left(B_1 + 1\right) p^{S_2+1,2} + \frac{1}{2}\frac{2B_1 - B_2 + S_2 + B_1 S_2 + 1}{B_2 + 1}$$

and by symmetry

$$S_2^{BR} = \frac{1}{2}\left(B_2 + 1\right) p^{S_1,1} + \frac{1}{2}\frac{2B_2 - B_1 + S_1 + B_2 S_1 + 1}{B_1 + 1}$$

Solving simultaneously the equilibrium target allocation then has

$$S_1^* = \frac{1}{3}\left(B_1 + 1\right)\left(p^{S_1,1} + 2p^{S_2+1,2} + \frac{3}{B_2 + 1}\right)$$

and

$$S_2^* = \frac{1}{3}\left(B_2 + 1\right)\left(p^{S_2+1,2} + 2p^{S_1,1} + \frac{3}{B_1 + 1}\right)$$

These can be implemented with *conditional* equilibrium charges

$$p^{S_1,1*} = \frac{\left(2B_1 + 2\right)S_2 - \left(B_2 + 1\right)S_1 + \left(B_1 - 2B_2 - 1\right)}{\left(B_2 + 1\right)\left(B_1 + 1\right)} \tag{20}$$

and

$$p^{S_2+1,2*} = \frac{\left(2B_2 + 2\right)S_1 - \left(B_1 + 1\right)S_2 + \left(B_2 - 2B_1 - 1\right)}{\left(B_1 + 1\right)\left(B_2 + 1\right)} \tag{21}$$

yielding

$$\Delta^{S*FP} = p^{S_2+1,2*} - p^{S_1,1*} = -3\frac{B_1 - B_2 - S_1 + S_2 + B_1S_2 - B_2S_1}{(B_2+1)(B_1+1)}$$

with rigid prices, as given above.

b) If prices are *flexible* (F) we have the Nash targets still as

$$S_1^* = \frac{1}{3}(B_1+1)\left(p^{S_1,1} + 2p^{S_2+1,2} + \frac{3}{B_2+1}\right)$$

and again by symmetry

$$S_2^* = \frac{1}{3}(B_2+1)\left(p^{S_2+1,2} + 2p^{S_1,1} + \frac{3}{B_1+1}\right)$$

These can be implemented by

$$p^{S_1,1} = -2p^{S_2+1,2} + \frac{3B_2S_1 + 3S_1 - 3B_1 - 3}{B_1 + B_2 + B_1B_2 + 1}$$

but now prices adjust so that

$$p^{S_2+1,2} = -2p^{S_1,1} + \frac{3B_1(S_2+1) + 3(S_2+1) - 3B_2 - 3}{B_1 + B_2 + B_1B_2 + 1}$$

and equilibrium charges are

$$p^{S_1,1*} = \frac{3B_1 - 2B_2 - S_1 + 2S_2 + 2B_1S_2 - B_2S_1 + 1}{B_1 + B_2 + B_1B_2 + 1}$$

and

$$p^{S_2+1,2*} = -\frac{3B_1 - B_2 - 2S_1 + S_2 + B_1S_2 - 2B_2S_1 + 2}{B_1 + B_2 + B_1B_2 + 1}$$

which are no longer symmetric. The charge difference for flexible prices is then

$$\Delta^{S*F} = -3\frac{2B_1 - B_2 - S_1 + S_2 + B_1S_2 - B_2S_1 + 1}{(B_2+1)(B_1+1)}$$

as given above.■

The equilibrium target prices have the expected comparative statics as:

$$\frac{\partial p^{S_1,1*}}{\partial S_1} = -\frac{1}{B_1+1} < 0, \frac{\partial p^{S_1,1*}}{\partial B_1} = \frac{S_1+2}{(B_1+1)^2} > 0$$

$$\frac{\partial p^{S_1,1*}}{\partial S_2} = \frac{2}{B_2+1} > 0, \frac{\partial p^{S_1,1*}}{\partial B_2} = -\frac{2S_2+1}{(B_2+1)^2} < 0$$

The comparative statics with flexible prices are identical except

$$\frac{\partial p^{S_1,1*}}{\partial B_2} = -\frac{2S_2+3}{(B_2+1)^2} < 0$$

which differs in magnitude but not in sign. We fill focus on the rigid price case in what follows.

3.1 Participation constraints

Note that (20) can be written as

$$p^{S_1,1*} = \frac{(2\beta B+2)(1-s)S - ((1-\beta)B+1)\,sS + (\beta B - 2(1-\beta)B - 1)}{((1-\beta)B+1)(\beta B+1)} \tag{22}$$

The participation constraints of a *seller on platform 1* is thus

$$U^{S1} = \frac{B_1 - S_1}{B_1+1} - p^{S_1,1*} = \frac{2B_2 - 2S_2 + B_1B_2 - 2B_1S_2 + 1}{(B_2+1)(B_1+1)} \geq 0 \tag{23}$$

which does not depend on S_1 anymore. The comparative statics reveal:

$$\frac{\partial U^{S1}}{\partial S_2} = -\frac{2}{B_2+1} < 0, \frac{\partial U^{S1}}{\partial B_1} = -\frac{1}{(B_1+1)^2} < 0, \frac{\partial U^{S1}}{\partial B_2} = \frac{2S_2+1}{(B_2+1)^2} > 0$$

so that taking price competition into account the seller utility now decreases in the number of sellers on the other platform, decreases in the number of buyers on its own but increases in the number of buyers on the other platform.

Expressing the constraint in terms of shares we find

$$\frac{\partial U^{S1}}{\partial S} = -\frac{2(1-s)}{B - B\beta + 1} < 0, \frac{\partial U^{S1}}{\partial s} = \frac{2S}{B - B\beta + 1} > 0$$

which clearly follows from the fact that U^{S1} does not depend on S_1 anymore. The remaining statistics are

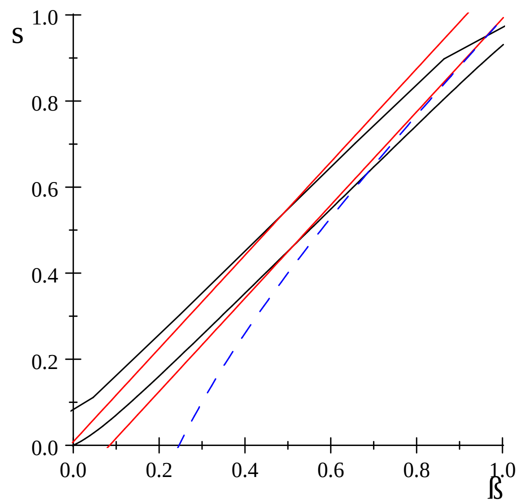
$$\frac{dU^{S1}}{dB} = \frac{\begin{array}{c}(\beta(1-\beta)(2\beta + 2S\beta(1-s) - 1))\,B^2 + \\ (4S\beta(1-\beta)(1-s))\,B + 2S(1-\beta)(1-s) + 1 - 2\beta\end{array}}{(B(1-\beta)+1)^2(B\beta+1)^2}$$

and

$$\frac{dU^{S1}}{d\beta} = B\frac{\begin{array}{c}(2\beta(1-\beta-S\beta(1-s)) - 1)\,B^2 + \\ (4S\beta(s-1) - 2)\,B + 2(S(s-1) - 1)\end{array}}{(B(1-\beta)+1)^2(B\beta+1)^2}$$

which are in- and decreasing in large markets respectively.

If we introduce the seller participation constraint (23) into the example above ($B = 10, S = 5$) we get the following picture with new switching constraints for endogenous prices (solid red) and the participation constraint for sellers on platform 1 (dashed blue line):



The example motivates the more general proof.

Lemma 5 *The participation constraint for sellers will be satisfied in any candidate equilibrium.*

Proof:
See Appendix.■

3.2 Profits

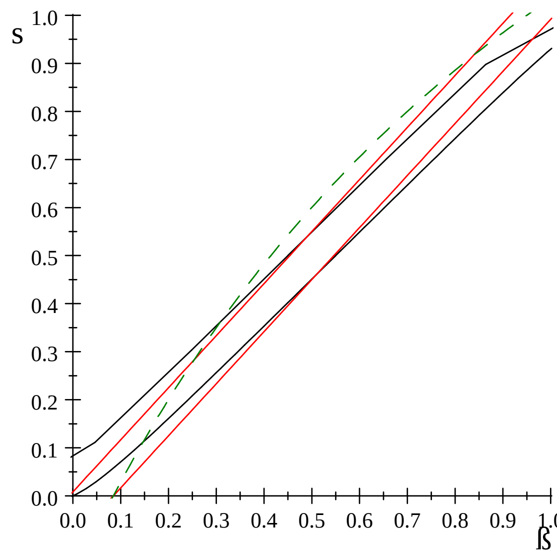
Profits for platform 1 are given by

$$\Pi^1 = p^{sS,1} sS = \left(\frac{(2\beta B + 2)(1-s)S - ((1-\beta)B + 1)\, sS + (\beta B - 2(1-\beta)B - 1)}{((1-\beta)B + 1)(\beta B + 1)}\right) sS$$

which is non-negative and thus feasible if the allocation in interior and platform equilibrium charges are non-negative. As seller prices on platform 1 (22) are decreasing in s, non-negative profits imply

$$s \leq \frac{2S - 2B + 3B\beta + 2BS\beta - 1}{3S + BS + BS\beta}$$

For the example above ($B = 10, S = 5$) we find the zero-profit line (dashed green line) as



Note that despite the fact that with endogenous prices, from the consumer perspective all equilibria along the diagonal are now feasible a non-negative-profit constraint on firms imply that extreme outcomes are still not possible and cannot be in the set of targeted allocations.

A general proof is conjectured to be possible.

4 Large platforms

The above analysis finds that equilibria of this game may have non-Bertrand outcomes (despite homogeneity of the product of the transaction) where pricing differences between the two platforms can prevail in subgame perfect equilibrium. We now investigate the robustness of this property of the model for large platforms, i.e. where the number of buyers and sellers in the market gets (very) large and we do not observe cornered market outcomes. Note that in the limit such an assumption implies that the essential *friction* of the EFM model (the fact that an individual buyer or seller switching has an impact on expected transaction prices) vanishes.

For applied purposes friction does not necessary imply that the *actual* number of agents remains bounded but that there remains the possibility of agents exerting "influence" (see Al-Najjar & Smorodinsky, 2000). This may well be true even in very large markets, e.g. in financial exchanges where a set of bulk traders that "move markets" are the rule rather the exception. An application of the model to very large markets then merely requires to narrow the set of agents to which it is meant to apply.

Proposition 6 *On large platforms any equilibrium is proportional and charges satisfy* $\Delta_B = \Delta_S = 0$.

Proof:

The buyer constraints are given above as (9) and (10). Letting the share of buyers to sellers on each platform be fixed at some $\bar{x}_i = S_i/B_i$ $i = 1, 2$ we find that the first constraint becomes

$$\frac{\bar{x}_1(\frac{1}{B_1} + \bar{x}_1)}{2(\frac{1}{B_1} + 1)} \geq \frac{\bar{x}_2(\frac{1}{B_2} + \bar{x}_2)}{2(1 + \frac{1}{B_2})(\frac{2}{B_2} + 1)} - \Delta_B \tag{24}$$

and on large platforms where $B_1, B_2 \to \infty$ we find that this reduces to

$$\frac{(\bar{x}_1)^2}{2} \geq \frac{(\bar{x}_2)^2}{2} - \Delta_B \tag{25}$$

for any share $\bar{x}_1$ as $u_B(1, \bar{x}_1) \to (\bar{x}_1)^2/2$. The second constraint can similarly be reduced to

$$\frac{(\bar{x}_2)^2}{2} - \Delta_B \geq \frac{(\bar{x}_1)^2}{2} \tag{26}$$

so that the only outcome that satisfies these constraints has $\Delta_B = 0$ and $\bar{x}_1 = \bar{x}_2$. Similarly for sellers we have from (15) that

$$1 - \bar{x}_1 \geq 1 - \bar{x}_2 - \Delta_S \tag{27}$$

and (16)

$$1-\bar{x}_2-\Delta_S \geq 1-\bar{x}_1 \tag{28}$$

which again can only be satisfied for $\Delta_S = 0$ and $\bar{x}_1 = \bar{x}_2$ as $u_S(1, \bar{x}_1) \to 1-\bar{x}_1$. The conclusion follows from noting that $\bar{x}_1 = \bar{x}_2 \Leftrightarrow \beta = s$.■

This finding mirrors Proposition 3 in Brown and Morgan (2009) who show that with vertical differentiation (i.e. a charge difference $\Delta_S > 0$ in our case) equilibrium in very large markets is impossible.

A version of their Proposition 4 holds that in addition:

Proposition 7 *In any quasi-equilibrium in which the sites coexist and eBay (here* 1*) enjoys an* exogenous *vertical differentiation advantage for sellers (here* $\Delta_S > 0$*) and a more than 50% market share, relatively more sellers are attracted to a given eBay auction than an Yahoo! auction for sufficiently many buyers.*

Proof:
The seller constraint (S2) from (16) can be transformed into

$$s_{S2} \geq \frac{1}{S(B+2)}(S-2B+\Delta_S+B\Delta_S-1)+B\beta\frac{S+B\Delta_S-B\Delta_S\beta+3}{S(B+2)} \tag{29}$$

Also given participation constraints hold the maximal advantage for sellers is bounded by

$$\Delta_S < Max_{\beta,s}\left\{\frac{(1-\beta)B-(1-s)S}{(1-\beta)B+1}\right\} \tag{30}$$

Now we show that if $\Delta_S > 0$ and $\beta > 1/2$ then $s > \beta$ for sufficiently many buyers.

Note that s_{S2} is strictly concave in β given that $\Delta_S > 0$. The difference between s_{S2} and the 45° line (where $\beta = s$) is

$$d \equiv s_{S2}-\beta = \frac{1}{S(B+2)}(S-2B+\Delta+B\Delta-1)+\beta\frac{3B-2S+B^2\Delta-B^2\Delta\beta}{S(B+2)} \tag{31}$$

and still strictly concave in β. Thus the difference d attains a maximum at

$$\beta_{\max} = \frac{3B-2S+B^2\Delta_S}{2B^2\Delta_S} > 0 \tag{32}$$

at $\beta = 0$ the difference is

$$d = \frac{1}{S(B+2)}(S-2B+\Delta+B\Delta-1)$$

which is strictly negative given (30). As derivatives are smooth there exists a unique intermediate value of β such that $d = 0$. This value can be found as

$$\beta_k = \frac{S - B(2 - \Delta_S) + \Delta_S - 1}{2S - B(B\Delta_S + 3)} \tag{33}$$

Note that

$$\frac{\partial \beta_k}{\partial \Delta_S} = -\frac{(3B - 2S + B\,(B+2)\,(2B - S))}{(\Delta B^2 + 3B - 2S)^2} \tag{34}$$

which given non-triviality $B > S + 1$ is negative. Hence the difference is falling in Δ_S. Then there is a critical level of seller advantage such that the critical level of the intersection of s_{s2} and the 45° line is exactly at $\beta = 1/2$. This level is

$$\Delta_k = \frac{B+2}{2B + B^2 + 2} \tag{35}$$

and is *falling* in B. With sufficiently many buyers for any $\Delta_S > \Delta_k (\to 0)$ we have given $\beta > 1/2$ that $s > \beta$, i.e. the seller switching constraint ($S2$) can only be satisfied strictly above the 45° line. Thus platform 1 faces $s > \beta$ and so a seller buyer ratio of

$$\frac{sS}{\beta B} > \frac{S}{B} \tag{36}$$

and by adding up

$$\frac{(1-s)S}{(1-\beta)B} < \frac{S}{B} < \frac{sS}{\beta B}.\blacksquare \tag{37}$$

The original Proposition 4 in Brown and Morgan claims that with an *exogenous* vertical differentiation advantage for sellers and a more than 50% market share, relatively more buyers are attracted to a Yahoo! than an eBay (platform 1) auction which contradicts their data. Once prices are endogenized using Proposition 7 we note that the theoretical implication is *exactly reversed* as sellers on eBay will actually face relatively more favourable buyer seller ratios as they will have to pay the higher seller charge and thus face an endogenous vertical differentiation *dis*advantage (i.e. $\Delta_S < 0$).

It turns out that endogenizing prices to sellers is also sufficient to reverse the result in Brown and Morgan (2009) about the relative *transaction prices* on both platforms and hence bring the model in line with their data in this respect too. We can show that:

Proposition 8 *In any quasi-equilibrium in which the sites coexist and eBay (here* 1*) enjoys an* endogenous *vertical differentiation* disadvantage for sellers *(here* $\Delta^{S*F} < 0$*) and a more than 50% market share, for sufficiently many buyers the transaction price on eBay is higher than that on Yahoo!.*

Proof:
See Appendix.■

Hence we find that with endogenous seller charges the transaction prices on eBay will be larger than those of Yahoo!, in line with the data findings in Brown and Morgan (2009).

Alternatively we can also use an exogenous vertical differentiation advantage for *buyers* to similarly show that:

Proposition 9 *In any quasi-equilibrium in which the sites coexist and eBay (here* 1*) enjoys an* exogenous *vertical differentiation* advantage for buyers *(here* $\Delta_B > 0$*) and a more than 50% market share, relatively more buyers are attracted to a given eBay auction than an Yahoo! auction for sufficiently many buyers and the transaction price on eBay is higher than on Yahoo!.*

Proof:
See Appendix.■

We have thus shown two alternatives by which the empirical results reported in Brown and Morgan (2009) can be brought in line with the theory. The first implies that the liquidity effects of a large market will dominate the effect of the endogenous seller charges on large platforms leading to a higher expected transaction price to the detriment of its buyers. Alternatively one may argue that if eBay has an exogenous vertical differentiation advantage for buyers in addition to being the dominant platform in a liquid market this is also sufficient to explain the more favourable buyers-seller ratio for its buyers and for it to have larger transaction prices than at Yahoo! auctions.

Note that endogenous seller charges satisfy

$$\lim_{B\to\infty}(\Delta^{S*F}) = \lim_{B\to\infty}(3\frac{(B-S)\,(1-2\beta)}{(B\beta+1)\,(B(1-\beta)+1)}) = 0. \tag{38}$$

The intuition for this limit result is straightforward: The possibility that the switching of either buyer or seller has a tangible impact on expectations decreases as the number of buyers and sellers increases so that in the limit as markets get very large all friction disappears from the model and we get a Bertrand type outcome with regard to the charge differences and proportional equilibria. This Proposition can be easily extended to an unspecified distribution of valuations and is thus robust.

We also have a result for welfare on large platforms: As total welfare of a platform goes out of bounds if the platform gets very large we look at total welfare per buyer and seller respectively

$$\frac{w(B,S)}{B} = u_B(B,S) + \bar{x}u_S(B,S) = \bar{x}(1-\frac{\bar{x}}{2}) \tag{39}$$

and

$$\frac{w(B,S)}{S} = \frac{1}{\bar{x}} u_B(B,S) + u_S(B,S) = 1 - \frac{\bar{x}}{2} \tag{40}$$

where $\bar{x}$ is the limit of the total seller to buyer ratio. By the non-triviality assumption the per capita welfare contribution of a buyer is thus always lower than that of a seller.

5 Conclusion

Often buyers cannot be charged for participating on a platform. For example on eBay seller-fee-shifting is not allowed. Alternatively the final transactions may not be observable as on used-car platforms. In these cases the strictness of the buyer switching constraints implies that independently of whether or not there are charges to the sellers, the equilibrium market structure of the platform duopoly will imply proportional equilibria. This strongly restricts the set of equilibria of the game compared to that in EFM.

The original Proposition 4 in the paper by Brown and Morgan (2009) exactly contradicts their data which finds that: "eBay sellers enjoy higher prices and more favourable buyer-seller ratios than do Yahoo! sellers." Endogenizing the platform's pricing decision for sellers using the motivation for an "insulating equilibrium" we are able to show that theory and practice actually reveal an endogenous vertical *dis*advantage for sellers on eBay being the dominant platform. This observation exactly reverses their theoretical findings bringing them in line with the data from their field experiments.

A similar finding pertains with respect to the predicted relative transaction prices on both platforms. Once charges to sellers are endogenized, being the dominant platform implies that transaction prices will indeed be larger on eBay, the more liquid platform, again as observed it their field experiments. An alternative theoretical derivation of these results can be derived for an exogenous vertical differentiation advantage for buyers on eBay.

In conclusion we have shown that our extension of the EFM model taking into account optimal platform pricing behaviour describes an equilibrium target allocation and, as its mirror image, a set of participation-contingent equilibrium seller charges that fits the data found in Brown and Morgan (2009) for the online auction market.

6 Appendix

Proof of Lemma (EFM):

Under the uniform distribution on [0,1] the ith *lowest* order statistic out of n draws is distributed Beta$(i, n-i+1)$ with probability density function

$$f^{i,n-i+1}(x) = \begin{cases} \frac{x^{i-1}(1-x)^{n-i}}{\int_0^1 u^{i-1}(1-u)^{n-i}du}, x \in (0,1) \\ 0 \text{ otherwise} \end{cases} \tag{41}$$

and expectation

$$\int_0^1 x f^{i,n-i+1}(x)dx = \frac{i}{n+1} \tag{42}$$

As the order statistic of the $S+1$ highest of B draws is also that of the $B-S$ *lowest,* the expectation of the price given by the $S+1$ highest buyer valuation can be rewritten as

$$\int_0^1 x f^{S+1,B}(x)dx = \frac{B-S}{B+1} \tag{43}$$

which is also expected seller surplus due to the normalized reservation value. Thus the density of the order statistic $v^{S+1,B}$ is

$$f^{S+1,B}(x) = \begin{cases} \frac{x^{B-S-1}(1-x)^{S}}{\int_0^1 u^{B-S-1}(1-u)^{S}du}, x \in (0,1) \\ 0 \text{ otherwise} \end{cases} \tag{44}$$

Total welfare $w(B,S)$ on one platform given uniformly distributed valuations can thus be written as

$$\begin{aligned} & w(B,S) = S\int_0^1 \left(\int_x^1 vf(v\,|v>x)dv\right) f^{S+1,B}(x)dx = \\ & \frac{S}{\int_0^1 u^{B-S-1}(1-u)^S du}\int_0^1 \left(\int_x^1 v\left(\frac{1}{1-x}\right)dv\right)(x^{B-S-1}(1-x)^S)dx = \\ & \frac{S}{2\int_0^1 u^{B-S-1}(1-u)^S du}\int_0^1 (x+1)(x^{B-S-1}(1-x)^S)dx = \\ = \quad & S(1-\frac{1}{2}\frac{1+S}{B+1}) = S\left(\frac{B-S}{B+1}\right) + B\left(\frac{S(1+S)}{2B(B+1)}\right).\blacksquare \end{aligned}$$

Proof of Lemma 1:

Given $\Delta_B = 0$ all equilibria will be on the diagonal (proportional) where $\beta = s$, $\forall S, B \in \mathcal{N}_0$. We look at the seller constraint and the buyer constraint in turns and show whether it is possible to have them satisfied for non-proportional equilibria. Constraint (S1) is

$$\frac{B\beta - Ss}{B\beta + 1} = \frac{B(1-\beta) - (S(1-s)+1)}{B(1-\beta)+1} - \Delta_S \tag{45}$$

or

$$s_{(S1)} = \frac{3B\beta - B + SB\beta + S + 1 + \Delta_S B^2\beta - \Delta_S B^2\beta^2 + \Delta_S(B+1)}{S\,(B+2)} \tag{46}$$

and (S2) is

$$\frac{B(1-\beta) - S(1-s)}{B(1-\beta)+1} - \Delta_S = \frac{B\beta - (Ss+1)}{B\beta + 1} \tag{47}$$

or

$$s_{(S2)} = \frac{-2B + 3B\beta + SB\beta + S + \Delta_S B^2\beta - \Delta_S B^2\beta^2 + \Delta_S(B+1) - 1}{S\,(B+2)} \tag{48}$$

with vertical difference between the two seller constraints

$$s_{(S1)} - s_{(S2)} = \frac{1}{S} \tag{49}$$

for any Δ_S. Thus it may be possible to have the seller constraint strictly satisfied at a non-proportional equilibrium by 'squeezing in' a non-proportional equilibrium candidate vertically.

For $\Delta_S = 0$ we have

$$\beta_{(S1)} = \frac{SsB + 2Ss + B - S - 1}{B\,(3+S)} \tag{50}$$

and

$$\beta_{(S2)} = \frac{2B - S + 2Ss + SsB + 1}{B\,(3+S)} \tag{51}$$

so that the horizontal difference between the two seller constraints is

$$\beta_{(S2)} - \beta_{(S1)} = \frac{B+2}{B\,(3+S)} > \frac{1}{B} \tag{52}$$

as $B > S + 1$. Thus it may be possible to have the seller constraint satisfied at a non-proportional equilibrium by 'squeezing in' non-proportional equilibrium candidate horizontally.

We now look at the critical buyer constraints: Now (B1) is

$$\frac{sS(1+sS)}{2\beta B(1+\beta B)} \geq \frac{(1-s)S(1+(1-s)S)}{2((1-\beta)B+1)(1+(1-\beta)B+1)} - \Delta_B \tag{53}$$

and (B2) is

$$\frac{(1-s)S(1+(1-s)S)}{2(1-\beta)B(1+(1-\beta)B)} - \Delta_B \geq \frac{sS(1+sS)}{2(\beta B+1)(1+\beta B+1)} \tag{54}$$

For $\Delta_B = 0$ the solution to (B1) is

$$\begin{aligned}\beta_{(B1)} &= \frac{1}{2\,(1+S)\,(2s-1)} \times \\ &\quad \frac{(4S+2SB)\,s^2 + (-2S+2+2B)\,s + 1 + S - \sqrt{\Psi}}{B}\end{aligned} \tag{55}$$

and the one for (B2) is

$$\begin{aligned}\beta_{(B2)} &= \frac{1}{2\,(1+S)\,(2s-1)} \times \\ &\quad \frac{(4S+2SB)\,s^2 + (2B-2-6S)\,s + 3 + 3S - \sqrt{\Psi}}{B}\end{aligned} \tag{56}$$

with

$$\begin{aligned}\Psi &= 4S^2\,(B+2)^2\,s^4 - 8S^2\,(B+2)^2\,s^3 + \\ &\quad \left(-16SB - 12 + 4S^2B^2 - 4SB^2 - 8S + 20S^2 - 16B + 16S^2B - 4B^2\right)s^2 - \\ &\quad 4\,(1+S)\left(S - 4B - B^2 - 3\right)s + 1 + S^2 + 2S\end{aligned} \tag{57}$$

The horizontal difference is then

$$\beta_{(B1)} - \beta_{(B2)} = \frac{1}{B} \tag{58}$$

Thus given that the two constraints with $\Delta_B = 0$ are always on opposite sides of the $\beta = s$ diagonal there will always be proportional equilibrium candidates and it is impossible to 'squeeze in' another non-proportional equilibrium candidate horizontally. Note that the result does not hold for $\Delta_B > 0$ although the horizontal difference remains the same.

The vertical difference between (B2) and (B1) is difficult to calculate directly. However we can use the fact that the distance between (B1) and the diagonal is monotone increasing in β and the mirror image, between (B2) and the diagonal is monotone decreasing in β which follows from buyers preference for the smaller platform. Hence if we can show that this distance for (B1) at $\beta = 1$ (or the distance for (B2) at $\beta = 0$ is smaller than $\frac{1}{S}$ again we can be sure that no non-proportional equilibrium can be 'squeezed in' next to the proportional equilibria on the diagonal.

Using (B1)

$$\beta_{(B1)} = \frac{1}{2(1+S)(2s-1)} \times \frac{(4S+2SB)s^2 + (-2S+2+2B)s + 1 + S - \sqrt{\Psi}}{B} \tag{59}$$

we solve this equation for the relevant root and evaluate it at $\beta = 1$ to find

$$s_{(B1)} = -\frac{1}{2}\frac{-2SB - 2SB^2 - B - B^2 - 2 + \sqrt{(4+4B+5B^2+B^4+2B^3+16SB+8B^2S^2+8BS^2+16SB^2)}}{S(B+B^2-2)} \tag{60}$$

The vertical distance to the diagonal is then given as

$$1 - s_{(B1)}(\beta = 1) = \frac{1}{2}\frac{-4S - B - B^2 - 2 + \sqrt{(4+4B+5B^2+B^4+2B^3+16SB+8B^2S^2+8BS^2+16SB^2)}}{S(B+B^2-2)} \tag{61}$$

Now

$$1 - s_{(B1)}(\beta = 1) < \frac{1}{S} \tag{62}$$

will hold if

$$B < -S \tag{63}$$

which cannot hold, or if

$$B > S - 1 \tag{64}$$

which holds by the non-triviality constraint that $B > S + 1$.

What remains to be shown is that we can rank all proportional equilibria in terms of their welfare. This is done as follows:

Any proportional equilibrium implies that $\beta = s$ and so total welfare given as

$$W(\beta, s, B, S) = \frac{1}{2}S\frac{\begin{array}{c}(2S+SB)\,s^2 + (B - 2S - 2SB\beta - 2B\beta)\,s \\ -2B + B\beta + S + SB\beta - 2B^2\beta - 1 + 2B^2\beta^2\end{array}}{(B\beta+1)\,(-B+B\beta-1)}$$

reduces to

$$W = \frac{1}{2}S\frac{\begin{array}{c}\left(2B + SB - 2B^2 - 2S\right)\beta^2 + \\ \left(2B^2 + 2S - 2B - SB\right)\beta + 2B - S + 1\end{array}}{(B\beta+1)\,(B(1-\beta)+1)} \tag{65}$$

with first derivative

$$\frac{\partial W}{\partial \beta} \overset{!}{=} 0 = \frac{1}{2}S\frac{(2\beta-1)\,(B+2)\,(B-S)}{(B\beta+1)^2\,(B(1-\beta)+1)^2} \tag{66}$$

with solution $\beta^* = 1/2$. See that $\partial W/\partial\beta < 0$ for $\beta < 1/2$ and $\partial W/\partial\beta > 0$ for $\beta > 1/2$ and any B, S.

The second order condition is

$$\frac{\partial^2 W}{\partial \beta^2}\,|_{\beta=\beta^*} = 16S\frac{B-S}{(B+2)^3} > 0 \tag{67}$$

as $B > S{+}1$ and hence β^* yields a minimum of the welfare function when we look at proportional equilibria, the welfare worst proportional equilibrium.Hence all equilibria on the diagonal can be ranked unambiguously and hence the efficient equilibrium is unique.■

Proof of Lemma 5:

We show that the participation constraint will remain outside the S2 switching constraint in general.Note that (23) can be written as

$$\frac{2(1-\beta)B - 2(1-s)S + \beta B(1-\beta)B - 2\beta B(1-s)S + 1}{((1-\beta)B+1)(\beta B+1)} \geq 0$$

which can be solved as

$$s \geq \frac{1}{2S + 2BS\beta}\left(-2B + 2S + B^2\beta^2 + 2B\beta - B^2\beta + 2BS\beta - 1\right)$$

The RHS is an increasing and strictly concave function in β as

$$\frac{\partial RHS}{\partial \beta} = \frac{1}{2}\frac{B}{S\left(B\beta+1\right)^2}\left(B^2\beta^2 + 2B\beta + B + 3\right) > 0$$

and

$$\frac{\partial^2 RHS}{\partial \beta^2} = -\frac{B^2\left(B+2\right)}{S\left(B\beta+1\right)^3} < 0$$

It thus takes its maximum value at $\beta = 1$ where the constraint becomes

$$s_{PC} \geq \frac{1}{2S + 2BS}\left(2S + 2BS - 1\right)$$

The $S2$ constraint is from above (16)

$$\frac{(1-\beta)B - (1-s)S}{(1-\beta)B+1} - \Delta_S \geq \frac{\beta B - (sS+1)}{\beta B + 1}$$

where, if we replace the equilibrium rigid prices from above and solve for the binding s we find:

$$s = \frac{1}{4S + 2BS}\left(2S - B + 3B\beta + 2BS\beta + 1\right)$$

which is linearly increasing in β. Again the RHS takes a maximum at $\beta = 1$ for which the condition becomes

$$s_{S2} = \frac{1}{4S + 2BS}\left(2B + 2S + 2BS + 1\right)$$

Now if we takes the vertical difference to the S1 constraint (16) we find:

$$s_{S2} - s_{PC} = \frac{1}{2}\frac{4B - 2S - 2BS + 2B^2 + 3}{S\left(B+2\right)\left(B+1\right)}$$

the sign of which depends on the numerator only. It is increasing in B and decreasing in S. As non-triviality () implies $B > S$ then at most $S = B$ and the numerator becomes $2B + 3 > 0$ so the term is always positive. Due to symmetry the result is sufficient for the participation condition to hold in any equilibrium.■

Proof of Proposition 8:

The transaction prices difference is

$$t_y - t_e = \frac{(1-\beta)B - (1-s)S}{(1-\beta)B+1} - \frac{\beta B - sS}{\beta B + 1} = \frac{B - S - 2B\beta + 2Ss - BS\beta + BSs}{(B(1-\beta)+1)(B\beta+1)} \tag{68}$$

which has the same sign as

$$\sigma \equiv B - S - 2B\beta + 2Ss - BS\beta + BSs = B - S + Ss(B+2) - B\beta(2+S) \tag{69}$$

which is increasing in s.The (S2) constraint gives

$$s_{S2} \geq \frac{1}{S(B+2)}(S - 2B + \Delta_S + B\Delta_S - 1) + B\beta\frac{S + B\Delta_S - B\Delta_S\beta + 3}{S(B+2)} \tag{70}$$

With endogenous and rigid prices the seller charge differential is

$$\Delta_S^* = 3\frac{(B-S)(1-2\beta)}{(B\beta+1)(B(1-\beta)+1)} < 0 \tag{71}$$

and with many buyers $\Delta_S^* \to 0$. Hence what remains is the condition

$$s_{S2} \geq \frac{1}{S(B+2)}(S - 2B - 1) + B\beta\frac{S+3}{S(B+2)} \tag{72}$$

Substituting in the above yields

$$\sigma = -B(1-\beta) - 1 < 0 \tag{73}$$

so this is not sufficient for σ to be positive we need a higher s. Still we know from Proposition 3 that the buyer switching constraints are forcing equilibria to be *proportional* so that $\beta = s$ has to hold. The transaction price differential then reduces to

$$t_y - t_e = (B-S)\frac{1-2\beta}{(B(1-\beta)+1)(B\beta+1)} \tag{74}$$

and as $B > S + 1$ and $\beta > 1/2$ we find that this difference is indeed negative.■

Proof of Proposition 9:

The transaction prices difference is

$$t_y - t_e = \frac{(1-\beta)B - (1-s)S}{(1-\beta)B+1} - \frac{\beta B - sS}{\beta B + 1} = \frac{B - S - 2B\beta + 2Ss - BS\beta + BSs}{(B(1-\beta)+1)(B\beta+1)} \tag{75}$$

which has the same sign as

$$\sigma \equiv B - S - 2B\beta + 2Ss - BS\beta + BSs = B - S + Ss(B+2) - B\beta(2+S) \tag{76}$$

which is decreasing in β.

Assuming a exogenous vertical buyer advantage for eBay (1), the (B2) constraint implies

$$\frac{(1-s)S(1+(1-s)S)}{2(1-\beta)B(1+(1-\beta)B)} - \Delta_B \geq \frac{sS(1+sS)}{2(\beta B+1)(1+\beta B+1)} \tag{77}$$

solving for β implicitly yields

$$2\beta_{B2} \geq 2 + \frac{1}{B} - \frac{\sqrt{\begin{array}{c} B^2(2(1+\beta B)(2+\beta B)\Delta_B + sS + sS^2)\times \\ (2(1+\beta B)(2+\beta B)(\Delta_B + 2S(1-s)(S(1-s)+1)) \\ +sS+sS^2 \end{array}}}{B^2(2(1+\beta B)(2+\beta B)\Delta_B + sS + sS^2)} \tag{78}$$

With sufficiently many buyers $2\beta_{B2} \to 2$ so that $\beta > s$. Also

$$\sigma \equiv B - S + Ss(B+2) - B(2+S) = 2Ss - S - BS - B + BSs. \tag{79}$$

The maximum this can take (at $s = 1$) is

$$\sigma = 2S - S - BS - B + BS = -(B-S) \tag{80}$$

which by non-triviality $B > S + 1$ is *always* negative.■

7 References

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