Selling Cookies*

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Abstract

We develop an integrated model of data pricing and targeted advertising. A monopolistic data provider determines the price to access "cookies," i.e. informative signals about individual consumers' preferences. The demand for data is generated by advertisers that seek to tailor their advertising spending to the match value with each consumer, and by a publisher that wishes to offer targeted advertising space. We characterize the demand for information from both sides of the advertising market, and we derive the optimal pricing policy for the data provider. The data provider profitably restricts information supply to raise the price of information. This may lessen or foster downstream competition, depending on the advertising technology. However, competition among data sellers need not reduce the price of information. Finally, we explore the implications of nonlinear pricing of information, and characterize the exclusive data sales that emerge as part of the optimal mechanism.

KEYWORDS: Data Providers, Online Data, Information, Media Markets, Advertising, Targeting.

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1 Introduction

As the cost of collecting, aggregating and storing data has dramatically decreased over the past decade, new markets for data have emerged in many forms and varieties. Individual firms amass both internal data and external data. The former is proprietary data, which is generated by the transactions of the firm, while the latter is made available by data providers who collect, mine, and resell detailed consumer-level information.[1] Both kinds of data enable firms to improve their competitive position, vis-à-vis potential customers and other firms. Such competitive advantages are mainly driven by an enhanced ability to segment their consumer population, and to more precisely tailor their products and prices to the customer's needs.

Data vendors offering access to databases and data analytics provide information with respect to virtually any economic transaction. To name a few examples, financial data providers, such as Bloomberg and Thomson Reuters, provide real-time and historical financial data. Credit rating agencies, whether for individuals, such as Equifax and Transunion, or for business and governmental agencies, such as Moody's and Standard & Poor's, provide detailed financial information. Data brokers, such as LexisNexis and Acxiom, maintain and grow large databases on individual consumers. Online aggregators such as Spokeo and Intelius mine publicly available data to compile personal data profiles.

The information purchased from these vendors is used by "downstream" firms to reach out to new or existing customers. This may occur through the direct marketing channels (e.g. phone, e-mail) or through intermediated channels (e.g. advertising). These channels are not mutually exclusive, yet they might differ in their implications for the value of precision, for the incentives to acquire information, and for potential violations of the users' privacy. The former channel is the main use of data purchased through vendors of detailed user profiles such as Acxiom or Linkedin.[2] The latter channel is especially relevant for online display advertising markets.

Several platforms, such as BlueKai, promote online marketplaces for the exchange of user data. While this has also led to severe concerns over users' privacy on the web, these platforms monetize the match-making potential by linking websites that monitor traffic to advertisers who are interested in information about individual users. This is possible in part because many advertising networks and exchanges allow advertisers to "bring their own data" when specifying their real-time bids for selected impressions.[3] This in turn enables advertisers to better target their messages on other websites, by tracking or following their desired users across different pages. Sometimes, user

[1] Data acquisition often takes place online through "cookies." For example, more than half of the sites examined by a 2010 Krux.com / Wall Street Journal study installed 23 or more "third party" cookies on visiting users' computers. Dictionary.com installed the most, placing 159 third-party cookies. (*The Web's New Gold Mine: Your Secrets,* the Wall Street Journal, July 30, 2010.)

[2] Through Acxiom's Consumer Data Products Catalog, "corporate clients can buy data to pinpoint households that are concerned, say, about allergies, diabetes or "senior needs." Also for sale is information on sizes of home loans and household incomes." (*You for Sale: Mapping, and Sharing, the Consumer Genome,* the New York Times, June 16, 2012.)

[3] Real-time bidding accounts for a growing share of internet advertising revenues: Forrester Research estimates that real-time bidding will constitute 18% of the online display-ad market this year, up from 13% last year. (*Real-Time Auctions Drive Rise in Online Tracking,* the Wall Street Journal, June 17, 2012.)

behavior data is already *de facto* incorporated in the advertising products, for example in the form of a *retargeting* option. This is often the case with large advertising networks who collect their own data, such as Yahoo or Google.

In the present paper, we investigate the role of data management platforms on the price and allocation of information about individual consumers. We seek to provide a framework to address questions about the data industry and in particular about its interaction with online and offline advertising markets. Thus, we develop a flexible model that applies to direct marketing strategies, while at the same time capturing the key features of online display advertising.

In our framework, we consider heterogeneous consumers and firms. Crucial to our analysis is a match-value function that captures the potential surplus that can be generated by a given firm and a given consumer. In our baseline model, we consider horizontal heterogeneity in match values. This could emerge, for example, from a market with horizontally differentiated products, and heterogeneous preferences of consumers over the product space. In order to realize their potential match value, firms must contact consumers, and they do so through a single publisher, which is best seen as a large advertising network (or a collection of websites). In the online-advertising interpretation, we abstract from the details of the auction mechanisms used to allocate each impression, and we summarize the marginal cost of advertising through a constant price.

Advertisers wish to tailor their spending to the match value with each consumer. Because they are unable to do so under prior information only, they are willing to pay for additional signals. The lack of information about individual users induces two kinds of risk: on the one hand, firms risk to waste advertising on low-value matches; on the other hand, they leave money on the table by not pursuing high-value matches as intensively as they could.

The key innovation of our framework is to allow for selective data purchases. In particular, firms may buy bits of information about specific user types, so to target their marketing efforts or their advertising spending. Online, targeting is made possible by cookie matching. In our model, we focus on the sale of information "by the unique user."[4] Formally, this means the set of information structures available to an advertiser consists of specific partitions of the space of match values, corresponding to the ability to "identify" specific users.

Information about consumers is generated by the data purchased from a fourth-party data provider (i.e., neither a competing firm nor the consumer). As a consequence, firms make their advertising decisions under an information structure that has been determined "upstream," *i.e.* in the market for data. Another key feature of our analysis is the pricing structure of information. We initially restrict the data provider to set a unit price for each information bit, or cookie, and later explore alternative data pricing mechanisms. The insights obtained under linear pricing are informative of other models of data sales with market power. These include: auctions with a reserve price; imperfectly competitive markets; and nonlinear monopoly pricing in the presence of vertical buyer heterogeneity.

[4] This closely follows the real-world online markets, where cookies are typically sold at a price per unit of data ("per unique," or "per stamp").

1.1 Overview of the Results

Our baseline model treats both prices of cookies and advertising as given, and characterizes advertisers' demand for information. We show that advertisers choose to purchase information about two convex sets of consumers, i.e. those with the highest and lowest match values. When advertisers encounter a consumer they have no information about, they cannot tailor their spending, and choose a constant action. Therefore, they try to minimize the variance of match values within the set of "unknown" consumers. Furthermore, under quite general conditions, the data-buying policy takes the form of a single cutoff match value, and advertisers buy information about all users above (or below) the cutoff. The optimality of each data-buying policy is related to the two sources of mismatch risk, and can be established based on properties of the complete-information profits alone: in particular, if the curvature of the profit function is increasing (positive third derivative) then the wasteful-ads risk is less prominent, and advertisers buy data about the highest-value consumers. The opposite is true for the case of decreasing curvature.

In our model, the socially optimal allocation of information involves providing full disclosure of all match values to the advertisers. Not surprisingly, a monopolist data provider maximizes profits by restricting the supply of information and raising the price of cookies. We focus on the monopolist's response to changes in the precision of the information. More precise information corresponds to mean-preserving spreads of the distribution of match values. We identify supermodularity conditions under which the monopolist chooses to profit from a larger supply of (very high- and very low-value) high-demand cookies, and thus increases the price of data. On the contrary, the comparative statics of the monopoly price with respect to the cost of advertising depend on the curvature of the match cost function. Intuitively, the cost of advertising reduces both the payoff advertisers can obtain through better information, and their uninformed payoff. The two contrasting effects on the demand for information are related to the optimal cookie-buying policy. A picture emerges whereby if advertisers choose to purchase the highest-value signals, they are also willing to pay less when the cost of advertising increases.

We then extend our model in three main directions: by allowing publishers to purchase information; by exploring different structures for the markets for data and for advertising; and by introducing more sophisticated data-pricing techniques.

In the online world, user-level information is often bundled with advertising space itself (though the billing is by and large separate). Thus, we move towards an integrated model of data pricing by allowing the publisher to buy the data as well. We assess the profitability of buying data for either side of the advertising market, and quantify the value of information based on its strategic use "downstream." The incentives to acquire information depend on the characteristics of the matching cost function. Most importantly, while advertisers have a positive willingness to pay for information, the publisher may not. In particular, the publisher wishes to buy data only when the complete-information demand for advertising is convex in the value of the match. Furthermore, the interaction between all three parties can lead to vivid segmentation patterns for the online advertising space. In particular, we find that segmentation of advertising space can occur in equilibrium, where premium impressions are sold by an informed publisher, and information about residual impressions is purchased by the advertisers.

We turn to the analysis of market structures by endogenizing the unit price of advertising space. We assume that each user has a fixed total attention span, which limits the aggregate intensity of firms' contacts with a given consumer. In the case of online advertising, this upper bound may be equivalently interpreted as the amount available advertising space. The equilibrium price of advertising is determined through market clearing. Perhaps not too surprisingly, the data provider's optimal strategy consists of raising the price of data. However, two completely different mechanisms may lead to the same conclusion, depending on how the cost of advertising is affected by the distribution of information across advertisers. Conditions on the advertising technology determine whether the data provider wishes to augment or reduce the "congestion" of advertising space. In the more intuitive case, the data provider can profitably restrict the supply of information to lessen downstream competition and appropriate a larger fraction of the surplus.

In a related model, we explore possibility of consumers selling their own information. Formally, this corresponds to a model with a fragmented population of sellers, each one having an exclusive over one signal. Surprisingly, we find that concentrating sales in the hands of a single data provider is not necessarily detrimental to social welfare, and that prices are higher under fragmentation, independently of any privacy concerns.

We conclude our paper by extending the scope of the pricing mechanisms available to the data provider. We introduce nonlinear pricing and show that the data provider can screen vertically heterogeneous advertisers by offering to sell subsets of the database at a decreasing marginal price. The optimal nonlinear price determines endogenous exclusivity restrictions on a set of "marginal" cookies: in particular, second-best distortions imply that some cookies that would be profitable for several advertisers are exclusively bought by a subset of high-value advertisers. Furthermore, if we restrict attention to binary actions for the advertisers (contact, no contact), we can show that a cookie-based information structure policy (perfectly revealing match values above a threshold only) is a revenue-optimal mechanism. In turn, this optimal mechanism can be decentralized by charging a nonlinear tariff for access to portions of the monopolist's database.

1.2 Related Literature

The issue of optimally pricing information in a monopoly, as well as a competitive market has been addressed in the finance literature since the seminal contributions by Admati and Pfleiderer (1988), Admati and Pfleiderer (1990) and Allen (1990). The main difference between the standard approach to pricing financial information and our paper's is in the bundled vs. unbundled information sales. In particular, previous papers have focused on the pricing an information structure that generates an informative signal for all realizations of the state of the world. This is also the case in the work of Sarvary and Parker (1997), who model competing consulting companies, and of Anton and Yao (2002), who model the sale of information to competing parties. In contrast, we focus on pricing of information bits that allow firms to recognize specific user types. Iyer and Soberman (2000) address the issues of selling information to firms competing in a differentiated-products duopoly. They focus on the implications for upstream and downstream revenue of selling heterogeneous pieces of information, corresponding to valuable product modifications. Our model also focuses on selling differentiated information bits, which can take different interpretations depending on our match value function.

The related literature on the optimal choice of information structures is recent. Bergemann and Pesendorfer (2007) consider the design of optimal information structures within the context of an optimal auction. There, the principal simultaneously controls the design of the information and the design of the allocation rule. More recently, Kamenica and Gentzkow (2011) consider the design of the information structure by the principal when the agent will take an independent action on the basis of the received information. Rayo and Segal (2010) examine a similar question in a model with multidimensional uncertainty and private information on the agent's cost of action.

The implications of specific information structures in auctions, and their implication for online advertising market design, are analyzed in recent work by Abraham, Athey, Babaioff, and Grubb (2012) and Celis, Lewis, Mobius, and Nazerzadeh (2012). Both papers are motivated by asymmetries in bidders' ability to access additional information about the object for sale. Consequently, they examine the role of the distributions of valuations resulting from the private acquisition of data by a single bidder. In particular, Abraham, Athey, Babaioff, and Grubb (2012) focus on second price auctions in a common value environment, while Celis, Lewis, Mobius, and Nazerzadeh (2012) propose an approximately optimal mechanism in a private values model. In a closely related contribution to these two papers, Kempe, Syrganis, and Tardos (2012) study the first-price, common-value auction with asymmetrically informed bidders.

In our earlier work, Bergemann and Bonatti (2011), we analyzed the role of (exogenous) information structures on the competition for advertising space. In this strategic environment, more precise information about consumers' locations allows for better targeting of advertisement messages, and improves firms' revenues. It may, however, be detrimental to the seller of the advertising space. With a related motivation, in the present paper we investigate further the role of information on industry profits, and the scope for profitable information intermediaries. We depart from the model in Bergemann and Bonatti (2011) along two major directions: first, we endogenize the information structure and investigate the price of the information itself; second, we consider firms operating on interdependent product markets. In particular, we relate the mode of product-market competition to the profitability of different information provision policies. The role of information in a specific model of competition is also analyzed in Ganuza (2004), while relevant notions of ranking of information structures are developed in Johnson and Myatt (2006) and in Ganuza and Penalva (2010).

In parallel work, we complement the individual tailoring of actions to a specific signal with a model of market tailoring, in which competing firms purchase information about market conditions. To the best of our knowledge, our paper is the first to address the question as to what happens if information can be made available (at a price) by a third party. To the extent that the database provides the competitor with a certain information structure, we might say that the database, by choice of the information structure, induces a specific game of incomplete information among the competitors (while being consistent with the prior information of the agents). The associated class of games is analyzed in Bergemann and Morris (2011b) and Bergemann and Morris (2011a) as a problem of robust prediction and an associated equilibrium concept, referred to as Bayes correlated equilibrium.

Finally, our work is related to model of noisy information in an oligopoly environment. In particular, the value of information has been extensively studied in the context of information sharing by oligopolists, see Raith (1996) for a general model and Vives (1999) for a summary of the results in the literature.

2 Model

2.1 Matching and Preferences

We consider a symmetric model with a unit mass of consumers i and advertisers j, a single publisher, and a monopolistic data provider. Each (i,j) consumer-firm pair generates a potential match value $v \in V = [v_L, v_H]$. Heterogeneity in match values is purely along an horizontal dimension. To capture this, we impose a symmetry restriction on the distribution of match values. In particular, we assume that for each consumer i, the distribution of match values $v\,(i,j)$ with firms j is given by $F\,(v)$. Likewise for each firm j, match values with consumers i are identically distributed according to $F\,(v)$. Examples of distributions that satisfy our symmetry assumption include: i.i.d. match values across consumer-firm pairs; and uniformly distributed firms and consumers around a unit-length circle, with match values given by a decreasing function of distance.

Firms must advertise to consumers in order to realize the potential match value. Advertising space can be purchased from the publisher at a constant marginal cost c. The publisher allows firms to target messages to each consumer i, thereby ruling out duplication risk, but the publisher has no information about pair-specific match values other than the prior distribution.

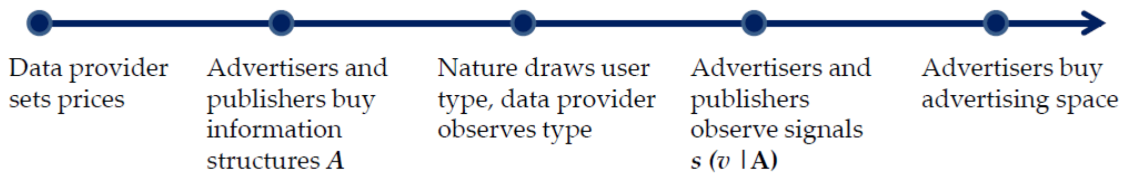
The advertising technology is summarized by the match cost function $m\,(q)$, which denotes the amount of advertising space m required to generate a match with probability q. The value of an actual match is then given by $v\,(i,j)$. Equivalently, we may think of q as the intensity of the contact achieved by firm j's advertising, i.e. q is not restricted to $[0,1]$. The complete-information profits of a firm generating a contact of intensity q with a consumer of value v are given by

$$\pi\,(v,q) \triangleq vq - cm\,(q)\,.$$

2.2 Information and Timing

We formulate the information acquisition problem of a firm purchasing data about its consumers as the choice of an information structure H from a set $\mathcal{H}$. Each information structure determines a distribution of signals $s \in S$ that are informative about v. The firm uses the signals to tailor

FIGURE 1: TIMING



its advertising effort q. Finally, each information structure H is sold at a price (exogenous for now) $p(H)$. We adopt the following canonical formulation for a decision maker's (i.e. a firm's) information acquisition problem:

$$\max_{H \in \mathcal{H}} \left[\mathbb{E}_H \left[\max_q \{ \mathbb{E}[v \mid s] q - cm(q) \} \right] - p(H) \right], \tag{1}$$

where the first expectation is taken with respect to signals s, conditional on the information structure H.

We now impose restrictions on the set of information structures and on the related price functions. In particular, we assume that data about individual users is sold in the form of cookies, i.e. consumer-specific signals. Thus, the set of available signals coincides with the set of match values. In turn, the set of available information structures coincides with the set of measurable subsets of V. If firm j chooses the set of signals $A_j \subset V$, then it is endowed with the following signal structure:

$$s(v; A) = \begin{cases} v & \text{if} \quad v \in A_j, \\ \emptyset & \text{if} \quad v \notin A_j. \end{cases}$$

Therefore, we are introducing a model where agents can choose which realizations of a payoff-relevant variable they want to acquire information about. In the rest of the paper, we shall denote by "cookie v" the signal $s = v$ that allow to identify consumers i such that $v(i, j) = v$. In other words, if firm j purchases cookie v, then the signal v belongs to the set A_j. Finally, we assume that cookies v are sold at a constant linear price $p(i) = p$.

The timing of the model is summarized in Figure 1.

3 Demand for Advertising and Information

When facing a prospective consumer i, each firm chooses the advertising intensity to maximize expected profits given the available information. Therefore, the demand for advertising space of a firm that acquired signals $A \subset V$ is given by

$$q^*(v; A) = \arg\max_q \mathbb{E}_A [\pi(v, q)] = \arg\max_q [\mathbb{E}_A[v] q - cm(q)]. \tag{2}$$

We denote the complete-information demand for advertising space by $q^*(v)$. Therefore, for all $v \in A$, the firm's demand for advertising satisfies

$$v = cm'(q^*(v)). \tag{3}$$

Whenever $v \notin A$, advertisers choose a constant advertising intensity $\bar{q}$ that satisfies

$$\mathbb{E}[v \mid v \notin A] = cm'(\bar{q}). \tag{4}$$

Therefore, under information structure A, the firm chooses

$$\bar{q} = q^*(\mathbb{E}[v \mid v \notin A]). \tag{5}$$

We can describe the firm's profits $\pi(v, q)$ under complete information and under prior information, by letting $A = V$ and $A = \emptyset$ respectively. In particular, we define the complete-information profits as

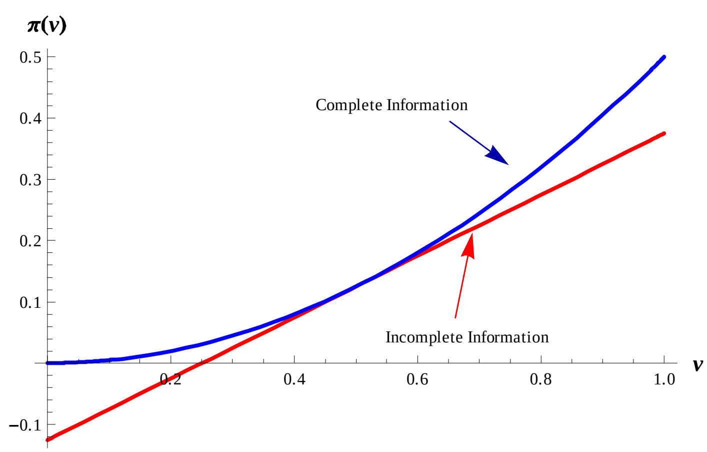
$$\pi(v) \triangleq \max_q \pi(v, q) = vq^*(v) - cm(q^*(v)).$$

Note that $\pi(v)$ is strictly convex, as we started with a linear objective. Under prior information, profits are given by the linear function

$$\pi(v, \bar{q}) = vq^*(\mathbb{E}[v]) - cm(q^*(\mathbb{E}[v])).$$

Figure 2 describes both profit functions for the case of quadratic match costs $cm(q) = q^2/2$ and match values uniformly distributed on the unit interval. As intuitive, under prior information, the

FIGURE 2: COMPLETE- AND INCOMPLETE-INFORMATION PROFITS



firm chooses excessive (wasteful) advertising vis-à-vis low-value consumers and insufficient advertising vis-à-vis higher-value consumers. The firm therefore has a positive willingness to pay for information.

To characterize the demand for cookies, we specialize the information acquisition problem in (1) to our setting. Given the prices of signals p and advertising space c, each firm solves the following problem:

$$\max_{A}\left[\int_{v_L}^{v_H}\left(vq^*\left(v;A\right)-cm\left(q^*\left(v;A\right)\right)-\mathbf{1}_{v\in A}p\right)dF\left(v\right)\right].$$

As Figure 2 suggests, the value of information is highest for extreme match values. Our first result establishes the optimality of excluding a connected set of cookies from purchase.

Lemma 1 (Cookie Intervals)

For any c and p, there exist thresholds $v_L \leq v_1 \leq v_2 \leq v_H$ such that each firm purchases all cookies in the set $A = [v_L, v_1] \cup [v_2, v_H]$.

We can now reformulate the firm's information-acquisition problem as follows:

$$\begin{aligned}&\max_{v_1,v_2}\int_{v_1}^{v_2}\left[p-v\left(q^*(v)-\bar{q}\right)+c\left(m\left(q^*(v)\right)-m\left(\bar{q}\right)\right)\right]dF\left(v\right), \qquad (6)\\ &\text{s.t. } cm'\left(\bar{q}\right)=\mathbb{E}\left[v \mid v\in[v_1,v_2]\right].\end{aligned}$$

In other words, the firm chooses the set of cookies to exclude, which yields a marginal benefit of p, taking into account the effect of cookie purchases on its own inference problem whenever uninformed. The average excluded type then determines the uninformed demand for advertising space $\bar{q}$, which in turn affects the value of information.

3.1 One-Sided Purchases

Note that a necessary condition for *exclusion* of an interior interval (i.e. $v_1 > v_L$ and $v_2 < v_H$) is that the marginal value of information is equal to the price of cookies at both extremes. The value of information about type v is given by

$$v\left(q^*\left(v\right)-\bar{q}\right)-c\left(m\left(q^*\left(v\right)\right)-m\left(\bar{q}\right)\right),$$

where $\bar{q}$ is the optimal intensity vis-à-vis the average type in the exclusion interval. For example, suppose types are uniformly distributed, so that the average type is the midpoint $(v_1+v_2)/2$. In order for the gains from information at v_1 and v_2 to be equalized, the curvature of the indirect profit function must be constant. Thus, as in the case of Figure 2, complete-information profits must be quadratic in v. The following result establishes sufficient conditions under which firms demand cookies in a single interval, by excluding prospects at the top or at the bottom of the distribution only.

Lemma 2 (Single Cookie Interval)

1. *If both $m'(q)$ and $F(v)$ are convex, then $A(c,p) = [v_L, v_1(c,p)]$.*

2. *If both $m'(q)$ and $F(v)$ are concave, then $A(c,p) = [v_2(c,p), v_H]$.*

Examples of match cost functions with concave marginal costs include power cost functions, $m(q) = q^a$ with $a < 1/2$. Examples of convex marginal costs include those derived from an exponential matching technology, i.e. $m(q) = -a \ln(1-q)$, with $a > 0$, and power cost functions $m(q) = q^a$, with $a > 1/2$. The knife-edge case of quadratic costs and uniformly distributed match values (which is covered in both parts (1.) and (2.) of Lemma 2) has the interesting property of being "location-free." That means the returns from exclusion of an interval of cookies are a function of its length only.

The main intuition for this the single-interval result can be traced back to the two sources of the value of information, i.e. wasteful advertising for low types, and insufficient advertising for valuable consumers. Lemma 2 relates the potential for mismatch risk to the properties of the match cost function. In particular, when the curvature of the cost function is increasing, it becomes very expensive to tailor advertising purchases to high-value consumers. In other words, the risk of insufficient advertising is not very high, given the cost of advertising space. The firm then purchases cookies related to lower-valued consumers. We can therefore specialize problem (6) to the low-v cookies purchases, and derive the inverse demand curve in terms of the marginal cookie v_1 as follows:

$$\begin{aligned} p(v_1) &= v_1 (q^*(v_1) - \bar{q}(v_1)) - c(m(q^*(v_1)) - m(\bar{q}(v_1))), \\ \bar{q}(v_1) &= q^*(\mathbb{E}[v \mid v > v_1]). \end{aligned} \tag{7}$$

Conversely, when the curvature of the cost function is decreasing, profit levels are "steeper" at the top, meaning the insufficient advertising risk is more relevant. Thus, firms purchase information about high-value users. The corresponding demand for information is given by

$$\begin{aligned} p(v_2) &= v_2 (q^*(v_2) - \bar{q}(v_2)) - c(m(q^*(v_2)) - m(\bar{q}(v_2))), \\ \bar{q}(v_2) &= q^*(\mathbb{E}[v \mid v < v_2]). \end{aligned} \tag{8}$$

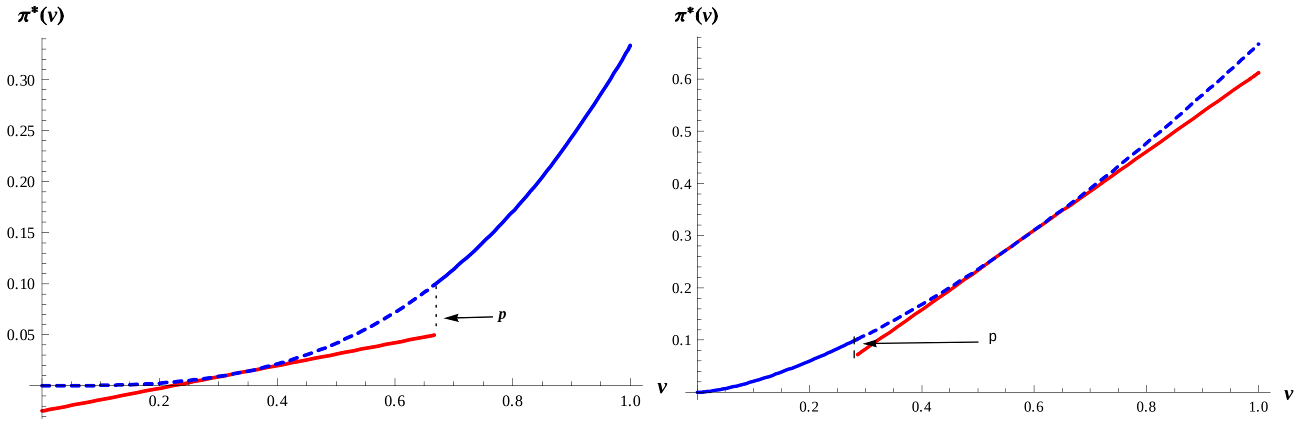
Figure 3 shows the complete-information profits and the actual profit levels under one-sided purchases. We assume uniformly distributed values, and consider power cost functions $cm(q) = q^b/b$, with $b \in \{3/2, 3\}$, respectively. In the first panel, firms purchase cookies in the set $A = [2/3, 1]$, while in the second panel $A = [0, 2/7]$.

3.2 Two-sided Purchases

We now derive conditions under which advertisers purchase both high- and low-v cookies. In particular, we find that quadratic complete-information profits and symmetric density $f(v)$ yield a rich set of examples. In this case, the set of excluded types is always an interval centered on the prior mean of v. We make the result formal in the following proposition.

Lemma 3 (Two-Sided)

Figure 3: High-v and Low-v Cookie Purchases



Suppose match values are distributed symmetrically with mean $\bar{v} = (v_L + v_H)/2$, *and matching costs are quadratic,* $m(q) = q^2/2$. *Advertisers purchase all cookies outside the set* $[\bar{v} - 2\sqrt{cp}, \bar{v} + 2\sqrt{cp}]$.

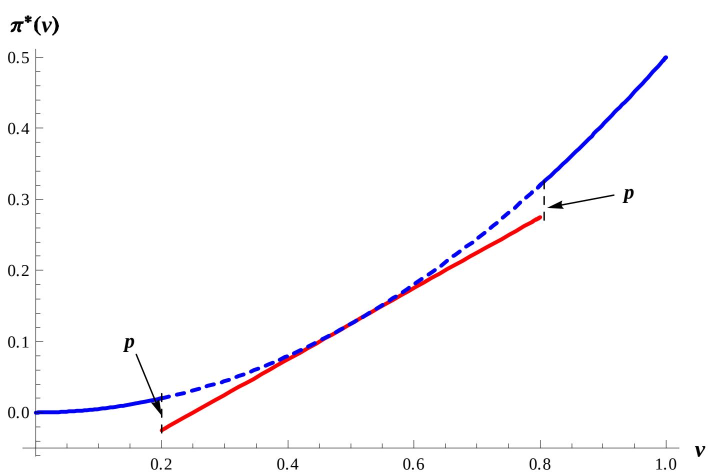
We can derive the inverse demand for information as an implication of Lemma 3. In particular, it is given by

$$p(x) = x^2/4c, \tag{9}$$

where $x = \bar{v} - v_1 = v_2 - \bar{v}$ is the half-range of the excluded cookies.

Figure 4 illustrates the demand for cookies and the resulting profit levels in the symmetric-quadratic environment.

Figure 4: Two-Sided Symmetric Purchases



Under the conditions of Lemma 3, the expected value of v when uninformed is equal to the prior mean $\bar{v}$, regardless of the measure of cookies bought by the firm. This is a key difference with the one-sided case, where the number of cookies influences the marginal value of information. In the two-sided case, the measure of signals purchased does not influence the uninformed action $\bar{q}$. Because for amount of data purchased the uninformed quantity is a constant $\bar{q} = q^*(\bar{v})$, the marginal value of information on type $\hat{v}$ is constant. A key implication is that the willingness to pay for any cookie v is independent of the distribution of types. The resulting demand curve for information is clearly affected by the distribution of types, but only through the quantity of cookies demanded at any price. We will take advantage of this separation when analyzing the comparative statics of the demand for cookies.

4 Comparative Statics of Demand

In this section, we analyze the response of the demand for information to changes in the distribution of match values, and in the cost of advertising space. We present our results in terms of the (direct and inverse) demand function for information, so to provide a basis to analyze for the role of alternative market structures later on (e.g. monopoly vs. competition in data sales, exogenous vs. endogenous price of advertising).

4.1 Information Precision

In order to explore the effects of the precision of the data provider's information, it is convenient to interpret v as a posterior mean. In particular, we assume the true match value is unknown to all, but the data provider has access to an informative signal. The data provider's signals induce posterior means v distributed according to $F(v)$. We may then relate the distribution F to varying degrees of information precision.

We model increasing precision as rotations around the mean of the match-value distribution. (This stochastic order implies mean-preserving spreads, see Johnson and Myatt (2006).) In particular let k denote the spread parameter. We then assume that $\mathbb{E}[F(v,k)] = \bar{v}$ for all k, and that $F(v,k)$ is increasing (decreasing) in k for all $v \leq (>)\, \bar{v}$. Let $A(p,k)$ denote the solution to the advertisers' problem (6) when cookies are distributed according to $F(v,k)$ and the price of cookies is p. The quantity of data demanded is then given by the measure of the set $A(p,k)$ under the distribution $F(v,k)$.

Proposition 1 (Information Precision)

1. *Under one sided purchases, advertisers' willingness to pay for the marginal cookie $v \in A(p,k)$ is increasing in k.*

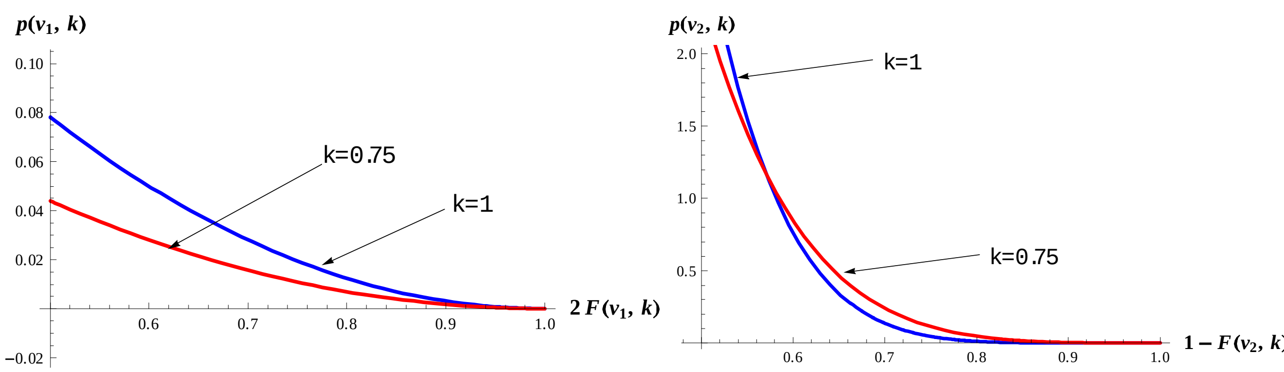
2. *Under one sided purchases, the quantity of data demanded at a price p is increasing in k if $\bar{v} \notin A(p,k)$.*

3. *Under two-sided purchases, advertisers' willingness to pay for the marginal cookie is constant and the quantity of data demanded is strictly increasing in k.*

Intuitively, an increase in the spread places more probability on the tails of the distribution. Let us consider one-sided and two-sided purchases separately. In the case of one-sided purchases, the uninformed quantity level will be much closer to the extreme v_L or v_H, thereby raising the willingness to pay for the marginal cookie v_1 or v_2. Whether this implies a higher or lower quantity of data sold, it will depend on the level of purchases. In particular, if firms are buying cookie $\bar{v}$, then a more spread out distribution implies less data sales, for a fixed v_2. Under two-sided purchases, as k increases, more probability mass is placed on the tails, and the marginal willingness to pay increases (so that the range of excluded cookies $[v_1, v_2]$ is decreasing in k). A fortiori, the total demand for information is higher.

We now consider an example where the support of match values varies with k. According to the informative-signals interpretation, suppose the true types are uniformly distributed on the unit interval, and that the monopolist observes truth-or-noise signals with precision k. The distribution of posterior means is then uniform on $[(1-k)/2, (1+k)/2]$, with a higher k implying a more spread-out distribution. In Figure 5, we plot the inverse demand curve for the case of power cost functions $m(q) = q^b$, with $b \in \{6/5, 2\}$, and $k \in \{3/4, 1\}$. Notice that, in the left panel, advertisers make two-sided purchases, whereas in the second panel, advertisers purchase cookies in $[v_2, v_H(k)]$.

Figure 5: Demand Curves for $v \sim U[(1-k)/2, (1+k)/2]$



4.2 Cost of Advertising

We now turn to the role of the price of advertising space c, and we ask whether the cost of advertising reduces the demand for data. On the one hand, a lower price of advertising space increases the advertisers' downstream surplus. On the other hand, it decreases the value of information (as advertising space is cheaper for the uninformed buyers as well). Let $A(p, c)$ denote the solution to the advertisers' problem (6) given the prices of cookies and advertising. Under both one-sided and two-sided purchases, we establish the following comparative statics result.

Proposition 2 (Cost of Advertising)

The advertisers' willingness to pay for the marginal cookie $v \in A(p, c)$ is decreasing (increasing) in the cost of advertising c if

$$\frac{d}{dq}\left(\frac{m''(q)}{m'(q)}\right) \leq (>)\, 0.$$

Combining this result with Lemma 2, we obtain sufficient conditions on the data purchasing strategy under which the cost of advertising has a monotone effect on the price of data.

Corollary 1 (Conflicting Interests)
If $\pi'(v)$ is strictly convex and $F(v)$ is weakly concave, the demand for cookies is decreasing in c.

In particular, if match values are uniformly distributed, when the highest-valued cookies are purchased by each firm the data provider benefits from a lower price of advertising space. Conversely the demand for data is *increasing* in c only if low-v cookies are purchased. The intuition is that when $m'''(q) \leq 0$, a higher cost c reduces the "outside options" for advertisers. In other words, a the mismatch risk is more relevant when high-v signals (which require high contact-intensity) are not purchased.

5 Monopoly Pricing

We now turn attention to the monopoly price of cookies. We analyze the case of one-sided and two-sided signal purchases separately. For both cases, we analyze the effect of information precision on the price and allocation of cookies. We then turn to the role of the price of advertising space.

Recall the advertisers problem specified in (6), and suppose the conditions of Lemma 2 for one-sided purchases apply. Thus, the inverse demand for data is given by (7) and (8). The data provider's profits as a function of the marginal cookies are then given by

$$\Pi(v_1) = p(v_1) F(v_1),$$

in the case of low-value purchases, and by

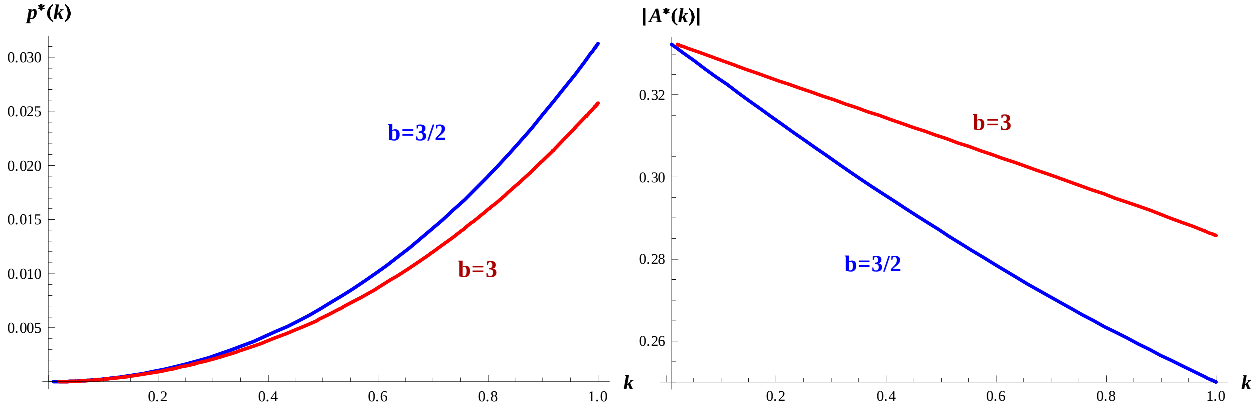
$$\Pi(v_2) = p(v_2)(1 - F(v_2)),$$

in the case of high-v purchases.

The effects of information precision on the monopoly price under one-sided purchases are quite complex. On the one hand, information precision affects advertisers' willingness to pay for the marginal signal. On the other hand, the spread of the values distribution directly impacts the quantity of data sold, for a fixed marginal signal. We therefore revisit the example with uniformly distributed types over a varying support $[(1-k)/2, (1+k)/2]$. For the case of power cost functions, the spread k has a positive impact on the monopoly price, and a negative impact on the quantity of data sold. This result does not depend on the low-value vs. high-v purchases. In Figure 6, we show both cases by letting $m(q) = q^b$, with $b \in \{3/2, 3\}$. In particular high-v cookies are purchased when $a = 3/2$ (i.e. when marginal costs $m'(q)$ are concave).

More precise results can be obtained for the two-sided case. We maintain the assumptions of Lemma 3, namely the *symmetry* of $F(v)$ and the *quadratic* profits. An important feature of the symmetric quadratic environment is that the distribution of types affects the price through the

FIGURE 6: MONOPOLY PRICE AND QUANTITY, $v \sim U\left[(1-k)/2, (1+k)/2\right], m(q) = q^b$



measure of cookies sold only, and not through the marginal value of information. To understand the role of precision, we again consider distributions ordered by rotations around a constant means $\bar{v}$, with a higher value of k corresponding to a more spread out distribution.

Because of the quadratic costs assumption, the marginal willingness to pay for a set of cookies that excludes the interval $[\bar{v} - x, \bar{v} + x]$ is given by $p\left(x\right) = x^2/4c$. Using the symmetry assumption, the data provider's profits may be written as

$$\Pi\left(c\right) = \max_x \left[F\left(\bar{v} - x, k\right) \frac{x^2}{2c}\right]. \tag{10}$$

Let $x^*\left(c, k\right)$ denote the solution to problem (10).

Proposition 3 (Rotation Order, Two-Sided Purchases)

1. *If the distribution $F\left(v, k\right)$ is log-submodular in v and k, the monopoly price of cookies is increasing in k.*

2. *If $F\left(v, k\right)$ is log-supermodular, the price of cookies is decreasing and the quantity of data sold is increasing in k.*

3. *If $x^*\left(c, k\right)$ is low enough, the monopoly price of cookies is increasing in k.*

Part (3.) follows from the observation that the rotation order implies that $F\left(v, k\right)$ is locally log-submodular close to the mean. Thus, if the price is ever low enough that the monopolist sells nearly every cookie (possibly when the distribution approaches a mass point at $\bar{v}$), then the price is increasing in the information precision. Clearly, when the monopoly price is decreasing in k, we know that a fortiori the quantity sold is increasing, but not viceversa.

Under our symmetry assumption, some examples of log-submodular distribution include: the symmetric Beta distribution with parameter $1/k$; the truncated normal distribution; and several "modified triangular" distributions, such as the two-sided (and truncated) exponential and Pareto distributions centered around $1/2$.

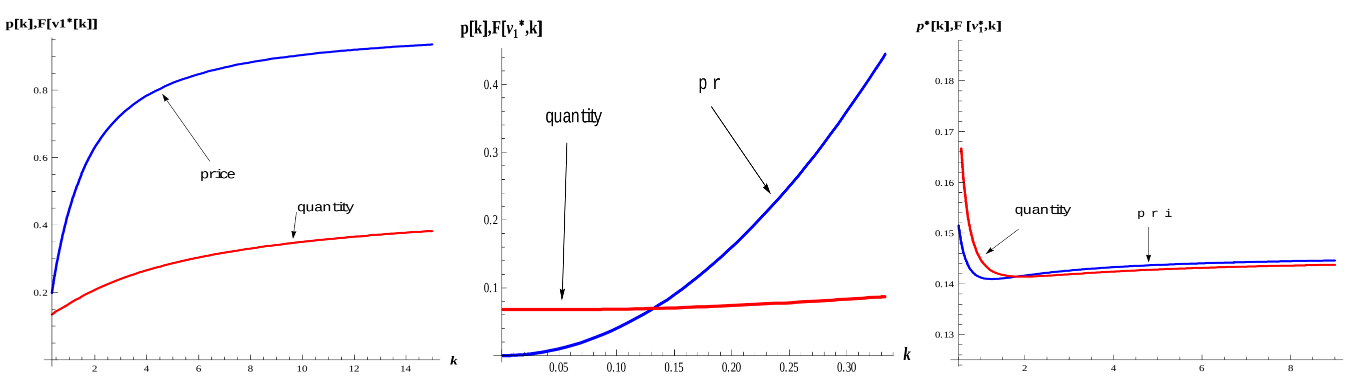
The normal distribution illustrates a sharper characterization that applies under a stronger stochastic ordering. Let $F(v,k)$ be ordered by increasing variance, i.e. $F(v,k) \triangleq G((v-\bar{v})/\sigma(k))$ with $\sigma'(k) > 0$. The following result then follows from the observation that $\partial F(v,k)/\partial k = -\sigma(k)^{-1}\,\partial F(v,k)/\partial v$ in the proof of Proposition 3.

Corollary 2 (Variance Order)
If $F(v,k)$ is ordered by increasing variance, the monopoly price of cookies is increasing in k if $G(\cdot)$ is log-concave.

Notice that the log-submodularity requirement differs from the standard result for comparative statics of the monopoly price. In particular, it applies to our data sales model under two-sided purchases only. This is due to both the independence of $p(v)$ from k, and to the quadratic costs, which imply the argument of $F(\cdot)$ in (27) is linear in x.

In Figure 7, we consider the monopoly prices and quantities as a function of the spread parameter k. In particular, we consider a symmetric Beta distribution with parameters $a = b = 1/k$; a truncated Laplace distribution with parameter $1/k$; and a modified triangular distribution on the unit interval.[5] In all three cases, we can interpret the spread of the distribution as the precision of the data provider's information when the true underlying match value is binary $v \in \{0,1\}$. The Beta and exponential distributions are log-submodular for all $k > 0$ and v, while the modified triangular is log-submodular if $v > 1/4k$. Notice that the monopoly quantity responds differently

Figure 7: Monopoly Price and Quantity, Rotation-Ordered $F(v,k)$



Symmetric Beta | Truncated Laplace | Modified Triangular

from the price, and it may increase or decrease even if the price is increasing.

[5] The latter distribution has the following density,

$$f(v,k) = \begin{cases} \left(\frac{1}{k} - \frac{2v}{k^2} + \frac{1}{v^2}\right) e^{-\left(\frac{1}{k} - \frac{2v}{k^2} + \frac{1}{v^2}\right)} B(k) & \text{for} \quad v \in \left[0, \frac{1}{2}\right], \\ \left(\frac{1}{k} - \frac{2(1-v)}{k^2} + \frac{1}{(1-v)^2}\right) e^{-\left(\frac{1}{k} - \frac{2(1-v)}{k^2} + \frac{1}{(1-v)^2}\right)} B(k) & \text{for} \quad v \in \left[\frac{1}{2}, 1\right], \end{cases}$$

where $B(k)$ is a constant that ensures $f(v,k)$ integrates to one.

We conclude this section by analyzing how the price of advertising affects the monopoly price of data.

Proposition 4 (Cost of Advertising)

1. *Under one-sided purchases with uniformly distributed match values, the monopoly price is decreasing (increasing) in the cost of advertising if*

$$\frac{d}{dq}\left(\frac{m''(q)}{m'(q)}\right) \leq (>)\, 0.$$

2. *Under two-sided purchases, the monopoly price is inversely proportional to the cost of advertising.*

Note that part (1.) of the last result requires that the composite function $F(v^*(p,c))$ is log-submodular. However, we can show (based on Proposition 2) that the cutoff function $v^*(p,c)$ is strictly submodular. Thus, the power transformation preserves the log-submodularity of the cutoff function.

6 Equilibrium Allocation of Cookies

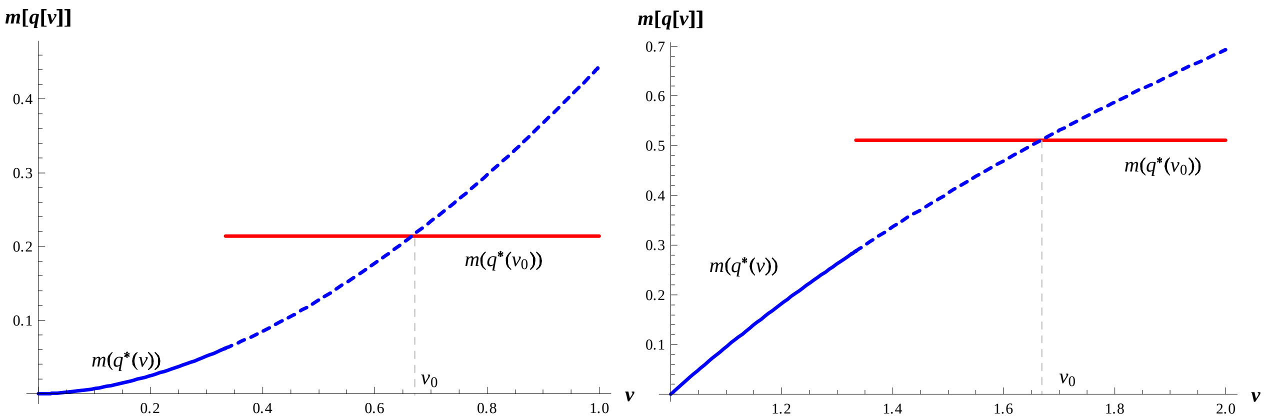
The developing markets for online data differ from traditional (offline) sales of consumer-level information along several dimensions. Perhaps the main distinguishing feature is that both sides of an advertising market can gain by acquiring consumer-level data. Thus, a data provider can sell to both advertisers and publishers, and depending on the technology and the contracting environment, it may choose which side to serve. In this section, we discuss the revenue-maximizing allocation of cookies by introducing the possibility for both sides of the market to buy data. An important distinction to be made is the one between horizontal match values (as in the rest of the paper), and vertical match values, where all advertisers have the same preferences over consumers.

Regardless of the nature of buyer heterogeneity, our first step consists of characterizing the publisher's demand for data. This requires analyzing the effect of cookie sales on the total demand for advertising space. If an advertiser wants to reach a user with intensity q, then it will have to spend $cm(q)$ to purchase the amount of space necessary for the contact. Now fix a set of *excluded* cookies $V \setminus A = [v_1, v_2]$. Consider then the total amount of space demanded by the advertisers

$$M(A) = \int_{v_L}^{v_1} m(q^*(v))\, dF(v) + \int_{v_1}^{v_2} m(q^*(\mathbb{E}[v \notin A]))\, dF(v) + \int_{v_2}^{v_H} m(q^*(v))\, dF(v)\,.$$

We first ask whether the total demand for advertising is increasing in v_1 and decreasing in v_2, i.e. increasing (in a specific sense) in the measure of cookies sold to the advertisers. To understand the publisher's trade-off, consider one-sided purchases of high-v prospects. As v_2 decreases, the publisher is substituting $m(\bar{q}(v_2))$ with $m(q(v_2))$, which is higher. At the same time, the average type below v_2 is decreasing, and so is the quantity of advertising sold for prospects $v \in [0, v_2]$. Figure 8 shows the demand for advertising $m(q(v))$ under one-sided purchases for a power cost function, and a logarithmic cost function. Not surprisingly, the publisher's preference for information is related to the convexity of the demand for advertising space.

Figure 8: Publisher Revenues under different cost functions



$m(q) = q^b/b,\ b = 9/4$ $\qquad$ $m(q) = -\ln(1-q)$

Lemma 4 (Demand for Advertising)

1. *The total demand for advertising space is increasing in v_1 and decreasing in v_2 if the complete-information advertising intensity $m(q^*(v))$ is convex in v. (The opposite is true if $m(q^*(v))$ is concave.)*

2. *The complete information intensity $m(q^*(v))$ is convex (concave) in v if*

$$\frac{m''(q)}{m'(q)} \text{ is decreasing (increasing) in } q.$$

When the demand for advertising is convex in v, the websites prefer to disclose horizontal-value information. In other words, the publisher benefits from an *indirect sale of information.* In fact, information about users is incorporated in the advertising product, which will sell at a higher price (here, in higher volume). Conversely, when the demand for advertising space is concave in v, publishers have a *negative* willingness to pay for the information. An example of such a cost function is derived from the Butters (1977) exponential technology, which leads to $m(q) = -\ln(1-q)$.

6.1 Horizontal Match Values

In this subsection, we maintain the assumption of horizontal heterogeneity among consumers. We consider technologies with $m(q^*(v))$ convex, and characterize the price of information that the data provider can charge to either side of the advertising market. A key question concerns which cookies will the publisher buy, if it decides to purchase information. In our horizontal match values framework, all cookies are symmetric from the point of view of the publisher. The following Lemma provides conditions under which it is immediate to translate the publisher's preference for information into marginal willingness to pay.

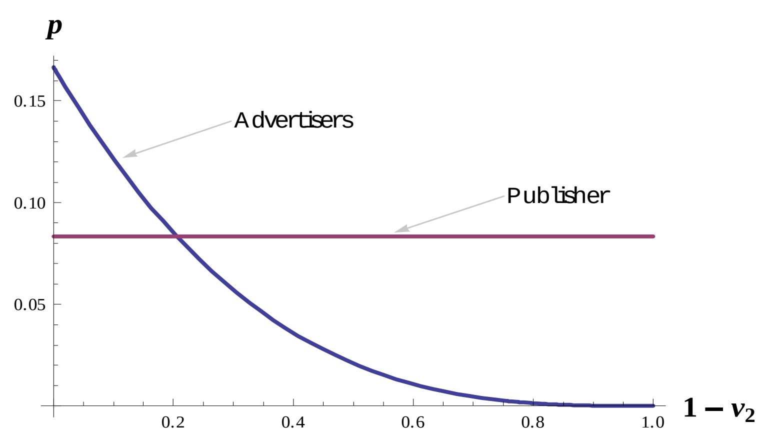
Lemma 5 (Constant Returns)
Assume the publisher can adopt randomized disclosure only. The publisher's revenue is linear in the measure of cookies bought.

Under the anonymous cookie purchases, assume the advertisers buy a fixed set of cookies A_0 . The publisher's marginal willingness to pay for data is constant, and given by

$$p_{PUB} = c\left(M\left(V\right) - M\left(A_0\right)\right).$$

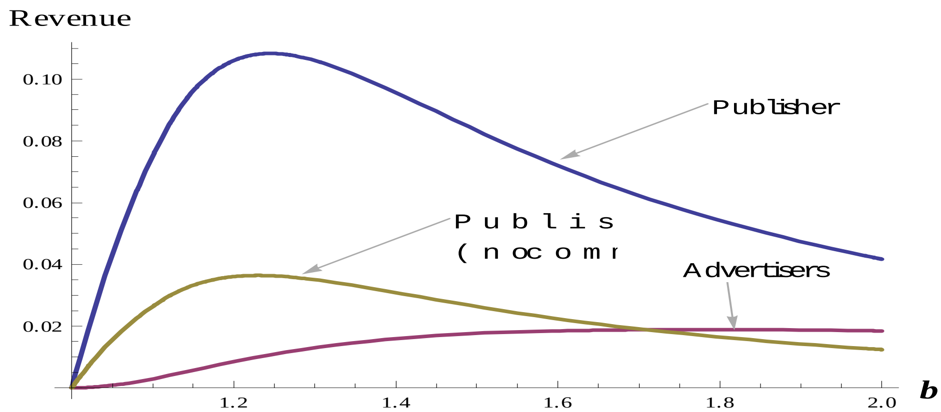
The data provider's preferred sales strategy depends, among other aspect, on the specific rules game between publishers and advertisers. Consider the following scenario: suppose the data provider sets a price p, and advertisers and the publisher simultaneously demand cookies. In the equilibrium of the game that most favors advertisers, the publisher will buy all the data, and release it to the advertisers. In this case, $A_0 = \emptyset$. The following picture compares the advertisers' and the publisher's marginal willingness to pay for uniformly distributed values and power cost functions $m\left(q\right) = q^b/b$, and $b = 3/2$.

FIGURE 9: PUBLISHER AND ADVERTISER WILLINGNESS TO PAY



The resulting profit levels are always higher if the data provider sets $p = p_{PUB}$ and advertisers do not buy information. Suppose instead that the data provider sets a price p_0 but cannot commit not to modify it after the publisher has purchased data. Then the publisher's willingness to pay is computed with reference point $A_0 = A\left(c, p^*\right)$. The next figure compares the profits from selling to advertisers with those from selling to the publisher under each alternative scenario.

Figure 10: Data Provider Revenue by Target Buyer



6.2 Vertical Match Values

In this subsection, we assume that each consumer i generates the same match value with all firms j, and that match values v are distributed according to $F(v)$. We maintain the linear pricing assumption, and ask whether the data provider will sell cookies to both sides, and if so, which will be the equilibrium allocation of data.[6] The main difference with the horizontal model is that the publisher can focus on which users (i.e. which match values v) to provide information about.

Furthermore, suppose the publisher releases whatever information it acquires to the advertisers. If the publisher buys cookies $v \in P \subset V$ and the advertisers buy cookies $v \in A \subset V$, the publisher's payoff is given by

$$\int_{A\cup P} m\left(q^*\left(v\right)\right) dF\left(v\right) + \int_{V\setminus(A\cup P)} m\left(q^*\left(\mathbb{E}\left[v \notin A \cup P\right]\right)\right) dF\left(v\right) - \int_{P} p dF\left(v\right).$$

Intuitively, from the point of view of the advertisers, any cookie purchased by the publisher is equivalent to a "free signal." From the point of view of the publisher, any signal bought by the advertisers shifts the demand for advertising space from incomplete to its complete-information level.

In what follows, we assume $m(v)$ is convex, and fix the unit price of advertising space c. We consider, a game with simultaneous data purchases by the advertisers and the publisher. It is immediate to see that equilibrium behavior imposes several restrictions on the endogenous data allocation. In particular, (a) no cookie v is purchased by both parties; and (b) for any $p > 0$, a positive measure of cookies is not purchased. Furthermore, as in Lemma 2, the third derivative of the publisher's complete-information profit function (in this case $m^*(q(v))$) determines the nature of the equilibrium purchases.

Proposition 5 (Publisher Demand)

[6] The linear pricing assumption is particularly important here, as it amounts to assuming that the publisher has a homogeneous group of advertisers (i.e. pure common values), but still the data provider does not price discriminate. In Section 9, we analyze nonlinear pricing in both the horizontal and the vertical model.

Let v be uniformly distributed, and assume advertisers purchase a cookies in a single interval A.

1. *The publisher purchases the highest (lowest) cookies $v \in V \setminus A$ if $r(v) \triangleq m(q^*(v))$ satisfies $r'''(v) \geq (<) 0$.*

2. *The function $r(v)$ satisfies $r'''(v) \geq 0$ if and only if*

$$\frac{d}{dq}\left(\frac{m''(q)^2 - m'(q)\,m'''(q)}{m''(q)^3}\right) \geq 0. \tag{11}$$

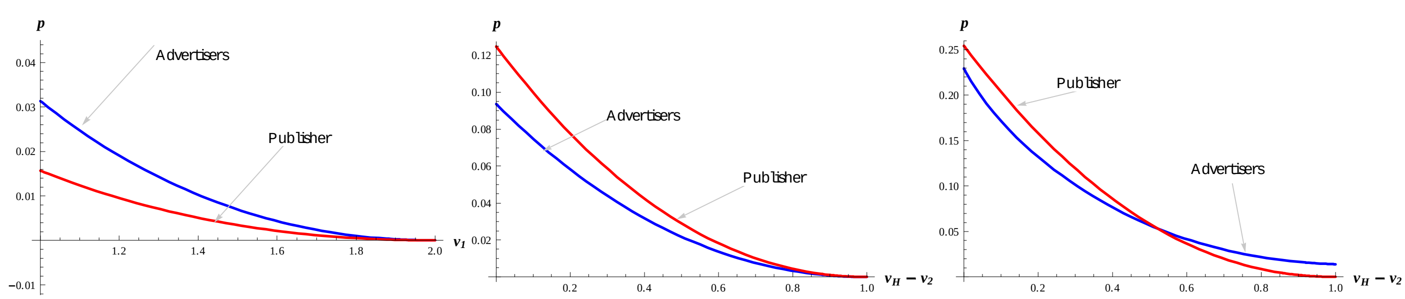
3. *Let the cost function $m(q) = q^b$, with $b > 1$. Then*

$$r'''(v) > 0 \iff m'''(q) < 0 \iff b < 2. \tag{12}$$

The condition (11) is hard to interpret, but (12) offers an easier example. With power cost functions, revenues $r(v)$ have a positive third derivative if and only if $q^*(v)$ is convex, which corresponds to $\pi'''(v) \geq 0$ (see Lemma 2). Thus, the advertisers and the publisher have the same order of preference for information about users of different value.

Furthermore, under power cost functions, the value of information for the advertisers and for the publisher can be ranked. In particular, if $b < 2$ so that high-v cookies are purchased, the publisher has a higher value of information. Conversely, for $b > 2$, low-v cookies are purchased and advertisers have a stronger preference for information. Not surprisingly, a convex combination of cost functions yields an intermediate result, in which the marginal willingness to pay is not uniformly ranked. Figure 11 displays the marginal value of information (for one-sided purchases) in three exemplary cases when values v are distributed uniformly and $c = 1$.

Figure 11: Demand Curve by Target Buyer



$m(q) = q^3$ $\quad$ $m(q) = q^{3/2}$ $\quad$ $m(q) = \alpha q^3 + (1-\alpha)\, q^{3/2}$

As we noted above, for a given price p the split of cookies between advertisers and publisher is arbitrary (subject to the restrictions in the results above). In particular, for low prices p, the data provider may be able to sell to both low prices of cookies. For example, in the left panel of Figure 11, the advertisers and the publisher can split the purchases of an interval of high-v cookies.

There clearly exists multiple solutions, subject to the constraint that advertisers will not purchase cookies below a certain threshold. While the sharing of data is indeterminate, it is clear that the advertisers and the publisher are the price-setters for $b > 2$ and $b < 2$, respectively.

To conclude, when consumers' match values are homogeneous across firms, the data provider's demand curve for cookies may be driven by the publisher's as well as by the advertisers' willingness to pay. The following is perhaps an intuitive equilibrium data allocation: when one-sided ("top") purchases are optimal, advertisers buying the very best cookies, and the publisher supplements their data buys. The price is determined by the publisher's demand. Conversely, when "bottom" purchases are optimal, the publisher buys the lowest-value cookies, and the advertisers purchase additional higher-value ones. The price is in this case determined by the advertisers' demand.

7 Advertising and Cookie Markets

We now examine the interaction of markets for data and for advertising. We focus, in particular, on the effects of market-clearing prices for advertising space on the monopoly price of cookies, and on the role of potential fragmentation in the data market.

7.1 Endogenous Cost of Advertising

We examine whether the data provider has an incentive to lower the price of advertising space c. The first step consists of asking whether the cost of advertising reduces the demand for data. Conditions under which this occurs were established in Proposition 2. The next step consists of ask whether cookie sales increase or decrease the demand for advertising space. But this was already established in Lemma 4. Finally, we endogenize the price of advertising space c by assuming a fixed supply of space M for each user i. This may correspond to a limit on the actual physical space on web pages that the user can access, or a limit on the user's attention span. The cost of advertising c is determined through a simple market clearing condition, which we interpret as a proxy for a uniform-price auction for web space. Thus, an equilibrium is given by an allocation of advertising space and cookies such that firms maximize profits and advertising markets clear.

The market-clearing condition is given by

$$M = \int_A m\left(q^*\left(v\right)\right) dF\left(v\right) + \int_{V\setminus A} m\left(q^*\left(\mathbb{E}\left[v \notin A\right]\right)\right) dF\left(v\right).$$

Thus, competing advertisers' demands for advertising impose a pecuniary externality through the unit price of advertising space. We apply our earlier results to establish whether the data provider can profitably reduce or increase congestion in the advertising market so to influence the demand for cookies through the equilibrium price of advertising space. The following result follows from Propositions 2 and Lemma 4.

Corollary 3 (Demand for Cookies and Advertising)

The demand for advertising space is increasing (decreasing) in the quantity of cookies sold if the demand for cookies is decreasing (increasing) in the cost of advertising.

We can compare cookie prices under exogenous vs. endogenous price of advertising space. Combining our previous results, we show that, if the cost of advertising is determined through market clearing, the data provider profitably increases the price of cookies, compared to the case of an exogenous c. In fact, the data provider wishes to reduce congestion when $\partial p/\partial c < 0$ and to create congestion when $\partial p/\partial c > 0$, because in that case c is decreasing in amount of information sold. We summarize our finding in the next proposition.

Proposition 6 (Endogenous Cost of Advertising)

1. *Let c denote the cost of advertising space, $p(c)$ the monopoly price given the (exogenous) cost c, and $M(c)$ the resulting total demand of advertising. Let $p(M)$ denote the monopoly price with endogenous c when the supply of advertising is given by M. Then it holds that*

$$p(M(c)) > p(c).$$

2. *Conversely, let $c(M)$ denote the equilibrium price of advertising space when total supply is given by M. Then it holds that*

$$p(M) > p(c(M)).$$

7.2 Concentration in the Market for Data

In this last section, we examine the role of individual users potentially selling their data (e.g. through enliken.com), and of the Data Exchange (e.g. promoted by Bluekai). Formally, we consider a continuum of data sellers, and we assume that each one has an exclusive over one consumer. Each seller sets the price of the corresponding cookie.

We look for a symmetric pricing equilibrium in which advertisers buy high-v cookies. Each seller chooses the marginal user v_2 to maximize profits given the advertisers' purchasing strategy, which is summarized by v_2^*. Thus, a symmetric equilibrium must solve the following problem:

$$v_2^* = \arg\max_v \left[p(v, v_2^*)(1 - F(v))\right],$$

where

$$p(v, x) = \pi(v) - \pi(v, q^*(\mathbb{E}[v < x]))$$

represents the marginal willingness to pay, given advertisers' purchasing strategies.

The key difference with the monopoly problem lies in the uninformed intensity $\bar{q}(v_2^*)$, which cannot be influenced by an individual seller's pricing decision. When we contrast the equilibrium $p(v_2^*)$ with the case of a data monopoly, we obtain the following comparison.

Proposition 7 (Equilibrium under Exclusivity)
The symmetric equilibrium price of cookies under exclusive sales is higher than the monopoly price with a single data provider.

The intuition for this result lies in a positive externality across cookie sales. When the monopolist sells the marginal cookie, this increases firms' willingness to pay for all others. For example, with high-v purchases, the more cookies bought, the lower the uninformed advertising level. Conversely, for low-value purchases, the higher the mismatch risk.

Of course, welfare implications of this result would need to take into account effects on information acquisition and user privacy. It is however, suggestive of the revenue implications of data exchanges that promote integration of 1st- and 3rd-party data.

8 Binary Advertising Levels

With the goal of studying nonlinear pricing for data, we consider a benchmark model with binary choice of advertising levels. Contact intensities are restricted to $q \in \{0, 1\}$. Given match values $v \in [0, 1]$, the complete-information expected profits are given by

$$\mathbb{E}_F [\pi (v)] = \int_c^1 (v - c)\, dF (v) .$$

Likewise, the incomplete-information profits are given by

$$\pi_0 = \max \{0, \mathbb{E}_F [v] - c\} .$$

The minimal amount of information that induces the complete-information profits is a binary signal indicating whether the match value is above or below the cost of advertising space c. Intuitively, advertisers can either acquire information about the most valuable users, and exclude all others, or acquire information about the least valuable ones, and contact all others.

We initially focus on linear prices for cookies, and characterize the advertisers' demand for signals.

Lemma 6 (Binary Action – Demand for Information)

1. *For all c and p there exists a threshold type $\hat{v}$ such that advertisers choose $q (v) = 1$ for all $v \geq \hat{v}$ and $q (v) = 0$ for all $v < \hat{v}$. The set of purchased cookies $A (c, p)$ is given by either $[\hat{v}, 1]$ or $[0, \hat{v}]$.*

2. *The marginal willingness to pay $p (\hat{v}, c)$ is increasing (decreasing) in c if $A (c, p) = [0, \hat{v}]$ or $A (c, p) = [\hat{v}, 1]$, respectively.*

Consider the willingness to pay for the marginal cookie. If advertisers buy "from the top," they select a cutoff $\hat{v}$ such that $\hat{v} - c = p$. Conversely, if advertisers buy "from the bottom," the threshold $\hat{v}$ is given by $\hat{v} = c - p$. Thus, the data-buying strategy determines the comparative statics of demand with respect to the cost of advertising. We now derive conditions under which advertisers will purchase the high- or the low-v cookies. In the former case, the advertisers' profits are given by

$$\pi_+ (\hat{v}) = \int_{c+p}^{1} (v - c - p) \, dF(v).$$

In the latter case, profits are given by

$$\pi_- (\hat{v}) = \int_{c-p}^{1} (v - c) \, dF(v) - pF(c - p).$$

In general, the advertisers' data-buying strategy is a function of the price of information, of advertising space and the distribution of match values. We relate the advertisers' demand to the two parameters c and p, and to the properties of the match-value distribution in the following result. In particular, we focus on distributions with a monotone density function $f(v)$ on $[0, 1]$, so that the difference between the median and $v = 1/2$ has an unambiguous sign.

Proposition 8 (Data-buying Policy)
Let v_M denote the median of $F(v)$.

1. *If $f(v)$ is decreasing, high- (low-) v cookies are purchased for $p \leq (>) \, \tilde{p}_D(c)$.*

   The threshold $\tilde{p}_D(c)$ is increasing, with $\tilde{p}_D(v_M) = 0$ and $\tilde{p}_D(v_D) = v_D$, where $v_D \in [v_M, 1/2]$ is the positive root of

   $$\int_0^{2x} F(v) \, dv = x.$$

2. *If $f(v)$ is increasing, high- (low-) v cookies are purchased for $p \geq (<) \, \tilde{p}_I(c)$.*

   The threshold $\tilde{p}_I(c)$ is decreasing, with $\tilde{p}_I(v_M) = 0$ and $\tilde{p}_I(v_I) + v_I = 1$, where $v_I \in [1/2, v_M]$ is the positive root of

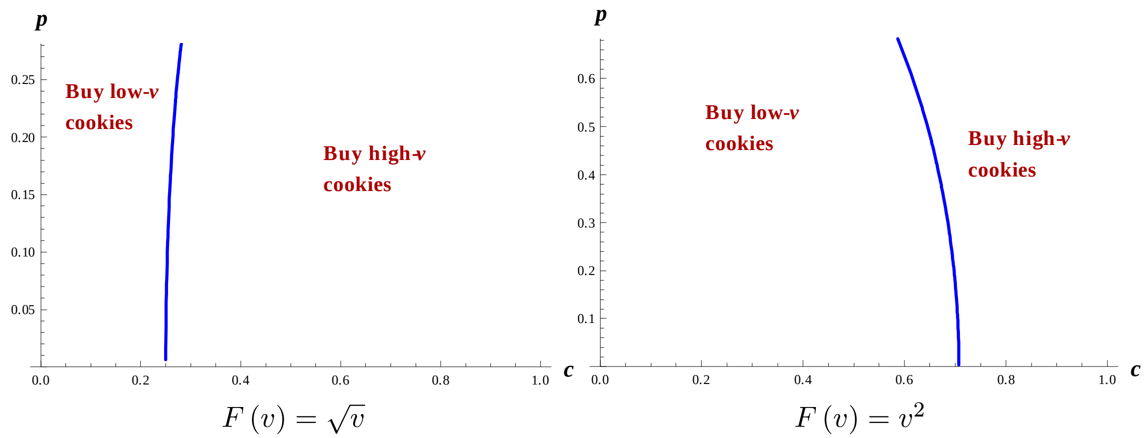
   $$\int_{2x-1}^{1} F(v) \, dv = 1 - x.$$

Figure 12 displays the advertisers' demand for signals for the case of increasing and decreasing density, respectively.

Evidently, when the cost of ads c is very high (low), advertisers purchase high- (low-) value cookies. While Proposition 8 provides sufficient conditions in terms of the distribution of values, this result should hold more generally. Intuitively, when c is high, only a small number of users are actually profitable. For any p, it is then optimal for advertisers to buy a small number of cookies and to contact those very high-value users only. The opposite intuition applies when c is very low: almost all users are profitable, advertisers buy a few low-v cookies, and exclude the corresponding users.

The optimal data-buying policy as a function of the price of cookies p is more involved. Our results suggest that, for low values of p, the skewness of the distribution is driving the advertisers'

Figure 12: Demand for Signals



choice. When p is low, the advertisers' choice is dictated by the comparison of $F(c)$ and $1 - F(c)$ (corresponding to the measures of the largest low- and high- value sets that can be demanded). As p increases, the set of cookies demanded shrinks. It may then become optimal to switch data-buying policy. For example, in the case of decreasing density, for low values of p, advertisers buy almost all cookies in $[c, 1]$. As the price increases, the high-v purchases strategy induces them to give up the least profitable users. These users may, however, be very numerous. If the cost c is sufficiently low, it is optimal to switch to a low-values purchases strategy. This entails wasting some messages on users with $v < c$, but ensures that advertisers are able to reach more consumers with intermediate values instead.

Having established the properties of the demand function, we now turn to monopoly pricing of cookies. We focus on the uniform distribution, which allows for a simpler characterization of demand. The following result is a direct consequence of Propositions 8 and Lemma 6 (in Appendix).

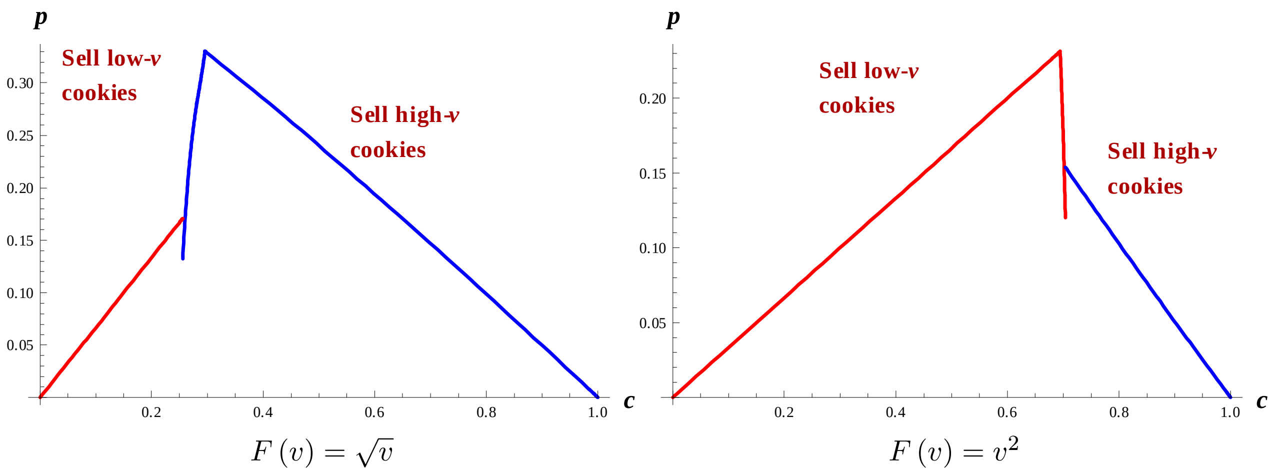
Corollary 4 (Uniform Distribution)
If $F(v) = v$, advertisers buy high- (low-) v cookies if $c > (<) 1/2$. More generally, if $f'(v)(c - 1/2) \leq 0$ the data-buying policy is independent of p.

Under the uniform, the profit-maximizing price of cookies $p^*(c)$ is then given by

$$p^*(c) = \frac{1}{2} \min\{c, 1 - c\}.$$

For more general distributions, the data-buying policy induced by the monopolist will be more complicated. In fact, the price of cookies need not be differentiable, or even continuous in c. Jumps may occur when the monopolist switches from inducing one data-buying policy to another. Typically, this occurs when moving from the unconstrained-optimal price under one strategy to constrained-optimal price that induces the other strategy. Figure 13 shows the monopoly price for the two separate cases of decreasing and increasing densities. In both cases, the monopolist induces high-v purchases for c above a threshold.

FIGURE 13: MONOPOLY PRICE AND COOKIES SOLD



9 Nonlinear Pricing

The solution to the monopolist's problem is indeed much simpler under nonlinear pricing. In this section, we consider a binary decision $q \in \{0, 1\}$ and match values v distributed on the interval $[0, 1]$. In addition, we introduce a private-information component to advertisers' willingness to pay for contacting users. We first explore the data provider's ability to screen advertisers by offering differently informative partitions of users, and pricing the amount of information in a nonlinear way. We then investigate the optimal nonlinear pricing of arbitrary information structures, and we show that partitional, revealing mechanisms are optimal in the binary action case.

With heterogeneous advertisers, the net value of a match is given by

$$\max \{\theta v - c, 0\}.$$

Thus for advertising to generate value at all, we have $\theta > c$. The marginal willingness to pay θ is private information of the advertisers. We assume that θ is distributed in the population of advertisers according to $G(\theta)$.

9.1 Binary Partitions

Under binary actions, the efficient disclosure rule can be induced by a binary partition given θ with cutoff

$$\overline{x}(\theta) = \frac{c}{\theta}. \tag{13}$$

An equivalent information structure discloses the value v perfectly and without noise if and only if

$$v \geq \overline{x}(\theta). \tag{14}$$

In other words, the data provider can attain the efficient allocation of information through a disclosure policy based on cookies. Under this policy, the advertisers contact all users they receive a signal about, and do not contact "unknown" users.

Regardless of its implementation, the expected value of the efficient partition is therefore

$$w(\theta) = \int_{c/\theta}^{1} (\theta v - c)\, dF(v).$$

9.1.1 Optimal Disclosure

Now consider an arbitrary binary disclosure rule with cutoff x. The value of this information structure is given by

$$w(\theta, x) = \int_{x}^{1} (\theta v - c)\, dF(v).$$

We therefore have

$$\frac{\partial w(\theta, x)}{\partial x} = -(\theta x - c) f(x), \tag{15}$$

with

$$\frac{\partial w(\theta, x)}{\partial \theta} = \int_{x}^{1} v dF(v)$$

and hence

$$\frac{\partial^2 w(\theta, x)}{\partial \theta \partial x} = -x f(x). \tag{16}$$

Because of this submodularity property, higher types θ should receive lower cut-offs x.

The optimal information allocation and pricing can then be solved via the virtual utility, and is given by:

$$\frac{\partial w(\theta, x)}{\partial x} = \frac{1 - G(\theta)}{g(\theta)} \frac{\partial^2 w(\theta, x)}{\partial x \partial \theta},$$

and after using (15) and (16) we obtain

$$x(\theta) = \frac{c}{\theta - \frac{1-G(\theta)}{g(\theta)}}. \tag{17}$$

Perhaps the surprising element is that the distributional information about the match values does not appear in the optimal information allocation. This results from linearity in all types θ's utility in the number of users contacts, with differences in willingness to pay originating from the match values v only. The next result follows immediately from the derivation of the optimal threshold (17).

Lemma 7 (Cookie Quantity)

The numbers of cookies sold, $q(\theta) = 1 - x(\theta)$ *is increasing in* θ *if the virtual utility*

$$\theta - \frac{1 - G(\theta)}{g(\theta)}$$

is increasing in θ*.*

9.1.2 Optimal Pricing

We can then derive the optimal pricing rule. The gross utility for the buyer is:

$$w(\theta, x(\theta)) = \theta \int_{x(\theta)}^{1} v dF(v) - c \int_{x(\theta)}^{1} dF(v) \tag{18}$$

and the indirect utility is then given by:

$$W(\theta) = \int_{\underline{\theta}}^{\theta} \frac{\partial w(\theta', x(\theta'))}{\partial \theta'} d\theta' = \int_{\underline{\theta}}^{\theta} \int_{x(\theta')}^{1} v dF(v) \, d\theta', \tag{19}$$

and the associated transfer is

$$t(\theta) = w(\theta, x(\theta)) - W(\theta).$$

On the one hand, we could insert $x(\theta)$ and get a more explicit expression, but one involving derivatives of the virtual utility function which might not be too helpful. On the other hand, we would like to make a number of statements about the transfers.

Lemma 8 (Transfers and Quantities)

1. *The total payment* $t(\theta)$ *is increasing in* θ *(and hence the number of cookies sold);*

2. *the cookie price* $p(x)$ *is increasing in* x *and can decentralize the direct optimal mechanism if* $(1 - G(\theta))/g(\theta)$ *is decreasing.*

Thus, if the we maintain the interpretation of an idiosyncratic distribution (i.e. of horizontally differentiated advertisers), the optimal pricing is simply a nonlinear quantity pricing. In particular, the data provider can decentralize the optimal mechanism by allowing advertisers to access a given portion of the database with volume discounts for those who demand a larger amount of cookies. If we extend the interpretation to a common value distribution (and hence with vertical differentiation), then we still have a cookie pricing scheme. In this scheme, the identity of the cookies sold matters, and the mechanism features a decreasing price for less valuable cookies. Furthermore, the most valuable cookies will be sold to all participating advertisers (i.e. those with sufficiently large virtual valuations), while intermediate cookies will feature a level of exclusivity which is decreasing in v.

Figure 14 shows the link between willingness to pay θ and the cut-off $x(\theta)$ for the case of the uniform distribution and $c = 1/2$.

FIGURE 14: OPTIMAL CUTOFF POLICY

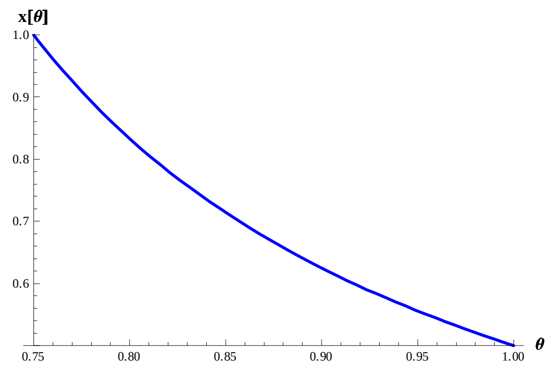
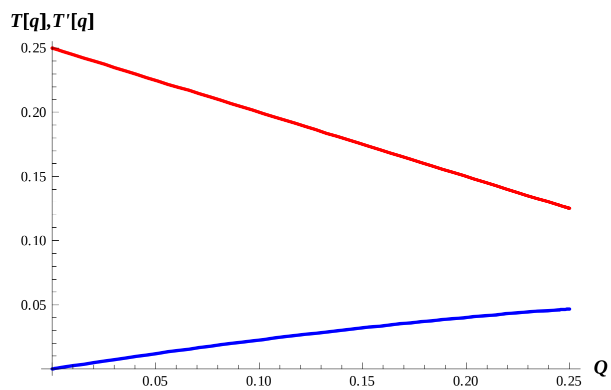


Figure 15 turns to the quantity of data sold, and shows the total pricing function $T(Q)$ and the marginal price $T'(Q)$, with

$$Q(\theta) = 1 - x(\theta).$$

In particular, we can interpret the marginal price also as the price of the additional cookie.

FIGURE 15: TOTAL AND MARGINAL PRICE OF DATA



9.2 General Information Structures

9.2.1 Signals and Values

Each advertiser i has a compact set $V_i = [0,1]$ of possible valuations for the contact with the customer, where a generic element is denoted by $v_i \in V_i$. The valuation v_i is independently distributed with prior distribution function $G(v_i)$, and the associated density function $g(v_i)$ is positive on V_i.

The signal space is denoted by $S_i \subseteq [0,1]$. The space S_i can either be countable, finite or infinite, or uncountable. Let $(V_i \times S_i, \mathcal{B}(V_i \times S_i))$ be a measurable space, where $\mathcal{B}(V_i \times S_i)$ is the class of Borel sets of $V \times S$. An *information structure* for advertiser i is given by a pair $\mathcal{S}_i \triangleq \langle S_i, F_i(v_i, s_i) \rangle$, where S_i is the space of signal realizations and $F_i(v_i, s_i)$ is a joint probability distribution over the space of valuations V_i and the space of signals S_i. We refer to this class of information structures as (Borel) measurable information structures. The joint probability distribution is defined in the usual way by

$$F_i(v_i, s_i) \triangleq \Pr(\widetilde{v}_i \leq v_i, \widetilde{s}_i \leq s_i).$$

The marginal distributions of $F_i(v_i, s_i)$ are denoted with minor abuse of notation by $F_i(v_i)$ and $F_i(s_i)$ respectively. For $F_i(v_i, s_i)$ to be part of an information structure requires the marginal distribution with respect to v_i to be equal to the prior distribution over v_i. The conditional distribution functions derived from the joint distribution function are defined in the usual way:

$$F_i(v_i | s_i) \triangleq \frac{\int_0^{v_i} dF_i(\cdot, s_i)}{\int_0^1 dF_i(\cdot, s_i)},$$

and similarly,

$$F_i(s_i | v_i) \triangleq \frac{\int_0^{s_i} dF_i(v_i, \cdot)}{\int_0^1 dF_i(v_i, \cdot)}.$$

The data provider can choose an arbitrary information structure $\mathcal{S}_i$ for every advertiser i subject only to the restriction that the marginal distribution equals the prior distribution of v_i. The cost of every information structure is identical and set equal to zero. The choice of $\mathcal{S}_i$ is common knowledge. At the interim stage every agent observes privately a signal s_i rather than her true match value v_i of the object. Given the signal s_i and the information structure $\mathcal{S}_i$ each advertiser forms an estimate about her true match value. The *expected value* of v_i conditional on observing s_i is defined and given by

$$w_i(s_i) \triangleq \mathbb{E}[v_i | s_i] = \int_0^1 v_i dF_i(v_i | s_i).$$

Every information structure $\mathcal{S}_i$ generates a distribution function $G_i(w_i)$ over *posterior expectations* given by

$$G_i(w_i) = \int_{\{s_i : w_i(s_i) \leq w_i\}} dF_i(s_i).$$

We denote by W_i the support of the distribution function $G_i(\cdot)$. Observe that the prior distribution $F_i(\cdot)$ and the posterior distribution over expected values $G_i(\cdot)$ need not coincide. It is helpful to illustrate some specific information structures.

The information structure $\mathcal{S}_i$ yields perfect information if $F_i(v_i) = G_i(v_i)$ for all $v_i \in V_i$. In this case, the conditional distribution $F(s_i | v_i)$ has to satisfy

$$F_i(s_i | v_i) = \begin{cases} 0 & \text{if} \quad s_i < s(v_i), \\ 1 & \text{if} \quad s_i \geq s(v_i), \end{cases} \tag{20}$$

where $s(v_i)$ is an invertible function.

The information structure $\mathcal{S}_i$ is said to be *positively revealing* if $F_i(v_i) = G_i(v_i)$ for all $v_i \geq$

$\widehat{v}_i \in V_i$, and with the conditional expectation given by $\widehat{w}_i = \mathbb{E}\left[v_i \left| v_i \leq \widehat{v}_i \right.\right]$

$$G_i(v) = \begin{cases} 0 & \text{if} \quad 0 < v \leq \widehat{w}_i; \\ F_i(\widehat{v}_i) & \text{if} \quad \widehat{w}_i \leq v \leq \widehat{v}_i; \\ F_i(v) & \text{if} \quad \widehat{v}_i \leq v \leq 1. \end{cases} \tag{21}$$

An information structure $\mathcal{S}_i$ which satisfies (20) without necessarily satisfying the invertibility condition is called *partitional*. An information structure is called *discrete* if S_i is countable and *finite* if S_i is finite.

After the choice of the information structures $\mathcal{S}_i$ by the auctioneer, the induced distribution of the agent's (expected) valuations is given by $G_i(w_i)$ rather than $F_i(v_i)$. The signal s_i and the corresponding expected valuation $w_i(s_i)$ remain private signals for every agent i and the auctioneer still has to elicit information by respecting the truthtelling conditions.

9.2.2 Revelation Mechanism

The data provider selects the information structures of the advertisers and a revelation mechanism. The objective of the data provider is to maximize his expected revenue subject to the interim participation and interim incentive constraints of the advertiser. The data provider can offer a menu of posterior expectations $G(w|\theta)$ at a price $t(\theta)$. In an incentive compatible mechanism the value function of an advertiser with a willingness to pay θ is given by

$$U(\theta) \triangleq \int_w \{\max\{\theta w - c, 0\}\, dG(w|\theta)\} - t(\theta) \tag{22}$$

and the interim incentive constraint requires that

$$\int_w \{\max\{\theta w - c, 0\}\, dG(w|\theta)\} - t(\theta) \geq \int_w \left\{\max\{\theta w - c, 0\}\, dG\left(w\left|\theta'\right.\right)\right\} - t\left(\theta'\right) \tag{23}$$

and the interim participation constraint requires that

$$U(\theta) \geq 0.$$

9.2.3 Revenue Maximizing Mechanism

For convenience, we shall restrict attention to a model with finite values and finite signals. We briefly discuss the extension to a continuum of types, values, and signal at the end. We present the finite model here as it allows to avoid additional qualification such as "almost surely" that arise in a model with a continuum of types, values or signals. A mechanism is then a transfer payment $t(\theta)$ and distribution $H_\theta : V \to \Delta(W)$ from values into expectations.

We denote by $W(\theta)$ the set of posterior expectations under distribution H_θ:

$$W(\theta) \triangleq \{w \in W\,|h_\theta(w) > 0\}$$

We say that $W(\theta)$ has binary support, in this case denoted by $B(\theta)$, if it contains only two elements:

$$B(\theta) = \{\underline{w}(\theta), \ \overline{w}(\theta)\} \tag{24}$$

and one of them leads to a contact, and the other one does not lead to contact:

$$\theta \underline{w}(\theta) - c < 0, \ \theta \overline{w}(\theta) - c \geq 0.$$

Lemma 9 (Binary Mechanism)
Every optimal revenue mechanism can be implemented by a binary mechanism.

By combining the posterior values into those with positive and those with negative value relative to the type θ of the agent, we do not change the value of the allocation for the agent. But, since the bundling/combination is performed with respect to the true type, it lowers the option value for all types other than the true type, because the binary mechanism forces them to take a constant action where before they might have chosen contingent actions. Thus, restricting the set of posterior realization only tightens the incentive constraints, and can only (weakly) improve the revenues for the principal.

Finally, we can show that every revenue-optimal mechanism can be implemented by a positively revealing mechanism. Therefore, the data provider can maximize revenues by revealing cookies above the optimal threshold in (17) and charging the corresponding prices. In our horizontal-differentiation interpretation, the optimal mechanism can also be decentralized by a nonlinear tariff for access to portions of the database.

10 Concluding Remarks

We now discuss some key directions for extending the current analysis.

Pricing General Information Structures Within the context of selling data with exogenous precision, we can extend our model to incorporate more general information structures. The question of how to then price them is closely related to the optimal bundling and nonlinear pricing. For example, the data provider may offer coarse (partitional) and potentially asymmetric information structures. These structures can be part of a profit-maximizing data sales policy if firms have heterogeneous preferences for precision. For example, with horizontal match values, firms may have prior information of differing precision, or different spreads in the distribution of match values.

Endogenous Precision We have so far treated the precision of the information available to the data provider as exogenous. However, by introducing a positive marginal cost for acquiring information about users, we can endogenize the level of precision of the data for sale. This endogenous precision level depends on the downstream mode of competition and on the direct and indirect externalities among advertisers. Both these factors affect the surplus that firms can create at the advertising stage, and hence determine how much of it the data provider can appropriate by selling information.

The issue of information acquisition clearly has important implications for user privacy online. We address these issues specifically in ongoing work, in which we focus on the dynamics of firm learning and pricing. The classic insights of dynamic pricing are now affected by the risk that the user will "clear her cookies" and reset the firm's learning process.

Heterogeneous Publishers We have explored the possibility of publishers purchasing information as a deviation from a candidate equilibrium in which data is sold to advertisers only. We can extend our analysis to a large number of small (homogeneous or heterogeneous) websites who sell space and buy information. This introduces two important themes: first, each website may wish to refine or garble its information depending on the thickness of advertiser demand for its space; and second, advertisers' demand for information is now driven by the opportunity of using the same information bit across several sites (by tracking the same consumers). Therefore, even if advertisers' demands for space may be separable across sites, the composition of each site will interact in determining the demand for data.

A related issue is the two-sided nature of data transactions. We seek to characterize both the equilibrium (bid) price charged to the publishers by the data provider, and the sales (ask) price charged by the publisher for the flow of information going to the data provider. A natural equilibrium condition is that the data provider pays for the marginal value of the information acquired from each website.

Appendix

Proof of Lemma 1. Suppose towards a contradiction that the set of *excluded* cookies $V \setminus A$ is not an interval. Let $\bar{q}$ denote the match intensity demanded by the firm for consumers $v \notin A$. Equation (5) establishes that $\bar{q}$ is the optimal match intensity for the average type $\bar{v}_A = \mathbb{E}[v \mid v \notin A]$. Suppose $\bar{v}_A \in A$. Now consider two consumers with $v'' > v'$ and $q^*(v'') > q^*(v') > \bar{q}$ such that the firm buys cookie v' but not v''. If $V \setminus A$ is not an interval, either such a pair exists, or there exists a pair with $v'' < v'$ and $q^*(v'') < q^*(v') < \bar{q}$ such that the firm buys cookie v' but not v''. Consider the former case, and compute the change in profits obtained by swapping cookies, i.e. purchasing (equal numbers of) cookies v'' instead of cookies v'. Define the difference between complete and incomplete-information profits as

$$\Delta(v, \bar{q}) = v(q^*(v) - \bar{q}) - c(m(q^*(v)) - m(\bar{q})),$$

and notice that $\Delta_v(v, \bar{q}) = (q^*(v) - \bar{q})$. Therefore $q^*(v'') > q^*(v') > \bar{q}$ implies $\Delta(v'', \bar{q}) > \Delta(v', \bar{q})$. Because the firm gains $\Delta(v'', \bar{q})$ and loses $\Delta(v', \bar{q})$, it follows that the swap strictly improves profits. An identical argument applies to the case of $q^*(v'') < q^*(v') < \bar{q}$. Finally, if $\bar{v}_A \notin A$, then a profitable swap involves not purchasing $\bar{v}_A$ and buying any other cookie instead. ■

Proof of Lemma 2. Consider the case of the uniform distribution, so that $F(v)$ is linear. The following are necessary conditions for exclusion of an interior interval $[v_1, v_2]$. Let

$$v_0 := \frac{v_1 + v_2}{2},$$

denote the mean excluded type, so that $\bar{q} = q^*(v_0)$, and by the envelope theorem $\bar{q} = \pi'(v_0)$. The value of the marginal signal v is then given by

$$\pi(v) - \left(\pi(v_0) + (v - v_0)\pi'(v_0)\right),$$

which is positive and convex because $\pi(v)$ is convex. Now let the set of excluded cookies have measure 2ε. Optimality requires that

$$p = \pi(v_0 + \varepsilon) - \left(\pi(v_0) + \varepsilon\pi'(v_0)\right) = \pi(v_0 - \varepsilon) - \left(\pi(v_0) - \varepsilon\pi'(v_0)\right),$$

or

$$\pi(v_0 + \varepsilon) - \pi(v_0 - \varepsilon) - 2\varepsilon\pi'(v_0) = \int_{v_0-\varepsilon}^{v_0+\varepsilon} \left(\pi'(v) - \pi'(v_0)\right) dv = 0.$$

The last expression is positive if $\pi'(v)$ is convex and negative if $\pi'(v)$ is concave. Suppose $\pi'(v)$ is convex. In a candidate interior solution, if the firm's marginal willingness to pay for the lowest excluded cookie equals the price, then it strictly prefers acquiring the highest excluded type. This rules out exclusion of an interior interval, and leaves exclusion of $[v_L, v_2]$ as the only candidate solution. The opposite result holds when $\pi'(v)$ is concave. In addition, if the density of types $f(v)$ is strictly monotone, then $v_0 \neq (v_1 + v_2)/2$. In particular, $v_0 \leq (v_1 + v_2)/2$ if $F(v)$ is concave. This increases the value of acquiring the highest excluded prospect even further, reinforcing the previous argument. Finally, we can relate the curvature of the profit function to that of the match cost function. The envelope theorem implies $\pi'(v) = q^*(v)$, and implicit differentiation of the first order condition yields

$$\pi''(v) = \frac{1}{cm''(q^*(v))}.$$

Because $q^*(v)$ is strictly increasing, we conclude that $\pi'''(v) > 0$ if and only if $m'''(q) < 0$. ■

Proof of Lemma 3. If costs are quadratic, so are the complete-information profits. By symmetry of the distribution, $\bar{v} = E[v \mid v \in [\bar{v} - \varepsilon, \bar{v} + \varepsilon]]$ for any $\varepsilon > 0$. The marginal value of information is then given by

$$p(v) = \pi^*(v) - (vq^*(\bar{v}) - cm(q^*(\bar{v}))) = (\bar{v} - v)^2/4c.$$

Solving for $\hat{v}$ yields the demand function. ■

Proof of Proposition 1. (1.) Consider the inverse demand for data in the low-value purchases case,

$$p(v_1) = v_1(q^*(v_1) - \bar{q}(v_1)) - c(m(q^*(v_1)) - m(\bar{q}(v_1))).$$

As k increases, by second-order stochastic dominance, the conditional expectation $\mathbb{E}[v \mid v > v_1]$

increases as well. Therefore, $\bar{q}(v_1)$ increases, and because $\bar{q}(v_1) > q^*(v_1)$, the willingness to pay $p(v_1)$ increases as well. An identical argument applies to the case of high-v purchases.

(2.) From part (1.) we know the threshold $v_1(p,k)$ is increasing in k. If in addition, $v_1 < \bar{v}$, then $F(v_1,k)$ is increasing in k, and therefore $F(v_1(p,k),k)$ is increasing a fortiori.

(3.) Under two-sided purchases, we know $\bar{q} \equiv q^*(\bar{v})$ for all k, and hence the willingness to pay for v is independent of the distribution. However, as k increases, both $F(v_1,k)$ and $1-F(v_2,k)$ increase, so the quantity of data demanded increases. ■

Proof of Proposition 2. Consider the case of low-value purchases first, i.e. let $A = [v_L, v_1]$. The inverse demand function $p(v_1)$ is given by

$$p(v_1) = \pi(v_1) - v_1\bar{q}(v_1) + cm(\bar{q}(v_1)).$$

Now for a fixed v_1, let

$$\bar{v} = \mathbb{E}[v \mid v > v_1],$$

so that

$$\bar{q}(v_1) = q^*(\bar{v}), \text{ with } \bar{v} > v_1.$$

Now consider the derivative

$$\frac{\partial p(v_1,c)}{\partial c} = -\left(m(q^*(v_1)) - m(q^*(\bar{v}))\right) - \left(v_1 - cm'(q^*(\bar{v}))\right)\frac{\partial q^*(\bar{v})}{\partial c},$$

where

$$\frac{\partial q^*(\bar{v})}{\partial c} = -\frac{m'(q^*(\bar{v}))}{cm''(q^*(\bar{v}))}.$$

Thus, using the first order condition $v = cm'(q^*(v))$ we obtain

$$\frac{\partial p(v_1,c)}{\partial c} = m(q^*(\bar{v})) - m(q^*(v_1)) + (v_1 - \bar{v})\frac{m'(q^*(\bar{v}))}{cm''(q^*(\bar{v}))}. \tag{25}$$

Notice that, as a function of v_1, the right-hand side of (25) is equal to zero at $v_1 = \bar{v}$, and its derivative with respect to v_1 is equal to

$$\frac{\partial^2 p(v_1,c)}{\partial v_1 \partial c} = -m'(q^*(v_1))\frac{dq^*(v_1)}{dv_1} + \frac{m'(q^*(\bar{v}))}{cm''(q^*(\bar{v}))}.$$

Because

$$\frac{dq^*(v_1)}{dv_1} = \frac{1}{cm''(q^*(v_1))}$$

we then obtain

$$\frac{\partial^2 p(v_1,c)}{\partial v_1 \partial c} = \frac{1}{c}\left(\frac{m'(q^*(\bar{v}))}{m''(q^*(\bar{v}))} - \frac{m'(q^*(v_1))}{m''(q^*(v_1))}\right). \tag{26}$$

Because $q^*(v)$ is strictly increasing in v, if $m''(q)/m'(q)$ is decreasing in q then the expression in (26) is positive, which implies $\partial p/\partial c$ is negative for all $v_1 < \bar{v}$.

An identical argument establishes this result for the case of $A = [v_2, v_H]$. The two-sided case under quadratic costs follows directly from equation (9). ■

Proof of Proposition 3. (1.) By symmetry and Proposition 3, the monopolist's profit as a function of price is given by

$$\Pi (p, k) = 2pF\left(\bar{v} - 2\sqrt{cp}, k\right). \tag{27}$$

The result then follows from implicit differentiation of the first order condition

$$F\left(\bar{v} - 2\sqrt{cp}, k\right) - \sqrt{p} F_v\left(\bar{v} - 2\sqrt{cp}, k\right) = 0,$$

with respect to k. Thus,

$$\begin{aligned}
\frac{\partial^2 \Pi}{\partial p \partial k} &= F_k\left(\bar{v} - 2\sqrt{cp}, k\right) - \sqrt{p} F_{vk}\left(\bar{v} - 2\sqrt{cp}, k\right) \\
&\propto F\left(\bar{v} - 2\sqrt{cp}, k\right) F_k\left(\bar{v} - 2\sqrt{cp}, k\right) - F_v\left(\bar{v} - 2\sqrt{cp}, k\right) F_{vk}\left(\bar{v} - 2\sqrt{cp}, k\right) \\
&\propto -\frac{\partial\left[\ln F\left(\bar{v} - 2\sqrt{cp}, k\right)\right]}{\partial v \partial k}.
\end{aligned}$$

(2.) Notice that the total quantity of cookies sold is $2F\left(\hat{v}(k), k\right)$, where $\hat{v} = \bar{v} - \sqrt{cp}$. Therefore

$$\frac{dF}{dk} = F_k\left(\hat{v}, k\right) + f\left(\hat{v}\right) \frac{\partial \hat{v}}{\partial k},$$

and if $\partial \ln F\left(\bar{v} - 2\sqrt{cp}, k\right) / \partial v \partial k > 0$ then the monopoly price is decreasing, which is sufficient for $dF/dk > 0$.

(3.) By the rotation order, we know $F_k > 0$ for $v < \bar{v}$, but because of the constant-mean assumption, it must be that $F_{vk} < 0$ around $\bar{v}$. The result then follows from part (1.). ■

Proof of Proposition 4. (1.) Consider first the case of uniformly distributed types and low-value cookie purchases. The data provider maximizes

$$p\left(v_1, c\right) F\left(v_1\right),$$

where $F\left(v_1\right) = \left(v_1 - v_L\right) / \left(v_H - v_L\right)$ and the inverse demand function is given by

$$p\left(v_1, c\right) = v_1\left(q^*\left(v_1\right) - \bar{q}\left(v_1\right)\right) - c\left(m\left(q^*\left(v_1\right)\right) - m\left(\bar{q}\left(v_1\right)\right)\right).$$

Rewrite the objective as a function of p, as follows:

$$\Pi (p) = \left(v_1 (p, c) - v_L\right) p.$$

Thus, log-submodularity of $v_1 (p, c) - v_L$ is sufficient for p to be decreasing in v_1. Therefore, consider

$$\frac{\partial^2 \ln\left(v_1 - v_L\right)}{\partial c \partial p} \propto \frac{\partial v_1}{\partial c} - \frac{v_1 - v_L}{\partial v_1 / \partial p} \frac{\partial^2 v_1}{\partial p \partial c}.$$

Clearly, $\partial v_1/\partial p < 0$. In addition, if $m''(q)/m'(q)$ is decreasing, we know from Proposition 2 that $\partial v_1/\partial c < 0$. Finally, we know from (26) that $\partial^2 p/\partial v_1 \partial c > 0$, which implies $\partial^2 v_1/\partial p \partial c < 0$. This establishes the first part of the result. Both inequalities are reversed if $m''(q)/m'(q)$ is increasing. Finally, notice that $m''(q)/m'(q)$ decreasing implies v_1 is also submodular (and supermodular for $m''(q)/m'(q)$ increasing). Therefore, any distribution (such as the power distribution) that preserves *log*-modularity guarantees that the monopoly price is decreasing in c.
(2.) This follows immediately from the proof of Proposition 3, through the change of variable $y = pc$ in the profit function for the data provider (27). ■

Proof of Lemma 4. Consider part (2.) first. Differentiate $m(q^*(v))$ with respect to v. We obtain

$$\frac{dm(q^*(v))}{dv} = m'(q^*(v))\frac{dq^*(v)}{dv} = \frac{m'(q^*(v))}{cm''(q^*(v))}.$$

Therefore, the demand for advertising space is convex in v if and only if $m''(q)/m'(q)$ is decreasing in q.
(1) To establish the first statement, focus on the case $A = [v_L, v_1]$ so that

$$M = \int_{v_L}^{v_1} m(q^*(v))\, dF(v) + (1 - F(v_1))\, m(q^*(\bar{v})).$$

Thus

$$\begin{aligned} \frac{\partial M}{\partial v_1} &= (m(q^*(v_1)) - m(q^*(\bar{v})))\, f(v_1) + (1 - F(v_1))\, m'(q^*(\bar{v}))\frac{\partial q^*(\bar{v})}{\partial \bar{v}}\frac{\partial \bar{v}}{\partial v_1} \\ &= f(v_1)\left(m(q^*(v_1)) - m(q^*(\bar{v})) + \frac{m'(q^*(\bar{v}))}{cm''(q^*(\bar{v}))}(\bar{v} - v_1)\right). \end{aligned}$$

As shown in (25)-(26), this expression is positive if and only if $m''(q)/m'(q)$ is decreasing in q, which corresponds to $m(q^*(v))$ being convex in v. ■

Proof of Lemma 5. If the publisher buys cookies at random, then regardless of the measure bought, the uninformed advertiser's demand is given by $m(q^*(\mathbb{E}[v \mid v \notin A_0]))$, where A_0 is the (possibly empty) set of cookies bought by the advertisers. Thus, the publisher's marginal willingness to pay is given by

$$c(\mathbb{E}[m(q^*(v))] - \mathbb{E}[m(q^*(v \mid A_0))]),$$

and thus it is constant in the measure of cookies bought.
Proof of Proposition 5 (1.) Let cookies be uniformly distributed, and assume without loss that $v \in [0, 1]$. Suppose that the publisher purchases a measure δ of cookies, i.e. $v \in [a, a + \delta]$. Consider the publisher's revenues as a function of a,

$$R(a; \delta) = \int_0^a r(v)\, dv + \delta r\left(a + \frac{\delta}{2}\right) + \int_{a+\delta}^1 r(v)\, dv.$$

Now consider the optimal choice of a, and compute the derivative

$$
\begin{aligned}
R_a(a;\delta) &= r(a) - r(a+\delta) + \delta r'\left(a + \frac{\delta}{2}\right) \\
&= -\int_a^{a+\delta} \left(r'(v) - r'\left(a + \frac{\delta}{2}\right)\right) dv
\end{aligned}
$$

and conclude that $R_a(a;\delta) \geq (\leq)\, 0$ if the function $r'(v)$ is convex (concave).
(2.) The expression follows from implicit differentiation (twice) of the first-order condition for the advertisers' choice of contact intensity

$$v = cm'(q^* v).$$

(3.) This follows directly from substitution of $m(q) = q^b$ into (11). ■

Proof of Proposition 6. When the cost of advertising is exogenous, the monopolist solves

$$\max_p pD(p,c),$$

where $D(p,c)$ denotes the total demand for cookies at a price of p when the cost of advertising is c. The first-order condition is then given by

$$D(p,c) + pD_p(p,c) = 0.$$

When c is endogenous, the monopoly price satisfies

$$D(p,c^*) + pD_p(p,c^*) + pD_c(p,c^*)\frac{\partial c^*}{\partial p} = 0.$$

Applying Corollary 3, we obtain that the two partial derivatives $F_c(p,c)$ and $\partial c^*/\partial p$ have the same sign, thus the monopoly price will be higher in the second case. Similarly, fixing the cost of advertising to its equilibrium level $c(M)$ results in an increase in the monopoly price.. ■

Proof of Proposition 7. Under monopoly, the data provider's chooses v_2 to solve

$$v_2^M = \arg\max_v \left[p(v,v)(1 - F(v))\right].$$

The first-order condition is given by

$$-p(v,v) f(v) + \partial p(v,v_2^*)/\partial v + \partial p(v,v_2^*)/\partial x = 0.$$

Conversely, in the symmetric equilibrium with a continuum of sellers, the threshold v_2^* solves

$$-p(v,v_2^*) f(v) + \partial p(v,v_2^*)/\partial v = 0.$$

However,

$$\frac{\partial p\left(v,v_2^*\right)}{\partial x}=-\frac{\partial \pi\left(v,q^*\left(\mathbb{E}\left[v<x\right]\right)\right)}{\partial q}\frac{\partial q^*}{\partial v}\frac{\partial \mathbb{E}\left[v<x\right]}{\partial x}<0,$$

because $q^*\left(v\right)$ is strictly increasing in v, and therefore $\partial \pi\left(v,q\right)/\partial q>0$ for all $q<q^*\left(v\right)$. Therefore, the price under competition is higher than under monopoly. ■

Proof of Lemma 6. (1.) Consider the advertisers' problem upon **not** receiving a signal. If they choose $q=0$, the value of the marginal cookie is given by $\max\left\{0,v-c\right\}$, which is increasing in v. If instead they choose $q=1$, the value of the marginal cookie is $\max\left\{0,c-v\right\}$, hence decreasing in v. Therefore, the value of information is strictly monotone in v, with the sign of its derivative determined by the uninformed action.
(2.) The marginal willingness to pay is therefore increasing (decreasing) in c if$q_0=1$ ($q_0=0$). ■

In the next results, we denote the difference in profits by

$$\Delta\left(p;c\right)\triangleq\pi_+\left(p,c\right)-\pi_-\left(p,c\right)=-\int_{c-p}^{c+p}\left(v-c\right)dF\left(v\right)-p\left(1-F\left(c+p\right)-F\left(c-p\right)\right). \tag{28}$$

The following lemma establishes the key properties of the function $\Delta\left(p;c\right)$.

Lemma 10 *The difference in advertiser profits between high- and low-value purchases may be written as*

$$\Delta\left(p;c\right)=\int_{c-p}^{c+p}F\left(v\right)dv-p.$$

It satisfies the following properties.

1. $\Delta\left(p;c\right)$ *is convex (concave) in* p *if* $F\left(v\right)$ *is convex (concave) in* v.

2. $\Delta\left(0,c\right)=0$.

3. $\partial\Delta\left(0;c\right)/\partial p\geq 0$ *if and only if* $F\left(c\right)\geq 1/2$.

Proof of Lemma 10. (1.) Integrating (28) by parts, we obtain

$$\begin{aligned}\Delta\left(p;c\right) &= -\left(pF\left(c+p\right)+pF\left(c-p\right)-\int_{c-p}^{c+p}F\left(v\right)dv\right)-p\left(1-F\left(c+p\right)-F\left(c-p\right)\right)\\ &= \int_{c-p}^{c+p}F\left(v\right)dv-p.\end{aligned}$$

Differentiating with respect to p we obtain

$$\Delta_p\left(p;c\right)=F\left(c+p\right)+F\left(c-p\right)-1, \tag{29}$$

hence evaluating at $p=0$ we obtain $2F\left(c\right)-1$. Differentiating once more, we obtain

$$\Delta_{pp}\left(p;c\right)=f\left(c+p\right)-f\left(c-p\right),$$

which establishes the result.
(2.) and (3.) Follow by inspection of (28) and (29) respectively. ■

Proof of Proposition 8. (1.) By Lemma 10, if $f(v)$ is decreasing, $\Delta(p,c)$ is concave in p. For $c \leq c_M$, $\Delta(p,c)$ is decreasing in p at $p=0$, hence negative everywhere. For $c > c_M$, the difference $\Delta(p,c)$ is increasing in p at $p=0$. Furthermore, $\Delta(p,c)$ is increasing in c, and so the solution $\tilde{p}(c)$ to $\Delta(p,c)=0$ must be increasing in c, because $\Delta(p,c)$ must be decreasing in p at a positive root. Such a root exists as long as $\tilde{p}(c) \leq c$ (otherwise buying from the bottom cannot be profitable). Substituting $c=p$ into the definition of $\Delta(p,c)$ we obtain the equation in the text.
(2.) By Lemma 10, if $f(v)$ is increasing, $\Delta(p,c)$ is convex in p. For $c \geq c_M$, $\Delta(p,c)$ is increasing in p at $p=0$, hence positive everywhere. For $c < c_M$, the difference $\Delta(p,c)$ is decreasing in p at $p=0$, which means (by the same logic as in part (1.)) that the solution $\tilde{p}(c)$ must be decreasing in c. However, a positive root to the equation $\Delta(p,c)$ exists as long as $c+\tilde{p}(c) \leq 1$. Substituting $p=1-c$ into $\Delta(p,c)=0$ yields the equation in the text. ■

Proof of Lemma 8. (1.) The transfer payment is given by

$$t(\theta) = -c\int_{x(\theta)}^{1} g(v)\,dv - \int_{\underline{\theta}}^{\theta} \theta' x(\theta')\, g(x(\theta'))\frac{dx(\theta')}{d\theta'}d\theta', \tag{30}$$

and differentiating (30) with respect to θ we find:

$$\begin{aligned} t'(\theta) &= cf(x(\theta))\frac{dx(\theta)}{d\theta} - \theta x(\theta) f(x(\theta))\frac{dx(\theta)}{d\theta}d\theta \\ &= -\frac{dx(\theta)}{d\theta} f(x(\theta))(\theta x(\theta)-c) \geq 0, \end{aligned} \tag{31}$$

where the inequality follows from (17) and Lemma 7.
Proof. (2.) We can rewrite the transfer also in terms of the threshold $x(\theta)$, and hence $t(x(\theta))$, and so using (31), we get

$$t'(x)\frac{dx(\theta)}{d\theta} = -\frac{dx(\theta)}{d\theta} f(x(\theta))(\theta x(\theta)-c) \Leftrightarrow t'(x) = -f(x(\theta))(\theta x(\theta)-c).$$

Now, the unit price per cookie sold at realization $x(\theta)$ is given by:

$$\frac{t'(x(\theta))}{f(x(\theta))} = -(\theta x(\theta)-c),$$

and using the solution of $x(\theta)$ from (17), we get

$$\frac{t'(x(\theta))}{f(x(\theta))} = -\left(\theta\frac{c}{\theta-\frac{1-G(\theta)}{g(\theta)}} - c\right) = -\frac{c\frac{1-G(\theta)}{g(\theta)}}{\theta-\frac{1-G(\theta)}{g(\theta)}}.$$

Thus if $\theta' > \theta$ and hence $x\left(\theta'\right) < x\left(\theta\right)$, then

$$\frac{t'\left(x\left(\theta'\right)\right)}{f\left(x\left(\theta'\right)\right)} < \frac{t'\left(x\left(\theta\right)\right)}{f\left(x\left(\theta\right)\right)},$$

and thus the price per cookie is decreasing. ■ ■

Proof of Lemma 9. Consider an arbitrary, and finite, optimal mechanism $\{H_\theta, t(\theta)\}$. By hypothesis it satisfies the interim incentive constraints, that is for all types θ, θ', we have

$$U(\theta) \triangleq \left[\sum_{w \in W(\theta)} \max\{\theta w - c, 0\} h_\theta(w)\right] - t(\theta) \geq \left[\sum_{w \in W(\theta')} \max\{\theta w - c, 0\} h_{\theta'}(w)\right] - t\left(\theta'\right) \triangleq U\left(\theta, \theta'\right).$$

We denote by $W^+(\theta)$ the set of posterior expectations that lead to a contact with the advertiser, or

$$W^+(\theta) \triangleq \{w \in W \,| h_\theta(w) > 0 \wedge \theta w \geq c\},$$

and by $W^-(\theta)$ the set of posterior expectations that do not lead to a contact with the advertiser, or

$$W^-(\theta) \triangleq \{w \in W \,| h_\theta(w) > 0 \wedge \theta w < c\}.$$

Now, we can clearly bundle all the posterior expectations in $W^+(\theta)$ and in $W^-(\theta)$ to obtain a binary support as described in (24). Now clearly, under the constructed binary support, the indirect utility remains constants, but the value of a misreport is (weakly) smaller, that is for all $\theta \neq \theta'$:

$$\left[\sum_{w \in W(\theta')} \max\{\theta w - c, 0\} h_{\theta'}(w)\right] - t\left(\theta'\right) \geq \left[\sum_{w \in B(\theta')} \max\{\theta w - c, 0\} h_{\theta'}(w)\right] - t\left(\theta'\right),$$

after all, in the original deviation the advertiser could have acted as in the binary support, but he had a possibly larger set of choices available to him, and hence is doing weakly worse in the binary mechanism, i.e. the value of a misreport has been (uniformly) lowered across all types. ■

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