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Abstract

We study the effect of different types of rational expectations on two-sided platforms equilibrium profits—under monopoly and competition. One side (developers) always forms rational and responsive expectations. In contrast, we distinguish several ways in which the other side (users) may form rational expectations. Two extremes are responsive expectations (expectations that adjust perfectly to price changes on both sides) and passive expectations (users do not observe prices charged to developers and have fixed expectations about the number of developers who join). We also consider expectation formation mechanisms in-between the extremes: a hybrid of passive and responsive expectations, and wary expectations (users do not observe prices charged to developers but assume they are set optimally by platforms, given the user prices observed).

We show that platforms with market power (monopoly) prefer more responsive over less responsive expectations. In contrast, platforms in strongly competitive environments have the opposite preference: they derive higher profits when users have less responsive expectations. The main reason is that more responsive expectations amplify the effect of price reductions. Platforms with market power benefit because this leads to demand increases, which they are able to capture fully. Competing platforms suffer because more responsive expectations intensify price competition.

Keywords: two-sided platforms, responsive expectations, passive expectations, wary expectations

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1 Introduction

In markets with network effects, the value that users derive from platforms is determined by the number of other users of the same type who join the same platform (direct network effects) or the number of users of a different type that join (cross-group network effects). Examples include social networks like Facebook or Google+, payment systems like PayPal or Visa, videogame systems like PlayStation 3 and Xbox 360, smartphone platforms like Apple's iPhone or Google's Android, etc. In most real-world settings, however, users are unable to calculate the effect of at least *some* platform prices on adoption by other users - on the same side or on the other side of the market. Thus, users may not take *all* prices into account when forming expectations. This may be for a number of reasons. In two-sided contexts, one side (typically consumers) may simply not be aware of the price charged to the other side. For instance, few videogame console users are aware of the royalties that console manufacturers charge to third-party game developers. Few iPhone users are aware of the fees charged by Apple to third-party app developers. And even when all prices are known to everyone, some users may not have sufficient information about aggregate demand on each side to compute each demand's responsiveness to price changes. Instead, users typically rely on external information (e.g., press announcements, market reports, word of mouth) to form *passive* expectations about the total number of developers that join a given platform.

The majority of the existing literature on platform pricing in the presence of network effects (one-sided or two-sided) typically assumes that users on *all* sides have full information about *all* prices and the ability to perfectly compute their impact on platform adoption. Thus, users are assumed capable to perfectly adjust their expectations in response to changes in platform prices. In this paper we demonstrate that different mechanisms through which users form *rational* expectations—which can be interpreted as different degrees of user information and/or sophistication—lead to economically meaningful differences in terms of market outcomes, firm profits in particular.

We focus on two-sided platforms. To fix ideas, we label the two sides as "users" and "developers" (e.g., smartphone operating systems, videogame consoles). Throughout the paper we assume for simplicity that the developer side has full information about all prices and forms *rational and responsive* expectations about user participation. In contrast, we allow the user side to be uninformed about developer prices and analyze the impact of different mechanisms for forming *rational* expectations (i.e., fulfilled in equilibrium) by the users.

First, we focus on contrasting responsive and passive expectations. When users are informed about developer prices and take them into account in forming expectations about platform adoption by developers, we say that users form *responsive rational* expectations (just like developers do). Conversely, when users are uninformed about developer prices and do not adjust their expectation of platform adoption by developers in response to *any* platform price changes, they form *passive rational* expectations. Users do, however, change their individual adoption decisions based on the price they are being charged. Next, we consider cases in-between the extremes of responsive and passive expectations. Specifically, we consider a hybrid where the market included users of both responsive and passive expectations, and *wary expectations*. Wary expectations are formed when users do not observe developer prices so cannot adjust their expectations about developer participation based on what the platform charges developers, yet they do adjust those expectations based on observed user prices. Specifically, we assume that users expect the platform to charge the optimal developer price from its perspective, *conditional* on the observed user price.

It is worth emphasizing that all types of expectations we use are *rational*, i.e., fulfilled in equilibrium. Thus, we do not introduce any behavioral biases. Still, both passive and wary expectations are typically "incorrect" off the equilibrium path, the difference being that wary expectations are incorrect "less often."

Studying these various mechanisms of rational expectation formation allows us to reach a better understanding of how different mechanisms of expectation formation affect economic outcomes. We focus mainly on changes in platform profits, though we also discuss changes in prices and quantities. In particular, we show that a monopoly platform's profits are maximized when all users hold responsive expectations, and minimized when all users hold passive expectations, with wary expectations and hybrid cases yielding profits in-between. In contrast, the ranking of equilibrium profits in a symmetric duopoly with fixed user market size is exactly reversed. Equilibrium platform profits are highest under passive expectations and lowest under responsive expectations. In the hybrid scenario—where some users hold responsive expectations, while the remaining users hold passive expectations—monopoly profits are increasing in the fraction of users holding responsive expectations, while duopoly profits are decreasing in this fraction.

The broader implication of our analysis is that platforms with market power prefer more responsive over less responsive expectations, which can also be interpreted as a preference for facing more informed users. In contrast, platforms in strongly competitive environments have the opposite preference: they derive larger equilibrium profits when users are less informed. The main reason is that more responsive expectations (or more information about prices and demand structure) amplify the effect of price reductions. Platforms with market power benefit because this leads to demand increases, which they are able to capture fully. Competing platforms suffer because more responsive expectations intensify price competition.

2 Related literature

The existing literature on platform pricing in the presence of network effects (one-sided and two-sided) contains two different approaches to modelling user expectations: responsive and passive.[1]

The majority of models assume responsive expectations. In other words, users are assumed to be informed of all prices and capable of perfectly computing the effect of price changes on overall demands for a given platform. This approach originated with Katz and Shapiro (1986), who study competition between incompatible technologies with direct (one-sided) network effects. Most of the recent literature on two-sided markets (e.g., Armstrong (2006), Armstrong and Wright (2007), Caillaud and Jullien (2003), Choi (2010), Hagiu (2009), Halaburda and Yehezkel (forthcoming), Rochet and Tirole (2006), Weyl (2010)) also assumes responsive expectations. The logic for a monopoly two-sided platform is as follows. For the two sides $i = 1$, 2, the demand functions are given by $n_i = D_i\left(n_j, p_i\right)$, n_j is realized demand on side $j \neq i$ and p_i is the price charged by the platform to side i; $D_i\left(.,.\right)$ is increasing in its first argument and decreasing in its second argument. Then all authors adopt one of two methods. They either solve directly for (n_1, n_2) as a function of (p_1, p_2) only, and then maximize the resulting profit expression over (p_1, p_2). Or they invert $n_i = D_i\left(n_j, p_i\right)$ to express p_i as a function of (n_i, n_j), then replace in the expression of profits and maximize over (n_1, n_2). Note that indeed (n_1, n_2) determines a unique (p_1, p_2). The problem (which arises with both methods) is that the reverse may not be true: there may be multiple equilibrium solutions (n_1, n_2) for a given (p_1, p_2). This issue is usually side-stepped by assuming platforms have the ability to coordinate users on the allocation (n_1, n_2) they prefer. Parker

[1] In models of platform adoption with network effects where platforms do not make pricing decisions (e.g., Church and Gandal (1992), Farrell and Saloner (1985) and (1986)), the distinction between various types of rational expectations is meaningless.

and Van Alstyne (2005) use a slightly different approach, by *directly* assuming two-sided demands that depend on both prices: $n_1 = D_1(p_1) + \alpha_1 D_2(p_2)$ and $n_2 = D_2(p_2) + \alpha_2 D_1(p_1)$, where $0 < \alpha_1, \alpha_2 < 1$.

Argenziano (2007) applies the global games methodology to the study of competition between one-sided networks. While each user receives a noisy signal regarding the standalone value of a given platform, she is still assumed capable of calculating the optimal adoption strategies for all other users as a function of platform price. Ambrus and Argenziano (2009) show that multiple asymmetric networks can coexist in equilibrium when agents are heterogeneous. In their model too, each individual agent observes all platform prices and calculates the resulting adoption decisions by all other agents. Thus, all of these papers rely on responsive rational expectations.

Passive expectations were first introduced in the economic literature on one-sided network effects by Katz and Shapiro (1985). This was also the first paper to explicitly distinguish passive rational expectations from responsive rational expectations.[2] In particular, Katz and Shapiro (1985) study Cournot competition between n firms (technologies) with direct network effects, where each firm sets price strategically. In the main text of the paper, the authors analyze the case of passive expectations in Cournot equillibrium: each firm chooses its output taking other firms' decisions and users' expectations regarding firms' outputs as fixed. In the appendix, the authors also analyze the case of responsive expectations, where users' expectations adjust (correctly) based on firms' output decisions. They confirm that most of their analysis applies to both cases but do not compare firms' equilibrium profits and prices under the two types of expectations. Nor do they treat the mixed case in which some users form responsive expectations, while others form passive expectations. The same Cournot model with fixed and fulfilled expectations is also used by Economides (1996) to study the incentives of a network leader to invite entry by competing followers.

Gabszewicz and Wauthy (2012) is the only two-sided model we are aware of that incorporates passive expectations. Moreover, they investigate the difference in equilibrium outcomes between passive and responsive expectations. In their paper, users on both sides are differentiated by the intensity of their indirect network effects. With fixed rational expectations, their two-sided demands are $n_i = D_i\left(n_j^e, p_i\right)$, where n_j^e is the demand on side j *expected* by agents on side i. The platform maximizes $\Pi = p_1 D_1(n_2^e, p_1) + p_2 D_2(n_1^e, p_2)$ over (p_1, p_2) treating (n_1^e, n_2^e) as exogenously given. Then rational expectations require $(n_1, n_2) = (n_1^e, n_2^e)$,

[2] Matutes and Vives (1996) do so in a model of financial intermediation.

which closes the loop by determining equilibrium demands and prices. The same approach is used for competing platforms. Gabszewicz and Wauthy (2012) show that responsive expectations lead to wider participation on the platform. In our paper, we undertake a systematic investigation of how different types of expectations affect the equilibrium outcome. Aside from passive and responsive expectations—investigated by Gabszewicz and Wauthy—we also analyze wary expectations and a hybrid model in which users with different expectation types are present in the market.

More recently, two papers have explicitly pointed out the impact of different user expectations (passive rational and responsive rational) on equilibrium allocations in markets with direct (i.e., one-sided) network effects. Griva and Vettas (2011) study price competition between two firms which are both horizontally and vertically differentiated. They find that competition under responsive expectations tends to be more intense and results in larger market shares captured by the high-quality firm, relative to the case with fixed expectations. Hurkens and Lopez (2012) study the impact of competition between communication networks. They show that replacing the standard assumption of responsive consumer expectations with a more realistic assumption of passive consumer expectations leads to radically different conclusions regarding firm preferences for termination charges. Specifically, with responsive consumer expectations, firms prefer lower (below-cost) termination charges. Instead, with passive consumer expectations, firms prefer above cost termination charges, consistent with the real-world tension between mobile operators and their regulators.

There are three key differences between these two papers and our analysis. First, both papers rely on models with one-sided network effects, whereas our main model is two-sided. Second, they focus on duopoly settings only, whereas we are interested in comparing the impact of expectations in monopoly and duopoly contexts. Indeed, a key insight that emerges from our analysis is that the effect of the mechanism through which expectations are formed is very different depending on the market structure. Third, Griva and Vettas (2011) and Hurkens and Lopez (2012) only consider two extreme cases —all users holding passive expectations and all users holding responsive expectations. In contrast, our model encompasses expectation formation mechanisms in-between the extremes: a continuous hybrid of passive and responsive expectations, and wary expectations. We are, thus, able to study the effect of small changes in the nature of user expectations on market equilibria and firm profits.

Evans and Schmalensee (2010) study platform adoption in the presence of network effects with imperfect, dynamic adjustments of user participation decisions. At a high level, our paper is related to theirs in the effort to formally capture imperfections in the mechanisms through which users form expectations—a prevalent phenomenon in real-world settings. The key difference is that in their model, platform prices are fixed and the focus is on determining conditions (critical mass) under which the imperfect dynamic adjustment process converges to positive levels of platform adoption.

Finally, we explicitly connect the various mechanisms of expectation formation in the context of two-sided network effects to the mechanisms of belief formation studied in the vertical contracting literature (Hart and Tirole 1990, McAfee and Schwartz 1994, Rey and Verge 2004). In that literature, a downstream firm D receiving an unexpected (out of equilibrium) offer from an upstream monopolist U must form beliefs about the changes in U's offers to D's rivals (which D does not observe). Passive expectations in our context correspond to the notion of passive beliefs in the vertical contracting context: D does not adjust its beliefs about contracts offered to rivals when D receives an unexpected offer from U. To the best of our knowledge, ours is the first paper to introduce the notion of *wary expectations*, a concept directly drawn from the notion of *wary beliefs* studied by McAfee and Schwartz (1994) and Rey and Verge (2004). In vertical contracting, D holds wary beliefs if, whenever it receives an unexpected offer, it anticipates that U also optimally adjusts its offers to D's rivals *given* the offer just received by D. In our context, a user who does not observe the price p_d charged by a two-sided platform to developers holds *wary expectations* if, when presented with a price p_u, she *assumes* that the platform's price p_d maximizes the platform's profits *given* p_u.

3 Responsive vs. Passive Expectations

Consider a monopoly two-sided platform: to fix ideas, suppose it is a videogame console connecting users (u) with game developers (d). We assume linear demand on both sides:

$$n_u = 1 + \alpha_u n_d^e - p_u \quad \text{and} \quad n_d = \alpha_d n_u^e - p_d \, . \tag{1}$$

Thus, demand on each side depends positively on the *expectation* of participation on the other side: users expect n_d^e developers to join and developers expect n_u^e users to join. The surplus derived by a user from the participation of each developer is $\alpha_u > 0$, while the profit made by each developer on every platform user is $\alpha_d > 0$. We assume the following condition holds throughout the paper, which ensures all monopoly maximization problems are well-behaved:[3]

$$\alpha_u + \alpha_d < 2\,.$$

In this formulation, the platform has standalone value normalized to 1 for every user but no standalone value for developers. Standalone values have no bearing on our results, therefore we have chosen the simplest possible formulation, with positive standalone value on one side (u) only. This is also quite realistic given the videogame console example we have in mind: the console may offer first-party games, web browsing and streaming movie services to users, but writing games for a console with no users is arguably worthless for developers.

Information and expectations. Throughout the paper we assume that all developers hold *responsive* expectations about user participation. In other words, they are fully informed about user prices p_u and their expectations about user participation *always* match *realized* user participation:

$$n_u^e = n_u \text{ for } \textit{any} \text{ given price pair } (p_u, p_d)\,.$$

In contrast, we allow users to be uninformed about developer prices and hold different types of rational expectations. Specifically, in this section we compare two polar cases:

- users have *responsive expectations*,
- users have *passive expectations*.

The asymmetry in information about prices on the other side between users and developers is realistic. Game developers are usually aware of console prices and have a good understanding of console user demand. On the other hand, most users have limited information about and understanding of the royalty arrangements between console manufacturers and game developers (or between Apple and iPhone app developers). These users most likely form their expectations of developer participation based on external information (e.g. news articles, word-of-mouth) and do not adjust them in response to the prices *actually* charged by platforms.[4]

[3] This condition also implies $\alpha_u \alpha_d < 1$.

[4] Throughout our analysis we only present the case where developers hold responsive expectations. From a formal perspective, introducing passive expectations on both sides does not change our analysis in any

Finally, is is worth emphasizing that all types of expectations we study are rational, i.e., fulfilled in equilibrium. The key difference lies in how users react to off-equilibrium prices, similarly to the distinctions in beliefs studied in the vertical contracting literature.

3.1 Monopoly

Users have responsive expectations. In this scenario, all users observe developer prices p_d and adjust their expectations accordingly, so that n_d^e matches realized developer participation n_d for any price pair (p_u, p_d) chosen by the platform. Two-sided demands from (1) can therefore be written:

$$n_u = 1 + \alpha_u n_d - p_u \quad \text{and} \quad n_d = \alpha_d n_u - p_d \, .$$

This is the standard formulation used in most of the two-sided market literature. It is straightforward to solve the last two equations for (n_u, n_d) as functions of (p_u, p_d) only:

$$n_u = \frac{1 - p_u - \alpha_u p_d}{1 - \alpha_u \alpha_d} \quad \text{and} \quad n_d = \frac{\alpha_d - p_d - \alpha_d p_u}{1 - \alpha_u \alpha_d} \, .$$

We can then optimize the platform's profits $p_u n_u + p_d n_d$ directly over (p_u, p_d), obtaining the following profit-maximizing prices and demands:

$$p_u^* = \frac{2 - \alpha_d (\alpha_d + \alpha_u)}{4 - (\alpha_d + \alpha_u)^2} \quad \text{and} \quad p_d^* = \frac{\alpha_d - \alpha_u}{4 - (\alpha_d + \alpha_u)^2} \, , \tag{2}$$

$$n_u^* = \frac{2}{4 - (\alpha_d + \alpha_u)^2} \quad \text{and} \quad n_d^* = \frac{\alpha_d + \alpha_u}{4 - (\alpha_d + \alpha_u)^2} \, . \tag{3}$$

This leads to optimal platform profits:

$$\Pi_M^* \,(\text{responsive}) = \frac{1}{4 - (\alpha_d + \alpha_u)^2} \, . \tag{4}$$

Users have passive expectations. In this scenario, users do not observe developer prices and therefore cannot adjust their expectations regarding developer participation (n_d^e) in response to *any* changes in platform prices (p_d or p_u). Expectations are fulfilled in equilibrium.

meaningful way. All results in the propositions below remain unchanged. (The corresponding analysis with passive expectations on both sides is available from the authors upon request.)

In turn, the platform has no choice but to treat users' passive expectations n_d^e as fixed when it sets its prices. From a modelling perspective, passive expectations are equivalent to assuming that users form fixed rational expectations n_d^e about developer participation *before* the platform sets prices (p_u, p_d). Thus, from the platform's perspective, two-sided *realized* demands now depend not just on prices but also on users' passive (fixed) expectations n_d^e, which the platform must take as given:

$$n_u = 1 + \alpha_u n_d^e - p_u \quad \text{and} \quad n_d = \alpha_d + \alpha_u \alpha_d n_d^e - p_d - \alpha_d p_u \,.$$

We can then optimize the platform's profits $p_u n_u + p_d n_d$ over (p_u, p_d), obtaining prices and realized demands $p_u^* (n_d^e)$, $p_d^* (n_d^e)$, $n_u^* (n_d^e)$ and $n_d^* (n_d^e)$, all of which depend on n_d^e. The final equilibrium is then obtained by imposing the rationality condition $n_d^* (n_d^e) = n_d^e$. It is straightforward to obtain the following optimal prices and demands:

$$p_u^* = \frac{2 - \alpha_d^2}{4 - (\alpha_d + \alpha_u)\,\alpha_d} \quad \text{and} \quad p_d^* = \frac{\alpha_d}{4 - (\alpha_d + \alpha_u)\,\alpha_d}\,, \tag{5}$$

$$n_u^* = \frac{2}{4 - (\alpha_d + \alpha_u)\,\alpha_d} \quad \text{and} \quad n_d^* = \frac{\alpha_d}{4 - (\alpha_d + \alpha_u)\,\alpha_d}\,. \tag{6}$$

The resulting optimal platform profits are:

$$\Pi_M^* \,(\text{passive}) = \frac{4 - \alpha_d^2}{\left[4 - (\alpha_d + \alpha_u)\,\alpha_d\right]^2}\,. \tag{7}$$

Comparison and discussion. We are interested in determining the effect of user expectations on platform profits. Comparing (4) and (7) leads to:

Proposition 1 *A monopoly platform makes higher profits when users hold responsive expectations than when users hold passive expectations:*

$$\Pi_M^* \,(\textit{passive}) < \Pi_M^* \,(\textit{responsive})\,.$$

This result is interpreted as follows. Users with passive expectations do not adjust their expectations in response to price changes, therefore they are less reactive to price changes (on either side). This means that, if we started in the responsive expectations equilibrium and transformed all users from responsive into passive, then the platform would be tempted to modify one or both of its prices in order to exploit users' lack of responsiveness. But users factor this behavior in their rational expectations formed *ex-ante*, which in turn reduces the profits that the platform can extract *ex-post*. Indeed, another way to interpret this result is to recall that users with passive expectations are in effect credibly committed to ignore the platform's price changes when forming their expectations. The platform's profits are therefore lower relative to the scenario when prices also influence user expectations.

It is in fact possible to prove generally (i.e., regardless of the shape of demand functions) that the scenario with responsive expectations maximizes monopoly platform's profits under all circumstances with rational expectations.

Proposition 2 *A monopoly two-sided platform achieves maximum profits in a rational expectations equilibrium when all agents on both sides hold responsive expectations.*

Proof. See Appendix A.1.

The logic behind this result is straightforward. Since expectations are fulfilled in equilibrium, the platform can replicate any rational expectations market allocation under the scenario with responsive expectations on both sides. The reverse is, however, not true: if expectations are not fully responsive then some rational expectations market allocations may not be attainable, which limits the platform's ability to attain the first-best outcome.

The result in Propositions 1 and 2 has an important implication. Whenever feasible, a monopoly two-sided platform would like to inform *all* users of the prices charged to developers and of the way in which developer demand responds to price changes.[5] As we will see in the next section, the opposite is true with competing platforms.

3.2 Competition

Let us now turn to the case of competition between two symmetric platforms. Users are distributed along a Hotelling segment $[0, 1]$ with density 1 and transportation costs $t > 0$. They are interested in joining at most one platform (i.e., they singlehome). From the perspective

[5] In this section we only compare the extreme cases where all users have responsive or all users have passive expectations. We study a hybrid model in Section 5.

of developers, the two platforms are identical, but developers are allowed to multihome, i.e., join both platforms.[6] Developers are differentiated by the fixed cost they incur when joining each platform. The fixed cost per platform is the same regardless of whether a developer joins one or both platforms, i.e., there are no economies of scope in joining multiple platforms (cf. Hagiu (2009)).

User demands are:

$$n_{u1} = \frac{1}{2} + \frac{\alpha_u \left(n_{d1}^e - n_{d2}^e\right) + p_{u2} - p_{u1}}{2t}$$

for platform 1, and $n_{u2} = 1 - n_{u1}$ for platform 2, where n_{d1}^e and n_{d2}^e are users' expectations about developer participation on each platform. Developer demands are $n_{d1} = \alpha_d n_{u1}^e - p_{d1}$ for platform 1, and $n_{d2} = \alpha_d n_{u2}^e - p_{d2}$ for platform 2.

In order to guarantee all optimization problems with competing platforms are well-behaved, we assume throughout the paper:

$$t > \alpha_u \alpha_d \, .$$

As in the previous section, all developers hold *responsive* expectations about user participation, i.e., $n_{ui}^e = n_{ui}$ for $i = 1, 2$. Once again, we focus on the two polar types of user expectations: responsive and passive expectations.

Users have responsive expectations. In this case, all users adjust their expectations n_{di}^e to match realized developer participation n_{di} $(i = 1, 2)$ for any prices chosen by the two platforms. Two-sided demands are then:

$$\begin{aligned} n_{u1} &= \frac{1}{2} + \frac{\alpha_u \left(n_{d1} - n_{d2}\right) + p_{u2} - p_{u1}}{2t} \quad \text{and} \quad n_{d1} = \alpha_d n_{u1} - p_{d1} \, , \\ n_{u2} &= 1 - n_{u1} \quad \text{and} \quad n_{d2} = \alpha_d n_{u2} - p_{d2} \, . \end{aligned}$$

It is straightforward to solve for user demands as functions of prices:

$$n_{u1} = \frac{1}{2} + \frac{p_{u2} - p_{u1} + \alpha_u \left(p_{d2} - p_{d1}\right)}{2\left(t - \alpha_u \alpha_d\right)} \, .$$

Platforms simultaneously choose prices to maximize profits, $p_{ui} n_{ui} + p_{di} n_{di}$ for $i = 1, 2$.

[6] In Appendix A.4, we also solve the model with singlehoming on both sides. The main result (ranking of profits under various types of expectations) is unchanged.

Taking the first-order conditions of platform i's profits in p_{ui} and p_{di} and solving for the symmetric equilibrium, we obtain the following equilibrium prices and demands:

$$p_u^* = t - \frac{3\alpha_d\alpha_u}{4} - \frac{\alpha_d^2}{4} \quad \text{and} \quad p_d^* = \frac{\alpha_d - \alpha_u}{4} \, ,$$

$$n_u^* = \frac{1}{2} \quad \text{and} \quad n_d^* = \frac{\alpha_d + \alpha_u}{4} \, .$$

Equilibrium profits are:

$$\Pi_C^* \left(\text{responsive}\right) = \frac{t}{2} - \frac{\alpha_d^2}{16} - \frac{6\alpha_d\alpha_u + \alpha_u^2}{16} \, . \tag{8}$$

Users have passive expectations. In this scenario, users do not adjust their expectations n_{di}^e to *any* changes in platforms' prices. Platform 1's profits are then simply:

$$p_{u1}n_{u1} + p_{d1}n_{d1} = \left(p_{u1} + \alpha_d p_{d1}\right)\left[\frac{1}{2} + \frac{\alpha_u\left(n_{d1}^e - n_{d2}^e\right) + p_{u2} - p_{u1}}{2t}\right] - p_{d1}^2 \, .$$

Taking the first order conditions in (p_{u1}, p_{d1}) with fixed (n_{d1}^e, n_{d2}^e) and imposing the symmetric equilibrium condition $n_{u1} = n_{u2} = \frac{1}{2}$ and the rational expectations condition $n_{d1} = n_{d2} = n_{d1}^e = n_{d2}^e$, we obtain the following equilibrium prices and demands:

$$p_u^* = t - \frac{\alpha_d^2}{4} \quad \text{and} \quad p_d^* = \frac{\alpha_d}{4} \, ,$$

$$n_u^* = \frac{1}{2} \quad \text{and} \quad n_d^* = \frac{\alpha_d}{4} \, ,$$

resulting in equilibrium profits:

$$\Pi_C^* \left(\text{passive}\right) = \frac{t}{2} - \frac{\alpha_d^2}{16} \, . \tag{9}$$

Comparison and discussion. Comparing equilibrium profit expressions (8) and (9) leads to:[7]

[7] In Appendix A.4, we show that this result also holds when both sides single-home and market size is fixed on both sides.

Proposition 3 *In the symmetric duopoly equilibrium with fixed Hotelling competition for users, platform profits are higher when all users hold passive expectations relative to the case when all users hold responsive expectations:*

$$\Pi_C^* \left(passive \right) > \Pi_C^* \left(responsive \right).$$

Remarkably, this is the reverse result relative to the monopoly platform case (cf. Proposition 1), where platform profits were higher under responsive expectations. The reason is as follows. When platforms compete for share in a market of fixed size on at least one side, each platform's incentives to lower price are strongest when users have responsive expectations. Indeed, such users respond to price decreases in two ways: they adjust their own demand, as well as their expectation of developer demand—both upwards. This creates intense price competition. With passive expectations, price competition is less severe because price cuts on the user side are less effective. Indeed, users holding passive expectations only consider the impact of a lower price on their own participation and ignore the impact of increased participation by users on developer participation.

Griva and Vettas (2011) obtain a similar result in a one-sided framework: they show that switching from passive to responsive expectations tends to lead to more intense competition in a duopoly with one-sided network effects.

Generalized competition. Proposition 3 implies that competing platforms with fixed market size on at least one side are better off when users are uninformed about developer prices—the opposite implication than for a monopoly platform (Propositions 1 and 2). One would then expect that, in a more general model of platform competition, equilibrium profits should be higher under passive expectations if and only if competition for users is sufficiently intense.

To confirm and illustrate this point, consider the following generalized duopoly model, which we explicitly solve in Appendix A.3. The user market is now composed of three segments: the Hotelling segment above and two symmetric "hinterlands"—one for each platform—on which user demand is identical to the demand for a monopoly platform from Section 3. The Hotelling segment has density x, while the monopolistic hinterlands have density $(1 - x)$, where $x \in [0, 1]$. The structure of developer demand remains unchanged, i.e., each platform acts as a monopolist vis-a-vis developers. Thus, demands for platform 1 are:

$$n_{u1} = x \left[\frac{1}{2} + \frac{\alpha_u \left(n_{d1}^e - n_{d2}^e \right) - (p_{u1} - p_{u2})}{2t} \right] + (1 - x) \left(1 + \alpha_u n_{d1}^e - p_{u1} \right) ,$$

$$n_{d1} = \alpha_d n_{u1} - p_{d1} ,$$

and symmetrically for platform 2, where $n_{d1}^e = n_{d1}$ and $n_{d2}^e = n_{d2}$ when users hold responsive expectations.

The parameter x is a measure of the intensity of competition for users between the two platforms. Indeed, one can interpret x as the probability perceived by each platform that it will be facing competition for users. When $x = 0$ each platform is a monopolist as in Section 3.1. When $x = 1$ we return to the competition scenario with fixed market size studied earlier in this section.

We solve this model explicitly in Appendix A.3. Figure 1 below shows an example of equilibrium platform profits under responsive expectations and passive expectations as functions of x. Both profit curves are decreasing in x as expected and they cross once at $x_0 \in [0, 1]$, such that profits with responsive expectations are higher if and only if x is below the cutoff x_0. In other words, equilibrium platform profits fall faster with the intensity of competition x when user expectations are responsive. This reinforces the point that responsive expectations lead to more severe price competition.

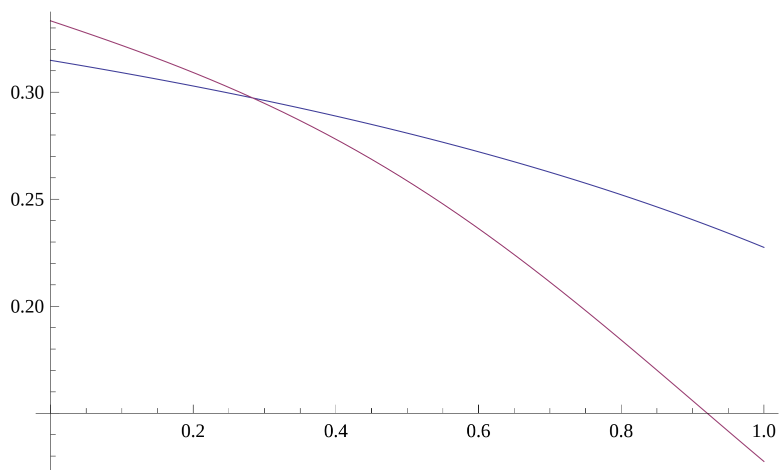


Figure 1: Equilibrium platform profits under responsive expectations (red line) and passive expectations (blue line), both as functions of x, with parameter values $t = 0.5$, $\alpha_u = 0.4$ and $\alpha_d = 0.6$.

This analysis confirms that equilibrium profits are higher under passive expectations if and only if the competition for users is sufficiently intense. Moreover, this section has established that the mechanisms through which expectations are formed have different effects on platform profits depending on the market structure (monopoly or competition). In what follows we further develop our analysis by introducing expectation configurations that lie in-between the two polar extremes of pure responsive and pure passive expectations: wary expectations, and a hybrid of passive and responsive expectations.

4 Wary expectations

Aside from the two extremes of passive and responsive expectations studied above, there are potentially many ways in which users can form *rational* expectations about developer participation. In this section, we focus on *wary expectations.* Our concept of wary expectations is directly adapted from the concept of wary beliefs from the vertical contracting literature (McAfee and Schwartz (1994), Rey and Verge (2004)). A user who does not observe developer prices is said to hold wary expectations if, when presented with any off-equilibrium price p_u, she *assumes* that the platform has also adjusted its price to developers, p_d, to maximize profits, *given* p_u. This leads the user to form her expectation of the number of developers who join the platform based on the price that she observes: $n_d^e(p_u)$.

As with passive expectations, when users hold wary expectations, they are assumed to be uninformed about developer prices. The difference is that when users hold wary expectations, they attempt to make the optimal inference about developer prices, based on the information available to them, i.e. p_u. Thus, in a sense, wary expectations lie somewhere in-between the two polar extremes of perfectly responsive and perfectly passive expectations in terms of responsiveness to platform prices. One can also view wary expectations as requiring an intermediate level of user information and/or sophistication. For this reason, studying wary expectations allows us to better understand how different degrees of information in expectation formation affect outcomes: prices and profits.

4.1 Monopoly

When all users have wary expectations, two-sided demands are:

$$n_u = 1 + \alpha_u n_d^e(p_u) - p_u \quad \text{and} \quad n_d = \alpha_d n_u - p_d\,,$$

where $n_d^e(p_u)$ is the wary expectation formed by users about the number of participating developers. The difference with passive expectations is that now n_d^e responds to changes in the price charged to users. In calculating $n_d^e(p_u)$, users *assume* the platform sets p_d optimally given p_u. That assumption is of course incorrect for *most* prices off the equilibrium path, but it is correct "more often" than passive expectations.

Denote by $p_d^e(p_u)$ the price that users with wary expectations anticipate the platform is charging. Thus, $p_d^e(p_u)$ and $n_d^e(p_u)$ are determined by maximizing profits $p_u n_u + p_d n_d$ over p_d (with p_u and n_d^e taken as exogenously given), and then imposing $n_d = n_d^e$. In other words, when users form wary expectations, they take into account that the platform cannot influence their expectations once p_u is fixed, but they impose rationality of those expectations, i.e., the developer demand resulting from the platform's *hypothetical* optimization must be consistent with (equal to) users' expectations.

The platform's profits can be expressed as:

$$p_u n_u + p_d n_d = (p_u + \alpha_d p_d)\left[1 + \alpha_u n_d^e - p_u\right] - p_d^2 .$$

Taking the first-order condition in p_d and imposing $n_d = n_d^e$, we obtain:

$$n_d^e(p_u) = p_d^e(p_u) = \frac{\alpha_d(1 - p_u)}{2 - \alpha_u \alpha_d} .$$

Thus, the level of developer participation expected by users is decreasing in the observed user price p_u. This is unsurprising: users understand that developer demand is increasing in user participation, which is in turn decreasing in the price charged to users. More interestingly, the price $p_d^e(p_u)$ that users expect the platform to charge developers is also decreasing in p_u. The reason is that, at fixed user expectations n_d^e, the platform's prices p_u and p_d are strategic substitutes (cf. expression of platform profits above).

With the expression of user wary expectations in hand, we can now turn to the actual optimization of platform profits:

$$p_u n_u + p_d n_d = \frac{2(p_u + \alpha_d p_d)(1 - p_u)}{2 - \alpha_u \alpha_d} - p_d^2 ,$$

which the platform maximizes over (p_u, p_d). It is straightforward to obtain:

$$p_u^* = \frac{2 - \alpha_u \alpha_d - \alpha_d^2}{4 - 2\alpha_u \alpha_d - \alpha_d^2} \quad \text{and} \quad p_d^* = \frac{\alpha_d}{4 - 2\alpha_u \alpha_d - \alpha_d^2}\,, \tag{10}$$

$$n_u^* = \frac{2}{4 - 2\alpha_u \alpha_d - \alpha_d^2} \quad \text{and} \quad n_d^* = \frac{\alpha_d}{4 - 2\alpha_u \alpha_d - \alpha_d^2}\,. \tag{11}$$

Note in particular that $n_d^* = n_d^e(p_u^*)$, i.e. wary expectations are rational, as noted from the outset. The resulting platform profits are:

$$\Pi_M^*(\text{wary}) = \frac{1}{4 - 2\alpha_u \alpha_d - \alpha_d^2}\,. \tag{12}$$

Comparing (12) with (4) and (7) yields:

Proposition 4 *Monopoly profits when all users hold responsive expectations are higher relative to monopoly profits when all users hold wary expectations, which in turn are higher than monopoly profits when all users hold passive expectations:*

$$\Pi_M^*(\textit{responsive}) > \Pi_M^*(\textit{wary}) > \Pi_M^*(\textit{passive})\,.$$

Thus, monopoly profits when users hold wary expectations are between monopoly profits under responsive and profits under passive expectations. This result is understood by noting that wary expectations are more responsive than passive expectations (wary users take one price into account when forming expectations), but they are of course less responsive than perfectly responsive expectations. The result confirms the intuition derived in Section 3 regarding the impact of user expectations on monopoly platform profits. A monopoly platform prefers users to hold more responsive expectations because they tend to enlarge the set of market allocations with rational expectations that the monopolist can achieve.

Second, note that the equilibrium two-sided prices are different under the three expectation mechanisms and have somewhat different comparative statics in the model parameters. This should not be too surprising: two-sided platforms typically re-adjust their two-dimensional pricing structures (e.g. increase one price and decrease the other) in response to changes in the external environment (e.g. type of expectations held by users).

Specifically, comparing (2), (5) and (10), it is straightforward to show that:

$$\begin{aligned} p_u^* \text{ (responsive)} &\leq p_u^* \text{ (passive)} \Longleftrightarrow \alpha_u \leq \alpha_d \\ p_d^* \text{ (responsive)} &\leq p_d^* \text{ (passive)} \Longleftrightarrow \alpha_d (\alpha_u + \alpha_d) > 2 \end{aligned}$$

$$\begin{aligned} p_u^* \text{ (responsive)} &\geq p_u^* \text{ (wary)} \text{ for all } \alpha_u, \alpha_d \\ p_d^* \text{ (responsive)} &\leq p_d^* \text{ (wary)} \Longleftrightarrow \alpha_d (3\alpha_u + \alpha_d) \leq 4 \end{aligned}$$

$$\begin{aligned} p_u^* \text{ (passive)} &\geq p_u^* \text{ (wary)} \text{ for all } \alpha_u, \alpha_d \\ p_d^* \text{ (passive)} &\leq p_d^* \text{ (wary)} \text{ for all } \alpha_u, \alpha_d . \end{aligned}$$

In particular, note that, unlike profits, prices with wary expectations need not be between the corresponding prices with responsive and passive expectations.

For all three types of expectations, the respective equilibrium demands (3), (6) and (11) are increasing in both network effects parameters α_u and α_d, as expected. On the other hand, the comparative statics of equilibrium prices (2), (5) and (10) in α_u and α_d are less trivial. We summarize them in the following table:

	Equilibrium user price p_u^*	**Equilibrium developer price p_d^***
Responsive expectations	negative iff $2 < \alpha_d (\alpha_d + \alpha_u)$ increasing in α_u iff $4\alpha_u > \alpha_d (\alpha_d + \alpha_u)^2$ decreasing in α_d iff $4\alpha_d > \alpha_u (\alpha_d + \alpha_u)^2$	negative iff $\alpha_d < \alpha_u$ increasing in α_d iff $(3\alpha_u - \alpha_d)(\alpha_u + \alpha_d) < 4$ decreasing in α_u iff $(3\alpha_d - \alpha_u)(\alpha_u + \alpha_d) < 4$
Passive expectations	negative iff $2 < \alpha_d^2$ increasing in α_u iff $2 > \alpha_d^2$ decreasing in α_d iff $4\alpha_d > \alpha_u \left(2 + \alpha_d^2\right)$	always positive always increasing in α_d always increasing in α_u
Wary expectations	negative iff $2 < \alpha_d (\alpha_d + \alpha_u)$ always decreasing in α_u always decreasing in α_d	always positive always increasing in α_d always increasing in α_u

These differences can have significant empirical implications. For instance, a monopoly two-sided platform facing users with passive expectations never subsidizes developers, but it might do so when users have responsive expectations (when $\alpha_u > \alpha_d$). Furthermore, different comparative statics in (α_u, α_d) imply that equilibrium prices respond differently to changes in the network effects parameters under different mechanisms of expectation formation. Thus, if one were to estimate network effects parameters based on observed prices, results would be different depending on the type of expectations that prevail.

4.2 Competition

Let us now turn to the Hotelling duopoly case, the same as the one studied in Section 3.2, except that now users hold wary expectations. Specifically, users form wary expectations n_{d1}^e and n_{d2}^e by assuming the two platforms' developer prices p_{d1} and p_{d2} are set optimally (Nash equilibrium) given (p_{u1}, p_{u2}) and (n_{d1}^e, n_{d2}^e). The expectations n_{d1}^e and n_{d2}^e, which depend on p_{u1} and p_{u2}, are determined by optimizing platform 1's profits over p_{d1}:

$$p_{u1}n_{u1} + p_{d1}n_{d1} = (p_{u1} + \alpha_d p_{d1}) \left[\frac{1}{2} + \frac{\alpha_u (n_{d1}^e - n_{d2}^e) + p_{u2} - p_{u1}}{2t} \right] - p_{d1}^2 .$$

Taking the first-order condition in p_{d1} (this is the platform optimization problem that users *assume* is taking place) yields:

$$p_{d1}^e = \frac{\alpha_d}{4} + \frac{\alpha_u \alpha_d (n_{d1}^e - n_{d2}^e) + \alpha_d (p_{u2} - p_{u1})}{4t} .$$

Similarly for platform 2 and p_{d2}^e.

At these prices:

$$n_{d1} - n_{d2} = \alpha_d (n_{u1} - n_{u2}) - (p_{d1}^e - p_{d2}^e) = \frac{\alpha_d \alpha_u (n_{d1}^e - n_{d2}^e) + \alpha_d (p_{u2} - p_{u1})}{2t} .$$

Finally, imposing the rationality condition $n_{d1} = n_{d1}^e$ and $n_{d2} = n_{d2}^e$, we obtain:

$$n_{d1}^e - n_{d2}^e = p_{d1}^e - p_{d2}^e = \frac{\alpha_d (p_{u2} - p_{u1})}{2t - \alpha_d \alpha_u} .$$

Thus, consistent with the monopoly case, the difference between the two platforms' developer prices expected by users is *decreasing* in the corresponding difference between user prices. Once again, this is because each platform's prices p_{ui} and p_{di} $(i = 1, 2)$ are strategic substitutes at fixed user expectations n^e_{d1} and n^e_{d2} (cf. platform 1's profit expression above).

Using the last equation, we can express user demand as a function of prices only:

$$n_{u1} = \frac{1}{2} + \frac{p_{u2} - p_{u1}}{2t - \alpha_u \alpha_d}.$$

We can now write platform 1's profits as:

$$p_{u1} n_{u1} + p_{d1} n_{d1} = (p_{u1} + \alpha_d p_{d1}) \left(\frac{1}{2} + \frac{p_{u2} - p_{u1}}{2t - \alpha_u \alpha_d} \right) - p_{d1}^2.$$

Taking the first-order conditions in p_{u1} and p_{u2} and imposing the condition for a symmetric equilibrium, we obtain:

$$p_u^* = t - \frac{\alpha_d \alpha_u}{2} - \frac{\alpha_d^2}{4} \quad \text{and} \quad p_d^* = \frac{\alpha_d}{4},$$

$$n_u^* = \frac{1}{2} \quad \text{and} \quad n_d^* = \frac{\alpha_d}{4}.$$

leading to equilibrium profits:

$$\Pi_C^* \left(\text{wary} \right) = \frac{t}{2} - \frac{\alpha_d \alpha_u}{4} - \frac{\alpha_d^2}{16}. \tag{13}$$

Comparing (13) with (8) and (9) yields:

Proposition 5 *In a Hotelling duopoly, equilibrium profits when all users hold passive expectations are higher than profits when all users hold wary expectations, which in turn are higher than profits when all users hold responsive expectations:*

$$\Pi_C^* \left(passive \right) > \Pi_C^* \left(wary \right) > \Pi_C^* \left(responsive \right).$$

Just like in the monopoly scenario, equilibrium duopoly profits with wary expectations are in-between equilibrium profits with responsive and passive expectations, except that the ordering is reversed. The explanation is very similar: the intensity of price competition under wary expectations falls between the two extremes of responsive and passive.

As in the monopoly case, equilibrium prices and comparative statics of those prices in the network effect parameters are quite different. In particular,

$$\begin{aligned} p_u^* \text{(passive)} &> p_u^* \text{(wary)} > p_u^* \text{(responsive)} \text{ for all } \alpha_u, \alpha_d, \\ p_d^* \text{(passive)} &= p_d^* \text{(wary)} > p_d^* \text{(responsive)} \text{ for all } \alpha_u, \alpha_d. \end{aligned}$$

The differences in comparative statics are summarized in the following table:

	Equilibrium user price p_u^*	**Equilibrium developer price p_d^***
Responsive expectations	negative iff $t < \left(3\alpha_d\alpha_u + \alpha_d^2\right)/4$ decreasing in α_u and α_d	negative iff $\alpha_d < \alpha_u$ increasing in α_d; decreasing in α_u
Passive expectations	negative iff $t < \alpha_d^2/4$ decreasing in α_d; constant in α_u	always positive increasing in α_d; constant in α_u
Wary expectations	negative iff $t < \left(2\alpha_d\alpha_u + \alpha_d^2\right)/4$ decreasing in α_d; constant in α_u	always positive increasing in α_d; constant in α_u

Once again, these differences can have important empirical implications. For instance, the developer side is not subsidized under passive and wary expectations, but it might be under responsive expectations (when $\alpha_u > \alpha_d$).

5 Hybrid responsive and passive expectations

A different way of modelling rational user expectations that lie in between fully passive and fully responsive is to consider a hybrid scenario in which some users hold passive expectations, while others hold responsive expectations. Indeed, up to now we have only considered "pure" forms of user expectations, i.e., scenarios in which *all* users held the same type of expectations. We are particularly interested in seeing how equilibrium platform profits respond to *small* changes in the type of user expectations, which can also be interpreted as the level of information held by users.

Specifically, we assume throughout this section that a fraction λ of users hold responsive expectations, while the remaining fraction $(1 - \lambda)$ of users hold passive expectations. All developers hold responsive expectations. Note that the extreme cases $\lambda = 1$ (all users hold responsive expectations) and $\lambda = 0$ (all users hold passive expectations) have been studied in Section 3.

5.1 Monopoly

Consider first a monopoly platform's problem. Two-sided realized demands (n_u, n_d) are:[8]

$$n_d = \alpha_d n_u - p_d\,,$$

$$n_u = \lambda\left(1 + \alpha_u n_d - p_u\right) + (1-\lambda)\left(1 + \alpha_u n_d^e - p_u\right),$$

where $\left(1 + \alpha_u n_d - p_u\right)$ is the part of user demand that comes from users with responsive expectations and $\left(1 + \alpha_u n_d^e - p_u\right)$ is the part of user demand coming from users holding passive expectations. Realized demands can be expressed as functions of prices and passive expectations:

$$n_u = \frac{1 + (1-\lambda)\,\alpha_u n_d^e - p_u - \lambda\alpha_u p_d}{1 - \lambda\alpha_u\alpha_d} \quad \text{and} \quad n_d = \frac{\alpha_d + (1-\lambda)\,\alpha_u\alpha_d n_d^e - p_d - \alpha_d p_u}{1 - \lambda\alpha_u\alpha_d}\,.$$

We use these expressions to optimize platform profits $\Pi = p_u n_u + p_d n_d$ over (p_u, p_d), obtaining prices and realized demands $p_u^*\left(n_d^e\right)$, $p_d^*\left(n_d^e\right)$, $n_u^*\left(n_d^e\right)$ and $n_d^*\left(n_d^e\right)$, all of which depend on n_d^e. The final equilibrium is then obtained by imposing the rationality condition $n_d^*\left(n_d^e\right) = n_d^e$. Relegating the remaining details to Appendix A.2, we obtain the following optimal prices and two-sided demands:

$$p_u^*(\lambda) = \frac{2 - \alpha_d\left(\alpha_d + \lambda\alpha_u\right)}{4 - \left(\alpha_d + \alpha_u\right)\left(\alpha_d + \lambda\alpha_u\right)} \quad \text{and} \quad p_d^*(\lambda) = \frac{\alpha_d - \lambda\alpha_u}{4 - \left(\alpha_d + \alpha_u\right)\left(\alpha_d + \lambda\alpha_u\right)}\,, \tag{14}$$

$$n_u^*(\lambda) = \frac{2}{4 - \left(\alpha_d + \alpha_u\right)\left(\alpha_d + \lambda\alpha_u\right)} \quad \text{and} \quad n_d^*(\lambda) = \frac{\alpha_d + \lambda\alpha_u}{4 - \left(\alpha_d + \alpha_u\right)\left(\alpha_d + \lambda\alpha_u\right)}\,. \tag{15}$$

The resulting optimal platform profits are:

$$\Pi_M^*(\lambda) = \frac{4 - \left(\alpha_d + \lambda\alpha_u\right)^2}{\left[4 - \left(\alpha_d + \alpha_u\right)\left(\alpha_d + \lambda\alpha_u\right)\right]^2}, \tag{16}$$

leading to the following proposition:

Proposition 6 *When a fraction λ of users holds responsive expectations and the remaining fraction $(1-\lambda)$ holds passive expectations, a monopoly platform's optimal profits and realized demands are* increasing *in λ.*

[8] We implicitly assume that the distribution of expectation types $(\lambda, 1-\lambda)$ is uncorrelated with the distribution of other user demand characteristics.

Thus, a monopoly platform is always better off when there are more users holding responsive expectations and fewer users holding passive expectations. This result confirms and reinforces the intuition derived in section 3 by comparing only the extreme cases $\lambda = 0$ and $\lambda = 1$.

The effect of λ on the platform's optimal prices is subtler. Users with responsive expectations are more responsive to price decreases relative to users holding passive expectations because the former take into account the effect of the price drop not just on their own demand, but also on expected demand by developers. Thus, in some sense, increasing λ is akin to increasing user demand elasticity, so one might expect that the optimal prices should be lower. The reason this intuition breaks down is that the platform can re-adjust its pricing structure: decrease one price and increase the other. Indeed:

- if $\alpha_u > \alpha_d$ then $p_u^*(\lambda)$ is increasing and $p_d^*(\lambda)$ decreasing;
- if $\alpha_u < \alpha_d$ and $\alpha_d(\alpha_d + \alpha_u) > 2$ then the effect is reversed: $p_u^*(\lambda)$ is decreasing and $p_d^*(\lambda)$ increasing;
- if $\alpha_u < \alpha_d$ and $\alpha_d(\alpha_d + \alpha_u) < 2$ then both $p_u^*(\lambda)$ and $p_d^*(\lambda)$ are decreasing.

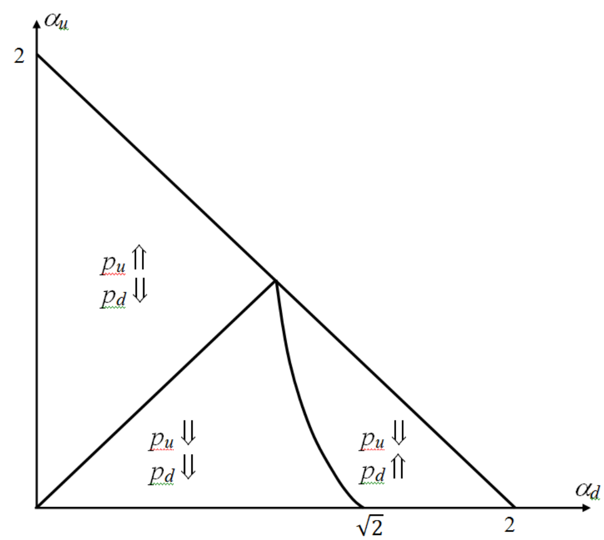


Figure 2: The effect of the fraction λ of users holding responsive expectations on the platform's optimal prices.

Still, the average price per user-developer interaction is decreasing in λ as expected:

$$\frac{p_u^*(\lambda)\, n_u^*(\lambda) + p_d^*(\lambda)\, n_d^*(\lambda)}{n_u^*(\lambda)\, n_d^*(\lambda)} = \frac{4 - (\alpha_d + \lambda\alpha_u)^2}{2(\alpha_d + \lambda\alpha_u)}.$$

5.2 Competition

We now turn to the case of Hotelling competition between two symmetric platforms on a fixed-size user market—similar to the one studied in Section 3.2, except for the hybrid structure of user expectations. User demand for platform 1 is now determined by:

$$n_{u1} = \lambda\left[\frac{1}{2} + \frac{\alpha_u(n_{d1} - n_{d2}) + p_{u2} - p_{u1}}{2t}\right] + (1-\lambda)\left[\frac{1}{2} + \frac{\alpha_u(n_{d1}^e - n_{d2}^e) + p_{u2} - p_{u1}}{2t}\right].$$

User demand n_{u2} for platform 2 is simply $1 - n_{u1}$.

Developer demands are:

$$n_{d1} = \alpha_d n_{u1} - p_{d1} \text{ for platform 1, and } n_{d2} = \alpha_d n_{u2} - p_{d2} \text{ for platform 2.}$$

It is then straightforward to solve for $(n_{u1}, n_{d1}, n_{u2}, n_{d2})$ as functions of prices $(p_{u1}, p_{d1}, p_{u2}, p_{d2})$ and fixed expectations (n_{d1}^e, n_{d2}^e). Platforms simultaneously choose prices to maximize profits, taking (n_{d1}^e, n_{d2}^e) as given. The final equilibrium is then determined by imposing the rational expectations conditions $n_{d1}^e = n_{d1}$ and $n_{d2}^e = n_{d2}$. Relegating details to Appendix A.3, we obtain that the symmetric equilibrium is characterized by the following prices and demands:

$$\begin{aligned} p_u^*(\lambda) &= t - \frac{3\alpha_d\alpha_u\lambda}{4} - \frac{\alpha_d^2}{4} \quad \text{and} \quad p_d^*(\lambda) = \frac{\alpha_d - \alpha_u\lambda}{4}, \\ n_u^*(\lambda) &= \frac{1}{2} \quad \text{and} \quad n_d^*(\lambda) = \frac{\alpha_d + \alpha_u\lambda}{4}, \end{aligned}$$

resulting in equilibrium profits:

$$\Pi_C^*(\lambda) = \frac{t}{2} - \frac{\alpha_d^2}{16} - \frac{6\alpha_d\alpha_u\lambda + \alpha_u^2\lambda^2}{16},$$

which leads to:[9]

[9] In Appendix A.4, we show that this result also holds when both sides single-home and market size is fixed on both sides.

Proposition 7 *When a fraction λ of users holds responsive expectations and the remaining fraction $(1-\lambda)$ holds passive expectations, platform profits in the symmetric competitive equilibrium with fixed user market size are* strictly decreasing *in λ.*

Once again, this is the opposite result relative to the monopoly case, in which platform profits were *increasing* in λ. This result generalizes the one obtained in Section 3.2. The interpretation is similar. If platforms compete for share in a market of fixed size on at least one side, the individual incentives to lower price are stronger when there are more users with responsive expectations, because these users are more responsive to price decreases. This creates more intense price competition, which leads to lower equilibrium platform profits since the user market does not expand.

Note also that equilibrium prices on both sides are decreasing in λ, unlike in the monopoly case, where one of the two prices could be increasing.

Generalized competition. Given the previous results, it is natural to expect that in a more general model of platform competition, equilibrium profits could be non-monotonic in λ. We confirm and illustrate this point using the model of Hotelling competition with hinterlands introduced in Section 3.2. The difference is that now user expectations are hybrid. Thus, using the same notation as in Section 3.2—x is the density of the Hotelling segment, while $(1-x)$ is the density of the monopolistic hinterlands—user demand for platform 1 is:

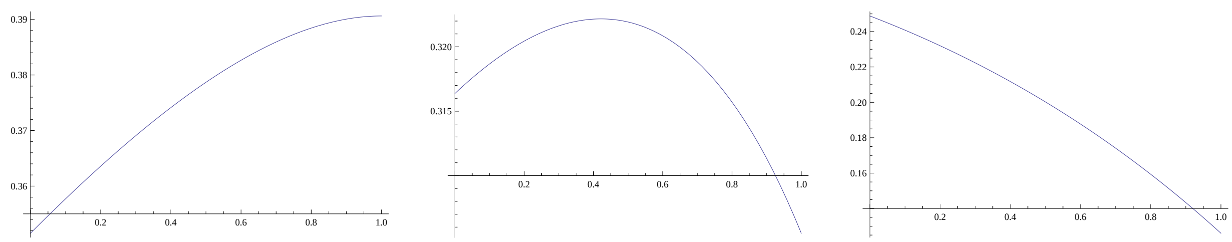
$$\begin{aligned} n_{u1} &= \lambda\left\{x\left[\frac{1}{2}+\frac{\alpha_u\left(n_{d1}-n_{d2}\right)-\left(p_{u1}-p_{u2}\right)}{2t}\right]+(1-x)\left(1+\alpha_u n_{d1}-p_{u1}\right)\right\} \\ &\quad +(1-\lambda)\left\{x\left[\frac{1}{2}+\frac{\alpha_u\left(n_{d1}^e-n_{d2}^e\right)-\left(p_{u1}-p_{u2}\right)}{2t}\right]+(1-x)\left(1+\alpha_u n_{d1}^e-p_{u1}\right)\right\}, \end{aligned}$$

whereas developer demand remains:

$$n_{d1}=\alpha_d n_{u1}-p_{d1}.$$

Demands for platform 2 are obtained by symmetry.

We explicitly solve this model and derive equilibrium platform profits $\Pi_C^*(x,\lambda)$ in Appendix A.3. Here let us simply emphasize the following points. When $x=1$, the model is equivalent to the competition scenario with fixed user market size studied above: equilibrium platform profits are *decreasing* in λ. When $x=0$, the model is equivalent to two independent two-sided monopolies, so that equilibrium profits are as determined in Section 5.1 and *increasing* in λ. As x increases from 0 to 1, equilibrium profits $\Pi_C^*(x,\lambda)$ are first increasing, then single-peaked, and lastly decreasing in λ. This is illustrated in Figure 3.



(a) $\Pi_C^*(0,\lambda)$ (b) $\Pi_C^*(0.35,\lambda)$ (c) $\Pi_C^*(0.8,\lambda)$

Figure 3: Shape of platform's profits in competition ($\Pi_C^*(x,\lambda)$), as a function of the share of users with responsive expectations (λ), for three different values of x. All graphs for parameters $t = 0.5$, $\alpha_u = 0.5$, $\alpha_d = 0.7$.

For general $x \in (0,1)$, the fraction of informed users λ has two conflicting effects: user market expansion through the Hotelling hinterlands tends to make increases in λ desirable for the two platforms, whereas competition for users on the interior of the Hotelling segment tends to make increases in λ undesirable (increased competitive pressure).

6 Conclusions

This paper explores differences between mechanisms through which expectations are formed in a market with indirect network effects. First, we have shown that the mechanism through which expectations are formed affects platform profits differently depending on market structures. Broadly speaking, platforms with market power prefer more responsive over less responsive (wary or passive) expectations. And platforms competing in a market of fixed size have the opposite preference: They derive higher profits when users have less responsive expectations. The main reason is that more responsive expectations amplify the effect of price reductions. For platforms with market power, this is good news because they can capture the demand increases and achieve higher profits. For competing platforms, more responsive expectations is typically bad news because they intensify price competition.

Second, from a methodological standpoint, our model encompasses two methods for modelling network effects—responsive and passive expectations—and adds a novel one, wary expectations. Needless to say, there exist other ways to model different degrees of user information and sophistication when it comes to forming expectations in markets with network effects. Our goal was to provide a first step towards clarifying the differences that mechanisms for expectation formation produce in terms of economic outcomes.

Finally, the differences in terms of responsiveness among the various mechanisms of expectation formation can be readily interpreted as different degrees of user information and/or sophistication. Thus, the results summarized above can also be understood as describing platform preferences over the extent to which users are informed. This suggests future work might endogenize the nature (or mix) of expectations and allow it to be influenced by platform actions (e.g., informing or educating users about prices and demands on the other side of the market).

A Appendix: Proofs

A.1 Proof of proposition 2

Let two-sided demands be $D_u(n_d^e, p_u)$ and $D_d(n_u^e, p_d)$ and define the corresponding inverse demand functions $\widetilde{D}_u(p_u, p_d)$ and $\widetilde{D}_d(p_u, p_d)$ as the solutions (n_u, n_d) to the system of 2 equations:

$$n_u = D_u(n_d, p_u) \quad \text{and} \quad n_d = D_d(n_u, p_d) \ .$$

Denote by (p_u^*, p_d^*) the profit-maximizing prices when all users on both sides hold responsive expectations:

$$(p_u^*, p_d^*) = \arg\max_{p_u, p_d} \left\{ p_u \widetilde{D}_u(p_u, p_d) + p_d \widetilde{D}_d(p_u, p_d) \right\} \ .$$

Consider now some other rational expectations equilibrium in which the platform's profit-maxizing prices are $\widehat{p}_u$ and $\widehat{p}_d$ and resulting two-sided demands are $\widehat{n}_u$ and $\widehat{n}_d$ (for example, this could be the profit-maximizing equilibrium when a fraction of users on each side holds passive expectations). Since expectations are fulfilled at the equilibrium prices, we must have :

$$\widehat{n}_u = D_u(\widehat{n}_d, \widehat{p}_u) \quad \text{and} \quad \widehat{n}_d = D_d(\widehat{n}_u, \widehat{p}_d) \ ,$$

so that:

$$\widehat{n}_u = \widetilde{D}_u(\widehat{p}_u, \widehat{p}_d) \quad \text{and} \quad \widehat{n}_d = \widetilde{D}_d(\widehat{p}_u, \widehat{p}_d) \ ,$$

which means that platform profits in this equilibrium are equal to $\widehat{p}_u \widetilde{D}_u(\widehat{p}_u, \widehat{p}_d) + \widehat{p}_d \widetilde{D}_d(\widehat{p}_u, \widehat{p}_d)$. But by definition, (p_u^*, p_d^*) maximizes $p_u \widetilde{D}_u(p_u, p_d) + p_d \widetilde{D}_d(p_u, p_d)$ over the (p_u, p_d) space, therefore the platform's optimal profits are always highest with fully responsive expectations on both sides.

The intuition for this result is straightforward. Since expectations are fulfilled in equilibrium, the platform can replicate any rational expectations market allocation $(\widehat{p}_u, \widehat{p}_d, \widehat{n}_u, \widehat{n}_d)$ under fully responsive expectations on both sides, simply by charging $(\widehat{p}_u, \widehat{p}_d)$. But by definition it can do at least as well in terms of profits by charging (p_u^*, p_d^*).

A.2 Hybrid responsive and passive expectations—Monopoly

We derive the expressions of optimal prices, demands and profits for a monopoly platform, when a proportion λ of users is holds responsive expectations and a fraction $(1-\lambda)$ holds passive expectations n_d^e. Two-sided demands are:

$$n_u = \lambda(1 + \alpha_u n_d - p_u) + (1-\lambda)(1 + \alpha_u n_d^e - p_u) ,$$

$$n_d = \alpha_d n_u - p_d .$$

Solving for realized demands (n_u, n_d) as functions of prices (p_u, p_d) and fixed expectations n_d^e we obtain:

$$n_u = \frac{1 + \alpha_u(1-\lambda) n_d^e - p_u - \alpha_u \lambda p_d}{1 - \alpha_u \alpha_d \lambda} ,$$

$$n_d = \frac{\alpha_d + \alpha_u \alpha_d (1-\lambda) n_d^e - \alpha_d p_u - p_d}{1 - \alpha_u \alpha_d \lambda} .$$

Plugging these two expressions in the expression of platform profits $\Pi = p_u n_u + p_d n_d$, taking the two first order conditions in (p_u, p_d) and imposing the rational expectations condition $n_d^* = n_d^e$ in equilibrium, we obtain:

$$\begin{aligned} n_u^* - \frac{p_u^* + \alpha_d p_d^*}{1 - \alpha_u \alpha_d \lambda} &= 0 , \\ n_d^* - \frac{\alpha_u \lambda p_u^* + p_d^*}{1 - \alpha_u \alpha_d \lambda} &= 0 . \end{aligned}$$

Furthermore, evaluating the expressions of (n_u, n_d) as functions of (p_u, p_d, n_d^e) in equilibrium, we also have:

$$(1 - \alpha_u \alpha_d \lambda) n_u^* - \alpha_u (1-\lambda) n_d^* = 1 - p_u^* - \alpha_u \lambda p_d^* ,$$

$$(1-\alpha_u\alpha_d)\, n_d^* = \alpha_d - \alpha_d p_u^* - p_d^* \,.$$

We can then solve the last 4 equations for the equilibrium prices and demands, obtaining:

$$p_u^* = \frac{2-\alpha_d\left(\alpha_d+\alpha_u\lambda\right)}{4-\left(\alpha_u+\alpha_d\right)\left(\alpha_d+\alpha_u\lambda\right)} \quad \text{and} \quad p_d^* = \frac{\alpha_d-\alpha_u\lambda}{4-\left(\alpha_u+\alpha_d\right)\left(\alpha_d+\alpha_u\lambda\right)}\,,$$

$$n_u^* = \frac{2}{4-\left(\alpha_u+\alpha_d\right)\left(\alpha_d+\alpha_u\lambda\right)} \quad \text{and} \quad n_d^* = \frac{\alpha_d+\alpha_u\lambda}{4-\left(\alpha_u+\alpha_d\right)\left(\alpha_d+\alpha_u\lambda\right)}\,.$$

Finally, equilibrium monopoly profits are:

$$\Pi_M^*\left(\lambda\right) = \frac{4-\left(\alpha_d+\alpha_u\lambda\right)^2}{\left[4-\left(\alpha_u+\alpha_d\right)\left(\alpha_d+\alpha_u\lambda\right)\right]^2}\,.$$

The last three expressions confirm (14), (15) and (16) in Section 5.

Setting $\lambda = 1$ we obtain (2), (3) and (4). Setting $\lambda = 0$, we obtain (5), (6) and (7).

A.3 Hybrid responsive and passive expectations—Competition

We directly treat the general duopoly case with hybrid expectations. The user market is composed of three segments: the Hotelling segment of density x and two symmetric “hinterlands” of density $(1-x)$, on which user demand is identical to the demand for a monopoly platform from Section 3. A fraction λ of users holds responsive expectations, while a fraction $(1-\lambda)$ holds passive expectations. The distribution $(\lambda, 1-\lambda)$ is independent of the position of a given user on the Hotelling line with hinterlands. The structure of developer demand remains unchanged, i.e. each platform acts as a monopolist vis-a-vis developers.

User demand for platform 1 is:

$$\begin{aligned}
n_{u1} &= \lambda\left\{x\left[\frac{1}{2}+\frac{\alpha_u\left(n_{d1}-n_{d2}\right)-\left(p_{u1}-p_{u2}\right)}{2t}\right]+\left(1-x\right)\left(1+\alpha_u n_{d1}-p_{u1}\right)\right\} \\
&\quad +\left(1-\lambda\right)\left\{x\left[\frac{1}{2}+\frac{\alpha_u\left(n_{d1}^e-n_{d2}^e\right)-\left(p_{u1}-p_{u2}\right)}{2t}\right]+\left(1-x\right)\left(1+\alpha_u n_{d1}^e-p_{u1}\right)\right\} \\
&= 1-\frac{x}{2}+x\left\{\frac{\alpha_u\left[\lambda n_{d1}+\left(1-\lambda\right)n_{d1}^e-\lambda n_{d2}-\left(1-\lambda\right)n_{d2}^e\right]-\left(p_{u1}-p_{u2}\right)}{2t}\right\} \\
&\quad +\left(1-x\right)\left\{\alpha_u\left[\lambda n_{d1}+\left(1-\lambda\right)n_{d1}^e\right]-p_{u1}\right\}
\end{aligned}$$

while developer demand for platform 1 is:

$$n_{d1} = \alpha_d n_{u1} - p_{d1}$$

and symmetrically for platform 2.

Note that $\lambda = 0$ corresponds to the generalized duopoly scenario discussed at the end of Section 3.2, while $x = 0$ and $x = 1$ correspond to the hybrid scenarios studied in Section 5.

To determine the pricing equilibrium, we need first to determine demands as functions of prices and constants (including passive expectations held by a fraction $(1 - \lambda)$ of users). To do that, we first determine (using $n_{d1} \pm n_{d2} = \alpha_d (n_{u1} \pm n_{u2}) - (p_{d1} \pm p_{d2})$):

$$n_{u1} + n_{u2} = \frac{2 - x + (1 - x)\left[(1 - \lambda)\, \alpha_u \left(n_{d1}^e + n_{d2}^e\right) - (p_{u1} + p_{u2}) - \lambda \alpha_u (p_{d1} + p_{d2})\right]}{1 - (1 - x)\, \lambda \alpha_u \alpha_d}, \quad (17)$$

$$n_{u1} - n_{u2} = \frac{[x + t(1 - x)]\left[(1 - \lambda)\, \alpha_u \left(n_{d1}^e - n_{d2}^e\right) - (p_{u1} - p_{u2}) - \lambda \alpha_u (p_{d1} - p_{d2})\right]}{t - [x + t(1 - x)]\, \lambda \alpha_u \alpha_d}. \quad (18)$$

Platform 1's profits are $p_{u1} n_{u1} + p_{d1} n_{d1}$, which can be re-written $(p_{u1} + \alpha_d p_{d1})\, n_{u1} - p_{d1}^2$. In this expression, n_{u1} is obtained by adding (17) and (18) and dividing by 2. In the symmetric equilibrium, expectations by uninformed users are fulfilled. Taking the first order conditions of the profit function in p_{u1} and p_{d1} and evaluating at equilibrium values, we obtain:

$$n_u^* = A(x, \lambda)\,(p_u^* + \alpha_d p_d^*)\ , \quad (19)$$

$$n_u^* = B(\lambda)\, p_d^*\,, \quad (20)$$

where we have denoted:

$$A(x, \lambda) \equiv \frac{1}{2}\left[\frac{1 - x}{1 - (1 - x)\, \lambda \alpha_u \alpha_d} + \frac{x + t(1 - x)}{t - [x + t(1 - x)]\, \lambda \alpha_u \alpha_d}\right] \quad \text{and} \quad B(\lambda) \equiv \frac{2}{\alpha_d - \lambda \alpha_u}.$$

From the initial demand functions and symmetry, we have:

$$n_u^* = C(x) - D(x)\,(p_u^* + \alpha_u p_d^*) \quad \text{and} \quad n_d^* = \alpha_d n_u^* - p_d^*\,, \quad (21)$$

where:

$$C(x) \equiv \frac{1 - x/2}{1 - (1 - x)\, \alpha_u \alpha_d} \quad \text{and} \quad D(x) \equiv \frac{1 - x}{1 - (1 - x)\, \alpha_u \alpha_d}.$$

It is then straightforward to solve (19), (20) and (21) in order to determine the equilibrium prices and demands:

$$p_u^* = \frac{C(x)\left[B(\lambda) - \alpha_d A(x,\lambda)\right]}{\Delta(x,\lambda)} \quad \text{and} \quad p_d^* = \frac{C(x) A(x,\lambda)}{\Delta(x,\lambda)},$$

$$n_u^* = \frac{C(x) A(x,\lambda) B(\lambda)}{\Delta(x,\lambda)} \quad \text{and} \quad n_d^* = \frac{\left[\alpha_d B(\lambda) - 1\right] C(x) A(x,\lambda)}{\Delta(x,\lambda)},$$

where:

$$\Delta(x,\lambda) \equiv A(x,\lambda) B(\lambda) + (\alpha_u - \alpha_d) D(x) A(x,\lambda) + D(x) B(\lambda) .$$

Finally, equilibrium platform profits are:

$$\Pi_C^*(x,\lambda) = \frac{C(x)^2 A(x,\lambda)\left[B(\lambda)^2 - A(x,\lambda)\right]}{\Delta(x,\lambda)^2} .$$

It is easily verified that:

- $A(0,\lambda) = \frac{1}{1-\lambda\alpha_u\alpha_d}$, $C(0) = D(0) = \frac{1}{1-\alpha_u\alpha_d}$ and $\Delta(0,\lambda) = \frac{4-(\alpha_d+\alpha_u)(\alpha_d+\lambda\alpha_u)}{(1-\lambda\alpha_u\alpha_d)(1-\alpha_u\alpha_d)(\alpha_d-\lambda\alpha_u)}$, leading to $\Pi_C^*(0,\lambda) = \Pi_M^*(\lambda)$ (expression (16) in the text) ;
- $A(1,\lambda) = \frac{1}{2(t-\lambda\alpha_u\alpha_d)}$, $C(1) = \frac{1}{2}$, $D(1) = 0$ and $\Delta(1,\lambda) = \frac{1}{(t-\lambda\alpha_u\alpha_d)(\alpha_d-\lambda\alpha_u)}$, leading to $\Pi_C^*(1,\lambda) = \Pi_C^*(\lambda)$ (expression (5.2) in the text) .

Calculating the derivatives of $\Pi_C^*(x,\lambda)$ in x and λ (for general x and λ) is inextricable. We have used Mathematica to graph $\Pi_C^*(x,0)$ and $\Pi_C^*(x,1)$ as functions of x in Section 3.2 (Figure 1) and $\Pi_C^*(x_0,\lambda)$ as a function of λ for various values of x_0 in Section 5 (Figure 3).

A.4 Duopoly with single-homing developers

Throughout the text of the paper we have assumed that developers multihome whenever there are two competing platforms. In this appendix, we derive equilibrium prices and profits under the various types of expectations in a platform duopoly with fixed market sizes and singlehoming on both sides.

A.4.1 Hybrid responsive and passive

We start with the hybrid model, in which a fraction λ of users holds responsive expectations, while the remaining fraction $(1-\lambda)$ holds passive expectations. User and developer demands for platform 1 are:

$$\begin{aligned} n_{u1} &= \lambda \left[\frac{1}{2} + \frac{\alpha_u (n_{d1} - n_{d2}) + p_{u2} - p_{u1}}{2t_u} \right] + (1-\lambda) \left[\frac{1}{2} + \frac{\alpha_u (n^e_{d1} - n^e_{d2}) + p_{u2} - p_{u1}}{2t_u} \right] \\ &= \frac{1}{2} + \frac{\alpha_u \lambda (n_{d1} - n_{d2}) + \alpha_u (1-\lambda) (n^e_{d1} - n^e_{d2}) + p_{u2} - p_{u1}}{2t_u} , \end{aligned}$$

$$n_{d1} = \frac{1}{2} + \frac{\alpha_d (n_{u1} - n_{u2}) - (p_{d1} - p_{d2})}{2t_d} ,$$

where the transportation costs t_u and t_d are positive and verify:

$$\begin{aligned} t_u t_d &> \alpha_u \alpha_d , \\ t_u + t_d &> \alpha_u + \alpha_d . \end{aligned}$$

Using the expressions above, we can solve for two-sided demands as functions of prices and fixed (passive) expectations only:

$$n_{u1} = \frac{1}{2} + \frac{t_d \alpha_u (1-\lambda) (n^e_{d1} - n^e_{d2}) - t_d (p_{u1} - p_{u2}) - \alpha_u \lambda (p_{d1} - p_{d2})}{2 (t_u t_d - \lambda \alpha_u \alpha_d)} ,$$

$$n_{d1} = \frac{1}{2} + \frac{\alpha_d \alpha_u (1-\lambda) (n^e_{d1} - n^e_{d2}) - \alpha_d (p_{u1} - p_{u2}) - t_u (p_{d1} - p_{d2})}{2 (t_u t_d - \lambda \alpha_u \alpha_d)} .$$

We then plug these expressions in platform 1's profits $p_{u1} n_{u1} + p_{d1} n_{d1}$, take the first-order conditions in p_{u1} and p_{d1} respectively and evaluate at the symmetric equilibrium to obtain equilibrium prices and demands:

$$p^*_u = t_u - \alpha_d \quad \text{and} \quad p^*_d = t_d - \lambda \alpha_u ,$$

$$n^*_u = n^*_d = \frac{1}{2} .$$

Equilibrium platform profits are:

$$\Pi^*_C (\lambda) = p^*_u n^*_u + p^*_d n^*_d = \frac{t_u + t_d - \alpha_d - \lambda \alpha_u}{2} ,$$

clearly decreasing in λ, just like in the case with developer multihoming (cf. Section 5).

In particular, we have:

$$\Pi^*_C (\text{responsive}) = \frac{t_u + t_d - \alpha_d - \alpha_u}{2} < \frac{t_u + t_d - \alpha_d}{2} = \Pi^*_C (\text{passive}) ,$$

which is the same result as the one obtained with developer multihoming (cf. Section 3).

A.4.2 Wary expectations

We now turn to the case when all users hold wary expectations. User and developer demands for platform 1 are:

$$n_{u1} = \frac{1}{2} + \frac{\alpha_u \left(n_{d1}^e - n_{d2}^e\right) + p_{u2} - p_{u1}}{2t_u},$$

$$n_{d1} = \frac{1}{2} + \frac{\alpha_d \left(n_{u1} - n_{u2}\right) - \left(p_{d1} - p_{d2}\right)}{2t_d},$$

where n_{d1}^e and n_{d2}^e are the wary expectations formed by users about developer participation and depend on observed user prices p_{u1} and p_{u2}.

To determine n_{d1}^e and n_{d2}^e, we optimize platform 1's profits $p_{u1}n_{u1} + p_{d1}n_{d1}$ over p_{d1} holding constant all other prices and expectations, and we impose that the resulting developer demand n_{d1} is equal to n_{d1}^e (rational expectations). This leads to:

$$p_{d1}^e = 2\, t_d\, n_{d1}^e\,.$$

The same holds for platform 2, therefore we have:

$$p_{d1}^e - p_{d2}^e = 2t_d \left(n_{d1}^e - n_{d2}^e\right),$$

where p_{d1}^e and p_{d2}^e are the prices that users expect the platform is charging developers given the observed user prices.

From the initial demand expressions we also have (expectations are rational):

$$\begin{aligned} n_{d1}^e - n_{d2}^e &= \frac{\alpha_d \left(n_{u1} - n_{u2}\right) - \left(p_{d1}^e - p_{d2}^e\right)}{t_d} \\ &= \frac{\alpha_d \alpha_u}{t_d t_u} \left(n_{d1}^e - n_{d2}^e\right) - \frac{\alpha_d}{t_d t_u} \left(p_{u1} - p_{u2}\right) - \frac{1}{t_d} \left(p_{d1}^e - p_{d2}^e\right)\,. \end{aligned}$$

We can solve the last two equations for $n_{d1}^e - n_{d2}^e$:

$$n_{d1}^e - n_{d2}^e = \frac{\alpha_d \left(p_{u2} - p_{u1}\right)}{3t_d t_u - \alpha_d \alpha_u}\,.$$

Using this expression, we can write demands for platform 1 solely as functions of prices:

$$n_{u1} = \frac{1}{2} - \frac{3t_d\left(p_{u1} - p_{u2}\right)}{2\left(3t_d t_u - \alpha_d \alpha_u\right)},$$

$$n_{d1} = \frac{1}{2} - \frac{3\alpha_d\left(p_{u1} - p_{u2}\right)}{2\left(3t_d t_u - \alpha_d \alpha_u\right)} - \frac{p_{d1} - p_{d2}}{2t_d}.$$

We can now take the first-order conditions of platform 1's profits in p_{u1} and p_{d1} and solve for the symmetric equilibrium. We obtain:

$$p_u^* = t_u - \alpha_d - \frac{\alpha_d \alpha_u}{3t_d} \quad \text{and} \quad p_d^* = t_d\,,$$

$$n_u^* = n_d^* = \frac{1}{2}.$$

Equilibrium platform profits are:

$$\Pi_C^*\left(\text{wary}\right) = \frac{t_u + t_d - \alpha_d}{2} - \frac{\alpha_d \alpha_u}{6t_d},$$

Comparing with the equilibrium profits under fully responsive and fully passive expectations derived above, we have (recall that $\Pi_C^*\left(\text{passive}\right) > \Pi_C^*\left(\text{responsive}\right)$):

$$\Pi_C^*\left(\text{wary}\right) < \Pi_C^*\left(\text{passive}\right)$$

$$\Pi_C^*\left(\text{wary}\right) > \Pi_C^*\left(\text{responsive}\right) \text{ if and only if } t_d > \alpha_d/3\,.$$

Thus, the only possible difference with respect to the developer multihoming case studied in the main text arises when $\alpha_d > 3t_d$, i.e., when competition for developers is very intense. In that case, wary expectations lead to lower platform profits than responsive expectations.

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