Multi-Sided Platforms*

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March 2015

Abstract

We study the economic tradeoffs that drive organizations to position themselves closer to or further away from a multi-sided platform (MSP) business model, relative to three traditional alternatives: vertically integrated firms, resellers or input suppliers. These tradeoffs lead to a comprehensive discussion of the defining features of MSPs. The formal model we develop focuses on the MSP versus vertical integration choice, which we interpret in the context of professional services. A key tradeoff emerges between the need to coordinate decisions that generate spillovers across professionals (best achieved by a vertical integrated firm) and the need to both motivate unobservable effort by professionals and ensure professionals adapt their decisions to their private information (best achieved by a MSP). We show how this baseline tradeoff is impacted by the nature of contracts available to the vertically integrated firm and the MSP, and by the possibility of professionals holding pessimistic expectations when deciding whether or not to join the vertically integrated firm or MSP.

JEL classification: D4, L1, L5

Keywords: Two-sided markets, multi-sided platforms, reseller, vertical integration.

1 Introduction

There is growing interest in the economics of multi-sided platforms (MSPs), which get two or more sides on board and enable interactions between them (e.g., Airbnb, eBay, Uber, XBox, etc). The pioneering

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*We thank participants in the various seminars and conferences where we have presented our related work, and EARIE 2014 in particular for the special session on Platforms and Policy. We have also benefited from individual discussions with Elizabeth Altman, Tom Eisenmann, Martin Peitz, Marc Rysman and Bo Shen. Julian Wright gratefully acknowledges research funding from the Singapore Ministry of Education Academic Research Fund Tier 1 Grant No. R122000215-112.

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models of MSPs introduced by Armstrong (2006), Caillaud and Jullien (2003), Parker and Van Alstyne (2005), and Rochet and Tirole (2003), as well as a large number of more recent contributions, all treat “multi-sidedness” as a given characteristic of the relevant industries and firms. It is important to recognize, however, that many real-world organizations make choices that determine how close or how far they are from a multi-sided economic model, and that these choices carry significant economic trade-offs.

For instance, Amazon started off as a pure retailer but has moved closer to a MSP model over time by enabling third-party sellers to trade directly with consumers on its website. Zappos, an online shoe retailer, went in the other direction, abandoning its initial model based on partnerships with shoe manufacturers that fulfilled customer orders directly. Increasingly, professional service firms are moving away from pure vertically integrated models in which all client services are provided by their employees (e.g. traditional staffing agencies, consulting firms and taxi companies), and towards the MSP model, in which they enable independent contractors or professionals to deal directly with clients (e.g. Elance-oDesk, the Gerson Lehrman Group, and Uber). There are interesting exceptions and nuances. In the private hospital market in Singapore, Raffles Hospital has bucked the trend by employing and managing its doctors as a vertically integrated firm rather than renting out its space to groups of independent specialists by way of clinics, as other private hospitals have done. Internet-enabled professional service intermediaries such as the Gerson Lehrman Group enable corporate clients to hire independent professionals from around the world for specific projects. Yet they maintain a significant degree of control over the contractual terms between clients and professionals, unlike pure MSPs such as Elance-oDesk, on which contractors have almost complete freedom to set their own terms or negotiate them with employers.

In this paper, we study the economic trade-offs that drive organizations to position themselves closer to or further away from a MSP model, relative to more traditional alternatives such as vertically integrated firms, resellers, or input suppliers. In so doing, we will provide a new definition of MSPs that clarifies what makes them special.

In terms of new modelling, we focus on one particular choice of business model, the choice a firm faces between operating in the MSP mode or operating in the vertically integrated (VI) mode. We have in mind markets for professional services, which clients can obtain directly from professionals through a MSP or from a VI firm. At a high level, the model emphasizes a key trade-off that arises between the coordination benefits of the VI mode when there are spillovers across the decisions of individual professionals/employees and the benefits of the MSP mode in making professionals residual claimants of their individual demand, which can better motivate them to provide unobservable individual effort and can ensure they better adapt their decisions to their private information.
In the specific model we propose, there are two decision variables at play, both of which affect demand—a transferable action and a non-transferable action. The non-transferable action is always chosen by the individual employee or professional, and we interpret it as unobservable costly effort. The transferable action is determined either by the firm (in VI mode) or by each individual professional (in MSP mode). It is thus the control rights over this transferable action which determine whether the firm acts in VI mode or MSP mode. We assume this transferable action generates demand spillovers across the services offered through the firm (in both modes). The transferable action could represent costly quality investments or marketing activities with respect to the service in question. The payment of commissions (bonuses) based on sales in VI mode can help motivate employees to address the moral hazard problem in which they invest too little in costly effort. However, commissions cannot perfectly solve this problem because they also distort the choice of the transferable action. Furthermore, commissions do not help the firm’s choice of transferable actions adapt to professionals’ private information. As a result, the MSP mode dominates, provided that coordination benefits are not too strong. We show how this trade-off is shifted if the MSP can charge fees based on the professionals’ sales, and by the nature of professionals’ expectations with respect to how many other professionals join the VI firm or MSP.

A few other authors have noted the possibility that platforms can sometimes choose whether or not to vertically integrate into one of their sides, although they have not modelled this choice (see Gawer and Cusumano, 2002; Evans et al., 2006; Gaware and Henderson, 2007; and Rysman, 2009). They discuss some of the economic drivers of these decisions. For instance, in the personal computer market, Apple produces its own hardware, whereas Microsoft leaves this to independent manufacturers. As a result, Apple manages only a two-sided platform between consumers and software providers, while Microsoft manages a three-sided platform between consumers, software providers, and hardware providers. These authors argue that Apple’s model leads to higher quality products, whereas Microsoft’s model generates more product variety and broader indirect network effects.

While we focus our formal modelling on the VI vs. MSP choice, we recognize that vertical integration is but one of several ways in which organizations can move away from a MSP mode. Specifically, we summarize the key insights from our recent work on the trade-offs faced by an intermediary choosing whether to be a MSP marketplace or a reseller (Hagiu and Wright, 2013, 2015). We also provide a discussion of the trade-offs between operating as a MSP or as an input supplier.

By equating the difference between MSPs and VI to the allocation of residual control rights between independent professionals and the firm, our work is loosely related to the voluminous literature on vertical integration and the theory of the firm (see Gibbons, 2005 for an overview). Rather than studying “make or buy” decisions, we study “enable (MSP) or employ (VI)” decisions, which involve
quite a different economic analysis. The key difference is the following: in the “make or buy” (vertical integration) decision, regardless of the choice, the focal firm contracts with and controls the sale to buyers. By contrast, in the “enable or employ” decision, the MSP mode involves contractual relationships between buyers and professionals, to which the focal firm is not a party, but merely an enabler of those contractual relationships.

Our analysis is a cross between what Gibbons (2005) calls “the incentive system theory of the firm” and “the adaptation theory of the firm.” The common feature with the incentive system theory of the firm is the existence of ex-post moral hazard issues (non-contractible effort provision by professionals), which is the rationale for incentives in the form of payment structures. The common feature with the adaptation theory of the firm is that one party (professionals) has superior information regarding the transferable decision variable relative to the other party (the firm). The novelty of our model relative to this literature is that we have two types of non-contractible decisions: one which is always controlled by the professionals and is the source of moral hazard; the other which can be controlled either by professionals or by the firm.

Our paper is also related to a more recent literature on organizational design which studies centralized vs. decentralized decision-making. At a high level, we share with this literature the focus on non-contractible decisions (ex-ante and ex-post) and the tradeoffs that arise from allocating the relevant decision rights to different parties. Loosely speaking, centralization corresponds to our VI mode and decentralization corresponds to our MSP mode. See, for example, Alonso et al. (2008) and (2014), although their focus on strategic communication (in their 2008 paper) and the tradeoff between information breadth and depth (in their 2014 paper) is very different from ours.

The rest of the article proceeds as follows. Section 2 gives a definition of MSPs and discusses what makes MSPs special. Section 3 provides a formal analysis of the trade-offs between the MSP mode and the VI mode. Section 4 discusses the trade-offs between MSPs and resellers. Section 5 discusses the trade-offs between MSPs and input suppliers. Section 6 concludes.

2 What makes multi-sided platforms special?

The purpose of this section is to clearly identify the elements that make multi-sided platforms (MSPs) different from regular firms and other intermediaries. Existing definitions of MSPs suffer from excessive specificity, over-inclusiveness, or being too vague to be of use. As a result there is disagreement among those in the literature about what constitutes an appropriate definition. The most common approach to date has focused on the presence of important cross-group or indirect network effects between the two or more customer groups participating on the platform (e.g., Caillaud and Jullien, 2003,
Armstrong, 2006).\footnote{A cross-group network effect arises if the benefit to users in at least one group (side A) depends on the number of users in the other group (side B) that join. An indirect network effect arises if there are cross-group network effects in both directions (from A to B and from B to A). In this case, the benefit to a user on side A depends on the number of participants on side B, which in turn depends on the number of participants on side A. Thus, the benefit to a user on side A depends (indirectly) on the number of users on side A.} Rochet and Tirole (2006) proposed an alternative definition, which focuses on whether the structure of prices set by the platform is non-neutral (i.e., whether the allocation of fees across the two sides matters for the total volume of transactions). Both approaches have limitations (see Rysman, 2009). For instance, suppliers of supermarkets and discount superstores that take on inventory risk care about the number of consumers visiting the stores (and vice-versa), so that indirect network effects exist. Furthermore, if supermarkets and discount superstores pay their suppliers more and increase their prices to consumers, this change is unlikely to be neutral for their sales. Thus, such stores along with most other retailers appear to be MSPs according to the definitions above, yet most economists would agree that they are not.

We believe that at the most fundamental level, MSPs have two key features beyond any other requirements (such as indirect network effects or non-neutrality of fees):

- They enable direct interactions between two or more distinct sides.

- Each side is affiliated with the platform.

Broadly speaking, by “direct interaction” we mean that the two or more distinct sides retain control over the key terms of the interaction, as opposed to the intermediary taking control of those terms. Where the interaction involves trading, the key terms of the interaction could be the pricing, bundling, marketing and delivery of the goods or services traded, the ability to determine the nature and quality of services offered, the terms and conditions, etc. By “affiliation” we mean that users on each side consciously make platform-specific investments that are necessary in order for them to be able to directly interact with each other. The investment could be a fixed access fee (e.g., buying a videogame console), expenditure of resources (e.g., spending time and money on learning how to develop applications using the iPhone’s APIs), or an opportunity cost (e.g., driving to a shopping mall, joining a loyalty program).\footnote{Note this does not rule out the possibility that the net payoff from affiliation might turn out to be positive due to some offsetting benefits users get when affiliating (e.g., rewards offered for signing up to a new credit card).}

Each of these dimensions helps distinguish MSPs from related but distinct business models. Direct interactions between multiple sides set MSPs apart from resellers and fully vertically integrated firms. Affiliation by all relevant customer types (sides) helps distinguish MSPs from input suppliers which are not “adopted” by all sides. This distinction holds even when the inputs are “essential”, which the
management literature calls “platforms” (see Gawer and Cusumano, 2008). Note that affiliation by multiple sides is necessary for MSPs to create cross-group network effects, the key defining property of MSPs put forward in much of the existing literature. Figure 1 illustrates these differences.

Figure 1: MSPs vs. alternative business models

![Diagram showing differences between MSPs and alternative business models](image)

Our definition makes clear that affiliation alone is not sufficient to create a MSP. We think this point underlies most of the controversy surrounding existing definitions built solely on indirect network effects. As discussed above, supermarkets and other old-fashioned retailers typically are subject to indirect network effects. However, according to our definition, they are more like resellers than MSPs since they control the relevant decision variables like marketing activities, prices, etc. A similar point applies to traditional consulting firms. Clients will be attracted to a consulting firm that has many other clients since this means it will have access to a greater number of qualified consultants (assuming consultants are attracted to a firm with many clients, for example, due to the ability to obtain a higher bonus). However, such a consulting firm is not a MSP if it controls the interaction between consultants and clients by employing the consultants and contracting with the clients, and ensuring employees do not directly contract with clients.

Our definition also allows for the possibility that some MSPs do not exhibit indirect network effects. Consider first the case of a newspaper (or other advertising supported media). A newspaper allows advertisers to directly interact with readers in the sense advertisers control the content of their ad. Each side makes a clear affiliation decision. According to our definition, and consistent with the practice in the literature, a newspaper is properly thought of as a MSP. However, if readers do not care about the number of adverts in a newspaper when deciding whether to subscribe, which seems
plausible, then even though advertisers value an increased number of readers, there will be no indirect network effect because the cross-group network effect will only apply in one direction. Thus, the most standard existing definition of MSPs based on the existence of indirect network effects is inconsistent with the widespread labelling of newspapers as MSPs regardless of whether readers care about the number of adverts in a newspaper.

A more subtle example arises when one side can coordinate the decision of which platform to adopt on behalf of the other side. Consider for example ScholarOne or other editorial software that enables editors, referees, and authors to handle the submission, refereeing, and publication processes for academic journals. Once an editor (or a publisher) adopts the editorial software, it automatically brings authors and referees on board as well. Thus, indirect network effects are absent and there is no coordination problem between the multiple sides of the platform because one side makes the affiliation decision on behalf of the other sides. The same is true for some other, similar MSPs, such as eVite (event organizers and invited guests) and Pigeonhole (an online service which allows conference organizers to offer participants the ability to post and vote on questions through their mobile devices, while speakers are able to filter questions and answer them).

Despite the fact that indirect network effects are neither necessary nor sufficient in our definition of MSPs, most MSPs do indeed create and capture value through indirect network effects, a point that has been made by previous authors. Thus, one could still use our definition in conjunction with a requirement of significant indirect network effects in order to focus on MSPs with indirect network effects.

3 MSP or vertically integrated firm?

As our definition of MSPs suggests, the distinction between a MSP and a vertically integrated (VI) mode rests on whether the services provided by A to B (as shown in Figure 1) constitute a direct interaction or are instead controlled by the firm. Either the firm produces the service itself by employing professionals to provide the service to its clients (the VI mode), or it enables independent professionals to provide the service directly to clients through its platform (the MSP mode). The choice of MSP mode vs. VI mode often arises for firms involved in the provision of professional services, which cannot be purchased and resold (e.g., errands, hair-cuts, programming projects, consulting or legal advice).

In the VI mode, the firm is directly responsible for and has residual control rights over the provision of the service to clients. Traditional consulting companies (e.g. the Boston Consulting Group, McKinsey), outsourcing firms (e.g. Infosys), taxi companies (e.g. Yellow Cab), and the original Atari VCS videogame console function in this way. Conversely, in the MSP mode, independent professionals
retain responsibility for and residual control rights over the services. Examples include: the Gerson Lehrman Group which allows independent consultants/experts to connect with and provide services to corporate clients; Elance-oDesk.com which allows independent contractors from anywhere around the world to connect with and perform their services (e.g., programming jobs) for customers; peer-to-peer transportation companies such as Lyft and Uber which enable independent drivers to offer rides to passengers in their own cars; and modern video-game console companies such as Microsoft, Sony and Nintendo which allow independent developers to produce and sell their games to console users. In these examples, the independent professionals control the nature and quality of services offered, as well as (sometimes) the prices charged.

An interesting example is hair salons, which make use of both modes. Some salons are run in the traditional fashion of hiring hairdressers. Other salons are run along the lines of a shopping mall, renting space (or booths) in their salon to independent hairdressers that sell their services directly to clients, on terms determined by the independent hairdressers, subject to any contractual restrictions imposed by the salon owner.

In this section, we develop a formal model to capture a fundamental trade-off faced by a firm that can choose where to position itself between the MSP model and the VI model. For conciseness, we focus on the case in which the firm either employs “professionals” to produce the goods or services provided to clients, or allows professionals to provide these independently over its platform. The fundamental trade-off in this choice of business model is between the coordination benefits that arise in a VI model and the benefits of motivating professionals’ effort and getting professionals to adapt their decisions to their private information that arise in a MSP model. On one hand, a VI firm can make decisions that internalize spillovers across the services provided by individual professionals. On the other hand, professionals typically have to exert costly effort that cannot be observed by the firm, and under the VI mode each employee will not put in sufficient effort given each employee does not capture the full returns to their investment in costly effort. Moreover, under the VI mode, the firm takes decisions that are not adapted to the professionals’ private information.

When cross-group network effects are present, different equilibria can arise depending on whether professionals coordinate on joining whenever all of them joining is an equilibrium (because they hold optimistic expectations about others joining) or coordinate on not joining whenever not joining is an equilibrium (because they hold pessimistic expectations about others joining). We first consider a model with optimistic expectations, and then extend the analysis to the case with pessimistic expectations. In each case, we consider what happens when the VI firm can use a bonus payment based on sales to incentivize the choice of effort by employees, and when the MSP can charge variable fees (or subsidies) to professionals based on sales in order to induce them to internalize spillovers.
3.1 The model

There are $N > 1$ independent professionals. Each professional $i = 1, ..., N$ supplies a unique service. There is a continuum of potential clients for each service. Each potential client incurs an “affiliation” cost $c$ to be able to purchase from the firm or MSP. This can capture travel costs or other fixed costs involved in dealing with the firm or MSP. Assume each potential client of professional $i$’s service obtains surplus above $c$, and so wishes to affiliate and purchase the service. The corresponding revenue generated per client is assumed to be exogenous and equal to $\pi > 0$. These assumptions allow us to focus on the key trade-offs between the two business models without introducing any pricing distortions on the buyer side. The number of clients for the service from professional $i$ is given by

$$D_i = m(n) + \theta_i a_i + x(a_i - a_i) + \gamma e_i,$$

which implies that the revenue generated by professional $i$ is $\pi D_i$.

In the expression of $D_i$ above, $m(n)$ is the baseline number of clients for each professional and is allowed to depend on the number $n$ of professionals who decide to join the firm/MSP. We assume $m(n) \geq 0$ and allow $m$ to increase (weakly) in $n$ to indicate that clients are more likely to come to the firm/MSP when more professionals are present (or when more services are available). Thus, we allow for a cross-group network effect to arise between professionals and client demand. This could arise because the more services/professionals are made available, the more clients will become aware of the firm/MSP (e.g., through word of mouth, reputation effects, or other sources of information and review) or the more likely a given client that is informed about the services available through the firm/MSP will be to become a repeat customer. In other words, we allow for positive agglomeration effects, contained in $m(n)$.

The variable $a_i$ captures the level of some costly, non-contractible, transferable action that raises the number of clients for the service offered by professional $i$. An example is the marketing and advertising of the professional services being offered. By “transferable” we mean that control over this action can be transferred between the firm and the professional. Specifically, it is undertaken by the firm under the VI mode and by professional $i$ under the MSP mode. A cost $\frac{1}{2}a_i^2$ is incurred to provide this activity by the firm in VI mode and by the professional $i$ in MSP mode.\(^4\) While the analysis\(^3\) These assumptions hold if professional $i$ faces a downward-sloping demand function $d(p_i)$ for how much of its service is consumed by each client. Assuming professionals and the firm cannot commit to $p_i$, so that the optimal $p_i$ under both modes is $p^m = \arg \max_p \{pd(p)\}$, then provided $\int_{p^m}^{\infty} d(p) dp > c$, all potential clients will consume and the revenue made on each individual client is $\pi = p^m d(p^m)$. Alternatively, $p^m$ could be exogenously determined, perhaps reflecting some industry norm or regulation, in which case $\pi = p^m$ if professionals face unit demand.

\(^4\)In an earlier version of this article we considered the case that $a_i$ represented the price of service $i$, in which case revenue is $a_i D_i$ and we need to assume $\theta_i < 0$.\n
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becomes considerably more complicated, the results and intuition obtained are similar.

The term $\theta_i$ measures the impact of $a_i$ on demand. We assume that $\theta_i$ is an i.i.d. random variable drawn from some distribution function with support $[\theta, \bar{\theta}]$ and whose realization is private information known only to the professional $i$ at the time $a_i$ is chosen. Assume $\theta > 0$. This private information can for example capture that each professional has a better idea of how much advertising in its services expands its client pool. The expected value of $\theta_i$ is denoted $E(\theta)$ and the variance is $Var(\theta)$.

The term $x$ is a spillover parameter, and captures the strength (and direction) of spillovers across professionals from their choice of activity $a_i$. The term $\pi_{-i}$ denotes the average level of activities across all services other than service $i$. Thus, the spillover affects the number of clients for service $i$ when the level of the activity $a_i$ differs from the average. We allow spillovers to be either positive or negative, but restrict attention to $x < \theta$ so that $D_i$ is always increasing in $a_i$. As an example, consider hair salons: the activity could be the extent to which professionals’ profiles and services are advertised. Salons that employ hairdressers (VI mode) make those advertising decisions themselves. Salons that rent out space/chairs allow each of their independent hairdressers to make their own decisions regarding advertising. In both cases, advertising is likely to raise the number of clients for each hairdresser’s services. However, the number of clients for an individual hairdresser’s service could either increase (positive spillovers captured by $x > 0$) or decrease (business stealing captured by $x < 0$) if other professionals advertise their profiles more.

The final determinant of client demand, $e_i$, is the unobservable effort made by professional $i$ at cost $\frac{1}{2}e_i^2$. This differs from $a_i$ in that it is non-transferable—it cannot be controlled by the firm under either mode. That is, the choice of effort and the costs associated with effort are always incurred by the professionals/employees themselves. Note that $\gamma$, which we assume is positive, captures the impact of the professional’s unobserved effort on the number of clients. Continuing with our hair salon example, $e_i$ could be the private efforts of hairdressers in providing good customer service (e.g. establishing a friendly relationship with customers that might lead to referrals and/or repeat visits).

Professionals or employees have access to an outside option (job) that pays them a fixed wage $w_0$. The VI firm pays employees a fixed salary $w$, possibly also a bonus (or commission) $b$ per client attracted, and chooses the transferable action $a_i$. The MSP charges independent professionals a fixed joining fee (e.g., a rent or a membership fee) $T$ and potentially also a variable fee $t$ proportional to the number of clients attracted if these are observed by the platform. The transferable action $a_i$ is now chosen by each professional. Note that the fixed salary and bonus set by the VI firm are equivalent to the MSP using a negative joining fee and a negative variable fee (i.e., a subsidy). This difference reflects that the firm and not the professional collects the revenue in the case of the VI firm, which is why it needs to make payments to the professionals to keep them willing to work.
The timing of decisions is as follows:

1. The firm chooses which mode to operate in—MSP or VI. Then it chooses its fees under the MSP mode or wages and commission rates under the VI mode.

2. Professionals make their affiliation decision.

3. Professionals’ private information regarding their respective $\theta_i$’s is realized. Transferable and non-transferable actions are chosen.

4. Clients make their affiliation and purchase decisions.\(^5\)

Note with this timing assumption, we are assuming that neither the VI firm nor the professionals (in MSP mode) can commit to their respective choices of the non-contractible transferable actions $a_i$ at the time the firm sets the fixed salary $w$ and bonus $b$ or fixed fee $T$ and variable fee $t$. We solve for the subgame perfect equilibrium of this game.

### 3.2 Results with optimistic expectations

With cross-group network effects, a coordination problem with multiple equilibria arises among professionals when they decide whether or not to join the firm/MSP. In particular, each professional’s expectation about the number of other professionals who join affects their individual decision. We assume in this section that professionals hold “optimistic expectations”—i.e., professionals always coordinate on joining if they obtain non-negative profit in the resulting equilibrium. Since $m$ is increasing in $n$, both the VI firm and the MSP want to sign up all $N$ professionals. The baseline demand will be determined by $m(N)$ in this case. For convenience, we define $m \equiv m(N)$.

Before proceeding, note that the total surplus created by the firm in this case is

$$\sum_{i=1}^{N} \left( \pi (m + \theta_i a_i + x (\bar{a}_{-i} - a_i) + \gamma e_i) - \frac{1}{2} a_i^2 - \frac{1}{2} e_i^2 - w_0 \right).$$

Thus, the first-best levels of transferable and non-transferable actions from the perspective of a hypothetical firm that could observe $\theta_i$ and directly control both $a_i$ and $e_i$ are

$$a_i^{FB} = \theta_i \pi$$

$$e_i^{FB} = \gamma \pi.$$

The left-hand side of these equalities is the marginal cost of one more unit of each type of action while the right-hand side is the corresponding marginal benefit. In what follows, a firm’s choice of mode (VI or MSP) depends on how close it gets to these first-best outcomes under each mode.

\(^5\)Nothing would change if clients and professionals made their affiliation decision at the same time.
3.2.1 Vertically integrated firm

Consider first the case in which the firm is vertically integrated and employs the professionals. An employee’s choice of unobservable effort is determined by the bonus payment $b$. The payoff to employee $i$ from working for the VI firm is $w + bD_i - \frac{1}{2}e_i^2$, so the level of effort chosen by each employee is $e^{VI}(b) = \gamma b$.

At the same time, the VI firm sets $a_1, ..., a_N$ to maximize its profit

$$\sum_{i=1}^{N} \left( (\pi - b) (m + E(\theta) a_i + x(\bar{a}_i - a_i) + \gamma e_i) - \frac{1}{2}a_i^2 - w \right),$$

implying the firm’s optimal choice is $a^{VI}(b) = E(\theta)(\pi - b)$. Note the VI firm coordinates its choice of transferable actions to take into account the externality, which is why $x$ does not affect its optimal choice of $a_i$. The bonus payment has two effects: it acts as a cost for the VI firm, lowering its margin and its choice of transferable action, and also acts as an incentive for employees to choose higher levels of unobservable effort.

Stepping back to the first stage (when the VI firm sets the salary and bonus), the fixed salary $w$ is set to render professionals indifferent between the option of joining and their outside option $w_0$. A higher bonus lowers the fixed salary that must be offered to attract the employee so that in the end, the VI firm’s profit is

$$N \left( \pi \left( m + E(\theta) a^{VI}(b) + \gamma e^{VI}(b) \right) - \frac{1}{2}a^{VI}(b)^2 - \frac{1}{2}e^{VI}(b)^2 - w_0 \right).$$

(2)

If bonuses are feasible, then maximizing (2) with respect to $b$ leads to the optimal bonus (a more detailed derivation is provided in the appendix)

$$b^* = \frac{\gamma^2}{\gamma^2 + E(\theta)^2 \pi},$$

(3)

which is positive but smaller than $\pi$. The optimal bonus is increasing in $\gamma$, so the firm stimulates more individual effort when effort matters more. However, since a positive bonus lowers the firm’s choice of transferable action below the first-best level (on average), the firm will want to limit its bonus payment, especially if the transferable action is an important demand driver. This is why the optimal bonus is decreasing in $E(\theta)$, the (average) importance of the transferable action.

3.2.2 Multi-sided platform

Consider next the case in which the firm allows individual professionals to interact directly with clients and to fully control their choices of $a_i$. The payoff to an individual professional joining the MSP is

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6Given $\pi$ is exogenous, it would make no difference to the analysis that follows if the bonus is paid on revenue $\pi D_i$ instead.
\( (\pi - t) D_i - \frac{1}{2} a_i^2 - \frac{1}{2} e_i^2 - T \), where \( T \) is the access fee charged by the firm in MSP mode (e.g., a fixed monthly booth rental fee in the case of a hair salon run as a MSP), and \( t \) is the variable fee charged by the firm in MSP mode for each client if it can monitor the number of clients (e.g. a fee based on sales in the case of a hair salon run as a MSP).\(^7\) Individual professionals maximize their payoff by choosing \( e_i^M(t) = \gamma (\pi - t) \) and \( a_i^M(t) = (\theta_i - x)(\pi - t) \). Given our assumption that \( x < \theta_i \), we have \( a_i^M(t) > 0 \) provided the variable fee does not try to tax all of the professional’s margin. In MSP mode, professionals will choose unobservable effort at the first-best level \( e_i^{FB} \) in the absence of any variable fee, but at a lower level to the extent they face a variable fee. Comparing transferable activities \( a_i^M(t) \) with the first-best level \( a_i^{FB} \), the expressions coincide when \( x = 0 \) and \( t = 0 \). However, if \( x > 0 \), so transferable activities create positive spillovers, then independent professionals choose an insufficient level of transferable activities. Similarly, if \( x < 0 \), so transferable activities create negative spillovers, then independent professionals choose an excessive level of transferable activities.

Stepping back to the first stage (when the MSP sets the fixed fee and variable fee), the MSP’s expected profit is \( N \left( t \left( m + E(\theta_i a_i^M(t)) + \gamma e_i^M(t) \right) + T \right) \). The MSP sets \( T \) to equalize each professional’s expected payoff from joining with their outside option \( w_0 \). As a result, the total profit of the firm in MSP mode is

\[
N \left( \pi \left( m + E(\theta_i a_i^M(t)) + \gamma e_i^M(t) \right) - \frac{1}{2} E \left( a_i^M(t)^2 \right) - \frac{1}{2} e_i^M(t)^2 - w_0 \right) .
\] (4)

If the MSP cannot charge a variable fee, then professionals keep the full profit from their costly actions, and the MSP’s profit comes only through the fixed fee. If the MSP can monitor the number of clients and charge a variable fee, then maximizing (4) with respect to \( t \) leads to the optimal variable fee

\[
t^* = \frac{-x (E(\theta)-x)}{(E(\theta)-x)^2 + Var(\theta) + \gamma^2 \pi} .
\] (5)

The optimal variable fee is positive if spillovers are negative (to reduce the excessive choice of transferable activities chosen by independent professionals) and is negative if spillovers are positive (to subsidize independent professionals so that they choose higher levels of transferable activities). Note also that using the assumption \( x < \theta_i \), it can easily be shown that \( t^* < \pi \).

3.2.3 Comparing modes

Comparing (4) with (2) and setting \( t = b = 0 \) to consider the case where bonus and variable fees are not feasible, we obtain the following result. The formal proof of this result and all others are collected in the appendix.

\(^7\)Again, given \( \pi \) is exogenous, it would make no difference to the analysis that follows if \( t \) is levied on sales revenue \( \pi D_i \) instead.
Proposition 1 When bonuses and variable fees are not feasible, the MSP mode is preferred to the VI mode if and only if

\[ x^2 < \gamma^2 + \text{Var}(\theta), \]  

i.e. if the magnitude of the spillover parameter \( x \) is sufficiently small relative to the importance of moral hazard (measured by \( \gamma \)) and professionals’ private information (measured by \( \text{Var}(\theta) \)).

Without the use of a sales bonus or variable fees, the trade-off between the MSP mode and VI mode is very straightforward in this framework. The trade-off is unambiguously shifted in favor of the MSP mode when unobservable effort becomes more important (\( \gamma \) increases) since this mode allows independent professionals to keep the full returns to their costly effort decisions and so eliminates the employees’ moral hazard problem. Increasing the importance of professionals’ private information (i.e. increasing \( \text{Var}(\theta) \)) also shifts the trade-off in favor of the MSP mode since this mode allows independent professionals to keep the full returns of their costly investment in their transferable activity, which gives them the correct incentives to adapt their decisions to their private information. In contrast, a VI firm without bonuses will not induce any positive effort on the part of employees, and the firm will also base its choice of transferable activities only on the expected value of \( \theta \), thereby creating a distortion. The trade-off is shifted in favor of the VI mode when spillovers become more important (\( |x| \) increases) since the firm can coordinate the choices of transferable activities to take spillovers into account. In contrast, a firm in MSP mode has no way to get professionals to internalize spillovers without the use of variable fees (or subsidies).

Given that the VI firm collects the proceeds of sales, it seems reasonable that it can monitor sales and condition payments to professionals based on their sales. In particular, suppose the VI firm can offer a bonus to its workers based on the number of clients they attract. Substituting (3) into (2) and comparing with (4) evaluated at \( t = 0 \), we obtain the following result.

Proposition 2 Compared to the case without bonuses or variable fees, the use of bonuses by the VI firm shifts the trade-off in favor of the VI firm. In this case, the MSP mode is preferred to the VI mode if and only if

\[ x^2 < \left( \frac{E(\theta)^2}{E(\theta)^2 + \gamma^2} \right) \gamma^2 + \text{Var}(\theta). \]  

With the VI firm using bonuses but the MSP not using variable fees, an increase in the importance of moral hazard and professionals’ private information shifts the trade-off towards the MSP mode, and an increase in the magnitude of spillovers shifts the trade-off towards the VI mode.

Allowing the VI firm to use bonuses shifts the trade-off in favor of the VI mode because it allows the VI firm to deal with the moral hazard problem, although at the cost of distorting its own choice
of the transferable action (the bonus reduces the margin the firm obtains on its sales). This effect
is captured by the additional term in brackets on the right-hand-side of (7) compared to (6). That
term is strictly less than one, which implies that the range of parameters over which the MSP mode
domains the VI mode is now smaller relative to the previous case with no bonuses.\footnote{The precise meaning of this statement is that any parameters \((x, \text{Var}(\theta), \gamma, E(\theta))\) that satisfy (7) also satisfy (6), but the reverse statement is not true.}

Note that \(E(\theta)^2\) measures the importance of transferable actions on demand. The larger is \(E(\theta)^2\)
relative to \(\gamma^2\), the closer is the fraction in brackets in (7) to one, and so bonus payments become less
effective in shifting the trade-off in favor of the VI mode. This is because a higher bonus payment
causes the VI firm to choose a suboptimal level of the transferable action: this distortion is worse the
more important the transferable action is for demand.

The parameters \(|x|, \text{Var}(\theta)\) and \(\gamma\) have the same impact on the trade-off between the two modes
as in Proposition 1. Namely, a higher \(|x|\) increases the relative payoff to the VI mode, and a higher
value of \(\gamma\) or \(\text{Var}(\theta)\) increase the relative payoff to the MSP mode.

So far we have assumed the MSP cannot monitor sales, and so can only make use of a fixed access
fee. In case the MSP can monitor the sales by professionals that use its platform, it will also be
able to make use of payments that depend on the demand for each service provided by professionals.
Substituting (5) into (4) and comparing with the VI firm’s profit when it uses its optimal bonus, we
obtain the following result.

**Proposition 3** Compared to the case with bonuses but no variable fees, the use of variable fees by the
MSP shifts the trade-off in favor of the MSP. In this case, the MSP mode is preferred to the VI mode
if and only if

\[
\left( \frac{\text{Var}(\theta) + \gamma^2}{\text{Var}(\theta) + \gamma^2 + (E(\theta) - x)^2} \right) x^2 < \left( \frac{E(\theta)^2}{E(\theta)^2 + \gamma^2} \right) \gamma^2 + \text{Var}(\theta).
\]  

With the VI firm using bonuses and the MSP using variable fees, an increase in the magnitude of
spillovers always shifts the tradeoff towards the VI mode; an increase in the importance of professionals’
private information shifts the trade-off towards the MSP mode if and only if \(|t^*| < \pi\); an increase
in the importance of moral hazard shifts the trade-off towards the MSP mode if and only if \(|t^*| < \pi \sqrt{1 - \frac{b^*}{\pi} (2 - \frac{b^*}{\pi})}\).

Allowing the MSP to charge variable fees (or pay variable subsidies) shifts the trade-off in favor
of the MSP mode relative to that in (7) as it allows the MSP to partially overcome spillovers. This
is captured by the additional term in brackets on the left-hand-side of (8) compared to (7). That
term is strictly less than one, which implies that the range of parameters over which the MSP mode dominates the VI mode is now larger relative to the previous case with bonuses but no variable fees.\footnote{This is also true if professionals have no informational advantage. This can be seen by setting \( \text{Var}(\theta) = 0 \) and \( E(\theta) = \theta \) in (7) and (8).}

On the other hand, moving from the case with no bonuses for the VI firm and no variable fees for the MSP to the present case with bonuses and variable fees has an ambiguous effect on the trade-off between the two modes (compare (6) and (8)). The net effect depends on whether or not having the MSP internalize spillovers using variable fees is more important than having the VI firm deal with the moral hazard problem using bonuses. One might have intuitively expected the two modes to become equivalent when bonuses and variable fees can be used: bonuses help the VI firm (partially) overcome the moral hazard problem, whereas variable fees help the MSP (partially) overcome the spillovers issue. The reason the two modes are not equivalent (i.e. the trade-off persists) is that using bonuses to offset moral hazard distorts the VI firm’s choice of transferable activity, while using variable fees to offset spillovers distorts the professionals’ choice of effort and transferable activities. Moreover, professionals’ private information also drives a wedge between the two modes.

Finally, consider the effect of parameters \(|x|\), \(\text{Var}(\theta)\) and \(\gamma\) on the relative profitability of the MSP mode in the case with bonuses and variable fees, which is represented by the inequality in (8). The standard intuition applies here for an increase in \(|x|\): stronger spillovers increase the relative payoff of the VI mode, which can directly control for spillovers through its choice of transferable activities. Things are less clear cut with respect to changes in \(\text{Var}(\theta)\) and \(\gamma\). Note that both sides of the inequality (8) are increasing in \(\text{Var}(\theta)\) and \(\gamma\) (holding \(E(\theta)\) constant), but it is not obvious which side increases faster.

Consider first the effect of \(\text{Var}(\theta)\). As shown in the proof of Proposition 3, the condition for the right-hand side to be increasing in \(\text{Var}(\theta)\) faster than the left-hand side is exactly \(|t^*| < \pi\). If \(t^* \geq 0\) then this necessarily holds given our assumption that \(x < \theta\), which implies \(t^* < \pi\). The only case in which the left-hand side increases faster than the right-hand side is \(t^* \leq -\pi\), which can happen only if \(x > E(\theta)/2\). In this case, positive spillovers from the transferable action are so strong that the optimal subsidy required to help correct for spillovers in the case of a MSP exceeds the exogenous revenue obtained per client. As a result, the subsidy per client makes professionals excessively sensitive to private information, so that an increase in the importance of this private information (i.e. an increase in \(\text{Var}(\theta)\)) exacerbates the distortions in the choices of effort and transferable actions and so leads to a decrease in the relative profitability of the MSP mode.

In the proof of Proposition 3 we show a similar result holds for an increase in \(\gamma\): this shifts the tradeoff towards the MSP mode if and only if \(|t^*| < \pi \sqrt{1 - \frac{b^*}{\pi} \left(2 - \frac{b^*}{\pi}\right)}\). Once again, the intuition is
that when spillovers are sufficiently strong (in either direction), variable fees (or subsidies) become excessively large and as a result professionals’ choices of effort levels are more distorted under the MSP mode than under the VI mode. In this case, increasing the importance of effort actually decreases the relative profitability of the MSP mode.

### 3.3 Results with pessimistic expectations

We have thus far assumed professionals hold optimistic expectations. Suppose instead professionals hold pessimistic expectations, i.e., they coordinate on not joining whenever this is an equilibrium. This scenario is particularly relevant for MSPs that are part of early-stage ventures. The nature of expectations also matters for the VI firm if it offers a bonus per client attracted. The bonus payment means the payoff to a professional depends on how many clients the professional attracts. Because of network effects, the number of clients attracted depends on how many other professionals supply the VI firm. Put differently, the VI firm can entirely avoid pessimistic expectations by only offering a fixed salary (no bonus). Similarly, the MSP can entirely avoid pessimistic expectations by only charging a transaction fee (no fixed fee). Of course, these extreme scenarios will usually not be optimal: offering no bonus eliminates professionals’ effort incentives, whereas collecting all revenues as transaction fees severely limits the incentives provided to professionals to invest in costly (transferable and non-transferable) activities.

Our treatment of pessimistic expectations follows Caillaud and Jullien (2003), Hagiu (2006), Hagiu and Spulber (2013) and Halaburda and Yehezkel (2013). We denote

\[
\Delta m \equiv m(N) - m(1),
\]

which is a parameter that reflects the strength of network effects and therefore the exogenous change in client numbers due to pessimistic expectations. (If \(\Delta m = 0\), i.e. if there are no network effects, then the number of clients under optimistic and pessimistic expectations would be the same.) To ensure that the VI firm does not find it optimal to set a negative bonus and the MSP does not find it optimal to charge variable fees in excess of \(\pi\) (if \(b^* < 0\) or \(t^* > \pi\) then professionals will exert no effort and it’s hard to justify they will be able to commit to working at all) we make the following additional assumption

\[
\Delta m < \pi \gamma^2. \tag{9}
\]

The analysis of the VI firm under pessimistic expectations is similar to the case with optimistic expectations. The key difference is that the fixed salary \(w\) that renders professionals indifferent between joining and their outside option \(w_0\) must now be higher (for any given bonus \(b\)) since each professional expects no other professionals to join whenever this is an equilibrium. Indeed, the total
bonus that each professional expects to receive is \( b(m(1) + \theta_i a + \gamma e_i) \) when \( a_i = a \) for all \( i \). Since the total bonus depends on \( m(1) \) rather than \( m(N) \), the fixed wage needs to be higher by \( b\Delta m \) to attract all professionals. With this fixed wage, all professionals join and the VI firm’s profit is equal to its profit under optimistic expectations minus the increase in wages that needs to be paid, which is \( Nb\Delta m \). Optimizing over \( b \) implies
\[
b_{PE}^* = \frac{\gamma^2 - \frac{\Delta m}{\pi}}{\gamma^2 + E(\theta)^2\pi}.
\] (10)

This can be compared to (3). The optimal bonus in (10) is strictly lower relative to that in (3), and decreasing in \( \Delta m \). The more important the network effects parameter, the lower the optimal bonus: this is because the bonus is constrained by pessimistic expectations whereas the fixed wage is not. Of course, this requires the VI firm to compensate professionals by offering higher fixed wages, which is why the VI firm’s profit will be decreasing in the network effects parameter. Note that the assumption (9) ensures that \( b_{PE}^* \geq 0 \).

The analysis of a MSP with pessimistic expectations is also similar to the case with optimistic expectations. The fixed fee \( T \) that renders professionals indifferent between joining and their outside option \( w_0 \) must now be lower (for any given variable fee \( t \)) because each professional expects no other professionals to join whenever this is an equilibrium. Indeed, the variable revenue that each professional expects to obtain is \( (\pi - t)(m(1) + \theta_i a + \gamma e_i) \) when \( a_i = a \) for all \( i \). Thus, the fixed fee that the MSP can charge needs to be lower by \( (\pi - t)\Delta m \) to attract all professionals. With this fixed fee, all professionals join and the MSP’s profit is equal to the profit under optimistic expectations minus the decrease in the fixed fee, which is \( N(\pi - t)\Delta m \). Optimizing over \( t \) implies
\[
t_{PE}^* = \frac{\Delta m - x(E(\theta) - x)\pi}{(E(\theta) - x)^2 + Var(\theta) + \gamma^2}.
\] (11)

This can be compared to (5). The optimal variable fee in (11) is strictly higher relative to that in (5), and increasing in \( \Delta m \). The more important the network effects parameter, the higher the optimal variable fee: this is because the fixed fee is constrained by pessimistic expectations, whereas the variable fee is not.

Comparing profits across the two modes, we obtain the following proposition.

**Proposition 4** If variable fees are not feasible, pessimistic expectations always shift the tradeoff between the two modes in favor of the VI mode. If the MSP can use variable fees and the VI firm can use bonuses, pessimistic expectations shift the tradeoff between the two modes in favor of the VI mode if and only if \( b_{PE}^* < \pi - t_{PE}^* \).

Without bonuses or variable fees, the intuition is simple. In this case, there are no network effects in VI mode but maximum network effects in MSP mode. As a result, pessimistic expectations have
no effect on the VI firm’s profit but do lower profit in MSP mode. One can reinterpreted this result to say that early stage start-ups may find it profitable to function in VI mode and then transition to the MSP mode later, once they have overcome pessimistic expectations. A similar intuition applies even if we introduce bonuses. Because the optimal bonus per client is less than the exogenous profit per client, equilibrium network effects will always be stronger under the MSP mode, and so pessimistic expectations reduce profit more for the MSP mode than for the VI mode.

When the VI firm can use bonuses and the MSP can use variable fees, the effect of pessimistic expectations on the trade-off is no longer so obvious, but has an appealing interpretation. Specifically, pessimistic expectations lower MSP’s profitability more than they lower VI firm’s profitability if and only if equilibrium network effects in MSP mode (which is proportional to professionals’ payoff per client \( \pi - t^*_P E \)) is larger than equilibrium network effects in VI mode (which is proportional to \( b^*_P E \)). In particular, this means that if unobservable effort is particularly important, then the VI firm will choose to offer a very high bonus, which can make equilibrium network effects stronger under the VI mode. As a result, pessimistic expectations may cause a larger decrease in profits under the VI mode.

Finally, we consider the effects of different parameters on the tradeoff between the two modes under pessimistic expectations and compare them to the effects under optimistic expectations. When variable fees for the MSP mode are not feasible, the effects of private information, moral hazard and the magnitude of spillovers are identical to those given in Propositions 1 and 2. When both bonuses and variable fees are feasible, the effects are no longer the same as in Proposition 3 but they remain similar. Specifically, using the envelope theorem in a similar way to the proof of Proposition 3, it can be shown that an increase in the importance of professionals’ private information, \( Var (\theta) \), shifts the trade-off towards the MSP mode if and only if \( |t^*_P E| < \pi; \) and an increase in the importance of moral hazard, \( \gamma \), shifts the trade-off towards the MSP mode if and only if \( |t^*_P E| < \pi \sqrt{1 - \frac{b^*_P E}{\pi} \left( \frac{2 - \frac{b^*_P E}{\pi}}{\pi \left( Var(\theta) + \gamma^2 \right)} \right)}. \)

Thus, the interpretation of these effects is the same as before. The effect of the spillover parameter, \( x \), is slightly different. Specifically, under pessimistic expectations, an increase in \( x \) shifts the trade-off towards the VI mode if and only if

\[
x > \frac{-E(\theta) \Delta m}{\pi (Var(\theta) + \gamma^2) - \Delta m},
\]

instead of \( x > 0 \) with optimistic expectations. The interpretation is the same as before when \( x \) is outside of the interval \( \left[ \frac{-E(\theta) \Delta m}{\pi (Var(\theta) + \gamma^2) - \Delta m}, 0 \right] \): an increase in the magnitude of spillovers, \( |x| \), makes the VI mode relatively more profitable, because the firm can internalize spillovers through its control over transferable activities. The only difference occurs on the interval \( \left[ \frac{-E(\theta) \Delta m}{\pi (Var(\theta) + \gamma^2) - \Delta m}, 0 \right] \), where an increase in the magnitude of (negative) spillovers shifts the trade-off towards the VI mode under optimistic expectations, but towards the MSP mode under pessimistic expectations.
3.4 Other trade-offs

Other trade-offs which seem relevant but that we have not formally modeled include:

*Two-sided private information:* While we only allowed for private information on the professionals’ side, in some contexts both sides—professionals and the firm—may have private information about the effectiveness of marketing activities to clients. For instance, online service platforms such as Elance-oDesk may extract novel insights from the aggregate data generated by the interactions between contractors and customers on their sites—insights that are not known to any individual contractor. Our model could be easily extended to allow for two-sided private information, as in Hagiu and Wright (2015). This would lead to additional results very similar to the ones obtained in that paper (summarized in the next section): the larger the difference between the variance of professionals’ private information and the variance of the firm’s private information, the more the trade-off is shifted towards the MSP mode.

*Scale and learning economies:* A professional service firm facing clients with similar and repetitive needs is likely better off using the VI mode, which allows it to create and capture value by investing in centralized capabilities that can be leveraged across multiple client projects (e.g., McKinsey’s standardized problem-solving methodologies). This concern is less relevant for a firm that faces clients with very idiosyncratic needs, which may prefer to function as a MSP.

*Two-sided moral hazard:* In addition to the professionals’ effort-related moral hazard, professional service firms can have a moral hazard of their own if they are unable to credibly commit to adequately train their employees (see Morrison and Wilhelm, 2004 and Garicano and Rayo, 2013). Depending on the value of internal training relative to professionals’ existing human capital and on the difficulty of solving the training-related moral hazard problem, firms may find it optimal to abandon the VI mode and function as a MSP instead.

*Intrinsic motivation and adverse selection:* While it may be possible for some VI firms to include performance-based bonuses in their payments to employees, independent professionals can be intrinsically more motivated to perform well and develop their human capital and reputation. This intrinsic motivation can be rationalized by taking into account long-run career concerns that lie outside the scope of the client projects that the firm in question can compensate its professionals for. Thus, the MSP mode should be better suited for contexts where individual human capital is the most important driver of clients’ willingness-to-pay. Indeed, a well-known problem in professional service industries is the difficulty of preventing “star professionals” from becoming independent “free agents” and taking their clients with them (note that this does not hold true when teams are important, as noted above; see also Groysberg, 2011). This means that vertically integrated professional service firms may sometimes face an adverse selection problem: the professionals willing to work for them are
not necessarily the highest quality professionals.

4 MSP or reseller?

The distinction between a MSP and a reseller rests on whether the sale of goods from A to B (as shown in Figure 1) constitutes a direct interaction (the pure MSP case) or is instead entirely controlled by the intermediary (the pure reseller case). It is direct to the extent that A and/or B retain residual control rights over the goods traded. This typically arises when A retains ownership of the goods being traded: examples include eBay and shopping malls. In contrast, a pure reseller holds all residual control rights over the goods sold to B. This typically arises when the reseller takes ownership of the goods from A. Old fashioned retailers are typical examples.

Sometimes, A may explicitly contract away some control rights over goods sold to B to the intermediary (e.g., a contract limiting the price A can set, or a contract specifying how the goods must be shipped from A to B). Provided A retains all residual control rights, in our terminology A and B are still engaged in a direct interaction. Thus, a shopping mall remains a MSP even though it may place contractual restrictions on the goods sold by retailers, including what range of items they are allowed to sell, how they may be displayed, and various types of non-compete clauses. In other cases, the intermediary holds residual control rights over some key decision variables, while residual control rights over other decision variables remain with A (or B). In such cases, the business model is intermediate between a pure MSP model and a pure reseller model. An example is when a retailer sells shelf space to some branded suppliers (e.g., Kellogg’s and Pepsi). Suppliers may retain a significant number of residual control rights over the goods sold (e.g., how they are stocked, in-store layout and advertising), while the retailer also holds some residual control rights (e.g., customer service and pricing). Intermediate forms also arise when some goods or services are traded directly between A and B while others are controlled by the intermediary. Best Buy took a step towards the MSP mode by allowing Apple, Samsung and Microsoft to launch their own mini-stores (complete with their own sales personnel) within Best Buy stores (Apple in 2011, Microsoft and Samsung in 2013), but the marketing and selling of other brands remains under the control of Best Buy in its stores.

In Hagiu and Wright (2015) we modelled the choice faced by an intermediary that facilitates trades between consumers and suppliers to function either as a MSP (which we called a marketplace) or as a reseller, or to operate in a hybrid mode, having some products offered under each of the two different modes. The model focused on a single, non-contractible decision variable, which was interpreted as the choice of some marketing activity that occurs through the intermediary and that is undertaken by the party holding residual control rights (i.e., the reseller, or each independent supplier in the case of
a MSP). Examples of such an activity include the way in which a product is displayed, or the extent to which its brand is promoted relative to its features (e.g., through in-store signage or sales staff). The intermediary and the suppliers were each assumed to have private information about the ideal choice of the marketing activity.

In the most basic version of the model in Hagiu and Wright (2015), the relative information advantage of the intermediary versus suppliers was the sole determinant of which party should be given control rights over the marketing activity, and therefore of whether the MSP mode dominates the reseller mode or vice versa. Specifically, we showed that the MSP mode is preferred to the reseller mode if and only if the variance of the suppliers’ local information exceeds the variance of the intermediary’s local information. In case the variances of supplier and intermediary information vary across products, the intermediary should use the reseller mode for all products where it has an information advantage over suppliers and the MSP mode for all products where the advantage lies with suppliers.

This prediction is in line with evidence from several different markets. Amazon resells a higher share of products in the categories in which it is more likely to have an information advantage over third-party suppliers. This may explain why for books Amazon sells more than 50% of new books listed on its site, whereas for electronics Amazon sells closer to 1% of the new items listed (see Hagiu and Wright, 2015). Department stores have traditionally offered cosmetics products through dedicated “counters,” as in the MSP mode, where displays are designed and controlled by individual brands. The most likely explanation is that cosmetics manufacturers possess specialized knowledge about how best to market their brands to consumers that is hard for stores and their sales staff to obtain. At the same time, products for which such specialized marketing information is less important (e.g., mass-market accessories) are offered on displays controlled by the store and serviced by generalist sales personnel as in the reseller mode.

In Hagiu and Wright (2015) this baseline trade-off is then extended to allow for several other factors, which we summarize here.

1. First, we allowed marketing activities to generate cross-product spillovers. This unambiguously shifted the trade-off in favor of the reseller mode, reflecting that the reseller takes into account the cross-product externalities from the promotion of product $i$ on other products, something independent suppliers on a MSP will not do. This version of the model helps explain why cable TV operators typically operate in the reseller mode, coordinating their pricing and marketing decisions across different channels (e.g., through bundling, and the cross-marketing of different channels).
2. Second, we supposed the MSP mode has a variable cost disadvantage relative to the reseller mode, and introduced demand heterogeneity across products. The variable cost disadvantage of the MSP can reflect that a reseller is more efficient at selling incremental units to consumers than third-party suppliers due to the reseller’s superior scale in stocking, distribution and marketing. This variable cost disadvantage shifts the trade-off in favor of the reseller mode for short-tail (popular) products and in favor of the MSP mode for long-tail (unpopular) products. Consistent with this prediction, in Hagiu and Wright (2015) we document that Amazon (which has particularly low variable costs) acts as a reseller rather than a MSP (i.e., through its marketplace) for a high proportion of popular DVDs and a low proportion of unpopular DVDs.

3. Third, we introduced indirect network effects by assuming that the more suppliers that offer products through an intermediary, the more buyers want to buy through that intermediary. Such network effects do not affect a reseller’s profitability, assuming the reseller simply buys products from suppliers and thereby assumes all inventory risk. However, suppliers’ expectations about how many other suppliers will join can matter to a MSP, since these expectations affect how many buyers each supplier can expect to attract for its products, and thus how profitable it is to join the MSP. Just like in the VI vs. MSP analysis above, we focused on the case in which suppliers hold pessimistic expectations. To overcome these pessimistic expectations, the MSP has to significantly lower the fee it charges each supplier to join. This makes the MSP less profitable and shifts the trade-off in favor of the reseller. A MSP can partially mitigate this effect by offering some products under the reseller mode or by making use of variable fees to soften the effects of pessimistic expectations. Amazon seems to have used both strategies. By first selling products (mainly books) as a reseller, it attracted many buyers to its website, after which it was able to attract third-party sellers of new product categories on its marketplace without facing the usual chicken-and-egg problem associated with a marketplace launch. Amazon also relies mainly on variable fees to extract revenue, with individual suppliers able to choose a “no monthly fee” option if they prefer (Stone, 2013).

In addition to the trade-offs formally modeled in Hagiu and Wright (2015), the presence of asymmetric information is another factor potentially driving the choice of MSP or reseller mode. If suppliers’ goods are of uncertain quality and the reseller can achieve economies of scale in verifying quality (i.e., verifying one of many identical goods), then the reseller mode is more valuable, because it can leverage reputation effects (as in Biglaiser, 1993). On the other hand, if the sources of asymmetric information are idiosyncratic across each individual good sold by the reseller, then no such benefit may be obtained. Of course, MSPs can develop their own solutions to the asymmetric information problem, including
most obviously a feedback system that helps pool users’ experience. However, a feedback system is vulnerable to a free-rider problem, in that users have little incentive to share their experiences in a way that benefits other users. Moreover, feedback is based on past sales, and this tends to create lock-in and market power for large suppliers (e.g., PowerSellers on eBay), making it harder for new suppliers to enter than to sell to a reseller that can verify the quality of their products.

Another potential weakness that MSPs face relative to resellers is that they cannot easily aggregate the bargaining power of their many users. A good example is Intellectual Ventures (IV), the world’s largest patent aggregator, which has acquired more than 70,000 patents. IV typically acquires patents from universities, small companies and individual inventors, then resells or licenses them, mostly to large operating companies. IV can help aggregate the bargaining power of many patent owners with respect to large operating companies seeking to license (or purchase) patents. A MSP could not achieve this, which is perhaps why virtually all attempts to create patent marketplaces have failed (Hagiu and Yoffie 2013). A similar example is that of book publishers, who typically act as resellers of individual authors’ works to libraries and book retailers. One of the main roles played by publishers is to aggregate authors’ bargaining power with respect to book distributors and thereby extract better terms than what the authors could negotiate individually.

5 MSP or input supplier?

We pointed out in section 2 that many MSPs create and capture economic value through indirect network effects. This is why many firms, particularly in industries supplying complex technology products (e.g., PCs and smartphones), vie to become MSPs as opposed to simple product or input suppliers. They can do this by requiring affiliation of customers on both sides, which helps distinguish MSPs from input suppliers.

For example, Microsoft’s Windows acts as a MSP enabling interactions between application developers and PC users, but Intel’s microprocessors are mere inputs to PC manufacturers. The direct interaction that consists of users installing and using PC applications is enabled by the microprocessor just as much as it is enabled by the operating system. The key difference, however, is that application developers affiliate with Windows by investing significant resources in using its APIs, but they typically make few if any investments or design choices specific to the microprocessor. As a result, the indirect network effects between users and application developers belong to Microsoft, not Intel. Things would be different if Intel introduced specific (software) extensions that have to be taken into account by application developers. Unsurprisingly, occasional attempts by Intel to wrest away some of those network effects by offering Intel-specific software extensions to application developers were
met with stiff opposition by Microsoft (Yoffie, 2003 and Yoffie et al., 2003).

More recently and for similar reasons, some manufacturers of smartphones running on Android and Windows Mobile (HTC, Samsung and Motorola) have developed proprietary software APIs, which they expose to third-party application developers. The latter still mostly affiliate with the underlying operating systems (Android or Windows Mobile), but they also have some degree of affiliation with a given handset maker to the extent that they must tweak their apps in order to optimize performance for that handset maker’s devices. Through these efforts, handset makers are attempting to gain some degree of two-sidedness and indirect network effects (though not to the same extent as Apple’s iPhone).

It is important to note, however, that MSPs are not always more profitable or efficient than input suppliers. Consider contexts in which a firm (P) sells technology inputs to customers (B), who in turn incorporate those inputs in products they sell to end users (A). Firm P may sometimes prefer not to require affiliation by end users in order to avoid the higher costs associated with getting two interdependent sides (A and B) on board.

An example is that of fast-food chains. Franchisors like McDonald’s and Subway enable direct interactions between their respective franchisees and end-consumers. Franchisees affiliate by entering contracts with corporate McDonald’s or Subway, contracts which are strictly necessary in order to sell McDonald’s or Subway food to consumers. On the other hand, in McDonald’s case consumers do not have to form any meaningful affiliation with the franchisor in order to patronize the franchisees’ restaurants. In particular, when consumers make their way to a McDonald’s restaurant, they are visiting that particular franchisee and not McDonald’s the franchisor, and nor does awareness of the McDonald’s brand constitute a form of affiliation. In other words, and in accordance with our earlier definition of affiliation, consumers do not make any investments specific to McDonald’s the franchisor that enables them to patronize franchisees’ restaurants. Such an investment could be joining a loyalty program offered by McDonald’s the franchisor and that would be valid at all franchisees’ restaurants. However, perhaps surprisingly, the only loyalty program offered by McDonald’s to U.S. consumers covers McCafes, which are company-owned, not franchised. McDonald’s does not have a loyalty program covering its U.S. franchisees similar to the Subway Card for Subway franchisees. In Subway’s case, consumers do affiliate with the franchisor (in the U.S.) by adopting the Subway Card, which means Subway operates more as a MSP. Why does McDonald’s prefer to remain an input supplier by not offering a loyalty card? One reason for which such a card may not be desirable is that it would create inter-dependencies between different franchisees (e.g., franchisee 1 having to provide products for free in exchange for points earned at franchisee 2), which would increase transaction costs.

\[10\] In the same way, the “Intel Inside” marketing campaign does not mean that end-users are affiliated with the Intel platform.
(transfer pricing) and undermine some franchisees’ incentives. The upshot would be that a loyalty card generates indirect network effects and increases consumer switching costs: this is presumably why Subway decided to provide one. In contrast, McDonald’s decided the cost was not worth it and therefore remains an input supplier that does not generate indirect network effects for consumers.

6 Conclusion

In this article, we have studied a firm’s strategic positioning decision between a multi-sided platform (MSP) mode and three alternative modes. This allowed us to delineate what makes MSPs special, and indeed, we provided a new definition of MSPs. Our main focus was on the choice between two modes—MSP and vertically integrated (VI) modes—and we provided a formal model of the choice between them. The model highlighted the key trade-off between the coordination benefits of the VI mode when there are spillovers across the decisions of individual professionals and the benefits of the MSP mode both in motivating unobservable effort on the part of professionals and ensuring professionals adapt their decisions to their private information. We also studied how this trade-off was shifted by the nature of contracts available under the two different modes, and the nature of the professionals’ expectations (optimistic or pessimistic). Finally, we have also highlighted some of the key trade-offs that arise in the choice between operating as a MSP or as a reseller, and between operating as a MSP or as an input supplier.

There are many promising directions in which this type of analysis can be extended. First, one can combine our previous work on MSPs vs. resellers with the formal work on MSPs vs. VI in this paper to obtain a make (VI) vs. buy (reseller) vs. enable (MSP) spectrum, which broadens the traditional make vs. buy choice studied by the vertical integration literature. Second, the choice between MSP vs. input supplier is one which naturally lends itself to formal modelling, which is something we are currently working on. Third, in the formal model presented here we have focused on a firm choosing between two extremes—whether control rights for a single transferable variable are assigned to the firm or to independent professionals. In reality, control rights might be allocated to the firm over some (transferrable) decision variables and not over others, or with respect to some goods (or services) and not others. Introducing either one or both of these dimensions would provide a way to capture that in reality the choices analyzed in this paper happen along a continuum. Taken together, the MSP vs. VI, MSP vs. reseller and MSP vs. input supplier models would map out a fine-grained spectrum of business models, from a pure MSP on the one hand, in which all decisions with respect to all goods (or services) are controlled by suppliers or professionals, to each of the three other alternatives.

Finally, an interesting direction for future modelling is to introduce customer heterogeneity into
the formal model in this paper or that in Hagiu and Wright (2015). The trade-offs between MSPs on one hand and resellers or VI firms on the other hand may be more relevant for certain customer segments than others. For example, exploiting economies of scale by aggregating transactions and verifying quality are less important for large and more experienced buyers, suggesting such buyers will be more inclined to deal directly with suppliers through MSPs. In contrast, small buyers might prefer to deal with resellers or VI firms. The heterogeneity of buyers can therefore explain why different (possibly competing) organizations in the same industry may choose different modes of operation, some choosing to operate as MSPs while others choosing to operate as VI firms, resellers, or input suppliers.

References


7 Appendix

The formal derivation of results stated in the text are presented here.

**Optimistic expectations**  We start by deriving the expected profits under each mode. Suppose the VI firm can pay a bonus. The payoff to professional $i$ from working for the VI firm is

$$w + bD_i - \frac{1}{2}e_i^2 = w + b(m + \theta_i a_i + x(\bar{a}_i - a_i) + \gamma e_i) - \frac{1}{2}e_i^2,$$

so $e^{VI}(b) = \gamma b$. The VI firm sets $a_1, \ldots, a_N$ to maximize its profit

$$\sum_{i=1}^{N} \left( (\pi - b)(m + E(\theta) a_i + x(\bar{a}_i - a_i) + \gamma e_i) - \frac{1}{2}a_i^2 \right),$$

implying the firm’s optimal choice of $a_i$ is $a^{VI}(b) = E(\theta)(\pi - b)$ for all $i$. Substituting $e^{VI}(b)$ and $a^{VI}(b)$ into (13), the firm’s expected profit in the first stage is

$$N \left( (\pi - b) \left( m + \frac{(\pi - b)E(\theta)^2}{2} + b\gamma^2 \right) - w \right).$$

The fixed salary $w$ is set to render professionals indifferent between joining and getting the expected value of (12) evaluated at $e^{VI}(b)$ and $a^{VI}(b)$, and getting their outside option $w_0$. This implies

$$w = w_0 - b \left( m + (\pi - b) E(\theta)^2 \right) - \frac{b^2\gamma^2}{2}.$$

Substituting the fixed salary $w$ into (14), the VI firm’s expected profit as a function of $b$ is therefore

$$\Pi^{VI}(b) = N \left( \pi m + \frac{(\pi^2 - \pi b^2)E(\theta)^2}{2} + \frac{(2\pi - b)b\gamma^2}{2} - w_0 \right).$$

If the VI firm cannot offer a bonus, then $b = 0$ and (15) becomes

$$\Pi^{VI}(0) = N \left( \pi m + \frac{\pi^2E(\theta)^2}{2} - w_0 \right).$$

If bonuses are feasible, then maximizing (15) with respect to $b$ implies the optimal bonus given by (3). With this optimal bonus, the resulting profits in VI mode are

$$\Pi^{VI}(b^*) = N \left( \pi m + \frac{\pi^2E(\theta)^2}{2} + \frac{\pi^2\gamma^4}{2\left(\gamma^2 + E(\theta)^2\right)} - w_0 \right).$$
Now consider the case of a MSP that can charge a variable fee $t$. The payoff to professional $i$ from joining the MSP is

$$(\pi - t) (m + \theta_i a_i + x (\bar{\pi} - a_i) + \gamma c_i) - \frac{1}{2} \sigma_i^2 - \frac{1}{2} \epsilon_i^2 - T,$$

so $e^M (t) = \gamma (\pi - t)$ and $a^M (t) = (\theta_i - x) (\pi - t)$. Given $e^M (t)$ and $a^M (t)$, the MSP’s expected profit is

$$N t \left( m + (E (\theta^2) - x E (\theta) + \gamma^2) (\pi - t) \right) + NT.$$

In the first stage, the MSP can set $T$ to equate the expectation of each professional’s payoff evaluated at $e^M (t)$ and $a^M (t)$, and getting their outside option $w_0$. This implies

$$T = (\pi - t) m + \frac{(\pi - t)^2}{2} \left( E (\theta^2) - x^2 + \gamma^2 \right) - w_0.$$

Substituting this $T$ into (19), the MSP’s expected profit as a function of $t$ is therefore

$$\Pi^M (t) = N \left( \pi m + \frac{(\pi - t)^2}{2} \left( E (\theta^2) + \gamma^2 \right) - \frac{(\pi - t)^2}{2} x^2 - t (\pi - t) E (\theta) x - w_0 \right).$$

If the MSP cannot use variable fees, then $t = 0$ and (20) becomes

$$\Pi^M (0) = N \left( \pi m + \frac{\pi^2}{2} \left( E (\theta^2) + \gamma^2 - x^2 \right) - w_0 \right).$$

Choosing $t$ to maximize MSP profits implies (5). The resulting MSP profit is

$$\Pi^M (t^*) = N \left( \pi m + \frac{\pi^2}{2} \left( E (\theta^2) + \gamma^2 - x^2 \right) + \frac{x^2 (E (\theta) - \theta^2)}{E (\theta) - x^2 + Var (\theta) + \gamma^2} \right) - w_0.$$

We are now ready to prove the propositions in the text.

**Proof of Proposition 1:** The difference between (21) and (16) is

$$\Pi^M (0) - \Pi^{VI} (0) = \frac{\pi^2 N}{2} \left( Var (\theta) + \gamma^2 - x^2 \right),$$

which implies (6).

**Proof of Proposition 2:** The difference between (21) and (17) is

$$\Pi^M (0) - \Pi^{VI} (b^*) = \frac{\pi^2 N}{2} \left( Var (\theta) + \left( \frac{E (\theta)^2}{\gamma^2 + E (\theta)^2} \right) \gamma^2 - x^2 \right),$$

which implies (7). Note $\Pi^M (0) - \Pi^{VI} (b^*)$ is strictly increasing in $\gamma$.

**Proof of Proposition 3:** The difference between (22) and (17) is

$$\Pi^M (t^*) - \Pi^{VI} (b^*) = \frac{\pi^2 N}{2} \left( Var (\theta) + \left( \frac{E (\theta)^2}{\gamma^2 + E (\theta)^2} \right) \gamma^2 - \frac{Var (\theta) + \gamma^2}{Var (\theta) + \gamma^2 + (E (\theta) - x)^2} \right) x^2,$$

which gives (8). Consider $\Pi^M (t) - \Pi^{VI} (b)$. As we change $x$, $Var (\theta)$ and $\gamma$, this changes the optimal value of $t^*$ and $b^*$ but the effect of this on the profit difference is zero due to the envelope theorem. As a result, we have

$$\frac{d \left( \Pi^M (t^*) - \Pi^{VI} (b^*) \right)}{d (x)} = -N (\pi - t^*) (x (\pi - t^*) + E (\theta) t^*) = -x^2 \frac{N (\pi - t^*) \pi (Var (\theta) + \gamma^2)}{(E (\theta) - x)^2 + Var (\theta) + \gamma^2}$$

$$\frac{d \left( \Pi^M (t^*) - \Pi^{VI} (b^*) \right)}{d (Var (\theta))} = \frac{N \left( \pi^2 - (t^*)^2 \right)}{2}$$

$$\frac{d \left( \Pi^M (t^*) - \Pi^{VI} (b^*) \right)}{d (\gamma^2)} = \frac{N \left( \pi^2 - (t^*)^2 - b^* (2 \pi - b^*) \right)}{2},$$

where recall $0 < b^* < \pi$ and $t^* < \pi$. These results establish the comparative static results in Proposition 3.
Pessimistic expectations  We start by deriving the expected profits under each mode. In VI mode, the payoff to a professional expecting \( n^e \) other professionals to join is

\[
v^{VI}(n^e, b, w) = w + bm(n^e + 1) + b(\pi - b)E(\theta)^2 + \frac{b^2\gamma^2}{2},
\]
given the same optimal choices of \( a_i \) (by the VI firm) and effort (by each professional) as in the VI case with optimistic expectations. Under pessimistic expectations, \( n^e = 0 \) if and only if \( v^{VI}(0, b, w) < w_0 \), so the minimum salary \( w \) that the VI firm can offer in order to attract all \( N \) professionals is

\[
w(b) = w_0 - bm(1) - b(\pi - b)E(\theta)^2 - \frac{b^2\gamma^2}{2}.
\]

With this salary, the unique equilibrium is that all \( N \) professionals join, so the VI firm’s profit as a function of \( b \) is

\[
\Pi^{VI}_{PE}(b) = -Nw(b) + N(\pi - b)(m(N) + E(\theta)a^{VI}(b) + \gamma e^{VI}(b)) - \frac{N}{2}a^{VI}(b)^2
\]

\[
= \Pi^{VI}(b) - Nb\Delta m.
\]

Note, if \( b = 0 \) then VI’s profit is the same as (16). If bonuses are feasible, then maximizing \( \Pi^{VI}_{PE}(b) \) with respect to \( b \) implies the optimal bonus in (10), which is positive given (9). With this optimal bonus, the resulting profits in VI mode are

\[
\Pi^{VI}_{PE}(\theta, \gamma) = N\pi m + \frac{N\pi^2}{2} \left( E(\theta)^2 + \frac{(\gamma^2 - \Delta m)^2}{E(\theta)^2 + \gamma^2} \right) - Nw_0.
\]

In MSP mode, the expected payoff to an individual professional joining the MSP when she expects \( n^e \) other professionals to join is

\[
v^{M}(n^e, t, T) = (\pi - t)m(n^e + 1) + \frac{(\pi - t)^2}{2}(E(\theta)^2 - x^2 + \gamma^2) - T,
\]
given the same optimal choices of \( a_i \) and effort (by each professional) as in the MSP case with optimistic expectations. Under pessimistic expectations, \( n^e = 0 \) if and only if \( v^{M}(0, t, T) < w_0 \), so the maximum fixed fee that the MSP can charge is

\[
T(t) = (\pi - t)m(1) + \frac{(\pi - t)^2}{2}(E(\theta)^2 - x^2 + \gamma^2) - w_0.
\]

With this fixed fee, the unique equilibrium is that all \( N \) professionals join, so the MSP’s expected profit as a function of \( t \) is

\[
\Pi^{M}_{PE}(t) = NT(t) + Nt(m(N) + E(\theta a^M(t)) + \gamma e^{M}(t))
\]

\[
= \Pi^{M}(t) - N(\pi - t)\Delta m.
\]

Note, if \( t = 0 \) then the MSP’s profit is less than \( \Pi^{M}(0) \) by \( N\pi\Delta m \). If variable fees are feasible, then maximizing \( \Pi^{M}_{PE}(t) \) with respect to \( t \) implies the optimal variable fee given by (11). The assumption (9) and \( E(\theta) > x \) imply that \( t^{PE} < \pi \). With this optimal variable fee, the resulting profits in MSP mode are

\[
\Pi^{M}_{PE}(t^{PE}) = N\pi m - N\pi\Delta m + \frac{N\pi^2}{2} \left( E(\theta)^2 + \gamma^2 - x^2 + \frac{x^2(E(\theta) - x - \Delta m)^2}{(E(\theta) - x)^2 + Var(\theta) + \gamma^2} \right) - Nw_0.
\]

We are now ready to prove Proposition 4.
Proof of Proposition 4: Since both $\Pi^M_{PE}$ and $\Pi^V_{PE}$ are decreasing in $\Delta m$, we want to know whether $\Pi^M_{PE} - \Pi^V_{PE}$ is increasing or decreasing in $\Delta m$. We can use the envelope theorem to obtain

$$d \left( \frac{\Pi^M_{PE} (t^*_{PE}) - \Pi^V_{PE} (b^*_{PE})}{d(\Delta m)} \right) = N \left( b^*_PE - (\pi - t^*_{PE}) \right).$$

Note, if $t = 0$ then the derivative equals $N \left( b^*_PE - \pi \right)$ instead, which is unambiguously negative since $b^*_PE = \frac{\gamma^2 - \Delta \pi}{\gamma^2 + E(\theta)} \pi < \pi$. 

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