DO ONLINE SOCIAL NETWORKS INCREASE WELFARE? *

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ABSTRACT

We consider a setting where agents select among competing products of unknown quality by costly sequential search. The agents are active on an online social network, and late movers benefit from possibly being able to see early movers' purchase decisions. In this setting we consider the impact of advertising by firms, and the incentives of the underlying social network platform. We consider display advertising, which is standard firm to consumer advertising, and social advertising, in which agents who purchased that firm's product are highlighted to their friends. We show that in equilibrium, the heterogeneous firms spend the same amount on advertising. Social advertising is more lucrative than standard banner advertising. However, both forms of advertising have no effect on consumer welfare, and are instead a transfer from the firms to the social network. A social network motivated by advertising revenues may limit the amount of information agents see about actions by other agents, since this will increase advertising revenue. This reduces consumer welfare relative to the first best, since early movers' purchases are informative about relative quality.

KEYWORDS: social networks, advertising, search

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1 Introduction

People are influenced by their friends and acquaintances. Influential literatures suggest that social networks affect important personal outcomes such as health,[1] economic outcomes such as income and employment,[2] and may distort market outcomes.[3] In a majority of this literature, the social network is modeled as an inert conduit rather than a strategic agent. The launching point of this paper, motivated by commercial online social networks such as Facebook and Twitter, is to study outcomes taking into account the motivations of the underlying social network.

Online social network platforms have developed a spectacular user base in recent years and provide a rich layer of social interaction for its users. For example, Facebook is reported to have over 1 billion active users, who spend several hours a month on the site, while Twitter is reported to have over 300 million active users. Due to the vast amount of information being generated by the online connections of a given user, online platforms use algorithms to select and filter what is displayed.[4] Online network platorms are monetized mainly through advertising, which may affect the information that is displayed. Economically, biasing the organic information displayed in favor of certain products or services might have significant welfare effects—for example, herds on inferior products might form. In this context, and due to the tremendous scale of online network platforms, it is paramount to understand the welfare effects of having a financially motivated firm controlling the dispersion of socially generated information. We take a first step towards understanding this in this paper.

We address two main questions: First, what are the welfare effects of allowing for firms to bias social information versus organic, unbiased social information? Second, how does a first best of social information dispersion compare to the profit maximizing approach from the online platform perspective? At a high level, we find that in equilibrium, advertising is a transfer from firms to the the platform, with no effect on overall economic welfare. However, we show that a revenue motivated platform may limit the amount of information users see organically, since this increases the revenue from advertising. This distortion reduces consumer welfare.

We conduct our analysis in a stylized model. Two firms produce goods which are substitutes and compete for consumers. The firms' products have different qualities which are common knowledge among them, but not known to the consumers. Consumers decide which

[1] See e.g. Christakis and Fowler (2007).

[2] See e.g. Montgomery (1991) or Calvo-Armengol and Jackson (2004).

[3] For example due to inefficient herding: see the seminal work of Banerjee (1992) and Bikhchandani, Hirshleifer, and Welch (1992) for the underlying theory, and Salganik, Dodds, and Watts (2006) for a celebrated experiment.

[4] For example, consider Facebook's Newsfeed.

product to buy through costly sequential search among the products. Consumers are of two sorts: early movers and late movers, and both are active on a social network platform. Early movers make their purchases after costly search, and announce their purchase decision on the network platform. Each late mover observes the decision of at most one randomly selected early mover. This observation influences his beliefs about the qualities of the two products, and therefore his search and purchase decision.[5] The probability with which a late mover observes the decision of an early mover is referred to as the baseline virality of the network.

There are two types of advertising that the platform offers to firms. The first, display advertising, is the conventional firm to consumer communication. This is the standard form of advertising across the internet, where the firm displays a banner containing a logo, message or image on a webpage the user is viewing. It is also referred to as banner advertising. We assume this is uninformative, but increases customer awareness in that an otherwise indifferent consumer will first sample the product he sees a display ad for. The second kind is social advertising, which influences the information late movers see about the early movers' actions. This form of advertising is unique to social networks—examples include Facebook's "Sponsored Stories" and Google's "social ads." Here, a firm pays the social network to make posts by consumers with relevant content more visible to the online "friends" of these consumers.

Our approach to display advertising is taken from the literature (see Friedman (1958)) and is essentially a Tullock contest. Each consumer observes exactly one ad. Both firms simultaneously choose how much to spend on advertising. Any consumer observes the ad of a given firm independently with a probability which equal to the proportion of the expenditure of that firm to the total advertising expenditures.

Social advertising by a firm distorts the information seen by late movers about the purchase decisions of the early movers. In the absence of social advertising, recall that late movers observe the purchase decision of a randomly chosen early mover. We assume that social advertising by a firm increases the probability that the late mover observes an early mover who purchased that firm's product.

A MOTIVATING EXAMPLE To motivate our model consider a simple example of duopolists who each make a consumer product (such as cellphones or cars). Every consumer is in the market for a single unit of this product. The duopolists each know the quality of both products. Early movers have the ex-ante beliefs about the qualities of these products. Late movers may observe the choices made by his predecessors—for example, he sees what brands of cars people drive or what kind of phones they carry etc, and update their beliefs based

[5] This is a variant of the social learning setting considered in Mueller-Frank and Pai (2013).

on this.

Given these beliefs a consumer then may choose to acquire more information, e.g. take the cars on a test drive, read reviews of specific phones online, and so on. Sampling an alternative (test driving a car, reading a review of a phone) is costly due to the time and effort involved, and reveals information about the quality of that alternative. After sampling the first alternative, the agent decides whether to sample the second (given his opportunity cost of sampling versus the expected benefit given his beliefs). If he chooses not to sample further he is concluding that the first alternative is "good enough" and purchases that. If he has sampled both he picks the higher quality product. An early mover's choice of product to buy is therefore noisily informative about the relative quality of these products (since he may have sampled both). A late mover who observes an early mover's purchase will update his beliefs, and this will affect his choice of which product to sample etc.

Display advertising in this setting is a standard advertisement. In our setting, in equilibrium, it will only influence consumers who are otherwise indifferent between the two products to sample that product first. This makes it more likely that this consumer purchases that product, since the consumer might have high search costs and therefore may not sample further. Social advertising by a firm biases the information seen by late movers by making it more likely that the late movers see an early mover who purchased that product. Since purchase decisions are informative, this makes it more likely that the consumer chooses that product. We study the influence of each.

Discussion of Results We first consider banner advertising and compare the case of our social network platform (with positive virality) to an "unsocial" platform benchmark where late movers receive no information from early movers. In either setting, we find that in equilibrium, both firms spend the same amount on advertising, and the banner ads do not introduce any informational distortion. This implies that banner ads cannot be used to signal quality and are a mere transfer from firms to the platform. A platform uniquely interested in maximizing display advertising revenue would set its baseline virality to zero. In the countervailing direction, the consumer welfare of the social platform is strictly greater than the "unsocial" benchmark—this is because the purchase decisions of early movers is an informative signal, which helps late movers make better informed search and purchase decisions.

Next we introduce social advertising. We show that social advertising is welfare neutral. That is, welfare is the same as in the case where the baseline virality is set equal to one and each consumer receives organic information. Additionally, we show that advertising revenue is maximized if the baseline virality is set equal to zero and consumers observe only sponsored social information. Nevertheless, no informational distortions occur. Further, a social network platform that offers a combination of banner and social ads to firms generates strictly higher advertising than an unsocial platform. Hence, the introduction of social advertising makes a social platform more profitable than an unsocial platform. In regards to welfare, while social network platforms are strictly better than unsocial ones and social advertising introduces no welfare distortions, they do not achieve the first best solution. We show that social welfare strictly increases with the number of early consumers that a given late consumer observes while the total advertising revenue of the social platform strictly decreases.

1.1 Related Literature

The broader literature on social networks is too large to comprehensively cite here, we refer the interested reader to Jackson (2010) for an overview. We restrict ourselves to more closely connected papers.

There has been a recent interest in understanding how social networks may affect commercial activity. For instance, a strand considers settings where a monopolist seller sells a good to agents on a network, and agents' purchases have (positive or negative) externalities on their neighbors. In this class of settings, these papers study pricing by the seller and the distortions this network introduces—see e.g. Candogan, Bimpikis, and Ozdaglar (2012), Bloch and Quérou (2013) or Feldman, Kempe, Lucier, and Paes Leme (2013) for recent papers in the area, and Cabral, Salant, and Woroch (1999) for a classic reference. Fainmesser and Galeotti (2013) explicitly consider the value of the underlying network to be in selling information to the monopolist so that it can price discriminate. Kircher and Postlewaite (2008) observe that firms may offer higher quality products to "influential" agents in the network so that they may influence their connections. Chatterjee and Dutta (2014) study the adoption of a new product in a network when there are both "innovators" who immediately adopt the product, and rational agents who adopt only when expected gains exceed costs. They characterize the structure of networks in which good new products are adopted.

The increasing amount of commerce conducted on the internet has led to some seminal investigations of the business models of firms on the internet. This literature broadly studies questions raised by the ability to use novel mechanisms on the internet (real time auctions), or gather specific information about individual consumers. Most notably Edelman, Ostrovsky, and Schwarz (2005) and Varian (2007) study the advertising auction used by major search engines and its properties. Bergemann and Bonatti (2011) and Bergemann and Bonatti (2014) study targeting, and the sale of consumer specific information on the internet. We add to this literature by considering the ability of firms on the internet (social network) to control communication between individual consumers.

In terms of papers related to our model, the idea of considering search in a social setting was first considered in Mueller-Frank and Pai (2013), here we consider a variant of the more general model there. That paper concerned itself with characterizing asymptotic learning. Our basic model of advertising is, as we pointed out, a Tullock contest, and was first seen in Friedman (1958).

2 Model

There are two competing firms, 1 and 2, each of which produces a product of quality $q_i \in Q = [0, 1]$, $i = 1, 2$. The product qualities q_i are independently drawn at time $t = 0$ according to a probability measure with cumulative distribution function F_Q and density f_Q. The set of possible pairs of quality realizations is denoted $\mathbf{Q} = [0, 1]^2$. The firms commonly learn the realized product qualities of both firms.[6]

There are two groups of consumers that differ in the timing of their purchase decision. A continuum early movers $j \in E$ decide among the two products in time period $t = 1$, their mass is normalized to 1. A continuum of mass λ of late movers $k \in L$ decide in time period $t = 2$. A consumer's utility of purchasing firm i's product is equal to its quality q_i. The gross profit Π_i of firm i is equal to the measure of consumers purchasing its product.

Finally, there is a social network platform on which late consumers might observe the purchase decisions of early movers and on which firms might communicate with consumers via advertising. The platform knows the realized product qualities but is assumed to have complete control of the communication taking place. That is, the platform controls whether or not early choices are observed by late movers, which choices are observed, by whom and whether and how firms can communicate to consumers.[7] Let $v_B \in [0, 1]$ denote the baseline virality of the social network platform, i.e. with probability v_B each late mover independently observers exactly one early mover that is drawn uniformly from the group of early movers. Later we will consider the case where a late mover sees the actions of a large number of early movers.

[6]This assumption is relatively standard in the literature studying online advertising, and is normally defended on the grounds that the repeated interaction between the firms would publicly reveal any private information. For a standard reference, see Edelman, Ostrovsky and Schwarz (Edelman, Ostrovsky, and Schwarz, 2005).

[7]The assumption of complete control is made mainly for notational convenience, but there are several examples of online systems that approximate this, notably the newsfeed of facebook.

2.1 Consumer Search

The decision of each consumer is based on costly sequential search among these products. The sequential search model is as in Weitzman (1979). Each consumer has a probability distribution on $\mathbf{Q}$. This might be the prior distribution or a Bayesian update based upon some additional information. For example, an early mover who observes no other information will view the products as ex-ante identical draws from F_Q. By contrast, a late mover might observe the purchase decision of some early mover or a firm's advertising, and will update on this information appropriately.

At time t each consumer acting in the given period decides which product to sample first $s_j^1 \in \{1, 2\}$. Sampling a product perfectly reveals its quality to the consumer. Let $q_{s_j^1}$ denote the observed quality of the product sampled first. After observing $q_{s_j^1}$ consumer j decides whether to sample the remaining product, $s_j^2 \in \{1, 2\}$, or to discontinue searching, $s_j^2 = n$. For simplicity, the first product is sampled at no cost while sampling the second involves a cost of $c_j \in C = [0, 1]$ to consumer j. The search costs c_j are independently drawn according to a probability measure with cumulative distribution function F_C and density f_C.

Consumer j then decides to purchase one of the products he sampled. The purchase decision of consumer j is denoted by $a_j \in A = \{1, 2\}$. The net-utility of agent i is therefore the quality of the product he selects less the search cost if he chooses to sample a second time. We consider two forms of advertising on the social network platform that differ conceptually as follows.

2.2 Advertising

Display Advertising Display advertising is a traditional form of advertising as it consists of firms communicating with consumers.[8] In this form each consumer sees exacly one ad for one of the competing products. An ad contains no direct information in regards to the quality of the product but intuitively might serve to raise the awareness for its product.

Let $m_i^b \in \mathbb{R}$ be firm i's expenditure on banner advertising. Both firms simultaneously select their baner advertising expenditures in time $t = 0$ after observing the product qualities. The banner advertising revenue of the platform is the sum of the amount spent by each. Given these chosen advertising levels, each consumer independently sees an ad for product 1 with probability

$$\frac{m_1^b}{m_1^b + m_2^b},$$

and otherwise an ad for product 2. If both banner advertising expenditures are equal to

[8] This is also called display advertising in some of the literature on online advertising.

zero, a given consumer observes no banner advertising. Let Θ denote the set of possible ads, $\Theta = \{1, 2, x\}$, where x describes the case of no ad, and let $\theta_j \in \Theta$ denote the ad seen by consumer j.

Social Advertising Social advertising of firm i influences the probability with which a late consumer observes an early consumer who purchased product i and as such centers around consumer-to-consumer communication rather than the traditional firm-to-consumer communication.

Let ϕ_i be the measure of early consumers that purchased product i. Absent social advertising, the (independent) probability of a late consumer i observing a purchase of product i is then given by $v_B \phi_i$. At time $t = 1$, both firms simultaneously decide on the amount social advertising.[9] Let $m_i^s \in \mathbb{R}$ be firm i's expenditure on social advertising. For social advertising expenditures m_1^s, m_2^s the probability of a late consumer i observing a purchase of product i is then given by

$$v_B \phi_i + (1 - v_B) v_S \frac{\phi_i m_i^s}{\phi_1 m_1^s + \phi_2 m_2^s}.$$

This term can be interpreted as follows. With the probability given by the baseline virality v_B a consumer receives organic social information, and conditional on not receving organic social information the consumer observes sponsored social information with probability v_S. Again, if both firms spend zero on social advertising the probability of observing a purchase of product i is equal to $v_B \phi_i$. To match the reality of social and search engine advertising we assume that the consumer knows whether the social information he observes is organic or sponsored.[10] The formal nature of our social advertising is inspired by the "Sponsored Stories" on Facebook. [11] Finally, we assume that a consumer sees at most one advertising. That is, if he observes a sponsored social ad, he does not observe a banner ad.

2.3 Strategies and Equilibrium

We assume that the structure of the game described above is commonly known among all participants. The strategy of firm i is given by

$$\sigma_i : \mathbf{Q} \to \mathbb{R}_+ \times \mathbb{R}_+.$$

That is, for each possible realization of the product qualities firm i decides how much to spend on banner and social advertising.

[9] Assumption made for simplicity.
[10] This is a critical assumption that affects the set of equilibria.
[11] See e.g. the two minute video introducing this product at: http://goo.gl/6bZyQ.

Next consider an early consumer j. His strategy σ_j is a three-tuple consisting of the first sampling decision, the subsequent decision to sample further or not and the purchase decision. The following approach to search and the notation is taken from the companion paper Mueller-Frank and Pai (2013). Consumer j's initial sampling strategy is given by

$$\sigma_j^1 : \Theta \times C \to \{1, 2\}.$$

The subsequent sampling strategy is formalized as

$$\sigma_j^2 : \Theta \times C \times Q \to \left\{\neg s_j^1, n\right\}$$

where $\neg s_j^1$ denotes the product not sampled initially. His purchase decision is mechanical, we omit the formal notation: if the consumer only samples one product, he purchases that product, if he samples both, he purchases the product with the higher quality.

Next consider a late consumer k. His strategy σ_k is again a three-tuple with the difference that the sampling decisions capture the possibility of consumer k having observed the purchase decision of an early agent. Let $H_k = \{1, 2, x\}$ denote the set of possible histories that consumer k can observe (x denotes the case where consumer k observe no purchase decision of an early agent). Consumer k's initial sampling strategy is given by

$$\sigma_k^1 : H_k \times \Theta \times C \to \{1, 2\}.$$

His subsequent sampling strategy satisfies

$$\sigma_k^2 : H_k \times \Theta \times C \times Q \to \left\{\neg s_j^1, n\right\}$$

where $\neg s_k^1$ denotes the product not sampled initially. The purchase strategy is identical to an early mover's and omitted.

We solve the game for its Perfect Bayesian equilibria. As is standard in the literature σ_{-i} denotes the set of strategies of all players but i.

Definition 1. *A strategy profile σ is a Perfect Bayesian equilibrium of this three stage game if*

1. *σ_i maximizes the expected profit of firm $i = 1, 2$ given the strategies σ_{-i},*
2. *σ_j maximizes the expected utility of consumer $j \in E$ given the strategies σ_{-j}, and*
3. *σ_k maximizes the expected utility of consumer $k \in L$ given the strategies σ_{-k}.*

3 Benchmark Result: Search in the Absence of Advertising

Some basic results about how rational consumers search in the absence of advertising will be useful, and are collected here for use in later analysis. The reduced sequential social search game is identical to the above with the unique difference being that firms are exluded as players. The following exposition is borrowed from the companion paper Mueller-Frank and Pai (2013).

First consider an early consumer j. In the absence of any additional information, the marginal distributions of the qualities of both products are identical. According to the optimal search strategy characterized by Weitzman (1979) either product might be sampled first. Let us assume that he randomizes uniformly over which of the two products to sample first. If he samples product i first, he learns the quality q_i of this product. Next, he must decide whether to sample further or not. He will only sample if it is rational to do so, i.e. if the expected additional gain from searching exceeds his cost of an additional search. Formally, he searches further if:

$$c_j \leq \int_{q_i}^{1} (q - q_i) dF_Q(q).$$

We denote the cutoff cost that just leaves an early consumer indifferent from searching further, given the observed quality of the product he sampled first by $c_e(q_i)$, i.e.:

$$c_e(q_i) = \int_{q_i}^{1} (q - q_i) dF_Q(q).$$

Note that the cost cutoff c_e is strictly smaller than 1 for all realized qualities smaller than 1. Let p_{σ_j} denote the ex-ante probability of early consumer j to buy the better product given his strategy σ_j and let π_{σ_j} the posterior probability that product i is optimal conditional on consumer j having bought i and his strategy being σ_j. The following lemmas will be used throughout the paper.

Lemma 1. *For any equilibrium strategy σ_j of an early consumer j we have $p_{\sigma_j} = p \in \left(\frac{1}{2}, 1\right)$. Further, we have $\pi_{\sigma_j} > \frac{1}{2}$.*

Next consider the case of a late consumer k who observes that a (randomly selected) early mover j has purchased product i, i.e. $a_j = i$. Based upon the observation the late consumer then updates his probability distribution on the space of product qualities. The Bayesian updating has the following implication for the equilibrium strategy of consumer k.

Lemma 2. *Let σ be an equilibrium of the reduced sequential social search game. If a late consumer k observes the action of an early consumer, then he first samples the observed product.*

The updated posterior distribution of the quality of the observed product first order stochastically dominates the updated distribution of the other product's quality, roughly because with a certain probability both products were sampled by the early consumer in which case the observed product is optimal. The lemma then follows from Weitzman's (1979) characterization of the optimal sampling strategy which implies that (first order stochastic) dominant options are sampled first. Finally, the cost cutoff $c_k(q_i)$ of a late consumer k who observed the choice of an early consumer has the following characteristic.

Lemma 3. *Let σ be an equilibrium of the reduced sequential social search game. If a late consumer k observes the purchase of product i of an early consumer, then the cost cutoff $c_k(q_i)$ satisfies $c_k(q_i) < c_e(q_i)$ for all $q_i \neq 0, 1$.*

4 Advertising on Online Social Networks

4.1 Banner Advertising

We begin our analysis focusing on banner advertising. In particular, we are interested in how the baseline virality v_B interacts with the incentives of firms to advertise, the banner advertising revenue of the platform and the overall social welfare. As we alluded to in the introduction, a first intuition suggests that banner advertising might be more valuable in a high virality environment. Getting the early consumer to purchase can cause the good to go 'viral' since late consumers who see this purchase decision sample the observed product first and are less likely to engage in further costly search. Following this intuition, a higher incentive to advertise would lead to a higher advertising revenue and hence a higher baseline virality would induce higher profits for the platform.

In order to understand the welfare effects for a given baseline virality, one needs to understand the bias that banner advertising introduces in the sampling decisions of consumers. As consumers are fully rational, they draw inferences from the observed banner ad on the underlying product qualities according to the strategy of firms. For example, the better firm might outspend the worse firm which would make an observed banner ad a signal for the respective product to be superior and hence influence welfare by having a larger proportion of (early) consumers sampling the superior product first.

Theorem 1. *In every equilibrium both firms advertise the same amount, $m_1^*(\mathbf{q}) = m_2^*(\mathbf{q})$. The advertising revenue is weakly decreasing with the baseline virality in any equilibrium. If* *the equilibrium advertising revenue is positive then the revenue is strictly decreasing with the baseline virality.*

Hence banner ads cannot be used by firms to signal quality, in any equilibrium. Effectively, the induces a transfer from firms to the online platform, depending on how consumers respond to ads. For example, consumers might sample first the observed product in which case the advertising revenue of the platform is maximized. In the contrary, consumers might sample the first product independent of the banner ad they observe which leads to an advertising revenue of zero. According to the theorem an increased social structure makes banner advertising less profitable. The network would want to shut down the baseline virality of product relevant information as "likes" or purchases. Another way to interpret it is that social network platforms are simply less profitable from a banner advertising perspective than unsocial platforms which matches the real world fact that Google and Facebook advertising revenues per unique visitor differed by a factor of 10 before FB introduced some type of social advertising. To understand the relation between the baseline virality and the advertising revenue of the platform, the following lemma, which is used in the proof of the theorem, is very helpful.

LEMMA 4. *In every equilibrium a late consumer samples first the action of an observed early consumer independent of the banner ad he observes.*

According to the lemma, in any equilibrium, social information overrules any information that might be possibly obtained from observing a banner ad. To provide intuition for our result, consider a simple social environment with two consumers, i.e. consumer j moves first and consumer k sees 1's purchase decision prior to his own search. Further suppose that the early consumer samples the product first for which he has seen an ad. Renaming firms if necessary, let us assume that firm 1 has the superior product, i.e. $q_1 > q_2$. A consumer who does not observe the purchase decision of another consumer buys product 2 if and only if his search costs were high *and* he sampled 2 first, i.e. saw an ad for product 2. A late consumer k who observes the early consumer j buys product 2 with the following probability

$$\mathbb{P}[a_j = 2 \,|q_1 > q_2] \,(1 - F_C(c_l(q_2)))\,.$$

Instead, if consumer k does not observe j he purchases product 2 with the identical probability as consumer j, i.e.

$$\mathbb{P}[a_k = 2 \,|q_1 > q_2, v_B = 0] = \mathbb{P}[a_j = 2 \,|q_1 > q_2] > \mathbb{P}[a_j = 2 \,|q_1 > q_2] \,(1 - F_C(c_l(q_2)))\,.$$

An increased baseline virality means that a larger fraction of the late movers will see an (informative) social signal about the product purchased by an early mover. These consumers will be uninfluenced by the banner ad directly. Of course they may be influenced indirectly, i.e. the early mover may have been influenced by the banner ad, who in turn influences this consumer. But this influence is imperfect—the late mover searches again if his search cost is low enough, which occurs with probability $F_C(c_l(q_2))$. Next let us consider the welfare implications of banner advertising on social network platforms.

THEOREM 2. *In every equilibrium expected social welfare is strictly increasing in the baseline virality.*

If the network platform is restricted to banner advertising then its incentives are diametrically opposed to social incentives. The welfare externality results from limiting, or in fact ommiting socially generated information. The intuition for the result can be seen from the discussion of the purchase decision of a late consumer provided above. When observing the decision of an early consumer, the optimal product is bought with strictly higher probability. To conclude, for a fixed base line virality banner advertising is welfare neutral. However, the profit maximization objectives induce the platform to set the baseline virality equal to zero which results in the same expected welfare as in a society with no social platform and no banner advertising.

4.2 Combined Banner & Social Advertising

In this section we consider the equilibria of the game when both banner and social advertising is possible for firms and the resulting advertising revenue for the platform. Again we are interested in the relation between profit maximizing incentives of the platform and the expected social welfare as a function of the baseline and sponsored virality.

THEOREM 3. *For any pair of viralities (v_B, v_S) consider the equilibrium that maximizes the total advertising revenue. The maximal advertising revenue over $(v_B, v_S) \in [0,1]^2$ is achieved for $(v_B, v_S) = (0,1)$ if*

$$1 + F_C(c_e(q_2)) > F_C(c_l(q_2 \,|a_e = 2)) + F_C(c_l(q_2 \,|a_e = 2))F_C(c_e(q_2)).$$

Otherwise the maximal advertising revenue is achieved if both the baseline and sponsored virality is set equal to zero, $(v_B, v_S) = (0,0)$.

5 Increasing the density of the social network

In our analysis we have so far assumed that each late mover observes the action of at most one early consumer. However, in observed real world social networks, users have, on average, a large number of friends. Our assumption that information on purchase decisions be restricted to only one early consumer is in contradiction with this stylized fact. In this section we address this question from two different angles. First, we consider the total advertising revenue of the platform when increasing the number of observations within our model of banner and social advertising. Second, we analyze the equilibrium welfare effects of increasing the social observation set of late consumers.

Each late consumer independently observes organic social information with probability v_B. Now the organic social information consists of the purchase decisions of k early consumers who are drawn independently from the set of early consumers. Let ϕ^* denote the measure of early consumers who select the superior product for a given strategy profile σ and realized qualities q_1, q_2. The social advertising set of a given late consumer is then drawn from a binomial distribution with distribution parameter ϕ^*. That is, with probability ϕ^* an early consumer is drawn who selected the superior product. Note that

$$\phi^* = \frac{m_1^b + m_2^b F_C(c_e(q_2 \,|\theta = 2))}{m_1^b + m_2^b}.$$

To simplify the exposition and subsequent analysis suppose that the number of draws k is odd. A given late consumers observes a majority for the superior product with probability p_k

$$p_k = \Pr\left[B(\phi^*, k) \geq \frac{k}{2}\right].$$

Without loss of generality let 1 be the superior product. For the sponsored social advertising observation set, suppose that the probability with which product 1 is drawn from the population of early consumers can be influenced by social advertising expenditures as follows

$$\phi^*_{\mathbf{m}^s} = \frac{\phi^* m_1^s}{\phi^* m_1^s + (1-\phi^*) m_2^s}.$$

Under social advertising, a given late consumer observes a majority for the superior product with probability $p_k^{\mathbf{m}^s}$

$$p_k^{\mathbf{m}^s} = \Pr\left[B(\phi^*_{\mathbf{m}^s}, k) \geq \frac{k}{2}\right]$$

To summarize the advertising exposure of a given late consumer, note that he observes a banner ad with probability $(1-v_B)(1-v_S)$, organic social information, i.e. k independent unbiased draws of the purchase decisions of early consumers, with probability v_B, and sponsored social information with probability $(1-v_B)v_S$. In order to analyze the advertising revenue as a function of increasing the observation set of agents, we focus attention on the equilibrium that maximizes the total advertising revenue for a fixed level of baseline virality

v_B and sponsored virality v_S.

THEOREM 4. *For any pair of viralities (v_B, v_S) consider the equilibrium that maximizes the total advertising revenue. As the size of the social observation set k grows large, the social advertising expenditures converge to zero.*

The basic intuition of this result is simple. Imagine a late mover who sees k randomly chosen early movers. If k is large, the product purchased by the majority of the early movers is the higher quality product with high probability. The late movers therefore "free-ride" on this information, sampling the higher quality product first (with high probability) and rarely searching further. On the margin, therefore, advertising has no impact on the late movers' choices, and therefore firms do not spend on it.

This theorem therefore summzarizes the central tension between a revenue-motivated social network and social welfare. A dense network is welfare improving—the free-riding late movers do not need to spend effort on search, and make better choices. However, the fact that they have so much information of early movers' choices leaves them uninfluenceable by advertising. As a result, the social network may wish to limit the amount of information about early movers that late movers "organically" see.

6 Discussion and Conclusions

In this paper, we took a first step toward understanding the distortions that may arise when a social network is modeled as having its own commercial interests, rather than an inert conduit. We considered a simple model where agents may conduct costly sequential search to choose between competing products of unknown quality. Information on the social network is thus economically valuable: the choices made by predecessors is informative about the qualities of the products, potentially saving an agent search costs and preventing them from purchasing inferior products.

We considered two forms of advertising the social network may allow. The first, banner advertising, is potentially valued by a firm because it may help a product go "viral," i.e. late moving agents may purchase the product purely based on observing that their friends have, rather than search on their own. However, we show that this intuition is not quite correct—advertising spend on banner advertising actually decreases relative to a benchmark in which there is no social network.

The second form, social advertising, is motivated by advertising products recently offered by major online social networks (such as Facebook and Google), and allows a firm to highlight activity taken by a user (e.g. buying a product by that firm) to the users friends. We show that this is a more lucrative type of advertising, since the fact that the user took the action is informative to other users.

Neither form of advertising directly impacts consumer welfare in our model. Advertising is solely a transfer from firms to the social network, with no resulting distortion. However, a social network focused on advertising revenues may want to limit the amount of information its users see about each others' activities. Users who see the choices of a lot of predecessors will perfectly discern which of the products is better, and therefore there will achieve first best welfare. However, these consumers also cannot be influenced by advertising, and therefore advertising revenues drop to zero. As a result, a social network may do better by limiting such information, so as to better monetize from advertising.

Similar concerns have also been present in search engines,[12] which has led to vigilant antitrust oversight. Such worries are more muted in the social networking space. This may partly be because large online social networks which have advertising as their core business model have emerged only recently. It may also be that the incentives we suggest are more subtle and less focal than those of search engines. However, recent worries voiced by several businesses who advertize on Facebook (the current largest social network) suggest that at the very least,[13] the results of this paper warrant further empirical investigation.

[12]Indeed the original paper describing the organization of Google, Brin and Page (1998), states " clear that a search engine which was taking money for showing cellular phone ads would have difficulty justifying the page that our system returned to its paying advertisers. For this type of reason ... we expect that advertising funded search engines will be inherently biased towards the advertisers and away from the needs of the consumers" (Appendix A).

[13]See e.g. the following article by entrepreneur Mark Cuban, http://blogmaverick.com/2012/11/19/what-i-really-think-about-facebook/ or this article by food delivery service Eat 24 http://blog.eat24hours.com/breakup-letter-to-facebook-from-eat24/.

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A Appendix

A.1 Proof of Lemma 1

The probability p_{σ_j} is defined as follows

$$\Pr\left(\cup_{i=1,2}\left\{a_j=i,q_i>q_{\neg i}\right\}\right)=\Pr\left(q_1>q_2\right)\Pr_{\boldsymbol{\sigma}}\left(a_j=1\,|q_1>q_2\right)+\Pr\left(q_1\leq q_2\right)\Pr_{\boldsymbol{\sigma}}\left(a_j=2\,|q_1\leq q_2\right).$$

We establish the three claims of the lemma in turn.

1. As both product qualities are identically distributed we have

$$\Pr\left(\cup_{i=1,2}\left\{a_j=i,q_i>q_{\neg i}\right\}\right)=\frac{1}{2}\left(\Pr_{\boldsymbol{\sigma}}\left(a_j=1\,|q_1>q_2\right)+\Pr_{\boldsymbol{\sigma}}\left(a_j=2\,|q_1\leq q_2\right)\right).$$

The first sampling strategy determines the probality of selecting product i conditional on $q_i>q_{\neg i}$. Abusing notation let σ_s^1 denote the probability with which agent j samples product 1 first. We have

$$\begin{aligned}\Pr_{\boldsymbol{\sigma}}\left(a_j=1\,|q_1>q_2\right)&=\sigma_s^1+(1-\sigma_s^1)\Pr\left(c_j<c_e(q_2)\,|q_1>q_2\right)\\ \Pr_{\boldsymbol{\sigma}}\left(a_j=2\,|q_1\leq q_2\right)&=(1-\sigma_s^1)+\sigma_s^1\Pr\left(c_j<c_e(q_1)\,|q_1\leq q_2\right).\end{aligned}$$

Since q_1 and q_2 are identically distributed we have

$$\Pr\left(c_j<c_e(q_2)\,|q_1>q_2\right)=\Pr\left(c_j<c_e(q_1)\,|q_1\leq q_2\right)$$

which implies

$$\Pr_{\boldsymbol{\sigma}}\left(\cup_{i=1,2}\left\{a_j=i,q_i>q_{\neg i}\right\}\right)=\frac{1}{2}+\frac{1}{2}\Pr\left(c_j<c_e(q_2)\,|q_1>q_2\right)=p$$

which establishes $p_{\sigma_j}=p$ for every equilibrium strategy σ_j.

2. By the reasoning above we have

$$p=\frac{1}{2}+\frac{1}{2}\Pr\left(c_j<c_e(q_2)\,|q_1>q_2\right).$$

Note that $q_1>q_2$ implies $q_2<1$. Since the support of c_j is equal to $[0,1]$ and further $c_e(q_2)>0$ for all $q_2<1$ we have

$$\Pr\left(c_j<c_e(q_2)\,|q_1>q_2\right)>0$$

concluding the proof of the claim $p>\frac{1}{2}$.

3. The posterior probability of product 1 being superior conditional on $a_j = 1$ is given by

$$\pi_\sigma = \Pr_{\boldsymbol{\sigma}}(q_1 > q_2 \,|a_j = 1) = \frac{\Pr(a_j = 1 \,|q_1 > q_2)}{\Pr(a_j = 1 \,|q_1 > q_2) + \Pr(a_j = 1 \,|q_1 \leq q_2)}.$$

Using the above and simplifying yields

$$\Pr_{\boldsymbol{\sigma}}(q_1 > q_2 \,|a_j = 1) = \frac{\sigma_s^1 + (1 - \sigma_s^1)\Pr(c_j < c_e(q_2) \,|q_1 > q_2)}{2\sigma_s^1 + (1 - 2\sigma_s^1)\Pr(c_j < c_e(q_2) \,|q_1 > q_2)}.$$

Minimizing the conditional probability for the initial sampling probability σ_s^1 yields an argmin of $\sigma_s^1 = 1$ and a minimum value of the posterior probability of

$$\frac{1}{2 - \Pr(c_j < c_e(q_2) \,|q_1 > q_2)}.$$

Finally, since $\Pr(c_j < c_e(q_2) \,|q_1 > q_2) > 0$ we have

$$\pi_\sigma = \Pr_{\boldsymbol{\sigma}}(q_1 > q_2 \,|a_j = 1) > \frac{1}{2}$$

for any equilibrium strategy $\boldsymbol{\sigma}$. □

A.2 Proof of Lemma 2

Denote by $F_{i,\sigma}(.\,|a_j = i)$ the conditional cumulative distribution function of the quality of product i given that agent j purchased i, and by $F_{\neg i,\sigma}(.\,|a_j = i)$ the conditional cumulative distribution function of the other product. The proof follows in four steps.

1. We need to establish that

$$F_{i,\sigma}(q \,|a_j = i) < F_{\neg i,\sigma}(q \,|a_j = i)$$

for all $q \neq 0, 1$. We have

$$\begin{aligned} F_{1,\sigma}(q \,|a_j = 1) = &\Pr_{\boldsymbol{\sigma}}(q_1 > q_2 \,|a_j = 1)\Pr_{\boldsymbol{\sigma}}([0,q] \times Q_2 \,|a_j = 1, q_1 > q_2) \\ &+ \Pr_{\boldsymbol{\sigma}}(q_1 < q_2 \,|a_j = 1)\Pr_{\boldsymbol{\sigma}}([0,q] \times Q_2 \,|a_j = 1, q_1 < q_2), \end{aligned}$$

and

$$\begin{aligned} F_{2,\sigma}(q \,|a_j = 1) = &\Pr_{\boldsymbol{\sigma}}(q_1 > q_2 \,|a_j = 1)\Pr_{\boldsymbol{\sigma}}(Q_1 \times [0,q] \,|a_j = 1, q_1 > q_2) \\ &+ \Pr_{\boldsymbol{\sigma}}(q_1 < q_2 \,|a_j = 1)\Pr_{\boldsymbol{\sigma}}(Q_1 \times [0,q] \,|a_j = 1, q_1 < q_2) \end{aligned}$$

Denote by $\pi_\sigma == \Pr_{\boldsymbol{\sigma}}(q_1 > q_2 \,|a_j = 1)$. We need to establish the following inequality for every $q \neq 0, 1$

$$\begin{aligned}\mathbf{I}: \quad & \pi_\sigma \Pr_{\boldsymbol{\sigma}}([0,q] \times Q_2 \,|a_j = 1, q_1 > q_2) + (1-\pi_\sigma) \Pr_{\boldsymbol{\sigma}}([0,q] \times Q_2 \,|a_j = 1, q_1 < q_2) \\ & < \pi_\sigma \Pr_{\boldsymbol{\sigma}}(Q_1 \times [0,q] \,|a_j = 1, q_1 > q_2) + (1-\pi_\sigma) \Pr_{\boldsymbol{\sigma}}(Q_1 \times [0,q] \,|a_j = 1, q_1 < q_2).\end{aligned}$$

2. Independence of q_1, q_2 implies

$$\Pr(c_j < c_e(q_2) \,|q_1 > q_2) = \Pr(c_j < c_e(q_2) \,|q_1 > q_2, q_1 < q).$$

Further, since $c_e(q)$ is decreasing in q we have

$$\Pr(c_j > c_e(q_1) \,|q_1 < q_2) > \Pr(c_j > c_e(q_1) \,|q_1 < q_2, q_1 < q)$$

and

$$\Pr(c_j < c_e(q_2) \,|q_1 > q_2) < \Pr(c_j < c_e(q_2) \,|q_1 > q_2, q_2 < q).$$

3. Applying the law of total probability and step 2) imply

$$\begin{aligned}\Pr_{\boldsymbol{\sigma}}([0,q] \times Q_2 \,|a_j = 1, q_1 > q_2) &= \Pr_{\boldsymbol{\sigma}}([0,q] \times Q_2 \,|q_1 > q_2) \\ \Pr_{\boldsymbol{\sigma}}(Q_1 \times [0,q] \,|a_j = 1, q_1 < q_2) &= \Pr_{\boldsymbol{\sigma}}(Q_1 \times [0,q] \,|q_1 < q_2) \\ \Pr_{\boldsymbol{\sigma}}([0,q] \times Q_2 \,|a_j = 1, q_1 < q_2) &< \Pr_{\boldsymbol{\sigma}}([0,q] \times Q_2 \,|q_1 < q_2) \\ \Pr_{\boldsymbol{\sigma}}(Q_1 \times [0,q] \,|a_j = 1, q_1 > q_2) &> \Pr_{\boldsymbol{\sigma}}(Q_1 \times [0,q] \,|q_1 > q_2).\end{aligned}$$

4. Step 3. implies that for inequality **I** to hold it is sufficient that the following inequality holds

$$\begin{aligned}& \pi_\sigma \Pr_{\boldsymbol{\sigma}}([0,q] \times Q_2 \,|q_1 > q_2) + (1-\pi_\sigma) \Pr_{\boldsymbol{\sigma}}([0,q] \times Q_2 \,|q_1 < q_2) \\ < & \pi_\sigma \Pr_{\boldsymbol{\sigma}}(Q_1 \times [0,q] \,|q_1 > q_2) + (1-\pi_\sigma) \Pr_{\boldsymbol{\sigma}}(Q_1 \times [0,q] \,|q_1 < q_2)\end{aligned}$$

which follows directly from $\pi_\sigma > \frac{1}{2}$ (by Lemma 1) and from first order stochastic dominance of $F_{i,\sigma}(q \,|q_i > q_j)$ over $F_{\neg i,\sigma}(q \,|q_i > q_j)$. □

A.3 Proof of Lemma 3

The lemma is established in two steps. In the first step we show that first order stochastic dominance of the quality distribution of the early consumer over the conditional distribution of the late consumer is sufficient. The second step establishes such stochastic dominance the prior distribution $F_j(q)$ first order stochastically dominates the posterior distribution of a late agent conditional on the early consumer having purchased i and the realized quality q_i.

1. Consider two cumulative distribution functions F, F' on $[0,1]$ and suppose that F is first order stochastic dominant. We establish the following claim

$$\int_{q_i}^{1} (q - q_i)\mathrm{d}F(q) > \int_{q_i}^{1} (q - q_i)\mathrm{d}F'(q).$$

   Equivalent transformations yield

$$\begin{aligned} \int_{q_i}^{1} q\mathrm{d}F(q) - \int_{q_i}^{1} q\mathrm{d}F'(q) &> q_i(F'(q_i) - F(q_i)) \\ (1 - F(q_i))E_F\left[q\,|q > q_i\right] - (1 - F'(q_i))E_{F'}\left[q\,|q > q_i\right] &> q_i(F'(q_i) - F(q_i)). \end{aligned}$$

   Since $E_{F'}\left[q\,|q > q_i\right] > q_i$ to prove the claim it is sufficient to establish the validity of the following inequality

$$(1 - F(q_i))E_F\left[q\,|q > q_i\right] - (1 - F'(q_i))E_{F'}\left[q\,|q > q_i\right] > E_{F'}\left[q\,|q > q_i\right](F'(q_i) - F(q_i))$$

   which is equivalent to

$$E_F\left[q\,|q > q_i\right] > E_{F'}\left[q\,|q > q_i\right]$$

   By first order stochastic dominance of F over F' we have that

$$E_F\left[q\,|q > q_i\right] > E_{F'}\left[q\,|q > q_i\right]$$

   concluding the proof of the claim.

2. We need to establish the following inequality for all $\hat{q} \neq 0, 1$ and $q_i \neq 0$

$$\Pr\left(q_{\neg i} < \hat{q}\right) < \Pr\left(q_{\neg i} < \hat{q}\,|a = i, q_i = q\right)$$

   which is equivalent to

$$\Pr\left(q_{\neg i} < \hat{q}\right) < \frac{\Pr\left(q_{\neg i} < \hat{q}\,|q_i = q\right)\Pr\left(a = i\,|q_{\neg i} < \hat{q}, q_i = q\right)}{\Pr\left(a = i\,|q_i = q\right)}.$$

Independence of q_1, q_2 yields the following equivalent transformation

$$\Pr\left(a = i \,|q_i = q\right) < \Pr\left(a = i \,|q_{\neg i} < \hat{q}, q_i = q\right).$$

We have to consider two cases. First where $\hat{q} < q$. Here the respective probabilities are

$$\begin{aligned} &\sigma\left(\Pr\left(c > c(q)\right) + \Pr\left(c < c(q)\right)\Pr\left(q_{\neg i} < q\right)\right) + (1-\sigma)\Pr\left(q_{\neg i} < q\right)\Pr\left(c < c(q_{\neg i})\,|q_{\neg i} < q\right) \\ <&\sigma + (1-\sigma)\Pr\left(c < c(q_{\neg i})\,|q_{\neg i} < \hat{q}\right) \end{aligned}$$

which is satisfied since

$$\Pr\left(c > c(q)\right) + \Pr\left(c < c(q)\right)\Pr\left(q_{\neg i} < q\right) < 1$$

and for $\hat{q} < q$

$$\Pr\left(c < c(q_{\neg i})\,|q_{\neg i} < q\right) < \Pr\left(c < c(q_{\neg i})\,|q_{\neg i} < \hat{q}\right).$$

Finally, for $\hat{q} > q$ we have

$$\begin{aligned} &\sigma\left(\Pr\left(c > c(q)\right) + \Pr\left(c < c(q)\right)\Pr\left(q_{\neg i} < q\right)\right) + (1-\sigma)\Pr\left(q_{\neg i} < q\right)\Pr\left(c < c(q_{\neg i})\,|q_{\neg i} < q\right) \\ < \;&\sigma\left(\Pr\left(c > c(q)\right) + \Pr\left(c < c(q)\right)\Pr\left(q_{\neg i} < q\,|q_{\neg i} < \hat{q}\right)\right) \\ &+(1-\sigma)\Pr\left(q_{\neg i} < q\,|q_{\neg i} < \hat{q}\right)\Pr\left(c < c(q_{\neg i})\,|q_{\neg i} < q\right) \end{aligned}$$

which follows from

$$\Pr\left(q_{\neg i} < q\right) < \Pr\left(q_{\neg i} < q\,|q_{\neg i} < \hat{q}\right).$$

□

A.4 Proof of Lemma 4

The banner ad introduces a conditional iid signal. Suppose that with full measure both firms advertise the same. Then the banner ad contains no information and as established in Lemma 2, the late consumer samples first the product chosen by the early consumer. Suppose now that for a positive measure subset of $\mathbf{Q}$ firms advertise different amounts. Integrating over $\{\mathbf{q} \in \mathbf{Q} : \mathbf{q}_1 > \mathbf{q}_2\}$ and $\{\mathbf{q} \in \mathbf{Q} : \mathbf{q}_1 < \mathbf{q}_2\}$ for a given strategy of the firms leads to the following conditional probabilities of banner ad realizations $\Pr\left(\theta\,|q_1 > q_2\right)$, $\Pr\left(\theta\,|q_1 < q_2\right)$ for $\theta \in \Theta$. Note that the banner ads are conditionally independent random variables. Let p_θ be the posterior probability of the event $\{\mathbf{q} \in \mathbf{Q} : \mathbf{q}_1 > \mathbf{q}_2\}$ conditional on θ. We have $p_\theta < \frac{1}{2} < p_{\theta'}$ for some $\theta, \theta' \in \{1, 2\}$, $\theta \neq \theta'$. In order to establish the claim of the lemma, it is sufficient to establish first order stochastic dominance of the posterior marginal distribution over Q_1 of a late consumer over his posterior marginal distribution over Q_2 given that the observed early consumer selected $a_j = 1$, for any banner ad that the late consumer might observe. We denote the banner ad that induces a $p_\theta > \frac{1}{2}$ by θ^1 and the banner ad that induces a $p_\theta < \frac{1}{2}$ by θ^2.[14] We need to establish that the following inequality holds for any $q \neq 0, 1$

$$\Pr{}_{\boldsymbol{\sigma}}\left(q_1 < q \left| a_j = 1, \theta^2\right.\right) < \Pr{}_{\boldsymbol{\sigma}}\left(q_2 < q \left| a_j = 1, \theta^2\right.\right)$$

which is equivalent to

$$\begin{aligned}
& \Pr{}_{\boldsymbol{\sigma}}\left(q_1 > q_2 \left| a_e = 1, \theta^2\right.\right) \Pr{}_{\boldsymbol{\sigma}}\left(q_1 < q \left| a_e = 1, q_1 > q_2\right.\right) \\
& + \left(1 - \Pr{}_{\boldsymbol{\sigma}}\left(q_1 > q_2 \left| a_e = 1, \theta^2\right.\right)\right) \Pr{}_{\boldsymbol{\sigma}}\left(q_1 < q \left| a_e = 1, q_1 < q_2\right.\right) \\
< \; & \Pr{}_{\boldsymbol{\sigma}}\left(q_1 > q_2 \left| a_e = 1, \theta^2\right.\right) \Pr{}_{\boldsymbol{\sigma}}\left(q_2 < q \left| a_e = 1, q_1 > q_2\right.\right) \\
& + \left(1 - \Pr{}_{\boldsymbol{\sigma}}\left(q_1 > q_2 \left| a_e = 1, \theta^2\right.\right)\right) \Pr{}_{\boldsymbol{\sigma}}\left(q_2 < q \left| a_e = 1, q_1 < q_2\right.\right)
\end{aligned}$$

Note that

$$\begin{aligned}
\Pr{}_{\boldsymbol{\sigma}}\left(q_1 < q \left| a_e = 1, q_1 > q_2\right.\right) &\leq \Pr{}_{\boldsymbol{\sigma}}\left(q_2 < q \left| a_e = 1, q_1 < q_2\right.\right) \\
\Pr{}_{\boldsymbol{\sigma}}\left(q_1 < q \left| a_e = 1, q_1 < q_2\right.\right) &\leq \Pr{}_{\boldsymbol{\sigma}}\left(q_2 < q \left| a_e = 1, q_1 > q_2\right.\right)
\end{aligned}$$

To prove the claim it is therefore sufficient to establish the following inequality

$$\Pr{}_{\boldsymbol{\sigma}}\left(q_1 > q_2 \left| a_e = 1, \theta^2\right.\right) > \frac{1}{2}$$

which is equivalent to

$$\begin{aligned}
2\Pr{}_{\boldsymbol{\sigma}}\left(q_1 > q_2, a_e = 1, \theta^2\right) &> \Pr{}_{\boldsymbol{\sigma}}\left(a_e = 1, \theta^2\right) \\
\Pr{}_{\boldsymbol{\sigma}}\left(a_e = 1, \theta^2 \left| q_1 > q_2\right.\right) &> \frac{1}{2}\left(\Pr{}_{\boldsymbol{\sigma}}\left(a_e = 1, \theta^2 \left| q_1 > q_2\right.\right) + \Pr{}_{\boldsymbol{\sigma}}\left(a_e = 1, \theta^2 \left| q_1 < q_2\right.\right)\right) \\
\Pr{}_{\boldsymbol{\sigma}}\left(a_e = 1, \theta^2 \left| q_1 > q_2\right.\right) &> \frac{1}{2}\Pr{}_{\boldsymbol{\sigma}}\left(a_e = 1, \theta^2 \left| q_1 > q_2\right.\right)
\end{aligned}$$

concluding the proof. □

A.5 Proof of Theorem 1

There are three cases to consider.

1. First, suppose that the advertising strategies m_1^*, m_2^* are such that seeing an ad indi-

[14] Note that $\theta_1 = 2$ cannot be excluded at this stage.

cates that the corresponding product is of lower quality, i.e. $\Pr(q_i > q_j | \theta = j) > \frac{1}{2}$. This directly implies that in expectation the lower qualitity firm spends more on advertising than the higher quality firm. In case of $\Pr(q_i > q_j | \theta = j)$ an early consumer who observes $\theta = j$ samples product i first in any equilibrium. Consider any product quality realization $\mathbf{q}$ such that the lower quality firm advertises more. WLOG suppose that in $\mathbf{q}$ we have $q_1 > q_2$. By Lemma 4, a late consumer samples the product first which he saw sampled by an early consumer which gives the following expected profits for firm 2

$$\frac{m_1^*}{m_1^* + m_2^*} (1 - F_C(c_e(q_2 | \theta = 1))) (1 + \lambda(1 - v_B) + \lambda v_B (1 - F_C(c_l(q_2 | a_2 = 2, \theta_l)))) - m_2^*.$$

Since decreasing the advertising expenditures increases the expected revenue and reduces the cost for any $\mathbf{q}$ where the lower cost firm advertises more, advertising strategies m_1^*, m_2^* where $\Pr(q_i > q_j | \theta = j) > \frac{1}{2}$ cannot be an equilibrium.

2. Next consider the case where the advertising strategies m_1^*, m_2^* are such that seeing an ad indicates that the corresponding product is of higher quality, i.e. $\Pr(q_i > q_j | \theta = i) > \frac{1}{2}$ which implies that an early consumer observing an ad for firm i samples firm i's 'product first in any equlibrium. Consider any product quality realization $\mathbf{q}$ such that the higher quality firm advertises more. WLOG suppose that in $\mathbf{q}$ we have $q_1 > q_2$. By Lemma 4, a late consumer samples the product first which he saw sampled by an early consumer which gives the following expected profits for firm 1

$$\begin{aligned} &\Pi_1(\mathbf{m}^* | \mathbf{q}) \\ =&1 + \lambda - \frac{m_2^*}{m_1^* + m_2^*} (1 - F_C(c_e(q_2 | \theta = 2))) (1 + \lambda(1 - v_B) + \lambda v_B (1 - F_C(c_l(q_2 | a_2 = 2, \theta_l)))) \\ &- m_1^*, \end{aligned}$$

and for firm 2

$$\begin{aligned} &\Pi_2(\mathbf{m}^* | \mathbf{q}) \\ =&\frac{m_2^*}{m_1^* + m_2^*} (1 - F_C(c_e(q_2 | \theta = 2))) (1 + \lambda(1 - v_B) + \lambda v_B (1 - F_C(c_l(q_2 | a_2 = 2, \theta_l)))) \\ &- m_2^*. \end{aligned}$$

As $\mathbf{m}^*$ is the equilibrium advertising level, it has to be a mutually best response.

Solving for the respective first order conditions yields

$$\frac{\partial \Pi_1(\mathbf{m}^* \,|\mathbf{q}\,)}{\partial m_1^*}$$
$$= \frac{m_2^*}{(m_1^*+m_2^*)^2}\left(1-F_C(c_e(q_2\,|\theta=2\,))\right)\left(1+\lambda(1-v_B)+\lambda v_B\left(1-F_C(c_l(q_2\,|a_2=2,\theta_l\,))\right)\right)-1 \overset{!}{=} 0$$
$$\frac{\partial \Pi_2(\mathbf{m}^* \,|\mathbf{q}\,)}{\partial m_2^*}$$
$$= \frac{m_1^*}{(m_1^*+m_2^*)^2}\left(1-F_C(c_e(q_2\,|\theta=2\,))\right)\left(1+\lambda(1-v_B)+\lambda v_B\left(1-F_C(c_l(q_2\,|a_2=2,\theta_l\,))\right)\right)-1 \overset{!}{=} 0$$

which implies $m_1^* = m_2^*$ contradicting that at $\mathbf{q}$ we have $m_1^* > m_2^*$. Hence it cannot be an equilibrium to have advertising strategies m_1^*, m_2^* where the ad indicates that the corresponding product is of higher quality, i.e. $\Pr(q_i > q_j\,|\theta = i\,) > \frac{1}{2}$.

3. Finally, consider the case where in expectation advertising provides no information regarding the underlying qualities, i.e. $\Pr(q_i > q_j\,|\theta = i\,) = \frac{1}{2}$. Here both products have identical distribution conditional on either advertising. Suppose that each consumer samples according to the ad he observes, i.e. he first samples product θ. Invoking Lemma 2 (or Lemma 4), and considering $\mathbf{q}$ where $q_1 > q_2$ the expected profits of firms are

$$\begin{aligned}
\Pi_1(\mathbf{m}^*\,|\mathbf{q}\,) &= 1+\lambda-\frac{m_2^*}{m_1^*+m_2^*}\left(1-F_C(c_e(q_2))\right)\left(1+\lambda(1-v_B)+\lambda v_B\left(1-F_C(c_l(q_2\,|a_2=2\,))\right)\right) \\
&\quad -m_1^* \\
\Pi_2(\mathbf{m}^*\,|\mathbf{q}\,) &= \frac{m_2^*}{m_1^*+m_2^*}\left(1-F_C(c_e(q_2))\right)\left(1+\lambda(1-v_B)+\lambda v_B\left(1-F_C(c_l(q_2\,|a_2=2\,))\right)\right)-m_2^*.
\end{aligned}$$

As $\mathbf{m}^*$ is the equilibrium advertising level, it has to be a mutually best response. Solving for the respective first order conditions yields

$$m_1^*(\mathbf{q}) = m_2^*(\mathbf{q}) = m^*(\mathbf{q}) = \frac{\left(1-F_C(c_e(q_2))\right)\left(1+\lambda(1-v_B)+\lambda v_B\left(1-F_C(c_l(q_2\,|a_2=2\,))\right)\right)}{4}. \tag{1}$$

Note that for any other sampling strategy of consumers the expected advertising expenditures are a convex combination of $m^*(\mathbf{q})$ and 0.[15] An advertising expenditure of zero, for example, is achieved in equilibrium if all consumers sample first the product whose ad they have not observed, or if they sample independent of the ad observed. In order to conclude the proof of the theorem, we need to show that in fact the advertising

[15] Here we might have to show this formally. In particular, that for any sampling strategy of consumers both firms would spend the same.

expenditures are (weakly) decreasing in the virality v_B. This follows immediately from the functional form of $m^*(\mathbf{q})$, in particular since

$$F_C(c_l(q_2 \,|a_2 = 2)) > 0$$

for any $q_2 < 1$, and the advertising expenditure being a convex combination of $m^*(\mathbf{q})$ and 0. □

A.6 Proof of Theorem 2

As shown in Theorem 1, the advertising expenditures of both firms are identical in any equilibrium. The early stage welfare is unaffected by the baseline virality. Hence we restrict attention to the late stage expected welfare. Without loss of generality, let 2 be the inferior product. For a given baseline virality v_B the expected total (late-stage) welfare is equal to

$$\lambda \left(v_B \left(p + (1-p)F_C(c_l(q_2 \,|a_2 = 2))\right) + (1 - v_B)p\right)$$

which is increasing in v_B as $p < 1$ and $F_C(c_l(q_2 \,|a_2 = 2)) > 0$ for all $q_2 < 1$. □

A.7 Proof of Theorem 3

Proof: Consider the set of equilibria of the game. If there exists an equilibrium with the property that observing a banner respectively social ad for a product induces the consumer to sample that product first, then this equilibrium induces higher advertising expenditures then those equilibria lacking this property. Denote by $\phi(\sigma)$ the measure of early consumers who selected the inferior product for a given strategy profile σ and realized qualities q_1, q_2. WLOG suppose that product 1 is superior to product 2, $q_1 > q_2$. The expected profits of firms under positive impact advertising strategies are given by

$$\begin{aligned}
&\Pi_1(\mathbf{m}^b, \mathbf{m}^s \,|\mathbf{q}\,) \\
=&1 + \lambda - \frac{m_2^b}{m_1^b + m_2^b}\left(1 - F_C(c_e(q_2 \,|\theta^b = 2))\right)\left(1 + \lambda(1 - v_B)(1 - v_S) + \lambda v_B\left(1 - F_C(c_l(q_2 \,|a_2 = 2, \theta_l^b))\right)\right) \\
&- (1 - v_B)v_S\lambda\frac{\phi_2(\sigma)m_2^s}{(1 - \phi_2(\sigma))m_1^s + \phi_2(\sigma)m_2^s}\left(1 - F_C(c_l(q_2 \,|\theta_l^s = 2))\right) - m_1^b - m_1^s \\
&\Pi_2(\mathbf{m}^b, \mathbf{m}^s \,|\mathbf{q}\,) \\
=&\frac{m_2^b}{m_1^b + m_2^b}\left(1 - F_C(c_e(q_2 \,|\theta^b = 2))\right)\left(1 + \lambda(1 - v_B)(1 - v_S) + \lambda v_B\left(1 - F_C(c_l(q_2 \,|a_2 = 2, \theta_l^b))\right)\right) \\
&+ (1 - v_B)v_S\lambda\frac{\phi_2(\sigma)m_2^s}{(1 - \phi_2(\sigma))m_1^s + \phi_2(\sigma)m_2^s}\left(1 - F_C(c_l(q_2 \,|\theta_l^s = 2))\right) - m_2^b - m_2^s
\end{aligned}$$

where

$$\phi_2(\sigma) = \frac{m_2^b}{m_1^b + m_2^b} \left(1 - F_C(c_e(q_2 \,|\theta = 2))\right).$$

We solve for the equlibrium via the first order conditions. The equilibrium social advertising levels are determined by the following first order conditions

$$\begin{aligned}
\frac{\partial \Pi_1(\mathbf{m}^b, \mathbf{m}^s \,|\mathbf{q}\,)}{\delta m_1^s} &= (1 - v_B) v_S \lambda \frac{\phi_2(\sigma)(1 - \phi_2(\sigma)) m_2^s}{\left((1 - \phi_2(\sigma)) m_1^s + \phi_2(\sigma) m_2^s\right)^2} \left(1 - F_C(c_l(q_2 \,|\theta_l^s = 2))\right) - 1 \overset{!}{=} 0 \\
\frac{\partial \Pi_2(\mathbf{m}^b, \mathbf{m}^s \,|\mathbf{q}\,)}{\delta m_2^s} &= (1 - v_B) v_S \lambda \frac{\phi_2(\sigma)(1 - \phi_2(\sigma)) m_1^s}{\left((1 - \phi_2(\sigma)) m_1^s + \phi_2(\sigma) m_2^s\right)^2} \left(1 - F_C(c_l(q_2 \,|\theta_l^s = 2))\right) - 1 \overset{!}{=} 0.
\end{aligned}$$

The equilibrium levels of social advertising are given by

$$m_1^s = m_2^s = \lambda \phi_2(\sigma)(1 - \phi_2(\sigma))(1 - v_B) v_S \left(1 - F_C(c_l(q_2 \,|\theta_l^s = 2))\right).$$

The first order conditions for the equilibrium banner advertising levels, using the fact that in equilibrium $m_1^s = m_2^s$, induce equilibrium advertising expenditures given by

$$\begin{aligned}
m_1^b =& m_2^b = \frac{1}{4} \left(1 - F_C(c_e(q_2))\right) \left(1 + \lambda(1 - v_B)(1 - v_S) + \lambda(v_B + (1 - v_B) v_S) \left(1 - F_C(c_l(q_2 \,|a_e = 2))\right)\right) \\
m_1^s =& m_2^s = \frac{1}{4} \lambda (1 - v_B) v_S \left(1 - F_C(c_e(q_2))\right) \left(1 + F_C(c_e(q_2))\right) \left(1 - F_C(c_l(q_2 \,|a_e = 2))\right).
\end{aligned}$$

Note that since the equilibrium banner and social advertising expenditures are the same, and $\phi_2(\sigma) < \frac{1}{2}$, in fact for the underlying strategies it is consistent for consumers to sample first according to the banner or social ad they observe. The total advertising expenditure of each firm is equal to

$$\begin{aligned}
& m_i^b + m_i^s \\
=& \frac{1}{4} \times \left(1 - F_C(c_e(q_2))\right) \times \\
& \left(1 + \lambda(1 - v_B)\left(1 - v_S + v_S \left(2 + F_C(c_e(q_2))\right) \left(1 - F_C(c_l(q_2 \,|a_e = 2))\right)\right) + \lambda v_B \left(1 - F_C(c_l(q_2 \,|a_e = 2))\right)\right)
\end{aligned}$$

which is increasing in v_S if and only if

$$\begin{aligned}
\left(2 + F_C(c_e(q_2))\right) \left(1 - F_C(c_l(q_2 \,|a_e = 2))\right) &> 1 \\
1 + F_C(c_e(q_2)) &> F_C(c_l(q_2 \,|a_e = 2)) + F_C(c_l(q_2 \,|a_e = 2)) F_C(c_e(q_2)).
\end{aligned}$$

Finally, the advertising revenue is decreasing in v_B if and only if

$$\begin{aligned} 1 - v_S + v_S\,(2 + F_C(c_e(q_2)))\,(1 - F_C(c_l(q_2\,|a_e = 2))) &> 1 - F_C(c_l(q_2\,|a_e = 2)) \\ F_C(c_l(q_2\,|a_e = 2)) &> v_S(1 - (2 + F_C(c_e(q_2)))\,(1 - F_C(c_l(q_2\,|a_e = 2))) \end{aligned}$$

Since the left hand side is positive, for the inequality to be satisfied the following is sufficient

$$F_C(c_l(q_2\,|a_e = 2)) > 1 - (2 + F_C(c_e(q_2)))\,(1 - F_C(c_l(q_2\,|a_e = 2)))$$

which is equivalent to

$$1 + F_C(c_e(q_2)) > F_C(c_l(q_2\,|a_e = 2)) + F_C(c_l(q_2\,|a_e = 2))F_C(c_e(q_2))$$

which follows from $F_C(c_l(q_2\,|a_e = 2)) < 1$ for all $q_2 < 1$. □

A.8 Theorem 4

Proof: Consider the set of equilibria of the game. If there exists an equilibrium with the property that observing a banner ad respectively a majority within the sponsored or organic observation set for a product induces the consumer to sample that product first, then this equilibrium induces higher advertising expenditures then other equilibria lacking this property. Consider a strategy profile σ with positive impact advertising and denote by ϕ^* the measure of early consumers who selected the superior product for a given strategy profile σ and realized qualities q_1, q_2. WLOG suppose that product 1 is superior to product 2, $q_1 > q_2$. Denote the expected proportion of late consumers that select product 2 given σ and conditional on observing a majority for product 2 within the organic respectively sponsored observation by

$$E_B\left[P_2\left|\mathbf{q}, x < \frac{k}{2}, \sigma\right.\right],$$

respectively

$$E_S\left[P_2\left|\mathbf{q}, x < \frac{k}{2}, \sigma\right.\right]$$

where x denotes the number of observed agents that selected product 1. Note that for large k, the number of superior product purchases within the social observation set is approximately normally distributed. Let

$$\phi_1^* = \frac{m_1^b + m_2^b F_C(c_e(q_2\,|\theta = 2))}{m_1^b + m_2^b}$$

be the measure of early consumers that select product 1. For a realization $\mathbf{q}$ such that $q_1 > q_2$ the expected profits of firms under positive impact advertising strategies are given by

$$
\begin{aligned}
&\Pi_1(\mathbf{m}^b, \mathbf{m}^s \left| \mathbf{q} \right.) \\
=&1+\lambda - \frac{m_2^b}{m_1^b + m_2^b}\left(1 - F_C(c_e(q_2 \left| \theta^b = 2 \right.))\right)(1+\lambda(1-v_B)(1-v_S)) \\
&+ \lambda v_B \Phi\left(\frac{\sqrt{k}\left(\frac{1}{2} - \frac{m_1^b + m_2^b F_C(c_e(q_2|\theta=2))}{m_1^b+m_2^b}\right)}{\sqrt{\frac{m_1^b + m_2^b F_C(c_e(q_2|\theta=2))}{m_1^b+m_2^b}(1-\frac{m_1^b + m_2^b F_C(c_e(q_2|\theta=2))}{m_1^b+m_2^b})}}\right) E_B\left[P_2 \left| \mathbf{q}, k < \frac{n}{2}, \sigma \right.\right] \\
&+ \lambda(1-v_B)v_S \Phi\left(\frac{\sqrt{k}\left(\frac{1}{2} - \frac{\phi_1^* m_1^s}{\phi_1^* m_1^s + (1-\phi_1^*)m_2^s}\right)}{\sqrt{\frac{\phi_1^* m_1^s}{\phi_1^* m_1^s + (1-\phi_1^*)m_2^s}\left(1 - \frac{\phi_1^* m_1^s}{\phi_1^* m_1^s + (1-\phi_1^*)m_2^s}\right)}}\right) E_S\left[P_2 \left| \mathbf{q}, k < \frac{n}{2}, \sigma \right.\right] - m_1^b - m_1^s \\
&\Pi_2(\mathbf{m}^b, \mathbf{m}^s \left| \mathbf{q} \right.) \\
=&\frac{m_2^b}{m_1^b + m_2^b}\left(1 - F_C(c_e(q_2 \left| \theta^b = 2 \right.))\right)(1+\lambda(1-v_B)(1-v_S)) \\
&+ \lambda v_B \Phi\left(\frac{\sqrt{k}\left(\frac{1}{2} - \frac{m_1^b + m_2^b F_C(c_e(q_2|\theta=2))}{m_1^b+m_2^b}\right)}{\sqrt{\frac{m_1^b + m_2^b F_C(c_e(q_2|\theta=2))}{m_1^b+m_2^b}(1-\frac{m_1^b + m_2^b F_C(c_e(q_2|\theta=2))}{m_1^b+m_2^b})}}\right) E_B\left[P_2 \left| \mathbf{q}, k < \frac{n}{2}, \sigma \right.\right] \\
&+ \lambda(1-v_B)v_S \Phi\left(\frac{\sqrt{k}\left(\frac{1}{2} - \frac{\phi_1^* m_1^s}{\phi_1^* m_1^s + (1-\phi_1^*)m_2^s}\right)}{\sqrt{\frac{\phi_1^* m_1^s}{\phi_1^* m_1^s + (1-\phi_1^*)m_2^s}\left(1 - \frac{\phi_1^* m_1^s}{\phi_1^* m_1^s + (1-\phi_1^*)m_2^s}\right)}}\right) E_B\left[P_2 \left| \mathbf{q}, k < \frac{n}{2}, \sigma \right.\right] - m_2^b - m_2^s.
\end{aligned}
$$

We establish the claim of the theorem in several steps.

1. We solve for the equlibrium via the first order conditions. Let

$$
g_{\phi_1^*}(\mathbf{m}^s) = \frac{\phi_1^* m_1^s}{\phi_1^* m_1^s + (1-\phi_1^*)m_2^s}\left(1 - \frac{\phi_1^* m_1^s}{\phi_1^* m_1^s + (1-\phi_1^*)m_2^s}\right)
$$

and

$$
f_{\phi_1^*}(\mathbf{m}^s) = \frac{\sqrt{k}\left(\frac{1}{2} - \frac{\phi_1^* m_1^s}{\phi_1^* m_1^s + (1-\phi_1^*)m_2^s}\right)}{\sqrt{\frac{\phi_1^* m_1^s}{\phi_1^* m_1^s + (1-\phi_1^*)m_2^s}\left(1 - \frac{\phi_1^* m_1^s}{\phi_1^* m_1^s + (1-\phi_1^*)m_2^s}\right)}}.
$$

The equilibrium social advertising levels are determined by the following first order conditions

$$\begin{aligned}
\frac{\partial \Pi_1(\mathbf{m}^b, \mathbf{m}^s \,|\mathbf{q}\,)}{\partial m_1^s} &= -\lambda(1-\upsilon_B)\upsilon_S \Phi'\left(f_{\phi_1^*}(\mathbf{m}^s)\right) \frac{\partial f_{\phi_1^*}(\mathbf{m}^s)}{\partial m_1^s} E_S\left[P_2 \,\middle|\, \mathbf{q}, k < \frac{n}{2}, \sigma\right] - 1 \overset{!}{=} 0 \\
\frac{\partial \Pi_2(\mathbf{m}^b, \mathbf{m}^s \,|\mathbf{q}\,)}{\partial m_2^s} &= \lambda(1-\upsilon_B)\upsilon_S \Phi'\left(f_{\phi_1^*}(\mathbf{m}^s)\right) \frac{\partial f_{\phi_1^*}(\mathbf{m}^s)}{\partial m_2^s} E_S\left[P_2 \,\middle|\, \mathbf{q}, k < \frac{n}{2}, \sigma\right] - 1 \overset{!}{=} 0.
\end{aligned}$$

which implies

$$-\frac{\partial f_{\phi_1^*}(\mathbf{m}^s)}{\partial m_1^s} = \frac{\partial f_{\phi_1^*}(\mathbf{m}^s)}{\partial m_2^s}.$$

We have

$$\begin{aligned}
\frac{\partial f_{\phi_1^*}(\mathbf{m}^s)}{\partial m_1^s} =& \frac{\sqrt{k}\left(\left(-\frac{m_2^s(1-\phi_1^*)\phi_1^*}{\left(m_1^s\phi_1^*+m_2^s(1-\phi_1^*)\right)^2}\right)\sqrt{g_{\phi_1^*}(\mathbf{m}^s)} - \left(\frac{1}{2} - \frac{m_1^s\phi_1^*}{m_1^s\phi_1^*+m_2^s(1-\phi_1^*)}\right)\frac{1}{2}\left(g_{\phi_1^*}(\mathbf{m}^s)\right)^{-\frac{1}{2}}\frac{\partial g_{\phi_1^*}(\mathbf{m}^s)}{\partial m_1^s}\right)}{g_{\phi_1^*}(\mathbf{m}^s)} \\
\frac{\partial f_{\phi_1^*}(\mathbf{m}^s)}{\partial m_2^s} =& \frac{\sqrt{k}\left(\left(\frac{m_1^s(1-\phi_1^*)\phi_1^*}{\left(m_1^s\phi_1^*+m_2^s(1-\phi_1^*)\right)^2}\right)\sqrt{g_{\phi_1^*}(\mathbf{m}^s)} - \left(\frac{1}{2} - \frac{m_1^s\phi_1^*}{m_1^s\phi_1^*+m_2^s(1-\phi_1^*)}\right)\frac{1}{2}\left(g_{\phi_1^*}(\mathbf{m}^s)\right)^{-\frac{1}{2}}\frac{\partial g_{\phi_1^*}(\mathbf{m}^s)}{\partial m_2^s}\right)}{g_{\phi_1^*}(\mathbf{m}^s)}.
\end{aligned}$$

Since

$$\begin{aligned}
\frac{\partial g_{\phi_1^*}(\mathbf{m}^s)}{\partial m_1^s} &= -\frac{m_2^s(1-\phi_1^*)\phi_1^*}{\left(m_1^s\phi_1^*+m_2^s(1-\phi_1^*)\right)^2}\left(1-2\frac{m_1^s\phi_1^*}{m_1^s\phi_1^*+m_2^s(1-\phi_1^*)}\right) \\
\frac{\partial g_{\phi_1^*}(\mathbf{m}^s)}{\partial m_2^s} &= \frac{m_1^s(1-\phi_1^*)\phi_1^*}{\left(m_1^s\phi_1^*+m_2^s(1-\phi_1^*)\right)^2}\left(1-2\frac{m_1^s\phi_1^*}{m_1^s\phi_1^*+m_2^s(1-\phi_1^*)}\right)
\end{aligned}$$

substituting in, we have$m_1^s = m_2^s$. Since social advertising leads to unbiased draws for the social observation set we have

$$E_B\left[P_2 \,\middle|\, \mathbf{q}, x < \frac{k}{2}, \sigma\right] = E_S\left[P_2 \,\middle|\, \mathbf{q}, x < \frac{k}{2}, \sigma\right]$$

2. Let

$$\begin{aligned}
z(\mathbf{m}^b) &= \frac{\sqrt{k}\left(\frac{1}{2} - \phi_1^*(\mathbf{m}^b)\right)}{\sqrt{\phi_1^*(\mathbf{m}^b)(1-\phi_1^*(\mathbf{m}^b))}} \\
y(\mathbf{m}^b) &= \frac{(\phi_1^*)^2 + \frac{1}{2}}{2\sqrt{\phi_1^*\left(1-\phi_1^*\right)}}
\end{aligned}$$

and recall that

$$\phi_1^*(\mathbf{m}^b) = \frac{m_1^b + m_2^b F_C(c_e(q_2\,|\theta=2))}{m_1^b + m_2^b}.$$

The first order conditions for the equilibrium banner advertising levels, using the fact that in equilibrium $m_1^s = m_2^s$, are given by

$$\begin{aligned}
&\frac{\partial \Pi_1(\mathbf{m}^b \,|\mathbf{q}\,)}{\partial m_1^b} \\
=&-\Bigg(-\frac{m_2^b}{\left(m_1^b+m_2^b\right)^2}\left(1-F_C(c_e(q_2\left|\theta^b=2\right.))\right)(1+\lambda(1-v_B)(1-v_S)) \\
&+\lambda\left(v_B+(1-v_B)v_S\right)\Phi'\left(z(\mathbf{m}^b)\right)\frac{\partial z(\mathbf{m}^b)}{\partial m_1^b}E_B\left[P_2\left|\mathbf{q},k<\frac{n}{2},\sigma\right.\right]\Bigg)-1\overset{!}{=}0 \\
&\frac{\partial \Pi_2(\mathbf{m}^b \,|\mathbf{q}\,)}{\partial m_2^b} \\
=&\frac{m_1^b}{\left(m_1^b+m_2^b\right)^2}\left(1-F_C(c_e(q_2\left|\theta^b=2\right.))\right)(1+\lambda(1-v_B)(1-v_S)) \\
&+\lambda\left(v_B+(1-v_B)v_S\right)\Phi'\left(z(\mathbf{m}^b)\right)\frac{\partial z(\mathbf{m}^b)}{\partial m_2^b}E_B\left[P_2\left|\mathbf{q},k<\frac{n}{2},\sigma\right.\right]-1\overset{!}{=}0.
\end{aligned}$$

Substituing in expressions for $\frac{\partial \phi_1^*}{\partial m_1^b}$, $\frac{\partial \phi_1^*}{\partial m_2^b}$, $\frac{\partial z(\mathbf{m}^b)}{\partial m_1^b}$ and $\frac{\partial z(\mathbf{m}^b)}{\partial m_2^b}$, we have that $m_1^b = m_2^b$ is an equilibrium.Finally, note that since both the banner advertising as well as the social advertising expenditures are equal, a consumer when observing a banner ad are indifferent between sampling either product first, and as a proportion greater than half of early consumer select the superior product it is uniquely optimal to sample the observed product first. For $m_1^b = m_2^b = m^b$ we have

$$\begin{aligned}
\phi_1^* =& \frac{1}{2}+\frac{1}{2}F_C(c_e(q_2\,|q_1>q_2\,))>\frac{1}{2} \\
z(\mathbf{m}^b) =& \frac{\sqrt{k}\left(-\frac{1}{2}F_C(c_e(q_2\,|q_1>q_2\,))\right)}{\sqrt{\left(\frac{1}{2}+\frac{1}{2}F_C(c_e(q_2\,|q_1>q_2\,))\left(\frac{1}{2}-\frac{1}{2}F_C(c_e(q_2\,|q_1>q_2\,)\right)\right.}}<0 \\
\frac{\partial z(\mathbf{m}^b)}{\partial m_1^b} =& -\frac{1}{4m^b}\left(1-F_C(c_e(q_2\,|q_1>q_2\,))\right)\frac{\sqrt{k}}{\phi_1^*(1-\phi_1^*)} \\
&\left(\sqrt{\phi_1^*(1-\phi_1^*)}-(\phi_1^*(1-\phi_1^*))^2\left(\frac{1}{2}-\phi_1^*\right)(2\phi_1^*-1)\right)
\end{aligned}$$

The first order condition for m_1^b together with $m_1^b = m_2^b = m^b$ gives

$$\frac{1}{4m^b}\Bigg(\left(1 - F_C(c_e(q_2 \left|\theta^b = 2\right.))\right)(1 + \lambda(1-v_B)(1-v_S))$$

$$+ \lambda\left(v_B + (1-v_B)v_S\right)\Phi'\left(z(\mathbf{m}^b)\right)\frac{\sqrt{k}}{\phi_1^*(1-\phi_1^*)}$$

$$\left(\sqrt{\phi_1^*(1-\phi_1^*)} - (\phi_1^*(1-\phi_1^*))^2\left(\frac{1}{2}-\phi_1^*\right)(2\phi_1^*-1)\right)E_B\left[P_2\left|\mathbf{q}, k < \frac{n}{2}, \sigma\right.\right]\Bigg) = 1$$

In order to determine the limit value of m_b for k going to infinity it is sufficient to determine the limit of

$$\Phi'\left(-\sqrt{k}\right)\sqrt{k}$$

As Φ' is the density function of the standard normal distribution we have

$$\Phi'\left(-\sqrt{k}\right)\sqrt{k} = \frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}k}\sqrt{k}$$

which goes to zero by L'Hopitals rule substituting $y = \sqrt{k}$. Hence m_b converges to

$$\frac{1}{4}\left(1 - F_C(c_e(q_2 \left|\theta^b = 2\right.))\right)(1 + \lambda(1-v_B)(1-v_S))$$

which is smaller than the advertising expenditures of either firm for $k = 1$ which equal

$$\frac{1}{4}\left(1 - F_C(c_e(q_2))\right)(1 + \lambda(1-v_B)(1-v_S) + \lambda(v_B + (1-v_B)v_S)\left(1 - F_C(c_l(q_2 \left|a_e = 2\right.))\right)).$$

Finally, solving for the equilibrium level m^s via the corresponding first order condition gives

$$m^s = \lambda(1-v_B)v_S\Phi'\left(\frac{\sqrt{k}\left(-\frac{1}{2}F_C(c_e(q_2 \left|q_1 > q_2\right.))\right)}{\sqrt{\left(\frac{1}{2}+\frac{1}{2}F_C(c_e(q_2 \left|q_1 > q_2\right.))\right)\left(\frac{1}{2}-\frac{1}{2}F_C(c_e(q_2 \left|q_1 > q_2\right.))\right)}}\right)\sqrt{k}$$

$$\left(\frac{\left(\frac{1}{2}+\frac{1}{2}F_C(c_e(q_2 \left|q_1 > q_2\right.))\right)^2 + \frac{1}{2}}{2\sqrt{\left(\frac{1}{2}+\frac{1}{2}F_C(c_e(q_2 \left|q_1 > q_2\right.))\right)\left(\frac{1}{2}-\frac{1}{2}F_C(c_e(q_2 \left|q_1 > q_2\right.))\right)}}\right)E_S\left[P_2\left|\mathbf{q}, k < \frac{n}{2}, \sigma\right.\right]$$

which converges to zero for k going to infinity as $\Phi'\left(-\sqrt{k}\right)\sqrt{k}$ goes to zero. □