Incentives to Innovate with Complementary Inventions

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Abstract

The paper introduces a model in which complementary monopoly inventors follow quantity-setting strategies and and engage in bilateral negotiation with licensees. The main result of the analysis is that non-cooperative equilibrium with quantity-setting inventors generates prices that correspond to the cooperative outcome. This differs from Cournot's classic result that non-cooperative competition with price-setting generates prices that exceed the cooperative outcome. I further show that with endogenous R&D, quantity competition among complementary inventors generates more inventions, higher-quality technology, and lower product prices than a multi-project monopoly inventor. The discussion considers policy implications of complementary inventions.

Keywords: Invention, R&D, complements, innovation, entry, incentives, antitrust, intellectual property, licensing, royalties, technology standards

JEL Codes: D40, O31, L10

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I Introduction

Complex production processes and innovative products such as smart phones apply multiple inventions. Commentators have raised public policy concerns that license royalties will be too high and coordination will be difficult when there are complementary inventions. Many studies of the "complements problem," also known as "royalty stacking," "patent thickets," and the "tragedy of the anti-commons," suggest that policy makers should regulate royalties or weaken intellectual property (IP) rights.[1]

These studies are based on Cournot's (1838) complementary monopolies model.[2] Cournot's model implies that *price-setting* inventors choose total royalties greater than the monopoly price. High royalties could discourage or prevent innovation. Yet, the theoretical and empirical economics and law literatures offer no evidence for the standard assumption of price-setting. Evidence shows that in practice licensing almost always involves *quantity-setting strategies* and *bilateral negotiation*.

The main contribution of the paper is to introduce quantity-setting competition among complementary monopolists. I show that total royalties do not exceed the monopoly level. I find that competition among complementary inventors increases invention and innovation. This suggests that public policy makers should consider the full range of competitive strategies.

I develop a general three-stage model of invention, licensing, and innovation with complementary monopolies. At the initial invention stage, inventors enter the market, engage in stochastic research and development (R&D), and make discoveries. Next, at the licensing stage, quantity-setting inventors offer technology licenses to selected producers. Finally, at the innovation stage, royalties are set through bilateral negotiations between inventors and producers. Also, producers make technology adoption decisions and use the complementary inventions to manufacture final outputs

[1] For an overview of policy issues and empirical evidence see Geradin et al. (2008).

[2] Patent studies applying the Cournot model of price competition with complementary inventions include Economides and Salop (1992), Buchanan and Yoon (2000), Shapiro (2001), Gilbert (2004), Lerner and Tirole (2004), Lemley and Shapiro (2007), Ménière and Parlane (2010), Gilbert and Katz (2011), Llanes and Trento (2012), Llanes and Poblete (2014), Schmidt (2014), and Llobet and Padilla (2014).

that are supplied to consumers. The main results of the analysis are as follows.

First, I find that there exists a unique weakly dominant strategy equilibrium with quantity-setting inventors. The quantity-setting equilibrium corresponds to the outcome when a monopoly inventor offers the bundle of complementary inventions. This means that quantity-setting mitigates the complementary inventions problem because it reduces equilibrium royalties in comparison to Cournot price-setting. With quantity setting, inventors recognize the effects of their quantities on final outputs so that complementarity generates tacit coordination. Quantity-setting generates greater profits, consumers' surplus, and social welfare than Cournot's price-setting game. This implies that incentives to invent and to innovate are greater with quantity-setting inventors than with price-setting inventors.

Second, I consider stochastic R&D and entry of inventors engaged in complementary projects. I assume that increases in the number of inventions improve the likelihood that the set of complementary inventions will achieve a given quality and that increasing the number of inventions is subject to diminishing marginal returns. I show that competition among complementary inventors increases incentives to invent in comparison to a monopoly inventor with complementary inventions. Even though the non-cooperative equilibrium with complementary inventions generates the monopoly outcome, a difference emerges in incentives for R&D. This difference appears because entry of inventors is determined by the average return to invention and the number of inventions chosen by a monopolist is determined by the marginal return to invention. Because the average incentive to invent is greater than the marginal incentive to invent, entry of inventors generates more inventions than does a monopoly inventor with multiple R&D projects.

Third, I show that quantity-setting with endogenous R&D results in increased quality of inventions. The improved quality of inventions leads to greater total inventor profits, lower downstream prices, and greater downstream output than with a multi-project monopoly inventor. This implies that competition among complementary inventors increases incentives to innovate in comparison to the multi-project monopoly inventor. I give conditions under which complementary inventions can increase social welfare in comparison to a multi-project monopoly inventor.

Fourth, I compare the outcome with downstream competition among producers with the outcome when the downstream market is monopolistic. I find that downstream competition among producers increases incentives to invent when inventors choose the quantity of licenses, thus extending Arrow's (1962) result to complementary inventions. This result contrasts with the outcome when endogenous R&D is introduced into the Cournot pricing model. With price-setting inventors, downstream competition increases incentives to invent for non-drastic inventions or not-too drastic inventions, but can reduce incentives to invent with sufficiently drastic inventions due to free riding.

A valuable study by Singh and Vives (1984) compares quantity and price strategies in a model with differentiated products that can be complements or substitutes.[3] Their analysis does not consider perfect complements as in Cournot's model and the present model. Singh and Vives (1984, p. 547) state that "Cournot (Bertrand) competition with substitutes is the dual of Bertrand (Cournot) competition with complements. Exchanging prices and quantities, we go from one to the other." The present non-cooperative game with quantity-setting strategies and bilateral negotiation differs from Cournot's quantity-competition model in which products are substitutes.[4] The present analysis is not the dual of Bertrand's (1883) price-setting model in which products are substitutes. In Bertrand's model, prices fall to players' marginal costs whereas in the present quantity-setting model, all players choose quantities equal the monopoly outcome. The four different models are listed in Table 1.

[3] On comparisons between Cournot and Bertrand competition, see also Vives (1985), and Vives (1999, ch. 6). On games with complementary strategies, see generally Topkis (1998) and Vives (1999, 2005). Complementary strategies are considered in some public goods models (Hirshleifer, 1983).

[4] It is well known that Cournot's complementary monopolies model is simply a relabeling of the Cournot duopoly model; see for example Sonnenschein (1968). In the Cournot complementary monopolies model, input suppliers choose prices and in the Cournot duopoly model firms choose quantities.

	Quantity-setting strategies	**Price-setting strategies**
Perfect substitutes	Cournot competition	Bertrand competition
Perfect complements	The present model (Spulber)	Cournot complementary monopolies

TABLE 1. Competition models with quantity and price strategies when products (outputs, inputs or inventions) are perfect substitutes or perfect complements.

A number of studies examine complementary inventions in different frameworks. Ménière and Parlane (2010) examine pricing using two-part tariffs with complementary inventions. Layne-Farrar and Schmidt (2010) suggest that a combination of vertical integration and cross licensing is needed to address the "complements problem." Gilbert and Katz (2011) consider the efficient division of profits from complementary inventions in R&D races when complementary invention occurs sequentially. Kominers and Weyl (2012) consider seller competition within clusters associated with complementary inputs and consider the possibility of "hold-outs." Llanes and Poblete (2014) examine complementary inventions with technology standard setting and formation of patent pools.

There is a large literature on innovation when inventions are substitutes. Vives (2008) examines competitive pressures in a general monopolistic competition model in which vertically-integrated firms conduct R&D to lower their costs. Arora and Fosfuri (2003) study a two-stage model in which inventors with substitute inventions choose the number of licenses to offer to downstream producers. In their model, some inventors can also be vertically-integrated firms. Spulber (2013a, b) considers competition among inventors with substitute inventions.

Industrial organization studies have considered two-stage models with quantity-setting strategies followed by price-setting when products are substitutes in demand. Firms initially choose capacity and then compete in prices. Kreps and Scheinkman (1983) show that the two-stage outcome resembles Cournot quantity competition. Davidson and Deneckere (1986) show that the two-stage outcome can be more competitive than Cournot quantity competition. The current analysis introduces

quantity-setting following by negotiation with complements.

The analysis is related to incentives in partnerships with sharing rules. Legros and Matthews (1993) show there is an efficient Nash equilibrium in a partnership with a Leontief production function, although the Nash equilibrium need not be unique. The present analysis shows that there is a unique weakly dominant strategy equilibrium that is efficient with complementary inventions and downstream firms.

The paper is organized as follows. Section II presents evidence that licensing royalties are set through quantity setting and bilateral negotiation. Section III presents the basic three-stage model of invention, licensing, and innovation. Section IV characterizes the equilibrium of the licensing stage with inventors who offer licenses for complementary inventions and compares the outcome with quantity-setting inventors with Cournot's price-setting model. Section V characterizes the invention stage with endogenous R&D and examines how competitive pressures affect incentives to invent. Section VI considers how competitive pressures in the downstream market affect incentives to invent. Section VII presents applications and extensions of the model and Section VIII considers public policy implications. Section IX concludes the discussion.

II Evidence for Quantity-Setting Strategies and Negotiation of Licensing Royalties

This section presents evidence that IP owners generally use quantity-setting strategies and do not use price-setting strategies. Practically all licensing royalties are set through bilateral negotiation between IP owners and producers. This implies that licensing royalties are not set through posted prices, in contrast to standard assumptions. Indeed, the extensive economics and law literatures on patent licensing offer no evidence that IP owners post prices. Furthermore, the possibility of an alternative assumption – quantity setting – had not even been considered. The extensive evidence for quantity-setting and bilateral negotiation suggests that this possibility should at least be considered.

Quantity-setting necessarily precedes bilateral negotiation. The identification of potential licensees involves a determination of which firms are currently using the technology and thus infringing on patents. Also, patent owners may identify firms not currently using the technology that might apply the technology to commercialize the invention. Industry evidence suggests that identifying current or potential technology users are the first steps in patent licensing.[5]

Patent owners follow quantity-setting strategies because the number of licenses offered affects downstream market competition among licensees. Offering too many licenses would reduce the willingness to pay of potential licensees. Patent owners thus will limit the quantity of licenses offered to potential infringers or firms that can commercialize their technology on the basis of equilibrium willingness to pay for the technology.

Patent owners also follow quantity-setting stategies because there are transaction costs in the market for inventions. This implies that patent owners will limit the quantity of licenses to reduce their transaction costs. There are transaction costs associated with negotiating and monitoring technology transfer contracts (Zeckhauser, 1996, Hagiu and Yoffie, 2013, Spulber, 2014). Patent owners also incur transaction costs in licensing because of the costs of transfering and absorbing tacit knowledge (Spulber, 2012).

Jensen and Thursby (2001) find that auctions are not a good description of licensing by universities because TTOs experience difficulties in locating licensees for early stage inventions, which is what is usually licensed by universities. A survey of firms seeking to license their inventions finds that companies consider the transaction costs of identifying potential licensees to be a potential obstacle to licensing (Zuniga and Guellec, 2009). Rice (2006) identifies the costs of marketing patent licenses including the costs of specialized brokers.

Patent owners practically always engage in license negotiations with potential licensees. So, patent owners do not post prices as suggested by the Cournot com-

[5] For example, according to the company IPNav (2012) "The first question they [patent owners] answer is whether anyone is infringing the patents. If we can't identify any infringers, we may advise commercialization, sale, or abandonment of the patents."

plementary monopolies model. Although corporate patent owners sometimes make announcements regarding *expected* royalties, these are initial bargaining positions that differ substantially from the outcomes of negotiations between patent owners and producers. Staski (2010, p. 116) points out that "an 'announced' royalty rate may be significantly different than the 'actual' royalty rate resulting from a bilateral negotiation."

Bilateral negotiation is required because licensing agreements depend on the characteristics of the parties involved. A European Commission Study (Radauer and Dudenbostel, 2013, p. 53) emphasizes the need for negotiation because "considerable interaction must take place between licensors and licensees, on a bilateral and rather informal level."

Epstein and Malherbe (2011, p. 8) emphasize that royalties are negotiated and observe that negotiated royalties depend on the availability of alternatives and design-arounds, cost savings, and investment in commercialization and innovation. They emphasize that negotiated royalties "may be part of a complex transaction that includes joint licensing of other patents (i.e., patent pooling), cross-licenses, know-how, and/or product support."[6]

Training manuals for inventors provide considerable evidence for bilateral negotiation with no mention of posted prices. The World Intellectual Property Organization (WIPO) and the International Trade Centre (ITC) provide a training manual entirely devoted to licensing negotiation. Referring to the royalty rate in licensing agreements, the manual (WIPO, 2005, p. 57) states "It is important that the rate results in a good business proposition for both parties, and so negotiation of the royalty rate is fundamental to the success of the agreement." The European Patent Office Handbook for Inventors does not discuss posted prices but rather explains how to negotiate licensing agreements.[7] Goldscheider (1995-1996) offers a legal guide that focuses on negotiation of royalty rates.

University patent licenses are also negotiated. A study of university licensing

[6] Epstein and Malherbe (2011, p. 8).

[7] http://www.epo.org/learning-events/materials/inventors-handbook.html, Accessed August 24, 2014.

by Siegel et al. (2004, p. 18) observes that once a patent has been awarded, the technology transfer office (TTO) markets the technology to potential corporate licensees: "The next stage of the model involves working with firms or entrepreneurs to negotiate a licensing agreement."

Government patents are negotiated as well. For example, the National Aeronautics and Space Administration (NASA) negotiates royalties for all of its licences: "All NASA licenses are individually negotiated with the prospective licensee, and each license contains terms concerning transfer (practical application), license duration, royalties, and periodic reporting."[8]

III The Basic Model

This section introduces a three-stage model of invention, licensing, and innovation. At the invention stage, n inventors enter the market and engage in stochastic R&D that produces a set of complementary inventions. At the licensing stage, each of the inventors i offers technology licenses to producers with output per producer y_i , $i = 1, ..., n$. At the innovation stage, inventors and producers engage in bilateral negotiation over per-unit royalties r_i, $i = 1, ..., n$, when the downstream market is perfectly competitive, and lump-sum royalties R_i, $i = 1, ..., n$, when the downstream market is monopolistic. Also, producers make technology adoption and product pricing decisions, and the product market clears.

A. Inventors

Each inventor pays an entry cost k to engage in R&D. The n inventions generate a new technology c that is a unit cost for producers. I also consider a monopoly inventor that conducts R&D projects to develop complementary inventions.

Each inventor has a licensing cost function $C(\cdot)$ that is twice differentiable, increasing and convex and $C(0) = 0$. Licensing costs $C(\cdot)$ depend on the equilibrium number of licenses purchased by producers q not on the basis of the licenses offered to producers y_i. This assumption is the same as in Cournot's price-setting model

[8] http://www.nasa.gov/ames-partnerships/opportunities/licensing/#.U_pbu_ldWSo, Accessed August 24, 2014.

where costs also depend on producer purchases that are observed in equilibrium. Licensing costs include the costs of implementing technology transfers, supplying additional training and services, monitoring compliance with the licensing terms, observing the output of the downstream industry, and collecting license royalties. I consider asymmetric licensing costs in a later section.

The n inventions are assumed to be *perfect complements* so that they must be used together. This restriction is made to address the problem of licensing with complements. The analysis can be extended to address a combination of complementary and substitute inventions.

It is useful to consider aggregate R&D. Let the new technology c be a draw from the cumulative distribution function $F(c; n)$ defined on the non-negative real line with a continuous density function $f(c; n)$. Developing the new technology requires at least one inventor, $F(c; 0) = 0$ for all c. After R&D is concluded, the new technology c is common knowledge among inventors and producers.

Assume that a greater number of inventors increases the probability that the new technology will improve on any quality level c, $F(c; n+1) \geq F(c; n)$ for all c and strictly for some c.[9] Assume also that there are diminishing incremental returns to increasing the number of inventors, $F(c; n) - F(c; n-1) \geq F(c; n+1) - F(c; n)$ for all c and strictly for some c. The appendix gives parameterizations of some common probability distributions that satisfy these assumptions: uniform, exponential, Pareto, power law, and lowest order statistic.

Every R&D project succeeds in producing an invention that contributes to the final technology although the quality of inventions can vary. The analysis allows for the possibility that aggregate R&D is not successful and the new technology does not displace the initial technology. The aggregate R&D process is successful if and only if the realization of the new technology c is less than or equal to a critical cut-off that will be derived in the next section.

The aggregate R&D process is sufficiently general that it allows for some redundancy in R&D projects and overlap in the functions of inventions. Inventors have

[9] This assumption is equivalent to the usual stochastic order, that is, first-order stochastic dominance, $F(c; n)$ stochastically dominates $F(c; n+1)$ see Shaked and Shanthikumar (2007).

different areas of expertise so that they produce complementary inventions. The complementary inventions can be different types of technologies that must be used in combination. For example, one inventor has expertise in display screens, another inventor has expertise in batteries, and another inventor has expertise in microprocessors.

The aggregage R&D process allows the underlying R&D projects to be statistically independent or correlated random variables. The aggregate representation also can include knowledge spillovers across R&D projects. The underlying discoveries can be made simultaneously or sequentially.

Each inventor has IP rights that allow them to exclude others from using their invention. IP rights can be protected by patents, trademarks, copyrights, trade secrets, contracts, or other legal mechanisms. A producer must obtain a license to use a patented technology. I consider the problem of limited appropriability and patent infringement in a later section.

B. Producers

Suppose for now that the downstream industry is perfectly competitive. The discussion also considers downstream monopoly. Let p denote the price of the final good and let q be the output of the downstream industry. The market demand for the downstream industry $q = D(p)$ is decreasing and continuously differentiable. The market inverse demand is $p = P(q)$.

Let c represents the unit cost when a producer uses the complementary inventions to manufacture the final product. An existing technology represented by a unit cost c_0 is a substitute for the new technology. A producer has unit cost $\widetilde{c}$ given by

$$\widetilde{c} = \{ \begin{array}{ll} c & \text{if the producer uses all } n \text{ complementary inventions,} \\ c_0 & \text{otherwise.} \end{array} \tag{1}$$

Because inventions are perfect complements it is sufficient to consider only the aggregate technology. If there are substitutes for any invention, then competition within that category necessarily lowers royalties and improves the outcome. For example, Layne-Farrar et al. (2007) explain that competition from substitute inven-

tions within categories of complementary technologies offsets some of the effects of complementary monopolies with price-setting inventors. However, our objective is to consider the effects of perfect complements on the market outcome.

The complementary inventions can represent different types of components that are assembled to produce a final product, steps in a production process used to make a final product, elements of production equipment used to manufacture a final product, or any combination of these things. For example, the inventions can represent advances in complementary computer-aided design (CAD) and computer-aided manufacturing (CAM) technologies. Improvements in the various complementary components of an industrial robot reduce production costs; these components include sensors, computer hardware and software, control mechanisms, and the mechanical manipulator.[10]

The new technology also can be interpreted in terms of product quality. Let unit cost c be the cost of producing a final product that conforms to particular performance specifications. A reduction in the unit cost c allows producers to improve product quality without increasing unit costs. Electronics technology platforms such as a mobile phones, computers, television sets, and game consoles contain many complementary components including memory chips, microprocessors, software, batteries, and screens. Technological advances in each of these components improve the performance of the final product. So, a reduction in the unit cost c can be interpreted as technological improvements in each of the set of components that make up the final product.

C. The Three-Stage Game

At the invention stage, n inventors enter the market, engage in R&D, and observe the realization of technology c. Inventors make entry decisions based on expected profits $V(k)$, which will be derived endogenously. The equilibrium number of competing inventors is the largest integer n such that $V(n) \geq k$.

Next, at the licensing stage, each inventor i offers a number of licenses y_i to pro-

[10] The speed of a computer can be limited by the slowest component (Gawer and Henderson, 2007). On complementary innovations, see generally Rosenberg (1982). For a general discussion of complementarities in production, see Milgrom and Roberts (1990, 1995).

ducers. A producer obtains licenses for all of the inventions because the technologies are perfect complements. Because each producer has a Leontief production function for which the inventions are complementary inputs, the *aggregate* production function of the downstream industry also is a Leontief production function,[11]

$$q = \min\{y_1, y_2, \ldots, y_n\}. \tag{2}$$

where q is the output of the downstream industry.

Finally, at the innovation stage, per-unit royalties $r_1, r_2, \ldots, r_n$ are set through bilateral negotiations between inventors and producers. When q producers license the inventions, producers' per-unit willingness to pay for the set of complementary inventions equals inverse market demand net of unit production costs $P(q) - c$.

Inventors have all of the bargaining power and make first-and-final offers to producers. This is without loss of generality because the quantity of licenses in the licensing stage is unaffected by bargaining power for Coasian reasons, as will be shown in a later section. A later section also will show that when inventors choose quantity-setting strategies, the analysis of the non-cooperative game is robust to other pricing formulas used in royalty negotiation. The analysis also is robust to allowing inventors to have portfolios consisting of multiple patents. A later section also will consider the effects of asymmetric costs on invention and licensing.

At the innovation stage, there is a continuum of equilibria such that total license offers sum to the willingness to pay of each producer, $\sum_{i=1}^{n} r_i = P(q) - c$, and royalties cover average licensing costs,

$$\frac{C(q)}{q} \leq r_i \leq P(q) - c - (n-1)\frac{C(q)}{q}, i = 1, ..., n.$$

We restric attention to the symmetric equilibrium so that bargaining generates royalties equal to

$$r = \frac{P(q) - c}{n}. \tag{3}$$

[11] Each producer has a basic Leontief production function $x = \min\{\delta_1, \delta_2, \ldots, \delta_n\}$ where x is the producer's output, $\delta_i = 1$ if the producer uses invention i, and $\delta_i = 0$ otherwise.

Another way to view the bargaining equilibrium is to suppose that every equilibrium is equally likely. Then, the expected royalty for each inventor equals the royalty at the symmetric equilibrium,

$$r = \frac{1}{n} \int_{\frac{C(q)}{q}}^{P(q)-c-(n-1)\frac{C(q)}{q}} dr + \frac{C(q)}{q} = \frac{P(q)-c}{n}. \tag{4}$$

Because inventor revenues are necessarily linear, the analysis of the invention and licensing stages is the same if we consider either the expected royalty with a continuum of equilibria or the symmetric equilibrium.

IV Licensing and Innovation with Complementary Inventions

This section considers the licensing and innovation stages of the model. The main result of the analysis is a characterization of the equilibrium with quantity-setting inventors who offer licenses for complementary inventions. The section begins with a monopoly benchmark that will turn out to be important for the analysis.

A. The Monopoly Benchmark

At the final innovation stage, a monopolist with n complementary inventions negotiates licensing royalties with each producer. If the monopolist has all of the bargaining power, the total royalty ρ for the bundle of inventions equals each producer's willingness to pay,

$$\rho = P(q) - c. \tag{5}$$

Downstream producers have the option of using the initial technology. Producers adopt the bundle of inventions if and only if $\rho \leq c_0 - c$. Given that the total royalty equals each producer's willingness to pay, this is equivalent to $P(q) \leq c_0$.

At the prior licensing stage, the monopolist offers a bundle of licenses to q pro-

ducers. The monopolist chooses q to solve the profit-maximization problem,

$$q^M(c) \in \arg\max_q (P(q) - c)q - nC(q)$$

subject to the technology constraint

$$P(q) \leq c_0.$$

The set of complementary inventions is *drastic* in the sense of Arrow (1962) if and only if the constraint on the number of licenses $P(q) \leq c_0$ is non-binding. Let $Q^M = Q^M(c)$ be the profit-maximizing industry output when the initial technology constraint is non-binding. If the set of complementary inventions is *non-drastic*, the initial technology constraint is binding. Then, the output Q^0 solves $P(Q^0) = c_0$ and the monopoly royalty for the bundle of inventions equals the improvement in unit cost $c_0 - c$. [12]

The number of bundles provided by the monopoly with complementary inventions is $q^M(c) = \max\{Q^0, Q^M(c)\}$. If the profit-maximizing monopoly output is not unique, let $q^M(c)$ denote the smallest profit-maximizing output. The monopolist's royalty for the bundle of inventions is $\rho^M(c) = P(q^M(c)) - c$ or

$$\rho^M(c) = \min\{c_0 - c, P(Q^M(c)) - c\}. \tag{6}$$

The profit of the monopoly inventor licensing the bundle of inventions equals $nv(c, n)$ where the return for each invention is

$$v(c, n) = \frac{P(q^M(c)) - c}{n} q^M(c) - C(q^M(c)). \tag{7}$$

The monopoly inventor licensing the bundle of inventions to a competitive downstream market is viable only if the improvement in the technology is greater than average licensing costs. A monopoly inventor with a non-drastic set of inventions is viable only initial unit costs are greater than average licensing costs at the critical

[12] Letting $c_D > 0$ be the solution to $Q^M(c_D) = Q^0$, an invention is drastic if $c \leq c_D$.

output level Q^0.The highest unit cost at which the new technology is viable equals

$$C_0 \equiv c_0 - C(Q^0)/Q^0. \tag{8}$$

This is a *critical cut-off* that determines whether the monopoly inventor's R&D projects are successful. Let $C_0 > 0$ so that R&D is successful if and only if $c \leq C_0$.

B. Complementary Inventors

I now present the main result of the analysis. This result is important because it introduces and characterizes the quantity-setting non-cooperative game with complementary monopolies. Proposition 1 shows that the non-cooperative equilibrium equals the number of licenses offered by the monopoly with complementary inventions.

At the innovation stage, the expected royalty per invention is $r = \frac{P(q)-c}{n}$. At the licensing stage, because inventions are perfect complements, the profit of inventor i depends on the number of licenses he offers and the number of licenses offered by other inventors y_{-i}. Each inventor i chooses the quantity of licenses y_i to offer to producers that maximizes profit

$$\pi(y_i, y_{-i}) = \frac{P(q) - c}{n} q - C(q) \tag{9}$$

subject to

$$P(q) \leq c_0,$$

$$q = \min\{y_1, y_2, \ldots\ , y_n\}.$$

Proposition 1 presents the equilibrium of the non-cooperative game in which inventors offer licenses and negotiate royalties with downstream producers.

PROPOSITION 1. *The weakly dominant strategy equilibrium with inventors choosing the quantity of licenses to offer to producers is unique and equivalent to the smallest profit-maximizing monopoly output,* $y_i^*(c) = q^M(c)$, $i = 1, ..., n$ *and total royalties equal the monopoly outcome,* $nr^*(c) = \rho^M(c)$, *for any* c.

PROOF. Suppose first that the profit-maximizing monopoly output q^M is unique.

For any y_{-i}, let $\overline{y} = \min\{y_{-i}\}$. Consider first the possibility that $\overline{y} \geq q^M$. Then, because the monopolist selling the bundle of inventions maximizes profits, it follows that $\pi(q^M, y_{-i}) \geq \pi(y_i, y_{-i})$ for all y_i. If $y_i = q$, $\pi(y_i, y_{-i}) = \frac{1}{n}[(P(q) - c)q - nC(q)]$. So, if $\overline{y} \geq q^M$, inventor i maximizes profit by choosing the monopoly output, $y_i^* = q^M$. Conversely, if $\overline{y} < q^M$, then because the monopolist selling the bundle of inventions maximizes profits it follows that $\pi(q^M, y_{-i}) \geq \pi(y_i, y_{-i})$ for all y_i and strictly for $y_i < \overline{y}$. Again, inventor i maximizes profit by choosing the monopoly output, $y_i^* = q^M$. This implies that the monopoly output is the weakly dominant strategy for each inventor i, and thus the weakly dominant strategy equilibrium is the monopoly output.

Now suppose that the profit-maximizing monopoly output is not unique and let q' and q'' be monopoly outputs, where $q' < q''$. If $q' < \overline{y} < q''$, then inventor i strictly prefers to offer the lower monopoly output to any other quantity of licenses, $y_i^* = q'$. If $q'' \leq \overline{y}$, then inventor i is indifferent between the two monopoly outputs. If $\overline{y} \leq q'$, then the inventor is indifferent between q' and $\overline{y}$ and strictly prefers q' to any $y_i < \overline{y}$. Therefore, the smallest monopoly output q^M is the weakly dominant strategy for each inventor i. $\square$

This result only depends on the assumptions that demand is downward sloping, licensing costs are not decreasing, and inventions are perfect complements. Each inventor chooses the number of licenses equal to the monopoly output so that the weakly dominant strategy equilibrium is the monopoly outcome. Notice that the equilibrium corresponds to the smallest number of licenses that maximizes monopoly profit. So, the equilibrium is unique whether or not the monopoly output is unique.

The intuition for this result is that if all other inventors offer quantities of licenses in excess of the monopoly output, an inventor can maximize profits by restricting the quantity of licenses offered to producers. Conversely, if the lowest quantity of licenses supplied by other inventors is below the monopoly output, an inventor will not restrict the quantity of licenses any further and will be indifferent among all quantities of licenses above than minimum, including the monopoly level.

Intuitively, because of complementarity, inventors cannot license more than the minimum of all the other quantities. An inventor trying to restrict his output to

increase price would feel the full effect on his share of total profits. Thus, with quantity setting strategies, an inventor cannot increase his share of industry profits by lowering his quantity. This contrasts with price-setting strategies, because then an inventor could increase his share of industry profits by raising his price.

The static non-cooperative equilibrium with quantity-setting inventors attains the cooperative outcome. An inventor strictly prefers the monopoly outcome to any other outcome. Every inventor understands that his offer of licenses could control the market outcome. This means that an inventor will choose the number of licenses that would be offered by a monopolist selling the bundle of complementary inventions regardless of what other inventors are offering. Inventors tacitly coordinate without the need for communication or repeated interaction.

Proposition 1 has another important implication. For any given technology c, the monopoly outcome is not affected by the number of inventions. This implies that with non-cooperative quantity setting by inventors, for any given technology c, total royalties and downstream output are not not affected by the number of complementary inventions.

C. Comparison with Cournot's Price-Setting Model

Applying Cournot's (1838) model to licensing, $n \geq 2$ inventors choose royalties r_i, $i = 1, ..., n$. The downstream industry is perfectly competitive so that the output price equals average costs, $p = c + \sum_{i=1}^{n} r_i$. Profit equals $\pi^C(r_i, r_{-i}) = r_i D(c + \sum_{j=1}^{n} r_j) - C(D(c + \sum_{j=1}^{n} r_j))$. The Cournot-Nash non-cooperative equilibrium in royalties r_i^C, $i = 1, ..., n$ solves $r_i^C = \arg\max \pi^C(r_i, r_{-i}), i = 1, ..., n$.

Suppose first that there is no initial technology constraint. The Cournot equilibrium is symmetric, $r_i^C = r^C(c, n)$. The first-order conditions simplify to

$$[r^C - C'(D(c + r^C))]D'(c + nr^C) + D(c + nr^C) = 0. \tag{10}$$

Cournot demonstrates that the total of the non-cooperative input prices exceeds what would be charged by the monopolist selling the bundle of inputs. This is readily seen from a comparison the first-order condition with that of the multi-project monopolist. So, non-cooperative price competition raises total royalties above the

bundled monopoly royalty, $nr^C > \rho^M$. This immediately implies that the royalty for each invention is greater with Cournot price competition than with quantity-competition, $r^C > r^*$.

Now extend Cournot's model to include the initial technology constraint. The royalty for each invention is greater with Cournot price competition than with quantity competition when the initial technology constraint is binding only on royalties with price-setting firms, $r^* < c_0 - c < r^C$. Also, when the monopolist faces a binding initial technology constraint, that is, when the set of inventions is non-drastic, it follows the constraint is binding with both price-setting or quantity-setting inventors so that $r^* = r^C = c_0 - c$. Therefore, $r^C \geq r^*$ when there is an initial technology constraint in both settings.

Define social welfare as the sum of consumers' and producers' surplus $W(p) = CS(p) + PS(p)$, where consumers' surplus is $CS(p) = \int_p^\infty D(z)dz$ and producers' surplus is $PS(p) = (p - c)D(p) - nC(D(p))$. Let $v^C(c, n) = r^C(c)D(c + nr^C(c)) - C(D(c + r^C(c)))$ be equilibrium inventor profit in the Cournot price-setting model.

Compare Cournot's model of price setting by complementary input monopolists with the present model of quantity setting by complementary input monopolists.

PROPOSITION 2. *(i) The weakly dominant strategy equilibrium with quantity-setting inventors yields royalties that are strictly less than those at the Cournot equilibrium with price-setting inventors, $r^* < r^C$, when the set of inventions is drastic and yields the same royalties $r^* = r^C = c_0 - c$ when the set of inventions is non-drastic. (ii) Inventors' profits, consumers' surplus, and social welfare with non-cooperative quantity setting are greater than or equal to inventors' profits, consumers' surplus and social welfare with non-cooperative price-setting.*

The proof appears in the Appendix. Proposition 2 implies that for any given technology c, inventors have a greater incentive to invent with quantity-setting strategies than with price-setting strategies. This implies that more invention will occur when inventors choose quantities rather than prices.

The Cournot pricing equilibrium is affected by the number of complementary inventions. More inventions increases the sum of prices because free-riding affects

the non-cooperative outcome.[13] This means that profit, incentives to invent, and welfare under price setting increasingly differ from profit, incentives to invent, and welfare under quantity-setting as the number of inventors increases. The relative benefits of quantity-setting by inventors thus increase with growth in the complexity of innovations.

Proposition 2 also suggests that cost pass-through effects differ depending on whether inventors with complementary inventions choose quantities or prices.[14] So, variations in the quality of the new technology will have different effects on the downstream market.

Singh and Vives (1984) show that with complementary inputs, suppliers will choose price-setting over quantity-setting. This result assumes price commitment. The result can be obtained in the present setting with perfect complements. With two inventors, if one inventor were to choose quantity-setting, and the other were to choose price-setting, the price-setting inventor would choose a royalty that extracts all of the surplus, so that the other inventor would earn a zero return. To avoid this possibility, both inventors would choose price-setting strategies.[15] This suggests that price setting strategies will be observed in markets with complements when there is price commitment.

The outcome would change in the absence of price commitment. Then, with bargaining over royalties, the inventors would again reach the quantity-setting outcome. This outcome also would change if inventors could coordinate over strategies. This would allow them to achieve the more profitable quantity-setting outcome. This

[13] When market demand $D(p)$ is log-concave, $\frac{d^2 \ln D(p)}{dp^2} \leq 0$ it is possible to characterize the Cournot-Nash equilibrium with price-setting firms and complementary inputs. This can be readily seen by applying standard analyses of Cournot-Nash competition with homogeneous outputs; see Vives (1999) and Amir and Lambson (2000).

[14] With quantity-setting inventors, the market price is $p^M(c) = nr^*(c) + c$ and the cost pass-through effect of increases in downstream costs c is the monopoly cost-pass through effect, $\frac{dp^M(c)}{dc} = n\frac{dr^*(c)}{dc} + 1$. With price-setting inventors, the market price is $p^C(c) = nr^C(c) + c$ and the cost pass-through effect is $\frac{dp^C(c)}{dc} = n\frac{dr^C(c)}{dc} + 1$. Weyl and Fabinger (2013) contrast tax incidence under monopoly and symmetric oligopoly including complementary monopolists, and also consider asymmetric oligopoly.

[15] It can be shown that the monopoly outcome is the limiting case of quantity setting in Singh and Vives (1984) as products approach perfect complementarity.

suggests that quantity-setting will be observed in markets with complements when there is no price commitment.

V Endogenous R&D and Incentives to Innovate

This section characterizes the initial invention stage of the model. The analysis will show that incentives to invent for competing inventors differ from incentives to invent for the monopoly inventor with multiple projects.

A. The Monopoly Benchmark

To examine the effects of competition among complementary inventors, consider a multi-project monopoly inventor that conducts N complementary R&D projects. The multi-project monopoly inventor's expected profit from licensing equals the sum of the profit of N individual inventions,

$$\Pi(N) = N \int_0^{C_0} v(c, N) f(c; N) dc = \int_0^{C_0} q^M(c) F(c; N) dc. \tag{11}$$

The monopoly inventor maximizes expected profit net of R&D costs, $N^* = \text{argmax}_N[\Pi(N) - kN]$. The monopoly inventor chooses the largest number of projects such that incremental returns to invention are greater than or equal to the cost per project, k.

The monopoly inventor's incremental expected profit is the weighted sum of marginal profits from improvements in the best invention,

$$\Pi(N) - \Pi(N-1) = \int_0^{c_0} q^M(c)[F(c; N) - F(c; N-1)] dc. \tag{12}$$

Because the distribution function is increasing in the number of complementary inventions, the monopoly inventor's incremental return is positive. Because of diminishing returns, the monopoly inventor's incremental expected profit $\Pi(N) - \Pi(N-1)$ is decreasing in the number of projects. This implies that the net profit-maximizing number of projects N^* is well defined.

B. Entry of Inventors

By Proposition 1, each inventor has revenues that correspond to the monopolist's revenue per invention, $v(c,n)$. Inventors enter the market in the first stage of the game based on expected profit net of entry costs. An inventor's expected profit from licensing is

$$V(n) = \int_0^{C_0} v(c,n) f(c;n) dc. \tag{13}$$

Differentiating the inventor's revenue with respect to c gives the "marginal revenue" generated by an increase in the quality of the invention. The inventor's marginal revenue exactly equals downstream industry output divided by the number of inventors,

$$-v_1(c,n) = \frac{q^M(c)}{n}. \tag{14}$$

To see why, notice that when the set of inventions is drastic, the competition constraint is non-binding. From the envelope theorem and the monopolist's optimization condition, the marginal effect of the new technology on the inventor's profit equals the downstream industry's output divided by the number of inventors. When the set of inventions is non-drastic, the competition constraint is binding and output does not depend on the quality of the set of inventions, $q^M(c) = Q^0$. This again implies that the marginal effect of the new technology on the inventor's profit equals downstream industry output divided by the number of inventors.

The expected profit for an inventor $V(n)$ can be viewed as the sum of the marginal profit effects of the quality of the set of complementary inventions multiplied by the probability of each quality level. The marginal profits from improvements in the quality of the technology are weighted by the probability $F(c;n)$. Integrating by parts, and noting that $v(C_0,n) = 0$, implies that the inventor's expected profit equals

$$V(n) = \int_0^{C_0} q^M(c) \frac{F(c;n)}{n} dc, \tag{15}$$

which is the weighted sum of industry outputs at different quality levels of the technology.

Diminishing returns to the number of inventions implies that average likelihood $\frac{F(c;n)}{n}$ is decreasing in the number of inventions, see Lemma 1 in the Appendix. This

implies that the inventor's expected profit $V(n)$ also is decreasing in the number of inventions. So, the market equilibrium with entry of inventors n^* is well defined.

Compare invention with entry of complementary inventors with invention by a multi-project monopoly inventor. The result shows that non-cooperative licensing increases incentives to invent in comparison to a multi-project monopoly inventor. The proof appears in the Appendix.

PROPOSITION 3. *(i) The equilibrium number of inventions with competing complementary inventors is greater than the equilibrium number of inventions chosen by a multi-project monopoly inventor, $n^* \geq N^*$. (ii) The expected quality of the set of complementary inventions is greater with competing complementary inventors than with a multi-project monopoly inventor, $E[\widetilde{c} \mid n^*] \leq E[\widetilde{c} \mid N^*]$. (iii) Total expected inventor profit from licensing is greater with competing complementary inventors than with a multi-project monopoly inventor, $n^* V(n^*) \geq \Pi(N^*)$. (iv) The expected final market price is lower and expected industry output is greater with competing complementary inventors than with a multi-project monopoly inventor, $p(n^*) \leq p(N^*)$* and $q(n^*) \geq q(N^*)$.

The average return to invention with complementary inventions is greater than the incremental return to invention with a multi-project monopoly inventor. This implies that entry of inventors is sufficient to generate more R&D and higher quality inventions that the multi-project monopoly inventor.

Another way to look at this result is to note that the monopolist's incremental profits can be written as

$$\begin{aligned} \Pi(N+1) - \Pi(N) &= \frac{\Pi(N+1)}{N+1} - N\left(\frac{\Pi(N)}{N} - \frac{\Pi(N+1)}{N+1}\right) \\ &= V(N+1) - N(V(N) - V(N+1)). \end{aligned}$$

The first term is the share of total profits for the $N+1$'st inventor. The second term is the replacement effect because inventors' profits are reduced by entry. Competitive inventors who enter the market do not consider the replacement effect, in contrast to the monopolist. This differs from Arrow's result because this is due to upstream competition among inventors.

Industry output is increasing in the realization of the quality of the set of complementary inventions. The expected market price is decreasing in the quality of the set of complementary inventions, so the expected market price is lower with competing complementary inventors than with a multi-project monopoly inventor.

Let n^C be the number of inventors that enter with Cournot price-setting. From Proposition 2, $v(c,n) \geq v^C(c,n)$ for all c and strictly for some c. So, expected profit with quantity-setting inventors is greater than with price-setting inventors.

PROPOSITION 4. *(i) The equilibrium number of inventions with quantity-setting inventors is greater than with price-setting inventors, $n^* \geq n^C$. (ii) The expected quality of the set of complementary inventions is greater with competing complementary inventors than with a multi-project monopoly inventor, $E[\widetilde{c} \mid n^*] \leq E[\widetilde{c} \mid n^C]$. (iii) Expected inventor profit from licensing is greater with quantity-setting inventors than price-setting inventors. (iv) The expected final market price is lower and expected industry output is greater with quantity-setting inventors than price-setting inventors.*

Increases in the quality of inventions with quantity-setting inventors as compared to price-setting inventors reinforce the effects of lower output prices with quantity-setting inventors.

B. Economic Efficiency

The welfare analysis is second-best because it takes non-cooperative licensing as a given. The welfare measure therefore is based on monopoly pricing subject to the initial technology constraint. Notice that the definition of expected social benefits from R&D allows for the possibility that R&D does not result in inventions that improve on the initial technology.

Expected social benefits from R&D equals the expected benefits of the new technology net of benefits under the initial technology, $\int_{c_0}^{\infty} D(z)dz$,

$$\begin{aligned} w(n) &= \int_0^{C_0} [\int_{P(q^M(c))}^{\infty} D(z)dz + (P(q^M(c)) - c)D(P(q^M(c))) - nC(q^M(c))]f(c,n)dc \\ &\quad + \int_{c_0}^{\infty} D(z)dz[1 - F(C_0,n)] - \int_{c_0}^{\infty} D(z)dz. \end{aligned}$$

After integrating by parts and simplifying, expected welfare equals

$$w(n) = \int_0^{C_0} [P'(q^M(c))q^{M\prime}(c) + 1]q^M(c)F(c,n)dc. \tag{16}$$

The optimal number of R&D projects maximizes expected welfare $n^0 = \text{argmax}_n[w(n) - kn]$. The socially optimal number of inventions is the largest number of projects such that incremental returns to invention are greater than or equal to the cost per project, k.

PROPOSITION 5. *The optimal number of complementary inventions is greater than or equal to the number of inventions chosen by the multi-project monopoly,* $n^0 \geq N^*$.

PROOF. The incremental effect of the number of inventions on social welfare is

$$w(n) - w(n-1) = \int_0^{C_0} [P'(q^M(c))q^{M\prime}(c) + 1]q^M(c)[F(c;n) - F(c;n-1)]dc. \tag{17}$$

Because $P'(q^M(c))q^{M\prime}(c) > 0$, it follows that $w(n) - w(n-1) \geq \Pi(n) - \Pi(n-1)$ so that $n^0 \geq N^*$. □

Proposition 5 shows that the multi-project monopolists with complementary inventions chooses fewer inventions that the second-best social optimum. Also, entry of complementary inventors increases invention and lowers product prices in comparison to the multi-project monopoly, as shown by Proposition 3.

These results run contrary to standard discussions of the complements problem, which argue that complementary inventions generate too little invention and innovation. The standard approach based on Cournot price-setting inventors implies that public policy makers need to encourage invention and innovation through policies that target complementary inventions. Policies directed at complementary inventions would include compulsory licensing, regulation of royalties, and encouraging the formation of patent pools. However, the present analysis shows that complementary inventions encourage R&D and innovation relative to the multi-invention monopolist. This suggests that public policies that are designed to induce pooling and price coordination are not needed to improve incentives for R&D with comple-

mentary inventions.

This raises the question of whether or not entry of complementary inventions improves social welfare in comparison to the multi-project monopoly. To address this question, consider first whether entry of competitors is greater than or less than the second-best social optimum. There is not a clear-cut comparison between incremental welfare $w(n) - w(n-1)$ with incentives for entry $V(n)$.

The term $P'(q^M(c))q^{M\prime}(c)$ in incremental benefits $w(n) - w(n-1)$ increases the marginal social returns to R&D in comparison to incentives for entry of inventors. Because entry of inventors reduces prices and increases output by improving the quality of the set of complementary inventions, there are social benefits from entry. This corresponds to the "business creation effect" identified by Ghosh and Morita (2007a, 2007b). This effect occurs when upstream input sellers enter a market and sell to downstream manufacturers that have market power. Input sellers do not capture some of the benefits of their entry which go to downstream producers, thus leading to insufficient entry. In the present setting, invention generates a "business creation effect" because entry of inventors increases total net benefits by improving the quality of the set of inventions. Final consumers obtain some of those returns to entry because the downstream industry is perfectly competitive. As a consequence, inventors do not obtain all of the benefits of R&D so that the number of inventions could be less than the social optimum.

However, entry of inventors also results in diminishing marginal returns to invention which generates social costs of entry. This corresponds to "business-stealing effect" observed by Mankiw and Whinston (1986).[16] Because of this effect, the entry of inventors reduces the effectiveness of R&D even thought more inventions increase the quality of the set of complementary inventions. This effect occurs because the incremental effects of more inventors $F(c;n) - F(c;n-1)$ is less than or equal to the average effects $\frac{F(c;n)}{n}$. Inventors do not fully take into account the effects of their

[16] See also von Weizsäcker (1980) and Suzumura and Kiyono (1987). With product variety, Spence (1976) finds examples for which entry may be insufficient or excessive. Dixit and Stiglitz (1977) show that entry of producers exactly equals the social optimum in the CES model with product differentiation when there are no entry subsidies. Mukherjee and Mukherjee (2008) find that entry may be excessive or insufficient with a Stackelberg leader, see also Mukherjee (2012).

entry on the incremental returns to R&D for other inventors. This effect implies that the number of inventions could be greater than the social optimum.

Entry of complementary inventors is subject to these two effects, which generates a trade-off between improvements in innovative technology and diminishing marginal returns to invention. Entry of inventors improves the expected quality of the best invention and reduces the incremental returns to R&D. Whether or not competitive invention is greater than or less than the second-best social optimum, we can still address the main question of whether or not competitive invention improves efficiency in comparison to the multi-project monopoly inventor.

Proposition 5 shows that the multi-project monopoly inventor chooses too few inventions relative to the social optimum. Because competitive entry generates more inventions, this has the following implication. If the entry of inventors is less than or equal to the socially optimal number of inventors $n^* \leq n^0$ or if the optimal number of inventors is greater than the socially optimal number of inventors $n^* > n^0$ but not by too much, then entry of inventors will improve social welfare relative to a multi-project monopoly inventors.

This discussion suggests that entry of complementary inventors should be preferable to a multi-project monopoly inventor. The costs of R&D and licensing and diminishing average returns to invention serve to limit entry of inventors. It seems unlikely that the number of inventors will exceed the second-best optimum, so erecting barriers to the entry of inventors should be avoided. If the policy objective is to encourage invention and innovation and to lower final product prices for consumers, then public policies, including IP, antitrust, and other types of regulation, should also avoid encouraging joint management of complementary R&D and joint licensing of complementary inventions.

VI Downstream Competition versus Downstream Monopoly and Incentives to Innovate

This section considers the effects of downstream competition on incentives to invent with complementary inventions. I compare the outcome when the downstream industry is perfectly competitive with the outcome when the downstream industry is a monopoly. When the downstream industry is a monopoly, inventors providing complementary technologies choose lump-sum royalties, R_i, $i = 1, ..., n$. Ménière and Parlane (2010) consider lump-sum royalties with complementary inventions.

As a benchmark, consider the monopolist offering the set of complementary inventions. The monopolist with technology c chooses the number of licenses $Q^M(c)$ that maximizes profit $(P(q) - c)q$. If the profit-maximizing monopoly output is not unique, let $Q^M(c)$ denote the smallest profit-maximizing output. The profit of the downstream monopolist using the bundle of complementary inventions is $b(c) = (P(Q^M(c)) - c)Q^M(c)$. The profit of the downstream monopolist using the initial technology is $b(c_0)$. The monopoly inventor's royalty for the bundle of inventions equals incremental benefits from the complementary technologies, $\Phi^M(c) = b(c) - b(c_0)$.

Consider the final innovation stage when inventors have complementary inventions. At a symmetric equilibrium, royalties evenly divide the downstream monopolist's total incremental returns from the new technology,

$$R = \frac{(P(q) - c)q - b(c_0)}{n}.$$

At the licensing stage inventors choose the quantity of licenses y_i to maximize profit $R - C(q)$ subject to $q = \min\{y_1, y_2, \ldots, y_n\}$. By arguments similar to those in Proposition 1 the following holds.

PROPOSITION 6. *The weakly dominant strategy equilibrium with inventors choosing the quantity of licenses to offer to producers is unique and equivalent to the smallest profit-maximizing monopoly output, $y_i^*(c) = Q^M(c)$, $i = 1, ..., n$ and total royalties equal the monopoly outcome, $nR^*(c) = \Phi^M(c)$, for any c.*

Consider the initial invention stage. Let C_{00} denote the critical quality of the set of complementary inventions such that $R \geq C(Q^M(c))$ for $c \leq C_{00}$. An inventor's expected profit is

$$B(n) = \int_0^{C_{00}} [R^*(c) - C(X^M(c))] f(c; n) dc, \tag{18}$$

The equilibrium number of competing inventors n^{**} when there is a downstream monopoly is the largest integer n such that $B(n) \geq k$. The proof appears in the Appendix.

PROPOSITION 7. *The equilibrium number of competing inventors with complementary inventors is greater with downstream competition than with a downstream monopoly,* $n^* \geq n^{**}$ *if for* $c > c_D$

$$\frac{1}{n} \int_c^{c_0} [Q^0 - X^M(z)] dz \geq C(Q^0) - C(X^M(c)).$$

This result extends Arrow's analysis to quantity competition among inventors with complementary inventions and endogenous and uncertain R&D. The inequality in the proposition guaranties that for non-drastic inventions the benefits of competition under the new technology outweigh additional licensing costs. The inequality holds whenever there are no licensing costs.

Competitive pressures from the downstream market increases incentives to invent for any realization of the quality of the set of inventions. Because greater competition downstream increases the number of complementary inventions, the expected quality of the set of inventions is greater with downstream competition than with downstream monopoly. Also, the final product price will be lower with downstream competition than with downstream monopoly as a result of lower prices in competitive markets and the higher quality of the set of inventions.

The result in Proposition 6 contrasts with the incentive effects of downstream competition when there is Cournot price-setting competition among inventors. To see why, suppose that there are no licensing costs. With price-setting inventors, downstream competition increases incentives to invent for non-drastic inventions or

close-to-non-drastic inventions. This is because when inventions are non-drastic, total royalties are $nr^C = c_0 - c$, which limits the effect of free riding on total royalties. So, downstream competition yields strictly greater incentives to invent than downstream monopoly.

However, with sufficiently drastic inventions, downstream competition can reduce incentives to invent. Total royalties rise and profits for each inventor are reduced by increases in the number of inventors. This means that with drastic inventions, the incentive to invent with downstream competition, $nv^C(c, n)$, can be less than the incentive to invent with downstream monopoly, $R^*(c) - C(X^M(c))$. The free-rider effects of price-setting competition among inventors overwhelm the benefits of downstream competition in comparison to downstream monopoly.

VII Applications and Extensions

This section considers various factors that affect the outcome of technology licensing with complementary inventions. The discussion examines how the market outcome is affected by bargaining power, the size of the market and elasticity of demand, the size of patent portfolios, heterogeneity of inventors and producers, and other methods of determining royalties.

A. Bargaining Power

The results are robust to limited bargaining power of inventors. Let α denote inventor bargaining power, where $0 < \alpha < 1$. Then, the expected royalty at the innovation stage equals

$$r^* = \alpha \frac{P(q) - c}{n} + (1 - \alpha) \frac{C(q)}{q}. \tag{19}$$

Substituting for the expected royalty, an inventor's expected profits evaluated at q equal

$$r^* q - C(q) = \alpha \left[\frac{P(q) - c}{n} q - C(q) \right]. \tag{20}$$

This implies that bargaining power does not affect the licensing equilibrium with

quantity-setting inventors, where $q = \min\{y_1, y_2, \ldots\ , y_n\}$. Regardless of how rents are divided, inventors have an incentive to choose the optimal outcome with coordination. This agrees with Coase's (1960) analysis of bargaining and social cost. So, at the licensing stage, each inventor will choose the same number of licenses that would be chosen by a multi-project monopoly inventor with all of the bargaining power. This means that Proposition 1 is robust to limits on inventor bargaining power.

A consistent comparison with the Cournot pricing model requires a corresponding adjustment of the assumptions of that model. Cournot's model of price commitment implicitly assumes that the complementary monopolists have all the bargaining power. To compare with the present model, it would be useful to assume that inventors and producers divide the rents from exchange.

Bargaining power does affect incentives for entry of inventors. Reductions in bargaining power will decrease the number of inventors that engage in R&D and decrease the expected quality of the new technology. If a multi-project monopoly inventor has greater bargaining power than competing inventors, this could lead to the multi-project monopoly producing more inventions than competing inventors if bargaining power effects overcome the difference between average and incremental incentives to invent.

B. The Size of the Market and Elasticity of Demand

IP rights foster the development of the market for inventions. This is important because competitive pressures increase incentives to invent and to innovate when inventions are complements. This section shows that increases in the size of the product market and in the elasticity of demand also increase incentives for invention when inventions are perfect complements.

The size of the product market and the elasticity of demand provide useful measures of competitive pressures. When inventions are perfect substitutes, increases in the size of the product market and in the elasticity of demand increase the quality of inventions; see Schmookler (1959, 1962), Vives (2008), and Spulber (2008, 2010, 2013b). This section shows that similar effects of competitive pressures continue to hold with complementary inventions.

Let the size of the market be a parameter $g > 1$ that shifts out the demand curve

for all price levels, $q = gD(p)$. The inverse demand function is thus $p = P(\frac{q}{g})$. The parameter g affects the monopoly output and the inventor's profit $v(c, n, g)$. By the envelope theorem, the monopolist's problem implies

$$v_g(c, n, g) = -P'\left(\frac{q^M(c)}{g}\right)\frac{(q^M(c))^2}{ng^2} > 0.$$

The inventor's expected profit $V(n, g)$ is decreasing in the number of inventors and increasing in the size of the market. This implies that the equilibrium number of inventors that enter the market is non-decreasing in the size of the market. The size of the market thus increases the expected quality of the set of complementary inventions.

Consider the demand function $q = D(p) = p^{-\eta}$ with constant elasticity η. The inverse demand function is thus $p = P(q) = q^{\frac{-1}{\eta}}$. By the envelope theorem, it follows that

$$v_\eta(c, n, \eta) = (q^M(c))^{\frac{-1}{\eta}}(\ln q^M(c))\frac{1}{\eta^2}\frac{q^M(c)}{n} > 0,$$

for $q^M(c) > 1$. The output of the downstream industry is greater than one because each producer supplies one unit of output and there are multiple downstream producers. The inventor's expected profit $V(n, \eta)$ is increasing in the elasticity of demand. This implies that the equilibrium number of inventors that enter the market is non-decreasing in the elasticity of demand. Increases in the elasticity of demand thus increases the expected quality of the set of complementary inventions.

C. Licensing Patent Portfolios and the Number of Inventions

Consider now the situation in which each inventor has a portfolio of inventions. There are I inventors and each inventor i owns $m_i \geq 1$ inventions, where the total number of inventions is given by $n = \sum_{i=1}^{I} m_i$. Each inventor i offers y_i licenses for each of the m_i inventions in their portfolio. Assume that the royalty for each invention is an equal share of the marginal producer's willingness to pay, $r = \frac{P(q)-c}{n}$.

This implies that inventor i's profit is given by

$$\frac{m_i}{n}rq - m_i C(q) = m_i[\frac{P(q)-c}{n}q - C(q)].$$

This implies that the number of patents owned by an inventor does not affect the profit-maximizing quantity of licenses that they will offer for each of their inventions. The weakly dominant strategy equilibrium with patent portfolios will be the same in the situation where each inventor only owns one patent. So, the preceding results continue to hold were inventors to hold patent portfolios of different sizes.

An increase in inventor i's number of projects m_i would increase that inventor's share of total profit $(P(q)-c)q - nC(q)$. This does not imply that an inventor could simply get more profits by increasing the number of inventions. Producing an additional invention requires costs of R&D per project. The invention stage of the model can be generalized to allow a number of inventors I to choose the number of R&D projects m_i, where each project has a cost k. A non-cooperative Nash equilibrium in the number of projects would generate inefficiencies in R&D.

D. Heterogeneous Inventors

Suppose that inventors have heterogeneous licensing cost functions $C_i(\cdot)$, and marginal costs are ordered $C_1'(\cdot) > C_2'(\cdot) > ... > C_n'(\cdot)$. Also, let $C_i(0) = 0$. As before, suppose that expected royalties are $r^* = \frac{P(q)-c}{n}$. Because inventions are strict complements, the equilibrium output with be constrained by the smallest number of licenses. This implies that the inventor with the highest marginal costs $C_1'(\cdot)$ will determine the equilibrium number of licenses q. The unique weakly dominant strategy equilibrium will consist of different licensing offers $y_1 < y_2 < ... < y_n$. The equilibrium royalties will cover the costs of all of the firms because the number of licenses is determined by the highest-cost inventor.

The number of licenses would be smaller than the number that would be offered by a multi-project monopolist with lower marginal licensing costs. The comparison with Cournot price setting complementary monopolists is not clear cut. The comparison will depend on the relative importance two effects. With heterogeneous costs, high licensing costs may cause prices to be higher with quantity-setting inventors.

However, high free-rider effects observed with price setting may keep prices higher with price-setting inventors.

Suppose now that royalties differ across inventors with higher royalties going to those inventors who have higher licensing costs. For example, suppose that costs differ depending on a linear scalar, $C_i(\cdot) = b_i C(\cdot)$, and let royalties be given by

$$r_i = \frac{b_i}{\sum_{j=1}^{n} b_j}(P(q) - c).$$

Then, the weakly dominant strategy equilibrium will be identical to that with a monopolist that has total licensing costs $\sum_{j=1}^{n} b_j C(q)$.

E. Heterogeneous Producers

The analysis can be extended to allow differences among producers, with royalties being determined by the number of licenses and the willingness to pay of the marginal producer. Inventors offer licenses sequentially to producers beginning with the producer that has the highest willingness to pay and continuing with the producer that has the next-highest willingness to pay and so on. The marginal producer is determined by the equilibrium number of licenses. The equilibrium royalty is determined by negotiation between inventors and the marginal producer. When inventors have all of the bargaining power, the marginal producer is indifferent between adopting and not adopting the complementary inventions. This approach is based on the model of decentralized exchange introduced by Böhm-Bawerk (1891).[17]

Suppose that inventors cannot price discriminate across producers. Price discrimination could be ruled out by antitrust laws or other regulations. Also, inventors may offer licensing contracts to producers with "most favored nations" clauses that

[17] Böhm-Bawerk (1891) introduces the "method of marginal pairs." Böhm-Bawerk (1891, p. 213) observes that "If all are to exchange at one market price, the price must be such as to suit all exchanging parties; and since, naturally, the price which suits the least capable contracting party suits, in a higher degree, all the more capable, it follows quite naturally, that the relations of the last pair whom the price must suit, or, as the case may be, the first pair whom it cannot suit, afford the standard for the height of the price." The market clearing price and quantity are determined by the marginal pair who trade, that is, the buyer–seller pair who have the smallest positive difference between the buyer's value and that of the seller, or by the marginal pair who are excluded from trade.

require the inventor to offer a producer the best terms offered to other producers. Alternatively, inventors may belong to SSOs that require members to license patented inventions without price discrimination. Many SSOs require licensing of patented inventions under terms that are Fair, Reasonable, and Non-Discriminatory (FRAND), which can be interpreted as requiring uniform pricing to similar producers. This applies particularly to patented inventions that are "essential" to a technology standard issued by an SSO.

Let the inverse demand function $P(q)$ represent the willingness to pay levels of producers arranging in decreasing order. For any given number of licenses, the royalty reflects the outcome of negotiation between inventors and the marginal producer. Then, the weakly dominant strategy equilibrium in the number of licenses with differences among producers will be the same as the situation with identical producers. The preceding results then apply to the situation with heterogeneous producers.

F. The Apportionment Rule versus the Entire Market Value Rule

This section shows that the main results of the analysis hold with different ways of determining royalties. The per-unit royalty discussed thus far is sometimes referred to as the "apportionment rule." This contrasts with the "entire market value rule," which is an ad valorem royalty based on a share of revenues. Llobet and Padilla (2014) compare these two rules when multiple inventors engage in Cournot price competition and show that these rules generate different royalties for inventors and have different effects on incentives to innovate. However, the per-unit royalty rule and the entire market value rule are equivalent when inventors choose the quantity of licenses to offer to producers.

At the symmetric equilibrium with negotiation between inventors and producers under the ad valorem rule, each inventor obtains a share $\frac{s}{n}$ of producers' revenue, where $0 < s < 1$. For any given quantity of licenses q and technology c, each producer breaks even,

$$(1 - s)P(q) = c. \tag{21}$$

Then, an inventor's revenue under the ad valorem rule equals revenue with the per-

unit royalty,

$$\frac{sP(q)q}{n} = \frac{P(q) - c}{n} q = rq. \tag{22}$$

This implies that industry output and incentives to innovate with complementary inventions under the entire market value rule are the same as those under the per-unit royalty rule. So, all of the preceding propositions continue to apply under the entire market value rule.

VIII Public Policy and IP

The section considers the effects of appropriability of IP and competitive pressures on incentives to invent. The discussion considers implications of the results for public policy.

A. Appropriability of Intellectual Property

The analysis thus far assumes that patent owners have full legal protections for their IP. However, there may be various limitations to appropriability of the returns to licensing patents. Licensees may differ in their willingness to purchase licenses because they can use or develop alternative technologies. Some licenses may choose to infringe the patent and others may choose to challenge the validity of the patent. In addition to the costs of locating potential licenses and negotiating licensing agreements, patent owners face legal and regulatory costs that can further limit appropriability.

Patent owners reveal the secret of their technologies by approaching producers with licensing offers. Arrow (1962) suggests that inventors seeking to market their inventions may result in users copying or stealing their inventions. Producers that do not license the invention compete with producers that choose to license the invention. Inventors face a trade-off between the benefits of revealing the invention to potential licensees and the possibility that some producers will learn the secret and use the invention.

To formalize the risks faced by inventors, let $a < 1$ be the probability that a producer will license the invention. In equilibrium, inventors offer licenses to q pro-

ducers all of whom use the invention, but only aq producers license the invention. This implies that the quantity of output in the product market equals q. Inventors only incur costs of providing services to those inventors who purchase licenses. Inventors thus earn profits equal to

$$v(c,n,a) = \frac{P(q^M(c)) - c}{n} aq^M(c) - C(aq^M(c)). \tag{23}$$

By the envelope theorem, the monopolist's optimization problem implies

$$v_a(c,n,a) = -P'\left(q^M(c)\right) \frac{(q^M(c))^2}{n} > 0.$$

Greater appropriability, that is a higher value of the likelihood of licensing a, increases an inventor's profit.

The inventor's expected profit $V(n,a)$ is decreasing in the number of inventors and increasing in appropriability. Increases in appropriability thus increase the entry of inventors n^* and increase the expected quality of the set of complementary inventions. This implies that greater appropriability increases final output and decreases product prices to consumers.

This result implies that antitrust and IP should not be viewed as conflicting policies when inventions are complements. The analysis shows that complementary inventions are a coordination device that lowers royalties when inventors choose the quantity of licenses in comparison to the situation in which inventors choose prices. This suggests that antitrust policy should not seek to weaken IP rights based on concerns about "royalty stacking" with complementary inventions. Complementary inventions help rather than hinder coordination among inventors and other patent owners.

Also, because patent owners attain the cooperative outcome when they choose the quantity of licensing, patent pools are not needed as a coordination device for avoiding "royalty stacking." Llanes and Poblete (2014) consider non-cooperative price setting with complementary inventions and find that ex ante agreements to form patent pools address both the Cournot pricing problem and efficient choice of technology

standards.

In addition, with complementary inventions and quantity-setting inventors, greater appropriability of IP increases inventor profits and increases the expected returns to invention. This increases incentives for entry of inventors and increases the number of complementary inventions that make up the new technology. Greater appropriability of IP thus generates greater innovation by increasing industry output and reducing product prices. In addition, market competition among owners of complementary inventions generates more inventive activity and better quality inventions than a monopoly inventor that supplies the set of complementary inventions.

These results imply that antitrust policy seeking to promote consumer welfare should not seek to weaken IP rights when there are complementary products. This also implies that antitrust policy should avoid compulsory licensing and regulating royalties when inventions are complements. Inventions are strict complements if a Standard Setting Organization (SSO) specifies particular inventions that must be used in combination to produce a product that satisfies a technology standard, that is, all of the technologies are "essential" to the standard.[18] The analysis further suggests that complementarities that result from SSOs identifying particular inventions as "essential" to a technology standard should not require heightened antitrust scrutiny.

B. *Public Policy Implications*

The present discussion suggests that public policy should not be based solely on the Cournot price-setting model but should take into account other strategies. Applications of the Cournot price-setting model would require evidence that markets for inventions involve posted prices and an absence of coordination. Among the proposed policy remedies based on the Cournot price-setting model are regulation of royalty rates, antitrust scrutiny of patent acquisitions with complementary inventions, compulsory licensing, encouragement of patent pools, and limits on enforcement of IP rights through injunctions and damages for infringement.[19] Such policy prescrip-

[18] Such patents are referred to as Standard Essential Patents (SEPs). For some SSOs, patents that are declared to be essential to a standard may not be necessary to conform to the standard. There may not be a formal determination by the SSO of what patents are necessary to satisfy the requirements of a technology standard.

[19] See Geradin et al. (2008) for an overview. See also Department of Justice and Federal Trade

tions are based on a particular characterization of the licensing process. In contrast, the present analysis suggests that decentralized bargaining and implicit coordination reduce royalties and increase efficiency in the market for inventions.

Standard applications of the Cournot complementary monopolies pricing model imply that total royalties exceed what a monopolist would charge for a bundle of complementary inventions. This has raised public policy concerns about *excess royalties* also known as "royalty stacking" and "double marginalization with complements."

The present analysis suggests that with decentralized bargaining the problem of excess royalties will not arise. Total royalties with complementary inventions will not exceed the cooperative level even when inventors have all of the bargaining power. If inventors do not have all of the bargaining power, royalties with decentralized bargaining will be strictly less than what a monopolist would charge for the bundle of inventions. Even if inventors have all of the bargaining power, the quality of inventions with endogenous R&D will improve relative to the multi-project monopoly inventor, further reducing royalties below the monopoly level. Policy remedies for excess royalties would not be necessary even when all inventions are complements.

The present analysis also suggests that policy makers should not be concerned about *excess entry* of inventors. In the Cournot pricing model, entry of additional inventors generates greater free-rider problems. In contrast, equilibrium royalties with quantity-setting inventors are not affected by the number of complementary inventions. So, entry of additional inventors does not generate any free-rider problems. The analysis sheds light on the market for inventions. It shows that inventors have an incentive to choose licensees selectively because they obtain greater profits from quantity-setting strategies. The analysis helps to explain why inventors and producers would engage in bargaining over royalties instead of relying on posted prices.

The Cournot complementary monopolies model also raises public policy concerns that the existence of many complementary technologies will create *coordination prob-*

Commission, 2007, *Antitrust Enforcement and Intellectual Property Rights: Promiting Innovation and Competition*, www.usdoj.gov/atr/public/hearings/ip/222655.pdf, Accessed September 2, 2014.

lems that discourage the production of complex products. The public policy concern that complementary technologies discourage innovation is known as the problem of "patent thickets." The present analysis shows that market coordination is feasible even with many complementary technologies because inventors will not choose excessive royalties, so that royalties will not discourage innovation.

The present analysis further shows that with endogenous R&D, there will be more R&D projects with competing inventors than with a multi-project monopoly inventor. This implies that competing inventors will produce a better set of inventions and lower expected royalties than what would be observed with a multi-project monopoly inventor. Again, the problem of "patent thickets" will not occur.

These results help address related antitrust concerns about technology standards and standards organizations. Technology standards that require a set of patented inventions are referred to as standard essential patents (SEPs). Complementary SEPs need not generate excessive royalties because complementary inventions provide incentives for inventors to attain the cooperative outcome and to invest in R&D.

The present results have implications for public policy towards IP. A widely-expressed concern, known as the "tragedy of the anti-commons," is that dispersed ownership of complementary inventions results in underuse of resources.[20] According to this view, patents and other forms of IP create *excess property rights* that lead to economic inefficiency. Heller and Eisenberg (1998, p. 700) argue that "When owners have conflicting goals and each can deploy its rights to block the strategies of the others, they may not be able to reach an agreement that leaves enough private value for downstream developers to bring products to the market." Heller and Eisenberg (1998, p. 701) express concerns about the privatization of biomedical research: "An anticommons in biomedical research may be more likely to endure than in other areas of intellectual property because of the high transaction costs of bargaining, heterogeneous interests among owners, and cognitive biases of researchers." The theoretical foundation of the "tragedy of the anti-commons" is again the Cournot price-setting model applies to patents; see Buchanan and Yoon (2000).

The present analysis shows that patents need not create excess property rights.

[20] See also Heller (1998, 2008), Heller and Eisenberg (1998).

Because royalties are lower with quantity-setting competition than with price-setting competition, final industry output is greater with quantity competition than with price competition. This contradicts the "tragedy of the anti-commons" result and shows that even with complementary inventions, there need not be underutilization of resources due to excessively high prices for complementary inventions.

The present analysis of endogenous R&D with complementary inventions further shows that market competition can lead to greater incentives to innovate than with a multi-project monopoly. This suggests that greater protections for IP rights and market competition among inventors are consistent with incentives for R&D and innovative applications of complementary inventions. The Cournot complementary monopolies model cannot be used as a theoretical support for weakening IP protections without considering alternative competitive strategies.

IX Conclusion

Public policy makers should consider the full range of competitive strategies in markets for invention. With complementary inventions, inventors who license their inventions to producers take into account the effects of their decisions on the final market prices and output. By marketing their licenses to producers, each inventor understands that the quantity of licenses they offer can constrain the downstream market thus driving up prices and reducing total output. Inventors "internalize" the effects of their licensing decisions when inventions are complementary.

This explains why inventors choosing to market their licenses to producers are able to attain the cooperative outcome, even though they choose the quantity of licenses non-cooperatively. The equilibrium with quantity-setting inventors contrasts with Cournot's price-setting model in which individual inventors do not fully realize the effects of their pricing decisions on the market.

Because quantity-setting competition among inventors is an alternative to price-setting competition, public policy makers cannot rely solely on predictions based on Cournot's price setting model. This means that complementary inventions need not raise concerns over "royalty stacking," "patent thickets," or the "tragedy of

the anti-commons." Even if complementary sets of patented inventions are essential to producers meeting a technology standard, they may have incentives to attain a cooperative outcome. Additional competitive pressures from inventors supplying substitute inventions will further lower royalties.

The analysis suggests that antitrust policy makers should continue to favor competition over monopoly, even with complementary inventions. Thus, public policy should continue to support lower entry barriers in innovative industries so that contestability can promote coordination. This also means that complementary inventions need not require the formation of patent pools as a coordination device, although patent pools may provide some transaction efficiencies.

The analysis showed that greater appropriability of IP increases incentives to invent and to innovate with complementary inventions. Appropriability of IP is critical for the formation of a market for inventions, which includes not only patent licensing, but technology transfers and products that depend on other forms of IP. Greater appropriability of IP increases incentives for inventors and producers to participate in the market for inventions. Competition among inventors with complementary inventions and competition among downstream producers increases incentives to invent when there is a market for inventions. This implies that antitrust policy is not at odds with IP rights, but instead antitrust should promote innovation and increase economic efficiency by supporting IP.

Quantity-setting competition among inventors of complementary inventions results in greater incentives to invent, better inventions, lower product prices, and greater final output than a monopoly inventor with complementary inventions. This is because the average incentive to invent with competing inventors exceeds the marginal incentive to invent for a monopoly inventor. Complementary inventions are not a barrier to innovation because they provide incentives for tacit coordination that help increase innovation.

Appendix

PROOF OF PROPOSITION 2. (i) The monopolist's first-order condition in terms of output,

$$P'(Q^M)Q^M - nC'(Q^M) + P(Q^M) - c = 0,$$

can be written as in terms of the prices of the bundle,

$$[\rho^M - nC'(D(c+\rho^M))]D'(c+\rho^M) + D(c+\rho^M) = 0,$$

where $\rho^M = P(Q^M) - c$. The monopolist's first-order condition implies that

$$[\frac{\rho^M}{n} - C'(D(c+\rho^M))]D'(c+\rho^M) + D(c+\rho^M) = (1-\frac{1}{n})D(c+\rho^M) > 0.$$

So, $r^* < r^C$ when the set of inventions is drastic and $r^* = r^C = c_0 - c$ when the set of inventions is non-drastic. (ii) When the set of inventions is drastic, complementary inventors have greater profits with quantity setting because they share monopoly profits,

$$\pi(y_i^*, y_{-i}^*) = \frac{1}{n}[(P(q^M) - c)q^M - nC(q^M)] > r^C D(c+nr^C) - C(D(c+nr^C)).$$

Non-cooperative quantity setting generates lower prices that than non-cooperative price setting. So, the market price for the final product is higher with price-setting inventors than with quantity-setting inventors,

$$p^C = nr^C + c \geq \rho^M + c = P(q^M).$$

This implies that consumers' surplus and profits are greater with quantity-setting inventors than with price-setting inventors. So, social welfare is greater with quantity-setting inventors than with price-setting inventors. □

LEMMA 1. (i) $\frac{F(c;n)}{n} \geq \frac{F(c;n+1)}{n+1}$, (ii) $\frac{F(c;n)}{n} \geq F(c;n) - F(c;n-1)$.

PROOF. (i) The proof is by induction. Note that $F(c;n) - F(c;n-1) \geq F(c;n+1) - F(c;n)$ implies $F(c;1) - F(c;0) \geq F(c;2) - F(c;1)$ so that the statement holds

for $n = 1$. Given $\frac{F(c;n-1)}{n-1} \geq \frac{F(c;n)}{n}$ for $n > 1$ and $F(c;n) - F(c;n-1) \geq F(c;n+1) - F(c;n)$, it follows that $F(c;n)(2 - \frac{n-1}{n}) \geq F(c;n+1)$. So, $\frac{F(c;n)}{n} \geq \frac{F(c;n+1)}{n+1}$ for all $n \geq 1$. (ii) $\frac{F(c;n-1)}{n-1} \geq \frac{F(c;n)}{n}$ implies that $nF(c;n-1) \geq (n-1)F(c;n)$ so that $\frac{F(c;n)}{n} \geq F(c;n) - F(c;n-1)$ for all $n \geq 1$. □

Table 2 presents parameterizations of the uniform, exponential, Pareto and power law distributions that satisfy the assumptions on $F(c;n)$. The table also includes the distribution of the lowest order statistic, which satisfies the assumptions on $F(c;n)$. The distribution of the lowest order statistic describes the situation in which the quality of the set of complementary inventions depends on the performance of the invention with the best quality. The distribution of the lowest order statistic is based on a general cumulative distribution $G(c)$, where $\overline{G}(c) = 1 - G(c)$.

Although the number of inventors is integer valued, it is useful to characterize examples of the cumulative distribution as a continuous function of the number of inventions n. For all the distributions in the table, $F_n(c;n) > 0$ and $F_{nn}(c;n) < 0$. Diminishing marginal returns $F_{nn}(c;n) < 0$ implies that $F(c;n) - F(c;n-1) > F(c;n+1) - F(c;n)$.

Distribution	**Support**	$\mathbf{F(c;n)}$	$\mathbf{F_n(c;n)}$	$\mathbf{F_{nn}(c;n)}$
Uniform	$[0, b + \frac{1}{n}]$, $b > 0$	$\frac{nc}{bn+1}$	$\frac{c}{(bn+1)^2}$	$\frac{-2bc}{(bn+1)^3}$
Exponential	$(0, \infty)$	$1 - e^{-nc}$	ce^{-nc}	$-c^2 e^{-nc}$
Pareto	$[1, \infty)$	$1 - c^{-n}$	$c^{-n} \ln c$	$-c^{-n}(\ln c)^2$
Power Law	$(0, 1 + \frac{1}{n}]$, $0 < a < 3$	$(1 + \frac{1}{n})^{-a} c^a$	$(1 + \frac{1}{n})^{-a-1} \frac{a}{n^2} c^a$	$(1 + \frac{1}{n})^{-a-2} \frac{a}{n^4} c^a$
Lowest order statistic	$[0, \infty)$	$1 - (\overline{G}(c))^n$	$-(\overline{G}(c))^n \ln \overline{G}(c)$	$-(\overline{G}(c))^n [\ln \overline{G}(c)]^2$

TABLE 2. Distribution functions that satisfy the assumptions on $F(c;n)$.

PROOF OF PROPOSITION 3. (i) The profit-maximizing number of projects N^* satisfies $\Pi(N^*) - \Pi(N^* - 1) \geq k > \Pi(N^* + 1) - \Pi(N^*)$. By diminishing returns to invention, it follows that $F(c;n) - F(c;n-1) \leq \frac{F(c;n)}{n}$, see Lemma 1 in the Appendix. This implies that $\Pi(n) - \Pi(n-1) \leq V(n)$ so that $n^* \geq N^*$. (ii) Expected unit cost

is

$$E[\widetilde{c} \mid n] = \int_0^{C_0} cf(c,n)dc + c_0[1 - F(C_0,n)] = c_0 - [c_0 - C_0]F(C_0,n) - \int_0^{C_0} F(c;n)dc,$$

which is decreasing in n. The expected quality of the set of inventions is increasing in n, so $n^* \geq N^*$ implies that the expected quality of the set of complementary inventions is greater with competing complementary inventors than with a multi-project monopoly inventor, $E[\widetilde{c} \mid n^*] \leq E[\widetilde{c} \mid N^*]$. (iii) Total expected inventor profit equals $nV(n) = \int_0^{C_0} q^M(c)F(c;n)dc$, which is increasing in n, so that $n^*V(n^*) \geq N^*V(N^*) = \Pi(N^*)$. (iv) The expected market price is

$$p(n) = \int_0^{C_0} P(q^M(c))f(c,n)dc + c_0[1 - F(C_0,n)] = c_0 - \int_0^{C_0} P'(q^M(c))\frac{dq^M(c)}{dc}F(c;n)dc.$$

The expected price is decreasing in the number of inventions because $\frac{dq^M(c)}{dc} \leq 0$ and $F_N(c;n) > 0$. So, $n^* \geq N^*$ implies that $p(n^*) \leq p(N^*)$. Expected industry output is increasing in the number of inventions,

$$q(n) = \int_0^{C_0} q^M(c)f(c;n)dc + Q^0[1 - F(C_0,n)] = Q^0 - \int_0^{C_0} \frac{dq^M(c)}{dc}F(c;n)dc. \quad (24)$$

So, $n^* \geq N^*$ implies that $q(n^*) \geq q(N^*)$. □

PROOF OF PROPOSITION 7. If the set of complementary inventions is drastic, $c \leq c_D$, incentives to invent are greater with downstream competition than with downstream monopoly,

$$R^*(c) - C(X^M(c)) \leq \frac{P(Q^M(c)) - c}{n}Q^M(c) - C(Q^M(c)) - \frac{b(c_0)}{n} < v(c,n).$$

The first inequality follows from profit maximization by inventors. Now suppose that the set of complementary inventions is non-drastic, $c > c_D$. Note that $R^*(c) = \int_c^{c_0} \frac{X^M(z)}{n}dz$ and $v(c,n) = \int_c^{c_0} \frac{Q^0}{n}dz - C(Q^0)$. Then, by hypothesis, $R^*(c) - C(X^M(c)) \leq v(c,n)$. This also implies that $C_{00} \leq C_0$. From the definitions of $V(n)$ and $B(n)$, it follows that $V(n) \geq B(n)$. The entry conditions imply $n^* \geq n^{**}$. □

References

[1] Amir, R. and V. E. Lambson, 2000, "On the Effects of Entry in Cournot Markets," R*eview of Economic Studies*, 67, pp. 235-254.

[2] Arora A. and A. Fosfuri, 2003, "Licensing the Market for Technology," *Journal of Economic Behavior & Organization*, 52, pp. 277–295.

[3] Arrow, K. J., 1962, "Economic Welfare and the Allocation of Resources for Invention," in National Bureau of Economic Research, *The Rate and Direction of Inventive Activity*, Princeton, NJ: Princeton University Press, pp. 609-626.

[4] Bertrand, J., 1883, "Théorie Mathématique de la Richesse Sociale," *Journal de Savants*, 67, pp. 499–508.

[5] Böhm-Bawerk, E. von, 1891, *Positive Theory of Capital*, Translated from German by W.E. Smart. Macmillan, London.

[6] Buchanan, J. and Y. J. Yoon, 2000, "Symmetric Tragedies: Commons and Anticommons," *Journal of Law and Economics*, 43, April, pp. 1-15.

[7] Coase, R., 1960, "The Problem of Social Cost," *Journal of Law & Economics*, 3, October, pp. 1-69.

[8] Cournot, A. A., 1838 (1897), *Recherches sur les Principes Mathématique de la Théorie des Richesse*, English edition, N. T. Bacon, trans., *Researches into the Mathematical Principles of the Theory of Wealth,* New York: Macmillan.

[9] Davidson and Deneckere, 1986, "Long-Run Competition in Capacity, Short-Run Competition in Price, and the Cournot Model," *RAND Journal of Economics*, 17 (3), Autumn, pp. 404-415

[10] Dixit, A. K and J. E. Stiglitz, 1977, "Monopolistic Competition and Optimum Product Diversity," *American Economic Review*, 67, pp. 297–308.

[11] Economides, N. and S.C. Salop, 1992, "Competition and Integration Among Complements, and Network Market Structure," *Journal of Industrial Economics*, 40, pp. 105–123.

[12] Epstein, R. J. and P. Malherbe, 2011, "Reasonable Royalty Patent Infringement Damages after UNILOC," *AIPLA Quarterly Journal*, 39, pp. 3-34.

[13] Gawer, A. and R. Henderson, 2007, "Platform Owner Entry and Innovation in Complementary Markets: Evidence from Intel," *Journal of Economics & Management Strategy*, 16, pp. 1–34.

[14] Geradin, D., A. Layne-Farrar, and J. Padilla, 2008, "The Complements Problem Within Standard Setting: Assessing the Evidence on Royalty Stacking," *Boston University Journal of Science & Technology La*w, 14, pp. 144-176.

[15] Ghosh, A. and H. Morita, 2007a, "Free Entry and Social Efficiency under Vertical Oligopoly," *RAND Journal of Economics*, 38, pp. 541–554.

[16] Ghosh, A. and H. Morita, 2007b, "Social Desirability of Free Entry: A Bilateral Oligopoly Analysis," *International Journal of Industrial Organization*, 25, pp. 925–934.

[17] Gilbert, R. J., 2004, "Antitrust for Patent Pools: A Century of Policy Evolution," *Stanford Technology Law Review*, 3, http://stlr.stanford.edu/STLR/Articles/04_STLR_3.

[18] Gilbert, R. J. and M. L. Katz, 2011, "Efficient Division of Profits from Complementary Innovations," *International Journal of Industrial Organization*, 29, 4, July, pp. 443-454.

[19] Gilbert, R. J. and D. M. G. Newbery, 1982, "Preemptive Patenting and the Persistence of Monopoly," *American Economic Review*, 72, pp. 514-526.

[20] Goldscheider, R., 1995-1996, "Negotiation of Royalties and Other Sources of Income from Licensing," *IDEA - The Journal of Law and Technology*, 36 (1), pp. 1-18.

[21] Hagiu, A. and D. B. Yoffie, 2013, "The New Patent Intermediaries: Platforms, Defensive Aggregators, and Super-Aggregators," *Journal of Economic Perspectives*, 27 (1), Winter, pp. 45-65.

[22] Heller, M. A., 1998, "The Tragedy of the Anticommons: Property in the Transition from Marx to Markets," *Harvard University Press Law Review*, Vol. 111, 3, January, pp. 621-688.

[23] Heller, M. A.. 2008, *The Gridlock Economy: How Too Much Ownership Wrecks Markets, Stops Innovation, and Costs Lives*, New York: Basic Books.

[24] Heller, M. A. and R. S. Eisenberg, 1998, "Can Patents Deter Innovation? The Anticommons in Biomedical Research," *Science*, Vol. 280, 5964, May, pp. 698-701.

[25] Hirshleifer, J., 1983, "From Weakest-Link to Best-Shot: The Voluntary Provision of Public Goods," *Public Choice*, 41, pp. 371-386.

[26] IPNav, 2012, "Best Practices in Patent Monetization," http://www.ipnav.com/linkservid/6BB9DE27-5056-9000-03661B5708AD47ED/ accessed October 25, 2014.

[27] Jensen, R. and M. Thursby, 2001, "Proofs and Prototypes for Sale: The Licensing of University Inventions," *American Economic Review*, 91, 1, March, pp. 240-259.

[28] Kominers, S. D. and E. G. Weyl, 2012, "Holdout in the Assembly of Complements: A Problem for Market Design," *American Economic Review*, 102, pp. 360-365.

[29] Kreps, D. M. and J. A. Scheinkman, 1983, "Quantity precommitment and Bertrand competition yield Cournot outcomes," *Bell Journal of Economics*, 14 (2), Autumn, pp. 326-337.

[30] Layne-Farrar, A. and K. M. Schmidt, 2010, "Licensing Complementary Patents: 'Patent Trolls,' Market Structure, and 'Excessive' Royalties," *Berkeley Technology Law Journal*, 25, pp. 1121-1143.

[31] Layne-Farrar, A., J. Padilla and R. Schmalensee, 2007, "Pricing Patents for Licensing in Standard-Setting Organizations: Making Sense of FRAND Commitments," *Antitrust Law Journal*, 74, 3, pp. 671-706.

[32] Legros, P. and S. A. Matthews, 1993, "Efficient and Nearly-Efficient Partnerships," *Review of Economic Studies*, 60 (3), July, pp. 599-611.

[33] Lemley, M. A. and C. Shapiro, 2007, "Patent Holdup and Royalty Stacking," *Texas Law Review,* 85 (7), pp. 1991–2049.

[34] Lerner J. and J. Tirole, 2004, "Efficient Patent Pools," *American Economic Review*, 94, pp. 691-711.

[35] Llanes, G., and J. Poblete, 2014, "Ex Ante Agreements in Standard Setting and Patent-Pool Formation," *Journal of Economics & Management Strategy*, 23, 1, Spring, pp. 50–67.

[36] Llanes, G. and S. Trento, 2012, "Patent Policy, Patent Pools and the Accumulation of Claims," *Economic Theory*, 50, 3, August, pp. 703-725.

[37] Llobet, G. and J. Padilla, 2014, "The Optimal Scope of the Royalty Base in Patent Licensing," Working Paper, CEMFI.

[38] Mankiw, N. G. and M. D. Whinston, 1986, "Free Entry and Social Inefficiency," *RAND Journal of Economics*, 17, pp. 48–58.

[39] Ménière, Y. and S. Parlane, 2010, "Decentralized Licensing of Complementary Patents: Comparing the Royalty, Fixed-fee and Two-part Tariff Regimes," *Information Economics and Policy*, 22, pp. 178–191.

[40] Milgrom, P. and J. Roberts, 1990, "The Economics of Modern Manufacturing,' *American Economic Review*, 80, pp. 511–528.

[41] Milgrom, P. and J. Roberts, 1995, "Complementarities and Fit: Strategy, Structure, and Organizational Change in Modern Manufacturing," *Journal of Accounting and Economics*, 19, pp. 179–208.

[42] Mukherjee, A., 2012, "Social Efficiency of Entry with Market Leaders," *Journal of Economics & Management Strategy*, pp. 431–444.

[43] Mukherjee, A. and S. Mukherjee, 2008, "Excess-Entry Theorem: The Implications of Licensing," *The Machester School*, 76 (6), December, pp. 675-689.

[44] Radauer, A. and T. Dudenbostel, 2013, *PATLICE Survey: Survey on Patent Licensing Activities by Patenting Firms*, European Commission, Directorate-General for Research and Innovation, EUR 26114 EN, http://ec.europa.eu/research/innovation-union/pdf/patlice-survey.pdf, Accessed September 1, 2014.

[45] Rice, B., 2006, "Selling Your Patent: What You Need To Know," September, http://www.legalzoom.com/intellectual-property-rights/patents/selling-your-patent-what-you, Accessed May 5, 2014

[46] Rosenberg, N., 1982, *Inside the Black Box: Technology and Economics*, Cambridge UK: Cambridge University Press.

[47] Schmidt, K. M., 2014, "Complementary Patents and Market Structure," *Journal of Economics and Management Strategy*, 23, 1, pp. 68-88.

[48] Schmookler, J., 1959, "Bigness, Fewness, and Research," *Journal of Political Economy*, 67, 6, December, pp. 628-632.

[49] Schmookler, J., 1962, "Economic Sources of Inventive Activity," *Journal of Economic History*, 22, pp. 1-20.

[50] Shaked, M. and J. G. Shanthikumar, 2007, *Stochastic Orders*, New York: Springer.

[51] Shapiro, C., 2001, "Navigating the Patent Thicket: Cross Licenses, Patent Pools and Standard Setting," in Adam Jaffe, Joshua Lerner, and Scott Stern, eds., *Innovation Policy and the Economy*, Cambridge, MA: National Bureau of Economic Research, vol. 1, pp. 119–150.

[52] Siegel, D. S., D. A Waldman, L. E Atwater, and A. N Link, 2004, "Toward a Model of the Effective Transfer of Scientific Knowledge from Academicians to Practitioners: Qualitative Evidence from the Commercialization of University Technologies," *Journal of Engineering and Technology Management*, 21, Issues 1–2, March–June, pp. 115-142.

[53] Singh, N. and X. Vives, 1984, "Price and Quantity Competition in a Differentiated Duopoly," *Rand Journal of Economics*, 15, pp. 546-554.

[54] Sonnenschein, H., 1968, "The Dual of Duopoly Is Complementary Monopoly: or, Two of Cournot's Theories Are One," *Journal of Political Economy*, 76, pp. 316-318.

[55] Spulber, D. F., 2008, "Innovation and International Trade in Technology," *Journal of Economic Theory*, 138, January, pp. 1-20.

[56] Spulber, D. F., 2010, "The Quality of Innovation and the Extent of the Market," *Journal of International Economic*s, 80, pp. 260-270.

[57] Spulber, D. F., 2012, "Tacit Knowledge with Innovative Entrepreneurship," *International Journal of Industrial Organization*, 30, 6, pp. 641-653.

[58] Spulber, D. F., 2013a, "Competing Inventors and the Incentive to Invent," *Industrial and Corporate Change*, 22 (1), February, pp. 33-72.

[59] Spulber, D. F. 2013b, "How do Competitive Pressures affect Incentives to Innovate when there is a Market for Inventions?," *Journal of Political Economy*, 121, 6, pp. 1007-1054.

[60] Spulber, D. F. 2014, *The Innovative Entrepreneur*, Cambridge: Cambridge University Press.

[61] Stasik, E., 2010, "Royalty Rates And Licensing Strategies For Essential Patents On LTE (4G) Telecommunication Standards," *Les Nouvelles*, September, pp. 114-119, http://www.lesi.org/les-nouvelles/les-nouvelles-online/2010, Accessed May 16, 2014.

[62] Suzumura, K. and K. Kiyono, 1987, "Entry Barriers and Economic Welfare," *Review of Economic Studies*, Vol. 54, pp. 157–167.

[63] Topkis, D. M., 1998, *Supermodularity and Complementarity*, Princeton: Princeton University Press.

[64] Vives, X., 1985, "On the Efficiency of Bertrand and Cournot Equilibria with Product Differentiation," *Journal of Economic Theory*, 36, pp. 166–175.

[65] Vives, X., 1999, *Oligopoly Pricing: Old Ideas and New Tool*s, Cambridge, MA: MIT Press.

[66] Vives, X., 2005, "Complementarities and Games: New Developments," *Journal of Economic Literature*, 43, June, pp. 437–479.

[67] Vives, X, 2008, "Innovation and Competitive Pressure," *Journal of Industrial Economic*s, 56 (3), September, pp. 419-469.

[68] von Weizsäcker, C. C., 1980, "A Welfare Analysis of Barriers to Entry," *Bell Journal of Economics*, 11, pp. 399–420.

[69] Weyl, E. G. and M. Fabinger, 2013, "Pass-Through as an Economic Tool: Principles of Incidence under Imperfect Competition," *Journal of Political Economy*, Vol. 121, No. 3, June, pp. 528-583.

[70] WIPO, 2005, *Exchanging Value, Negotiating Technology Licensing Agreements: A Training Manual*, WIPO publication No. 906(E), January, Geneva, Switzerland.

[71] Zeckhauser, R., 1996, "The Challenge of Contracting for Technological Information," *Proceedings of the National Academy of Sciences*, 93, November, pp. 12743-12748.

[72] Zuniga, M. P. and D. Guellec, 2009, "Who Licenses Out Patents and Why Lessons From a Busines Survey," Statistical Analysis of Science, Technology and Industry, Working Paper 2009/5, Paris: OECD, http://www.oecd.org/science/inno/42477187.pdf, Accessed August 2014.