# Horizontal Mergers of Online Firms: Structural Estimation and Competitive Effects

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**Abstract**

This paper (1) presents a general model of online price competition, (2) demonstrates that it is possible to structurally estimate the underlying parameters of the model when the number of competing firms is unknown or in dispute, (3) provides structural estimates based on UK data for personal digital assistants, and (4) uses these estimates to simulate the competitive effects of horizontal mergers. Our results suggest that competitive effects in this online market are more closely aligned with the simple homogeneous product Bertrand model than might be expected given the observed price dispersion and number of firms. Our estimates indicate that so long as two firms remain in the market post merger, the average transaction price is roughly unaffected by horizontal mergers. However, there are potential distributional effects; our estimates indicate that a three-to-two merger raises the average transaction price paid by price sensitive "shoppers" by 2.88 percent, while the average transaction price paid by consumers "loyal" to a particular firm *declines* by 1.37 percent.

# 1 Introduction

Empirical competitive effects analysis is the heart of modern economic assessments of the impact of horizontal mergers. Frequently, such analysis is used to "rationalize" definitions of relevant antitrust markets that outside observers sometimes view as being too narrow. For example, in *FTC v. Staples-Office Depot*, the FTC defined a "narrow" relevant product market that included only office superstores (Staples, Office Depot, and Office Max), while the merging parties argued that the relevant product market included a number of other retail outlets (including Walmart) that also sold office supplies. Ultimately, Judge Hogan agreed with the narrow definition advanced by the FTC. Based on empirical evidence that Staples and Office Depot charged significantly lower prices in markets where they competed head-to-head compared to markets where the only competitors were non-superstore players such as Walmart, Judge Hogan's decision ultimately blocked the merger because of empirical competitive effects analysis indicating that prices would rise as a result of the merger. Virtually identical economic issues arose in the recent Whole Foods-Wild Oats matter, where the FTC argued that "premium natural organic supermarkets" are a separate product market that excludes traditional supermarkets. While the FTC initially lost in District Court, an appeal ultimately led to a consent agreement in which Whole Foods agreed to divest 32 stores. These examples illustrate the controversial nature of and economic challenges associated with market definition and competitive effects analysis in even fairly simple brick-and-mortar retail environments.

The present paper is motivated by our view that new-economy e-retail markets further complicate market definition and competitive effects analysis in horizontal mergers. One complication is the question of the relevant number of competitors, including whether online and traditional retailers selling similar products are in the same product market. Unfortunately, this analysis requires fairly detailed information; antitrust agencies (and private parties) expend considerable resources to obtain (or to comply with requests for) these data.

In a number of instances, agencies have concluded that online and offline markets are separate antitrust markets based on these data. For instance, on 13 February 2008 the Australian Competition and Consumer Commission concluded that online and offline markets for books were separate antitrust markets.[1] Likewise, Christie and Terry (2002) suggest that in the FTC's review of a merger between Monster.com and HotJobs.com, the FTC "...determined that the 'relevant market' within which to evaluate the competitive impact of the transaction included only online job services," and that "...FTC staff appears to have concluded that bricks (traditional methods) do not sufficiently compete with clicks (online methods), so as to be in the same relevant market, at least in some instances."

Even in environments where the online channel comprises a separate relevant antitrust market, to the best of our knowledge there are no ready tools available to assess potential competitive effects of mergers involving online retailers. The absence of such tools or analyses stems, in part, from the fact that (1) e-retail sales are still relatively modest; (2) online prices display considerable price dispersion, which substantially complicates predicting the price effects of mergers; and (3) the number of (potential) competitors in the online channel is typically an unknown. Finally, as is the case in traditional markets, (4) there are a variety of models one might reasonably use to assess competitive effects, and it is known that competitive effects analysis can be sensitive to the model being used.[2]

This paper represents a first attempt to empirically examine the competitive effects of horizontal mergers in an online retail market. We stress at the outset that, like the early literature on mergers in traditional markets that heavily relied on implications of the symmetric Cournot model, our framework is best viewed as a "benchmark" that is based on stylized assumptions about the nature of online competition. These assumptions, which are

[1] See Australian Competition and Consumer Commission (2008).

[2] For instance, it is well-known that the competitive effects of horizontal mergers in traditional markets depend not only on the number of firms and whether they are symmetric, but whether the relevant oligopoly model is Cournot, Bertrand, or Hotelling. Even for a given market model, merger simulation results frequently reveal that competitive effects are sensitive to the assumed functional form for demand. See Weinberg and Hosken (2009).

discussed in more detail in the next section, include (1) online firms are symmetric, pure-play e-retailers; (2) the number of (potential) online competitors at any point in time is known to firms but not to the econometrician; and (3) there are two types of online buyers: price sensitive "shoppers," who rely on a price comparison site to find the best deal, and price insensitive "loyals," who simply visit their preferred online firm's website. It is worth noting that this benchmark environment is the standard framework for modeling e-retail competition; see Baye, Morgan, and Scholten (2006) for a survey of this literature.

Section 2 presents a general model of online price competition that nests standard models ranging from Varian (1980) to Iyer *et al.* (2005) as special cases. The model enriches existing models of online price competition, including Baye and Morgan (2001), by adding two realistic features: (1) firms pay platforms for clicks; and (2) not all clicks result in sales. Section 3 describes our econometric methodology which permits us to structurally estimate the parameters of the model using only price data when (as is typically the case in antitrust investigations) the actual number of competing firms is unknown or in dispute. Section 4 reports structural estimates of the model based on UK data for personal digital assistants, while Section 5 uses these estimates to simulate the competitive effects of horizontal mergers in an online market.

Our results suggest that, at least in some instances, competitive effects in online markets are more similar to those predicted by the simple homogeneous product Bertrand model than might be expected given the price dispersion observed in (and predicted by theoretical models of) e-retail markets. More specifically, our estimates indicate that so long as two firms remain in the online market post merger, the average transaction price is roughly unaffected by horizontal mergers. However, there are potential distributional effects; our estimates indicate that a three-to-two merger raises the average transaction price paid by price sensitive "shoppers" by 2.88 percent, while the average transaction price paid by consumers "loyal" to a particular firm *declines* by 1.37 percent. Section 6 discusses the implications of our findings for market definition and competitive effects, and highlights a number of caveats.

## 2 Model of Online Price Competition

As discussed in the introduction, we assume that the online channel comprises the relevant product market. To fix ideas, suppose this market consists of a commonly known number of firms ($N > 1$) that produce at a constant marginal cost of $m \geq 0$. Firms offer identical products for sale through their individual websites, which may have different characteristics or provide different types of service. Some consumers, who we call "loyals," value these services and purchase by directly visiting the website of their preferred firm. Other consumers, who we call "shoppers," care only about price. They first access a price comparison site to obtain a listing of the prices charged by sellers advertising at the site and click through to the firm offering the lowest price. If no prices are listed, they visit the website of a randomly selected firm.[3] All consumers have unit demand and a maximal willingness to pay of $r$.

It is widely recognized that conversion rates in online markets are low—only a fraction of consumers that click on a price at a comparison site follow through by making a purchase. To account for this, we assume that consumers are in the mood to buy with probability $\gamma \in (0, 1]$ and in the mood to merely "look" with probability $(1 - \gamma)$. Thus, $\gamma$ may be interpreted as the conversion rate—the fraction of clicks that are converted into sales. Finally, we assume that each firm attracts $L \geq 0$ loyals and that there are a total of $S > 0$ shoppers.

We now turn to the details of firm behavior. To advertise at the comparison site, a firm must pay an (explicit or implicit) amount $\phi > 0$ to list its price, plus a cost per click (CPC) of $c \geq 0$ each time a consumer clicks on its price advertisement (listing). Thus, firm $i$'s strategy consists of a continuous pricing decision ($p_i$) and a zero-one decision to advertise its

[3] See Proposition 1 in Baye and Morgan (2001) for the sorts of arguments required to ensure that this is an optimal decision rule for shoppers.

price at the comparison site. Let $\alpha_i$ denote the probability that firm $i$ chooses to advertise on the comparison site. A firm that does not advertise its price on the comparison site avoids paying listing and clickthrough fees, but at the potential cost of failing to attract the shoppers visiting the comparison site.

We first characterize the symmetric equilibrium pricing and advertising strategies of firms competing in this online environment (see the Appendix for a proof).

**Proposition 1** *Suppose* $0 < \phi < S\left((r-m)\gamma\frac{N-1}{N} - c\right)$ *and* $0 \leq c < (r-m)\gamma\frac{N-1}{N}$. *Then in a symmetric Nash equilibrium:*

*(a) Each firm lists its price on the comparison site with probability*

$$\alpha^* = 1 - \left(\frac{\phi}{S\left((r-m)\gamma\frac{N-1}{N} - c\right)}\right)^{\frac{1}{N-1}} \in (0,1)$$

*(b) Conditional on listing a price at the comparison site, a firm's advertised price may be viewed as a random draw from*

$$F^*(p) = \frac{1}{\alpha^*}\left(1 - \left(\frac{(r-p)\lambda L + \frac{N\phi}{((r-m)\gamma(N-1)-Nc)}((r-m)\gamma - c)}{S((p-m)\gamma - c)}\right)^{\frac{1}{N-1}}\right) \tag{1}$$

*on* $[p_0, r]$, *where* $p_0 = m + \frac{1}{(S\gamma + L\lambda)}\left(\lambda L(r-m) + \frac{N\phi}{((r-m)\gamma(N-1)-Nc)}((r-m)\gamma - c) + Sc\right) \in (m, r)$.

*(c) A firm that does not advertise on the comparison site charges a price of* $p_i = r$ *on its own website.*

*(d) Each firm earns expected profits of*

$$E\pi = (r-m)\lambda L + \frac{\phi}{N\left(1 - \frac{c}{(r-m)\gamma}\right) - 1}$$

Notice that this model extends the original Baye and Morgan (2001) model to an environment in which all transactions take place online, and accounts for clickthrough fees as well as conversion rates that are potentially less than unity. Consistent with the empirical literature,[4] the model implies that prices listed at the comparison site are necessarily dispersed

[4] See Baye et al. (2006) for a survey of about twenty studies documenting price dispersion of 10 to 50 percent in online markets.

in equilibrium, and that the number of firms actually listing prices at the comparison site on any given date is generally less than the total number of firms in the market. Thus, the actual number of firms ($N$) at a point in time is unobservable to the econometrician. This model nests a variety of other models as special cases, including Rosenthal (1980), Varian (1980), Narasimhan (1988), Iyer and Pazgal (2003), Baye, *et al.* (2004), and Iyer, *et al.* (2005).

Under the maintained hypothesis that firms' prices are distributed according to equation (1), it is, in principle, possible to estimate the underlying parameters of the model. Unfortunately, data from price comparison sites indicate $A$, the number of firms choosing to list prices at the site at a given time, but not $N$, the total number of firms in the market. The model indicates that $A$ is a binomially distributed random variable with parameter $\alpha$, whereas $N$ is a constant. The extant literature mostly finesses this problem. For example, Baye, Morgan and Scholten (2006) as well as Moraga-Gonzalez and Wildenbeest (2008) use the number of observed prices as a proxy for $N$, in effect assuming that $N = A$. Hong and Shum (2006) assume that $N = +\infty$ in their identification of price dispersion models.

The problem of the unobservability of $N$ presents econometric challenges, especially when it varies over time or across products. Perhaps more importantly, it poses a serious problem for performing competitive effects analysis of mergers where the number of "potential" competitors is often disputed by the antitrust agency and the merging parties. The next section offers a two-step estimation procedure that explicitly accommodates the unobservability of $N$.

# 3 Identification and Estimation

The above model of online price competition is essentially a low-bid auction in which the firm offering the lowest price secures the price sensitive shoppers when it lists on the comparison site. As such, we can adapt the techniques of structural estimation of auction models to our setting. Specifically, in this section we show that the equilibrium distribution of prices in Proposition 1 (along with one additional but rather mild condition) implies the identification conditions for standard auctions pioneered by Hu (2008) and An, Hu and Shum (forthcoming).

Following these authors, suppose the maximum number of (potential) firms is $K$, and is known to the econometrician. The actual number of firms $(N > 1)$ at any point in time is common knowledge to the firms, but is unknown to the econometrician. Let $A$ denote the number of price listings on a given date and consider only dates in which two or more firms listed prices. For these dates, randomly select one of the listed prices. Partition prices into $K-1$ bins, and let $Z$ denote a discretization of the randomly selected price. Thus, $Z = K-i$ means that the randomly selected price lies in the $i$th highest bin.

From the econometrician's point of view (a) $N$, $A$, and $Z$ share the same support $\{2, ..., K\}$; (b) $r, m, \lambda, \phi, c, \gamma, L$, and $S$ are unknown parameters; and (c) $N$ is unobservable or in dispute. If we let $\theta \equiv (r, m, \lambda, \phi, c, \gamma, L, S)$, then under the hypothesis that the price data at the comparison site are generated according to $F^*$ in equation (1), we may write the underlying (undiscretized) distribution of prices as $F(p|N)$ and the associated density as $f(p|N)$.[5] The lemma below shows that the equilibrium density of listed prices is independent of $A$ and $Z$. Formally,

**Lemma 1** $f(p|N) = f(p|A, Z, N)$.

**Proof.** Follows directly from the fact that firms' prices are determined prior to their knowing realizations of $A$ and $Z$. ■

Next, notice that, given the data and the model, the probability there are exactly $A \geq 2$ listings at the comparison site is

$$g(A|N) = \frac{\binom{N}{A}(\alpha)^A(1-\alpha)^{N-A}}{1-(1-\alpha)^N - N\alpha(1-\alpha)^{N-1}}$$

[5] To ease the notational burden, we have suppressed $\theta$ in this notation.

It immediately follows that

**Lemma 2** $g(A|N) = g(A|Z,N)$

Lemma 1 implies that auxiliary variables $A$ and $Z$ only affect the equilibrium density of prices through the unobservable number of firms, $N$. Analogously, Lemma 2 implies that the instrument $Z$ affects the number of listed prices only through $N$.

Let $h(p,A,Z)$ denote the observed joint density of $p$, $A$ and $Z$. Let $\psi(N,Z)$ denote the joint density of $N$ and $Z$, which is unobserved because $N$ is unobserved. This specification allows for the possibility that the true number of firms $N$ might vary across products and over time without placing parametric restrictions on the data-generating process in this respect. Now, the law of total probability implies the following relationship between the observed and latent densities:

$$\begin{aligned} h(p,A,Z) &= \sum_{N=2}^{K} f(p|N,A,Z)g(A|N,Z)\psi(N,Z) \\ &= \sum_{N=2}^{K} f(p|N)g(A|N)\psi(N,Z), \end{aligned} \tag{2}$$

where the second equality follows from Lemmas 1 and 2.

Equation (2) can be written in matrix form as follows. Let

$$\begin{aligned} H_{p,A,Z} &= [h(p,A=i,Z=j)]_{i,j} \\ G_{A|N} &= [g(A=i|N=k)]_{i,k} \\ \Psi_{N,Z} &= [\psi(N=k,Z=j)]_{k,j} \end{aligned}$$

and

$$F_{p|N} = \begin{pmatrix} f(p|N=2) & 0 & 0 \\ 0 & ... & 0 \\ 0 & 0 & f(p|N=K) \end{pmatrix} \tag{3}$$

All of these are $K-1$-dimensional square matrices. Then equation (2) becomes:

$$H_{p,A,Z} = G_{A|N}F_{p|N}\Psi_{N,Z} \tag{4}$$

Next, consider the observed joint density of $A$ and $Z$. Again, the law of total probability together with Lemma 2 implies that

$$b(A,Z)=\sum_{N=2}^{K} g(A|N;\theta)\psi(N,Z) \tag{5}$$

or, using matrix notation analogous to that above,

$$B_{A,Z}=G_{A|N}\Psi_{N,Z} \tag{6}$$

Identification requires that the following rank condition be satisfied:

**Condition 1** Rank $(B_{A,Z})=K-1$.

Since both $A$ and $Z$ are observable, Condition 1 may be verified from the data. Equation (6) implies

$$\text{Rank}\left(B_{A,Z}\right)\leq \min\left\{\text{Rank}\left(G_{A|N}\right),\text{Rank}\left(\Psi_{N,Z}\right)\right\}, \tag{7}$$

and hence, Condition 1 implies that $G_{A|N}$ and $\Psi_{N,Z}$ are invertible. This induces our key identifying equation:

$$H_{p,A,Z}\left(B_{A,Z}\right)^{-1}=G_{A|N}F_{p|N}\left(G_{A|N}\right)^{-1} \tag{8}$$

The matrix on the left-hand side can be formed from the data. The right-hand side matrix represents an eigenvalue-eigenvector decomposition of the left-hand side matrix since $F_{p|N}$ is diagonal (cf. equation (3)). This representation allows us to estimate the unknown matrices $F_{p|N}$ and $G_{A|N}$.

The theory model implies:

**Lemma 3** *The eigenvalue-eigenvector decomposition in equation (8) is unique.*

**Proof.** Since, for all $N$, the distribution of equilibrium prices contains a common interval in the neighborhood of $r$, it then follows that for any $i,j\in\mathcal{N}$, the set $\{(p):f(p|N=i)\neq f(p|N=j)\}$ has nonzero Lebesgue measure whenever $i\neq j$. This immediately implies the uniqueness of the eigenvalue-eigenvector decomposition. ■

With Lemma 3 in hand, it then follows that an eigenvalue decomposition of the observed $H_{p,A,Z}\left(B_{A,Z}\right)^{-1}$ matrix recovers the unknown $F_{p|N}$ and $G_{A|N}$ matrices. Here, $F_{p|N}$ is the diagonal matrix of eigenvalues, while $G_{A|N}$ is the corresponding matrix of eigenvectors. Of course, $F_{p|N}$ and $G_{A|N}$ are only identified up to a normalization and ordering of the columns of the eigenvector matrix $G_{A|N}$. There is a clear, appropriate choice for the normalization of the eigenvectors because each column of $G_{A|N}$ should add up to one. The model also implies a natural ordering for the columns of $G_{A|N}$, since in the model $A \leq N$ with probability one. This implies that for any $i < j \in \mathcal{N}$, $g\left(A = j|N = i\right) = 0$. In other words, $G_{A|N}$ is an upper-triangular matrix, which, since it is invertible, has non-zero diagonal entries, i.e. $f\left(A = i|N = i\right) > 0$ for all $i \in \mathcal{N}$.

Finally, having recovered $G_{A|N}$, from equation $(6)$, we have

$$\Psi_{N,Z} = \left(G_{A|N}\right)^{-1} B_{A,Z}$$

and hence $\Psi_{N,Z}$ is also recovered. To summarize, we have shown:

**Proposition 2** *Suppose Condition 1 holds. Then $F_{p|N}$, $G_{A|N}$ and $\Psi_{N,Z}$ are identified (the former pointwise in $p$).*

We now describe how one may use the identification argument to estimate the model, given data from a price comparison site. Let $t$ index each set of price observations. For each $t$, we observe $A_t$, the number of firms choosing to list their prices at the comparison site, and let $p_i,\ i = 1, \ldots, A_t$ denote the $A_t$ listed prices of product $i$. Our estimation procedure accounts for the fact that $N_t$ is known to the competing firms at time $t$ but is, in effect, a random variable from the perspective of the econometrician. While we cannot recover the specific value of $N_t$ pertaining to each set of prices at each point in time, we are able to recover its marginal distribution.[6]

[6] Indeed, if we knew that $N$ is fixed across all the price observations, we could just set $N = E[A_t]/\alpha$ in Eq. (1).

To estimate the vector of parameters $\theta$, we use the following two-step estimation procedure: In the first step, we use our key equation (8) to nonparametrically estimate $G_{A|N}$. In the second step, based on the parametric form of $F(p|N;\theta)$ in equation (1) and the estimation in the first step, we recover the vector of parameters $\theta$ by MLE.

**Step One** We first describe how to use observable data on prices $(p)$ and the number of listing firms $(A)$ to estimate $G_{A|N}$. Our methodology closely parallels the approach taken in An, Hu and Shum (forthcoming). While the key identification equation (2) is stated in terms of the joint density $h(p, A, Z)$, faster convergence is achieved if instead we take the expectation over all prices given $(A, Z)$. Specifically, let $E[p|A, Z] = \int p\frac{h(p,A,Z)}{b(A,Z)}dp$, i.e. the expected price conditional on some realization $A$, $Z$. It then follows from equation (2) that

$$E[p|A, Z]\, b(A, Z) = \sum_{N=2}^{K} E[p|N] \times g(A|N)\psi(N, Z)$$

where $E[p|N] = \int pf(p|N)\, dp$.

Now define the matrices:

$$H_{Ep,N,Z} \equiv [E(p|A = i, Z = j)\, b(A = i, Z = j)]_{i,j}\,, \tag{9}$$

and

$$F_{Ep|N} \equiv \begin{pmatrix} E[p|N=2] & 0 & 0 \\ 0 & ... & 0 \\ 0 & 0 & E[p|N=K] \end{pmatrix}.$$

Then, we have

$$H_{Ep,A,Z} = G_{A|N}F_{Ep|N}\Psi_{N,Z}$$

which is analogous to equation (4). Similarly, we can obtain the estimating equation by postmultiplying both sides of this equation by $B_{A,Z}^{-1}$. This yields the analogous identification equation:

$$H_{Ep,A,Z}\,(B_{A,Z})^{-1} = G_{A|N}F_{Ep|N}\left(G_{A|N}\right)^{-1} \tag{10}$$

Consequently,

$$G_{A|N} = \zeta\left(H_{Ep,A,Z}\left(B_{A,Z}\right)^{-1}\right),$$

where $\zeta(\cdot)$ denotes the mapping from a square matrix to its eigenvector matrix.[7] Following Hu (2008), we may estimate the relevant matrices using sample averages:

$$\widehat{G}_{A|N} \equiv \zeta\left(\widehat{H}_{Ep,A,Z}\left(\widehat{B}_{A,Z}\right)^{-1}\right), \tag{11}$$

where

$$\widehat{H}_{Ep,A,Z} = \left[\frac{1}{T}\sum_t \frac{1}{A_j}\sum_{i=1}^{A_j} p_{it}\mathbf{1}(A_t = A_j, Z_t = Z_k)\right]_{j,k}. \tag{12}$$

Finally, let $\mathbf{g}(A)$ be a vector of marginal probabilities over the number of listings and let $\mathbf{\Gamma}_N$ denote the vector of the unknown frequency distribution of $N$. Then

$$\mathbf{g}(A) = G_{A|N}\mathbf{\Gamma}_N$$

and we may estimate the unknown distribution $\mathbf{\Gamma}_N$ using the data as follows:

$$\widehat{\mathbf{\Gamma}}_N = \left(\widehat{G}_{A|N}\right)^{-1}\hat{\mathbf{g}}(A) \tag{13}$$

where $\hat{\mathbf{g}}(A)$ denotes the empirical frequency of the number of listings.

**Step Two** In the first step, we obtained estimates of $G_{A|N}$ and $\Gamma_N$ nonparametrically. In the second step, we combine these estimates with the equilibrium restrictions on the price distribution from Proposition 1 to obtain estimates of the model's structural parameters, $\theta$.

Let $l(p, A; \theta)$ denote the joint density of prices and number of listings, and let $\Gamma(N)$ represent the unknown frequency distribution of $N$. Since $g(A|N;\theta)$ and $f(p|N;\theta)$ are conditionally independent, this density may be written as

$$\begin{aligned} l(p, A; \theta) &= \sum_{N=2}^{K} g(A|N;\theta)f(p|N;\theta)\Gamma(N) \\ &= e_A G_{A|N} F_{p|N} \mathbf{\Gamma}_N \end{aligned}$$

[7] Note that if the distribution of listed prices is such that the average price is monotonically ordered in $N$, then an analog of Lemma 3 holds for expected prices as well. This guarantees that $\zeta$ is a unique mapping. While Lemma 3 guarantees this for each $p$, technically, we are assuming that an analogous condition holds for expected prices. That is, $E[p|N]$, which are the eigenvalues from this decomposition, are distinct for every $N$.

where $e_A = (0, 0, ..., 1, ..., 0)$ is a row vector where the 1 appears as the $A$th element. Hence the likelihood function $\mathcal{L}$ for the $t$-th set of prices is

$$\begin{aligned}\mathcal{L} &= \prod_{i=1}^{A_t} l(p_i, A_t; \theta) \\ &= \prod_{i=1}^{A_t} e_{A_t} G_{A_t|N} F_{p_i|N} \mathbf{\Gamma}_N\end{aligned}$$

Using the first step estimates, we can write this as

$$\begin{aligned}\ln \mathcal{L} &= \ln \widehat{l}(p_i, A_t; \theta) \\ &= \sum_{i=1}^{A_t} \ln \left( e_{A_t} \widehat{G}_{A_t|N} F_{p_i|N;\theta} \widehat{\mathbf{\Gamma}}_N \right)\end{aligned}$$

where $F_{p_i|N;\theta}$ is a diagonal matrix with diagonal element $f\,(p|N;\theta)$. From equation (1), it may be shown that the density associated with $F^*\,(p)$ is given by

$$\begin{aligned}f^*\,(p|N;\theta) &= \frac{1}{N-1}\left(F^*\,(p|N;\theta) - \frac{1}{\alpha^*}\right) \\ &\quad \times \left( \frac{\lambda L}{(r-p)\,\lambda L + \frac{N\phi}{((r-m)\gamma(N-1)-Nc)}\,((r-m)\,\gamma - c)} - \frac{\gamma}{(p-m)\,\gamma - c} \right)\end{aligned}$$

for $p \in [p_0, r]$ and zero otherwise. Note that $\widehat{G}_{A_t|N}$ and $\widehat{\mathbf{\Gamma}}_N$ are estimated using the data, whereas $F_{p_i|N;\theta}$ is based on the theory model (we have added $\theta$ to the subscript of $F_{p_i|N}$ to emphasize the dependence on $\theta$, which will be selected so as to maximize the likelihood function).[8]

# 4 Data and Parameter Estimates

We apply the above estimation procedure to UK online price data obtained from Kelkoo.com for firms selling personal digital assistants. These data, which are described in detail in Baye *et al.* (2009), include the daily prices charged by firms selling 18 models of personal digital assistants (PDAs) over the period from 18 September 2003 through 6 January 2004. During

[8] We do not estimate $c$ because we have data on clickthrough fees, as discussed below.

this period, an average of four firms sold each product at the comparison site, so on the surface this market might appear to be fairly concentrated. Our estimation is based on clickthrough fees at Kelkoo.com of $c = .20$ (20 pence per click).

Since the first step of our estimation procedure (estimating $G_{A|N}$ nonparametrically) requires a large number of observations, we pool across all 18 products in both the first and second step of our estimation procedure to estimate a common parameter vector, $\theta$. Owing to a paucity of observations where the number of listings exceeds 10, we combine observations where more than 10 firms list prices into a single bin.[9] Hence, $G_{A|N}$ is a $10 \times 10$ matrix for purposes of estimation, with the first 9 columns corresponding to $N = 2, ...10$ and the last bin corresponding to $N > 10$.[10]

Table 1 reports the results of the first-stage estimation. Each cell in the table corresponds to the estimated probability that there are $A$ firms listing prices on the comparison site when the population of firms is $N$. In theory, each cell represents the probability of a given conditional outcome, although the estimation procedure places no constraints that require this. Notice that all of the cells in this matrix are well-defined probabilities.

We now turn to the step 2 results. Recall that $c$ is known data and not a parameter to be estimated. Following Baye and Morgan (2001), we set $L \equiv M/N$, so that $M$ (the parameter to be estimated) represents the total number of loyal consumers in the market, and $L$ is the (unobserved) number of loyals per firm on a given product-date. The resulting parameter estimates, along with bootstrapped standard errors, are reported in Table 2. The monetary parameters ($r, m$ and $\phi$) are denominated in GBP. As the table reveals, all of the parameters are precisely estimated. The parameter estimates indicate that, on an average day, a total of $M = 26.04$ consumers in the UK who are loyal to some online firm were interested in purchasing a PDA online, while $S = 13.16$ consumers were interested in purchasing online from the firm charging the best price. These estimates imply that about 34 percent of

[9] The maximum number of listings observed is 15.

[10] Baye and Morgan (2009) show that in an analogous model, the equilibrium price distribution as $N \to \infty$ is similar to that for finite values of $N$ near the lower end of this last bin.

consumers in this online market are price-sensitive shoppers, while 66 percent are loyals. It is interesting to contrast our estimates with those of Brynjolfsson, Montgomery, and Smith (2003), who find that around 13% of consumers in US e-retail markets are shoppers. Given the relatively less-developed state of e-retail in the UK compared to the US at the time our data was collected, it is not altogether surprising to find that fewer UK customers had become "attached" to a particular online retailer.

The estimated conversion rate, $\gamma = .15$, implies that a firm listing on Kelkoo.com has to receive, on average, 6.67 clicks in order to generate one sale. At a cost of 20 pence per click, this translates into an average cost per sale of 1.33 GBP in addition to the fixed listing fee of $\phi = 4.88$ GBP. Finally, notice that the estimated monopoly markup for a PDA, $(r - m)/m$, is about 66 percent.

# 5 Competitive Effects Analysis

The econometric framework described above, along with the structural estimates of the model of online price competition, permits us to address a number of issues that arise in the evaluation of the competitive effects of a potential horizontal merger of online firms. As discussed above, the data obtained from Kelkoo.com might lead one to conclude that the PDA market on Kelkoo.com is highly concentrated, since only four firms list prices for a given product on an average day. The heart of many disagreements between antitrust agencies and merging parties centers around the "correct" number of potential competitors in the relevant market, as well as the "correct" definition of the relevant market to use in conducting an analysis of the merger. In the case at hand, discussions between parties and the agency would involve the extent to which other retailers—those not presently listing at the site or utilizing other platforms (such as other comparison sites, pure-play brick and mortar firms, or the websites of individual firms)—should be included in the set of potential competitors. It is, of course, costly for parties and agencies to document the exact number of competitors in the market at the time of a proposed merger, to obtain historical information on the number of competitors and to predict potential entry. Our econometric framework permits us to recover deep structural parameters without this information.

Of course, were the UK e-retail market well-approximated by the homogenous product Bertrand model of competition, uncertainty regarding the number of firms would be moot for purposes of merger analysis. So long as at least two firms remain in the market post-merger, consolidation produces no unilateral competitive effects. This begs the obvious question—is Bertrand competition a good model for the UK e-retail market? Recall that a key implication of the homogeneous product Bertrand model is the law of one price. This prediction, however, is grossly at odds with the data. Indeed, a robust finding in global e-retail markets is that price dispersion is ubiquitous and persistent; see Baye, Morgan, and Scholten (2006) for a survey. Moreover, the degree of price dispersion is known to vary with the number of firms in the market. Thus, one might expect a merger to impact price dispersion, market power, and consumer welfare.[11] Our structural approach permits us to quantify competitive effects with limited data, as is the case when an antitrust agency does not wish to burden third parties with significant data requests.

To accomplish this, we first substitute the parameter estimates reported in Table 2 into the expressions summarizing equilibrium behavior in Proposition 1; below we use carets to denote the resulting estimates. Next, we calculate the implied average prices conditional on a given number of firms and display them in Table 3. Column (a) in Table 3 indicates the total number of firms in the relevant market ($N$), which is potentially in dispute. Column (b) provides the estimated average price listed at the comparison site conditional on different numbers of competitors, where the average listed price is

$$E\left[p\right] = \int_{\widehat{p_0}}^{\widehat{r}} p d\widehat{F^*}\left(p\right).$$

As would be expected, Table 3 shows that the estimated average listed price declines as the

[11] See Baye, et al. (2004) for evidence of the relationship between various measures of price dispersion and the number of competing e-retail firms.

number of firms increases—rather abruptly as one moves from monopoly to a duopoly, and modestly thereafter. Column (c) reports the estimated average minimum listed price, which is given by

$$E\left[p_{\min}\right] = \frac{1}{1-\left(1-\widehat{\alpha^*}\right)^N} \sum_{A=1}^{N} \binom{N}{A} \left(\widehat{\alpha^*}\right)^A \left(1-\widehat{\alpha^*}\right)^{N-A} \int_{\widehat{p_0}}^{\widehat{r}} pA \left(1-\widehat{F^*}(p)\right)^{A-1} d\widehat{F^*}(p)$$

Notice that this calculation takes into account the effect of a change in $N$ on the equilibrium distribution of prices, firms' propensities to advertise prices at the comparison site, and the impact of a larger number of listings on the minimum order statistic. Accounting for this, Column (c) of Table 3 shows that the estimated average minimum listed price also declines as the number of firms increases.

While it might be tempting to base competitive effects analysis on these average prices (presuming the average prices are relevant for loyals and the average minimum prices are relevant for shoppers), this would be incorrect: Neither of these averages represents average transaction prices. To calculate the average transaction price paid by loyals, one needs to account for a firm's propensity to list prices on the comparison site. In particular, when a firm does not list on the comparison site, it charges the monopoly price at its own website. Thus, the average transaction price paid by a loyal customer is

$$E\left[p^L\right] = \widehat{\alpha^*} E\left[p\right] + \left(1-\widehat{\alpha^*}\right)\widehat{r}.$$

Column (d) of Table 3 reports the estimated average transaction prices of loyal consumers. Notice that it declines abruptly as one moves from monopoly to duopoly, but then rises as the number of firms increases further.

Likewise, the average transaction price for shoppers must also account for listing decisions: The average transaction price paid by a price-sensitive shopper is given by

$$E\left[p^S\right] = \left(1-\left(1-\widehat{\alpha^*}\right)^N\right) E\left[p_{\min}\right] + \left(1-\widehat{\alpha^*}\right)^N \widehat{r}$$

Column (e) of Table 3 reports the estimated average transaction price of shoppers, which declines as the number of firms increases.

Columns (d) and (e) highlight that shoppers and loyals are impacted differently by heightened competition: So long as there are at least two firms in the market, loyal consumers are harmed by heightened competition, while shoppers are unambiguously made better off by increased competition. The overall transaction price, reported in Column (f) of Table 3, is merely a weighted average of the shoppers' and loyals' estimated transaction prices, where the weighting factor is determined by the estimated fraction of consumers who are shoppers and loyals:

$$E\left[p^T\right] = \frac{\widehat{M}}{\widehat{S}+\widehat{M}} E\left[p^L\right] + \frac{\widehat{S}}{\widehat{S}+\widehat{M}} E\left[p^S\right]$$

In summary, the estimates in Table 3 reveal that the average listed price and the average minimum listed price both decline as the number of firms declines. This is consistent with standard reasoning, which suggests that heightened competition leads to lower prices. However, this ignores the endogenous listing decisions of firms, which is, of course, relevant for the *transaction* prices paid by consumers. Here, a more subtle story emerges. Both shoppers and loyals pay lower average transaction prices as the online market moves from monopoly to duopoly. Thereafter, the effects of increased competition diverge: Loyal consumers are harmed (pay higher average transaction prices) as the number of firms further increases, while shoppers benefit from heightened competition.

Table 4 uses the results in Table 3 to simulate the competitive effects of a merger from $N$ to $N-1$ firms, where column (a) represents the post-merger number of firms. Obviously, the direction of the price changes is identical to that in Table 3, but it is instructive to examine the implied percentage changes in prices to highlight the potential value of our methodology. Suppose first that the antitrust agency and the parties agree that the appropriate welfare standard is one that aggregates shoppers and loyals, such that the average transaction prices displayed in column (f) of Table 4 are relevant. Then so long as there is agreement that the merger does not result in monopoly, a merger between two online firms will not harm the “average” online consumer. This conclusion is based on the assumption that firms in the online channel do not compete against firms in other channels. Thus, there is no need for the agency to examine claims by the parties that “there are many potential online firms” or that “brick and mortar firms are also in the relevant market.” In effect, column (f) reveals that—even though models of online competition are more complex than standard homogenous product Bertrand competition and the “law of one price” does not hold online—the conclusions based on our estimates are similar to what one would have concluded based on the simple Bertrand model, at least in this particular online market: There are no adverse competitive effects of a horizontal merger in this online market so long as it stops short of merger to monopoly.

Notice that since our analysis takes as its maintained hypothesis that the relevant market is the online channel, our approach is “biased” in favor of finding competitive effects. Since the evidence suggests there are none, there would appear to be little value to an antitrust agency (or to the parties) of expending resources collecting the additional information needed to determine whether offline firms discipline the prices charged by online firms.

The results in Table 4 also highlight a potential problem that could arise in the evaluation of horizontal mergers, owing to differences in price effects for shoppers and loyals. Recall that, in the estimated model, loyals never frequent the comparison site, while shoppers always shop there first. As a consequence, the plot thickens when an antitrust agency opts for a more narrowly defined relevant market (the price comparison site only) or focuses on harm to a subset of consumers (shoppers only). In these circumstances, the estimates in columns (c) and (e) of Table 4 become relevant.

Notice that column (c) represents the average price paid by a shopper that buys through the comparison site, and this rises as the number of (post-merger) firms shrinks. It follows that if the agency and the parties agree that the merger is not a merger to monopoly, so long as the agency’s tolerance for a price increase is 3 percent, there is again no value in examining whether other channels also compete or in conducting a detailed analysis of the actual number of firms in the market—even if the agency's focus is on a narrow market definition that includes only transactions through the comparison site. This is because, regardless of the actual number of firms, there is agreement that the estimated transaction price at the comparison site would rise by no more than 2.93% post merger.

Similarly, if the agency focused on consumer harm to one consumer group—price sensitive shoppers—the estimates in column (e) become relevant. In this case, so long as the agency's tolerance for a price increase is at least 2.88% and there is agreement that at least one competitor remains in the market post merger, there is no need for either party to spend resources attempting to resolve uncertainty regarding the actual number of competitors or whether other channels are in the relevant market.

# 6 Discussion

It is important to stress that the results highlighted above—(1) evidence that online markets are less vulnerable to adverse competitive effects from horizontal mergers than one might expect given the plethora of papers documenting significant price dispersion in online markets, and (2) mergers in online retail markets have opposite effects on the price sensitive shopper and loyal segments of consumers, and (3) harm to shoppers is no greater than 3 percent and is almost exactly offset by benefits to loyals—are based on data from one e-retail market in the UK. While we believe that the models and techniques developed in this paper are useful more generally, it would be a mistake to infer from our analysis that horizontal mergers are never problematic in online retail markets. We conclude by mentioning just a handful of caveats that highlight directions in which we hope to ultimately stretch our analysis.

First, the econometric technique developed in this paper is very data intensive, which precluded us from incorporating additional parameters to account for downward sloping demand and product/firm asymmetries. Second, the model assumes that, post merger, all firms reposition such that they each get an identical share of the (fixed) number of loyal consumers. As a purely theoretical matter, it is quite difficult to obtain closed-form solutions for equilibrium price distributions in the presence of such asymmetries.[12] Thus, our analysis does not account for competitive effects that might arise as a result of a firm gaining a disproportional share of loyals through mergers. Our analysis should be viewed as a first step to better understanding the competitive effects of horizontal mergers in online retail markets rather than the final word on the subject.

Apart from asymmetries, the model and estimation are subject to several other limitations or restrictions. Since we do not have data on consumer choice behavior outside the price comparison site, we are relying strongly on the theory model and structural estimation to infer the total number of loyals in the market. Moreover, we are inferring firm pricing behavior at the comparison site the pricing behavior of firms at their own websites. The structural model rules out the possibility that the firm might price discriminate between consumers visiting the site directly and those routed there through the price comparison site. While, in principle, the firm could offer different prices for these two different channels, in practice firms do not do this, probably for reputational reasons. Allowing for price discrimination would obviously affect equilibrium pricing behavior on the comparison site and hence affect our estimates.

[12] See Arnold et al. (forthcoming) for an asymmetric version of the Baye and Morgan (2001) model with two firms.

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# A Appendix

**Proof of Proposition 1:**

As in Baye and Morgan (2001), it is readily seen that equilibrium has the following two key properties: (1) A firm must be indifferent between listing its price at the clearinghouse or not; and (2) a firm must earn the same expected payoff from posting any price $p \in [p_0, r]$ at the clearinghouse.

A firm that eschews the comparison site earns profits of

$$\pi_0 = (r-m)\,\lambda L + (r-m)\,(1-\alpha)^{N-1}\,\frac{\gamma S}{N} \tag{14}$$

A firm that advertises a price $r$ on the site earns

$$\pi = (r-m)\,\lambda L + (r-m)\,(1-\alpha)^{N-1}\,\gamma S - c\,(1-\alpha)^{N-1}\,S - \phi$$

First, we equate $\pi$ and $\pi_0$ to obtain a closed-form expression for $\alpha$:

$$\begin{aligned}
(r-m)\,\lambda L + (r-m)\,(1-\alpha)^{N-1}\,\frac{\gamma S}{N} &= (r-m)\,\lambda L + (r-m)\,(1-\alpha)^{N-1}\,\gamma S - c\,(1-\alpha)^{N-1}\,S - \phi \\
(r-m)\,(1-\alpha)^{N-1}\,\frac{\gamma S}{N} &= (r-m)\,(1-\alpha)^{N-1}\,\gamma S - c\,(1-\alpha)^{N-1}\,S - \phi \\
\phi &= (1-\alpha)^{N-1}\,S\left((r-m)\,\gamma\frac{N-1}{N} - c\right)
\end{aligned}$$

Hence, we obtain

$$\begin{aligned}
(1-\alpha)^{N-1} &= \frac{\phi}{S\left((r-m)\,\gamma\frac{N-1}{N} - c\right)} \\
&= \frac{N\phi}{S\left((r-m)\,\gamma\,(N-1) - Nc\right)}
\end{aligned}$$

or

$$\alpha^* = 1 - \left(\frac{\phi}{S\left((r-m)\,\gamma\frac{N-1}{N} - c\right)}\right)^{\frac{1}{N-1}}$$

Notice that this is a well-defined probability if the fixed listing fee ($\phi$) and the clickthrough fee ($c$) are not too large:

$$0 < \phi < S\left((r-m)\,\gamma\frac{N-1}{N} - c\right)$$

and

$$0 \leq c < (r-m)\gamma \frac{N-1}{N}$$

These conditions ensure that $0 < \alpha^* < 1$.

This information alone is sufficient to determine profits in equilibrium. Substituting for $(1-\alpha)^{N-1}$ in equation (14) we obtain

$$\begin{aligned}
\pi_0 &= (r-m)\lambda L + (r-m)\frac{\phi}{S\left((r-m)\gamma\frac{N-1}{N}-c\right)}\frac{\gamma S}{N} \\
&= (r-m)\lambda L + (r-m)\frac{\gamma\phi}{((r-m)\gamma(N-1)-Nc)} \\
&= (r-m)\lambda L + \frac{\phi}{(N-1)-N\frac{c}{(r-m)\gamma}} \\
&= (r-m)\lambda L + \frac{\phi}{N\left(1-\frac{c}{(r-m)\gamma}\right)-1}
\end{aligned}$$

It remains to determine the equilibrium distribution of prices. Recall that a firm pricing at $p$ earns expected profits of

$$\pi(p) = (p-m)\lambda L + (p-m)(1-\alpha F(p))^{N-1}\gamma S - c(1-\alpha F(p))^{N-1} S - \phi$$

Such a firm must be indifferent between charging $p$ and not advertising at all. Hence

$$(p-m)\lambda L + (p-m)(1-\alpha F(p))^{N-1}\gamma S - c(1-\alpha F(p))^{N-1} S - \phi = (r-m)\lambda L + (r-m)(1-\alpha)^{N-1}\frac{\gamma S}{N}$$

We will now solve for $(1-\alpha F(p))^{N-1}$.

$$(1-\alpha F(p))^{N-1} S((p-m)\gamma - c) = (r-p)\lambda L + (r-m)(1-\alpha)^{N-1}\frac{\gamma S}{N} + \phi$$

Solving

$$\begin{aligned}
(1-\alpha F(p))^{N-1} &= \frac{(r-p)\lambda L+(r-m)(1-\alpha)^{N-1}\frac{\gamma S}{N}+\phi}{S((p-m)\gamma-c)}\\
&= \frac{(r-p)\lambda L+(r-m)\frac{N\phi}{S((r-m)\gamma(N-1)-Nc)}\frac{\gamma S}{N}+\phi}{S((p-m)\gamma-c)}\\
&= \frac{(r-p)\lambda L+(r-m)\frac{\phi}{((r-m)\gamma(N-1)-Nc)}\gamma+\phi}{S((p-m)\gamma-c)}\\
&= \frac{(r-p)\lambda L+\phi\left((r-m)\frac{\gamma}{((r-m)\gamma(N-1)-Nc)}+1\right)}{S((p-m)\gamma-c)}\\
&= \frac{(r-p)\lambda L+\frac{N\phi}{((r-m)\gamma(N-1)-Nc)}((r-m)\gamma-c)}{S((p-m)\gamma-c)}
\end{aligned}$$

Or rewriting

$$F=\frac{1}{\alpha}\left(1-\left(\frac{(r-p)\lambda L+\frac{N\phi}{((r-m)\gamma(N-1)-Nc)}((r-m)\gamma-c)}{S((p-m)\gamma-c)}\right)^{\frac{1}{N-1}}\right)$$

First, let us verify that $F(r)=1$, or equivalently, $(1-\alpha F(r))^{N-1}=(1-\alpha)^{N-1}$. To see this

$$\begin{aligned}
(1-\alpha F(r))^{N-1} &= \frac{\frac{N\phi}{((r-m)\gamma(N-1)-Nc)}((r-m)\gamma-c)}{S((r-m)\gamma-c)}\\
&= \frac{N\phi}{S((r-m)\gamma(N-1)-Nc)}
\end{aligned}$$

Next, we will calculate $p_0$, the price where $F(p)=0$. Equivalently, $(1-\alpha F(p_0))^{N-1}=1$

$$(1-\alpha F(p_0))^{N-1}=\frac{(r-p_0)\lambda L+\frac{N\phi}{((r-m)\gamma(N-1)-Nc)}((r-m)\gamma-c)}{S((p_0-m)\gamma-c)}=1$$

Cross-multiplying

$$(r-p_0)\lambda L+\frac{N\phi}{((r-m)\gamma(N-1)-Nc)}((r-m)\gamma-c)=S((p_0-m)\gamma-c)$$

Collecting the $p_0$ terms

$$\lambda Lr+\frac{N\phi}{((r-m)\gamma(N-1)-Nc)}((r-m)\gamma-c)+S\gamma m+Sc=p_0(S\gamma+L\lambda)$$

Hence

$$\begin{aligned}
p_0 &= \frac{1}{(S\gamma + L\lambda)}\left(\lambda Lr + \frac{N\phi}{((r-m)\gamma(N-1)-Nc)}((r-m)\gamma - c) + S\gamma m + Sc\right) \\
&= \frac{1}{(S\gamma + L\lambda)}\left(S\gamma m + \lambda Lm + \lambda Lr - \lambda Lm + \frac{N\phi}{((r-m)\gamma(N-1)-Nc)}((r-m)\gamma - c) + Sc\right) \\
&= m + \frac{1}{(S\gamma + L\lambda)}\left(\lambda L(r-m) + \frac{N\phi}{((r-m)\gamma(N-1)-Nc)}((r-m)\gamma - c) + Sc\right)
\end{aligned}$$

and this is more than $m$.

Last, we need to show that $F$ is increasing. Equivalently, we will show that $(1-\alpha F(p))^{N-1}$ is strictly decreasing in $p$. Recall that

$$(1-\alpha F(p))^{N-1} = \frac{(r-p)\lambda L + \frac{N\phi}{((r-m)\gamma(N-1)-Nc)}((r-m)\gamma - c)}{S((p-m)\gamma - c)}$$

Define $num \equiv (r-p)\lambda L + \frac{N\phi}{((r-m)\gamma(N-1)-Nc)}((r-m)\gamma - c) > 0$ and $den \equiv S((p-m)\gamma - c) > 0$. Differentiating with respect to $p$ reveals

$$\begin{aligned}
\frac{\partial(1-\alpha F(p))^{N-1}}{\partial p} &= -\frac{\lambda L(den) + S\gamma(num)}{(den)^2} \\
&< 0
\end{aligned}$$

## Table 1: Estimated $G_{A/N}$ Matrix

| Number of Listings ($A$) | Number of Firms ($N$) 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | > 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| (a) | (b) | (c) | (d) | (e) | (f) | (g) | (h) | (i) | (j) | (k) |
| 2 | 1.0000 | 0.9068 | 0.7141 | 0.7526 | 0.6775 | 0.3143 | 0.7010 | 0.6528 | 0.2585 | 0.2532 |
| 3 | 0 | 0.0932 | 0.2358 | 0.1311 | 0.1180 | 0.1425 | 0.0458 | 0.0427 | 0.1245 | 0.1220 |
| 4 | 0 | 0 | 0.0501 | 0.0459 | 0.0413 | 0.1293 | 0.0002 | 0.0002 | 0.1749 | 0.1713 |
| 5 | 0 | 0 | 0 | 0.0704 | 0.0634 | 0.1517 | 0.1172 | 0.1092 | 0.1381 | 0.1353 |
| 6 | 0 | 0 | 0 | 0 | 0.0998 | 0.1243 | 0.0225 | 0.0209 | 0.1040 | 0.1018 |
| 7 | 0 | 0 | 0 | 0 | 0 | 0.1380 | 0.0939 | 0.0874 | 0.1077 | 0.1055 |
| 8 | 0 | 0 | 0 | 0 | 0 | 0 | 0.0194 | 0.0181 | 0.0419 | 0.0411 |
| 9 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.0687 | 0.0279 | 0.0273 |
| 10 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.0225 | 0.0221 |
| > 10 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.0205 |

## Table 2: Parameter Estimates and Bootstrapped Standard Errors

| Parameter | Estimate | Standard Error |
|---|---|---|
| (a) | (b) | (c) |
| $\varphi$ | 4.88 | *0.139* |
| $r$ | 415.26 | *48.880* |
| $m$ | 250.09 | *12.286* |
| $M$ | 26.04 | *1.703* |
| $S$ | 13.16 | *0.648* |
| $\gamma$ | 0.15 | *0.006* |

## Table 3: Estimated Transaction Prices

| Number of Firms | Estimated Avg. Listed Price | Estimated Avg. Minimum Listed Price | Estimated Avg. Transaction Price Loyals | Estimated Avg. Transaction Price Shoppers | Estimated Avg. Transaction Price |
|---|---|---|---|---|---|
| **(a)** | **(b)** | **(c)** | **(d)** | **(e)** | **(f)** |
| 1 | 415.26 | 415.26 | 415.26 | 415.26 | 415.26 |
| 2 | 366.57 | 354.60 | 368.06 | 354.65 | 363.56 |
| 3 | 365.71 | 344.49 | 373.19 | 344.73 | 363.64 |
| 4 | 363.23 | 336.43 | 377.40 | 336.87 | 363.79 |
| 5 | 360.46 | 329.93 | 380.79 | 330.53 | 363.92 |
| 6 | 357.77 | 324.59 | 383.56 | 325.33 | 364.01 |
| 7 | 355.25 | 320.11 | 385.87 | 320.96 | 364.08 |
| 8 | 352.92 | 316.29 | 387.84 | 317.25 | 364.14 |
| 9 | 350.76 | 312.99 | 389.53 | 314.03 | 364.18 |
| 10 | 348.77 | 310.10 | 391.00 | 311.23 | 364.22 |
| 11 | 346.93 | 307.56 | 392.30 | 308.75 | 364.25 |
| 12 | 345.22 | 305.30 | 393.45 | 306.54 | 364.27 |
| 13 | 343.63 | 303.26 | 394.48 | 304.57 | 364.30 |
| 14 | 342.14 | 301.43 | 395.41 | 302.78 | 364.31 |
| 15 | 340.75 | 299.77 | 396.25 | 301.16 | 364.33 |

## Table 4: Percentage Change in Post-Merger Transaction Prices

| Number of Firms | Estimated Change in Average Listed Price | Estimated Change in Avg. Minimum Listed Price | Estimated Change in Average Transaction Price Loyals | Estimated Change in Average Transaction Price Shoppers | Estimated Change in Average Transaction Price |
|---|---|---|---|---|---|
| (a) | (b) | (c) | (d) | (e) | (f) |
| 1 | 13.28 % | 17.11 % | 12.82 % | 17.09 % | 14.22 % |
| 2 | 0.24 | 2.93 | -1.37 | 2.88 | -0.02 |
| 3 | 0.68 | 2.40 | -1.12 | 2.34 | -0.04 |
| 4 | 0.77 | 1.97 | -0.89 | 1.92 | -0.03 |
| 5 | 0.75 | 1.65 | -0.72 | 1.60 | -0.03 |
| 6 | 0.71 | 1.40 | -0.60 | 1.36 | -0.02 |
| 7 | 0.66 | 1.21 | -0.51 | 1.17 | -0.02 |
| 8 | 0.61 | 1.05 | -0.43 | 1.02 | -0.01 |
| 9 | 0.57 | 0.93 | -0.38 | 0.90 | -0.01 |
| 10 | 0.53 | 0.83 | -0.33 | 0.80 | -0.01 |
| 11 | 0.50 | 0.74 | -0.29 | 0.72 | -0.01 |
| 12 | 0.46 | 0.67 | -0.26 | 0.65 | -0.01 |
| 13 | 0.43 | 0.61 | -0.24 | 0.59 | -0.01 |
| 14 | 0.41 | 0.56 | -0.21 | 0.54 | 0.00 |