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Platform Competition under Partial Belief Advantage

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Abstract

This paper considers platform competition in a two-sided market that includes buyers and sellers. One of the platforms benefits from a partial belief advantage, in that each side believes that it is more likely that the other side will join the advantaged platform. We find that the degree of the platform's belief advantage affects its decision regarding the business model (i.e., whether to subsidize buyers or sellers), the access fees and the size of the platform. A slight increase in the platform's belief advantage may induce the advantaged platform to switch from subsidizing sellers to subsidizing buyers, or induce the disadvantaged platform to switch from subsidizing buyers to subsidizing sellers.

Keywords: platform competition, two-sided markets, belief advantage

1 Introduction

In platform competition in a two-sided market, a platform's ability to attract consumers depends not only on the consumers' beliefs regarding its quality, but also on consumers' beliefs regarding the platform's ability to attract the other side of the market.

Consider for example the market for smart-phones. The recent introductions of Apple's iPhone 4S with the improved operating system, and Samsung's Galaxy II with the improved Android 4, open a new round in the competition between the two platforms. Here, the ability of each platform to attract users depends not only on its perceived quality, but also on users' beliefs regarding the number of application developers that would be willing to develop new applications for the

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platform. Likewise, the ability to attract application developers to the platform depends on their beliefs regarding the number of users that will join the platform. Similarly, in the battle between HD DVD and BluRay, it mattered not only which format provides better experience of high-definition movies, but also how many movies will be released by the movie studios in a given format.

Naturally, even if one of the platform has some "beliefs advantage"—in that agents believe that it is more likely that other agents will join this platform, and not the rival—the platform still needs to identify how to translate this advantage into a competitive advantage over its rival.

This paper considers platform competition in a two-sided market that includes buyers and sellers. The main feature of our model is that one of the platforms has α-beliefs advantage over the competing platform. The idea of α-beliefs advantage is that if buyers and sellers do not know which platform other buyers and sellers are going to join, they will make their individual decisions based on the assumption that all other players coordinate on joining the advantaged platform with probability α.

We ask two main research questions. First, platforms usually compete by setting different prices to the two sides of the market. In particular, a platform may offer a low, perhaps negative price to one of the sides, and then charge a high price to the other side. For example, video-games consoles like Xbox or PlayStation, often sell at a loss in retail, but they make profits by charging the game developers who sell games to be played on the consoles. We therefore ask how a belief advantage or disadvantage affects the platforms pricing strategies in terms of (i) the side to attract and (ii) the number of sellers to attract. Notice that we raise this question for both the platform with a belief advantage and the platform with a belief disadvantage. This is because, as we show, a platform with a belief disadvantage can still win the market if it has sufficiently high quality, and if the platform correctly chose its pricing strategies in accordance with its belief disadvantage.

Second, in some cases, platforms can manipulate consumers' beliefs regarding the participation of other consumers. Each platform can launch an advertising campaign, aimed to help consumers on both sides of the market to coordinate on joining its platform instead of the competing platform. Since both platforms can launch such an advertising campaign, platforms also compete in advertising. We therefore ask what is the optimal advertising strategy for each platform, and how it is affected by market conditions. Moreover, how a platform should respond to an increase in advertising by its competing platform.

To answer these questions, we consider a model with the following features. There are two sides of a market, buyers and sellers. The number of buyers is limited, and there is large number of potential sellers that can enter the market, but each seller has fixed entry costs. Buyers want to buy one unit from each seller, and have a decreasing marginal utility with the number of sellers they buy from. The two sides cannot interact without a platform. Once they join a platform, sellers compete among themselves for buyers. This means that sellers can make positive profit from joining a platform only if buyers indeed joined the same platform and only if not too many other sellers joined the platform, such that competition among sellers fully dissipates their profits.[1]

For example, buyers can represent smart-phones users, that wish to buy smart-phone applications, or gamers that wish to buy videogames. Sellers can represent developers that can develop, for a given fixed development costs, a smart-phone application or a videogame. A platform is then a smart-phone operating system or a videogame console.

There are two competing platforms that differ in two respects. First, they may differ in their qualities. We allow for cases where each platforms is of higher or lower quality than the other. Second, one platform benefits form an "α-beliefs advantage" in that if there are two equilibria—one in which buyers and sellers join the advantaged platform, and another in which they join the disadvantaged platform—each side believes that all other players are going to play the first equilibrium with probability $\alpha > \frac{1}{2}$. Since the two platforms do not differ horizontally and since there are positive externalities between sides, we focus on equilibria in which one of the platforms wins the market. The two platforms compete by setting a different access fees to sellers and buyers, which can be positive or negative. Setting a negative fee to one side of the market may be a profitable tactics in two-sided markets, because the subsidized side attracts the other side to the platform, from which the platform can recuperate the lost revenue. Each platform needs to choose its business model, in that it needs to decide which side will be the subsidized side, and which will be the side bringing in the revenue. In the context of our model, the business models are characterized by access fees that a platform would charge. In reality, the choice of a business model is related to important decisions on the structure of firm, architecture of supply chain, or investment in marketing. Changing a business model may be costly and time consuming.

We establish the following main results. First, we show that the platforms' pricing strategies are affected by the degree of the belief advantage, α, in two distinct ways. First, if the sellers' fixed costs are very low and both platforms are symmetric in their beliefs advantage ($\alpha = \frac{1}{2}$), then both platforms will choose to fully cover the seller's fixed costs and offer a positive access price to buyers. That is, both platforms choose a business model that relies on the revenue from the buyers while subsidizing the sellers. Intuitively, whenever the platforms fully compensate the sellers for their fixed costs, sellers will join the platform regardless of their beliefs, because they know that they can never make looses. If $\alpha = \frac{1}{2}$, then no platform has an advantage in exploiting the two sides' beliefs, and therefore will prefer to choose the option of attracting sellers. In this case, the identity of the winning platform depends on its quality: the platform with the highest quality wins.

[1] Our results depend on such an asymmetry of the two sides. This asymmetry is common in many real-life two-sided markets. However, there are also two-sided markets to which our model does not apply, e.g., on-line dating.

We then show that given a same low fixed fees, if α becomes large enough, then the platform with belief advantage prefers to changes its business model, by charging higher access fees to sellers, and then using the presence of sellers and the belief advantage to attract the buyers. In this case however, the advantaged platform can win the market even if it offers a lower quality than the disadvantaged platform, because of its ability to exploit its beliefs advantage. It is also possible for the disadvantaged platform to win, if it offers substantially higher quality.

The platform with belief disadvantage may want to change its business model when increases under intermediate development costs. Here, if $\alpha = \frac{1}{2}$, both platforms will compete on attracting buyers, because fully subsidizing sellers is too costly. In this case again the platform with the highest quality wins the market. Now, for the same given fixed costs, as α increases, the disadvantaged platform changes its business model to one where it subsidizes sellers and relies on the revenue from the buyers. Again, the advantaged platform wins even if it offers a lower quality. Intuitively, if both platforms compete on the buyers, then they both rely on the buyers' beliefs regarding the probability that sellers join. If $\alpha = \frac{1}{2}$, then these beliefs are the same for both platforms. However, as α increases, one platform gains higher belief advantage over the other. Therefore, it is no longer profitable for the disadvantaged platform to subsidize buyers, which rely on beliefs, and will prefer to subsidize sellers.

We also find that whenever a platform subsidizes buyers, it will attract fewer sellers than the trade-maximizing level. However, if a platform subsidizes sellers, it will attract more sellers than the trade- maximizing level. Since an increase in α may alter a platform's decision regarding which side to subsidize, we find that a small increase in α can induce the advantaged platform to substantially decrease the size of its platform, by shifting from attracting more sellers than the trade-maximizing level, to attracting fewer sellers than this level. Likewise, a small increase in α can induce the disadvantaged platform to substantially increase the size of its platform, by shifting from attracting fewer sellers than the trade-maximizing level, to attracting more sellers than this level.

We then move to the case of endogenous beliefs. We consider a preliminary stage in which the two platforms compete in investing in advertising, that aimed to affect α. The higher the investment by the advantaged (disadvantaged) platform, the higher (lower) is α. We assume that in this preliminary stage both platforms do not know their relative quality, such that each platform believes that it has a positive probability for winning the market. For this part we focus on the case where the two sides compete on attracting the buyers.

We find that the best response of the advantaged platform is always downwards sloping. That is, the advantage platform will always be defensive in reducing its own level of advertising as a response to an increase in the disadvantaged platforms level of investment. For the latter platform, however, the best response is also downwards sloping if the "base beliefs", i.e, the value of α for identical level of advertising, is close to $\frac{1}{2}$, but can be upwards sloping otherwise, in which case the disadvantaged platform becomes an offensive player. Intuitively, as the competing platform increases its level of advertising, then the probability that the competing platform will win the market decreases, which will motivate the platform to invest less. However, the effect on the platform's profit in case it does win the market is different among the two platforms, which will he result in different responses.

The economic literature on competing platforms extends the work of Katz and Shapiro (1985) on competition with network effects. Spiegler (2000) considers an "extractor," such as a platform who can extract positive externalities from two agents. Caillaud and Jullien (2001) and Caillaud and Jullien (2003) consider competition between undifferentiated platforms, where one of them benefits from favorable beliefs. Hagiu (2006) considers undifferentiated platform competition in a setting where sellers join the platform first, and only then buyers. Lopez and Rey (2009) consider competition between two telecommunication networks when one of them benefits from "customers' inertia," such that in the case of multiple responses to the networks' prices, consumers choose a response which favors one of the networks. Jullien (2011) consider undifferentiated platform competition in a multi-sided market. Halaburda and Yehezkel (2011) consider undifferentiated competition where the two sides of the market are ex-ante uniformed about their utilities, and are ex-post privately informed. A common feature in the above literature is the assumption that one platform fully benefits from a belief advantage. This is equivalent to assuming $\alpha = 1$ in our model. We make two contributions to this literature. First, we consider the case where the advantaged platform benefits from only a partial belief advantage, in that $\frac{1}{2} < \alpha < 1$. As we explained above, this distinction turned out to be important because a platform may choose a different business model depending on whether $\alpha = 1$ or $\frac{1}{2} < \alpha < 1$. The choice of the business model has implication for the access fees, which side is subsidized, and what is the size of the platform (and whether its below and above trade-maximizing size). The second contribution of our paper is considering endogenous belief advantage, through the platforms' advertising strategies.

Our paper also contributes to the literature on business models. Ghemawat (1991) and Casadesus-Masanell and Ricart (2010) refer to firms strategy as its choice of a business model: the business model is a set of committed choices that lays the groundwork for the competitive interactions between the firms. The choice of the business model enables or limits particular tactical choices (e.g., prices). Specifically, our paper analyzes the choice of the business model in the context of two-sided platforms. As pointed by Rochet and Tirole (2003) and by Casadesus-Masanell and Zhu (2010), in the context of two-sided markets, one of the most important aspects of the business model is which side of the market is the primary source of the revenue.

2 Characteristics of the Market

We consider an environment with two competing platforms. Each platform needs to serve two groups of customers, the buyers and the sellers. These are called two sides of the market. Each buyer wishes to buy a product that a seller sells. The goods offered by the sellers and demanded by the buyers are homogeneous. However, a buyer and a seller cannot trade unless they have joined the same platform.

Buyers. There are N_B identical buyers. Buyers can be smartphone users, who have a demand for smartphone applications. Likewise, buyers can be gamers, who have a demand for videogames. The consumption utility of each buyer from buying n products is $u_B(n)$. The number n can represent the number of applications, videogames, etc. This consumption utility is positive for any $n > 0$ and increasing with n, but it reaches a saturation point at $\widehat{n}$, i.e., $u'_B(n) > 0$ for $n < \widehat{n}$ and $u'_B(n) < 0$ for $n > \widehat{n}$. Moreover, $u''_B(n) < 0$. To make sure that the second order conditions are satisfied, we assume that the third derivative is either negative, or positive but not too large. Specifically, $u'''_B < -\frac{u''_B}{n}$. The total buyer's utility also incorporates the cost of purchasing the products. The price for each product is the same, p. Then the total buyers utility is

$$U_B(n) = u_B(n) - p\,n\,.$$

Sellers. There is a large number, N_S, of identical sellers ready to enter the market, where $N_S > 2\widehat{n}$. Sellers can be developers of smartphone applications, developers of videogames, etc. Each seller offers one product (a smart-phone app, a videogame for a console, a movie in a given format), but he can sell multiple copies of it to multiple buyers. A seller receives p for every copy of the product he sells. Sellers have a fixed cost of developing the product, $K > 0$, which is the same for all sellers. We normalize marginal production costs to 0.[2] If n sellers join a platform, they provide n products and they behave competitively. Hence, products are sold at the price equal to the marginal consumption utility of the n-th product, $u'_B(n)$. If $n > \widehat{n}$ such that $u'_B(n) < 0$, buyers will not pay a positive price. As sellers will not sell at a loss, we assume that if $n > \widehat{n}$, then only $\widehat{n}$ are sold at $p(\widehat{n}) = u'_B(\widehat{n}) = 0$. Therefore, the equilibrium price is $p(n) = \max\{u'_B(n), 0\}$. As a tie-braking rule, we assume that in case sellers are exactly indifferent between joining a platform or not, they will enter as long as they expect to make positive sales (but stay out otherwise). This assumption enables us to eliminate unreasonable equilibria.

After incorporating the development costs, the sellers total payoff is $N_B\,p(n) - K$. Let $k = \frac{K}{N_B}$.

[2] Indeed, in most of our examples (a smart-phone app, a video game for a console, etc.) involve negligible marginal costs.

Then this payoff can be represented by $N_B(p(n) - k)$. As $p(n)$ is decreasing with n, we assume that $p(0) > k$, such that k is low enough such that sellers' total payoff is positive for some $n > 0$.

Network effects and the asymmetry between the sides. The above model has two main features that will play an important role in the analysis. First, there are positive network effects between the two sides of the market: buyers (sellers) gain higher utility from joining a platform the more sellers (buyers) join the same platform. In the market for smartphones, for example, a consumer decides which platform to join (iOS, Android, etc.,) based on the expected applications that would be developed to this platform. In the market for videogames, a gamer will buy a console based on the expected games that will be developed to this console. The same argument follows to developers (of smartphone applications or videogames). In the case where there is more than one platform in the market, network effects often lead to tipping of the market towards one of the platforms. This is also the feature of this model. Since both sides want to join the same platform, it creates also a coordination problem. In consequence, it may lead to multiplicity of equilibria.

The second main feature of our model is asymmetry between the two sides of the market. Buyers' participation is non-rivalous. The number of other buyers on the same platform does not affect each buyer's utility. This is not true for the sellers. Larger number of sellers on the same platform increases competition and decreases each seller's payoff. We believe that this asymmetry in rivalry reflects many (but not all) two-sided markets: Consumption of smart-phone apps is non-rivalous, even if the developers compete for the users. Similar statement is true about video games released for consoles, or movies released for a given format.[3]

Trade-maximizing outcome (first-best). To solve for the first-best outcome, notice first that it is cost-reducing for all buyers to join the same platform. This is because the sellers' fixed costs, K, are spread among a large number of buyers. Given that all buyers join the same platform, the number of sellers that maximizes total gains from trade between sellers and buyers, n^*, is the solution to

$$n^* = \arg\max_n \left\{ N_B \Big(U_B(n) + n(p(n) - k) \Big) \right\} .$$

Given our assumptions, above, n^* is unique, with $\widehat{n} > n^* > 0$ for any $k > 0$ and $n^* = \widehat{n}$ for $k = 0$. Moreover, n^* is decreasing with k.

Platforms. There are two competing platforms, which we call platform A and platform D. They differ in two aspects: in terms of the expectations they face in the market, and in that they are

[3] Our model does not apply, for example, to the dating market, where there is competition on both sides.

vertically differentiated.[4] We discuss the issue of market expectations later in this section and in Section 3. We measure the vertical differentiation with Δ—additional utility that a buyer gains by joining platform A. Variable Δ captures, for example, difference in the quality of service between the platforms; or an additional stand-alone service that one platform offers but the other one does not. We allow for both positive and negative Δ, i.e., platform A may be perceived as better or worse than platform D. We assume that platforms do not incur costs, so the revenues are equivalent to profits.[5]

Strategies and business models. Platforms compete by setting access fees to buyers and sellers, which can be positive or negative: (F_B^A, F_S^A), (F_B^D, F_S^D). As platforms aim at attracting two groups of "customers," the buyers and the sellers, they may find it optimal to offer lower access fee to one side than the other. In fact, it is well known that it may be optimal for a platform to subsidize one of the sides in order to charge higher fees to the other side. The business model identifies the side which is the primary source of revenue. In the context of our model, we find that platforms' equilibrium pricing strategies involve choosing one of two distinct business models. In one of the business models the sellers are the primary source of revenue, in that the platform fully extracts the sellers' profit, by charging sellers a high access fees, and charge a low—possibly negative access fees—to attract buyers. We call it the Sellers-Revenue-Based (SRB) business model. In the other business model, the buyers are the primary source of revenue, and the sellers may be subsidized. We call it the Buyers-Revenue-Based (BRB) business model.

In our model, the business models are distinguished only by different prices. In reality, the choice of a business model is often related to important choices in infrastructure and supply chain architecture. In the market for smartphones, for example, a platform that chooses a BRB may provide the developers with software development tools, technical training and guidance, and perhaps making the operating system open. A platform that chooses a SRB, however, may need to develop a strong marketing network for selling smartphones, and also incorporate in the operating system elements to control and restrict developers access. Such infrastructure differences require time to build, and may be expensive—or sometimes impossible—to change.[6] Therefore, firms need to decide on the business model before the actual pricing decisions. Moreover, the choice of the

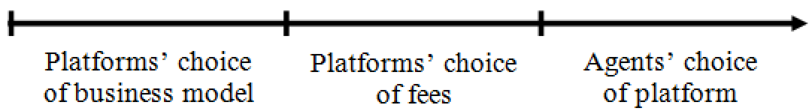
[4]In particular, we do not assume horizontal differentiation. In many markets both competing platforms have loyal following resulting from horizontal differentiation. But there always exists a segment of customers that do not have prior preference for one platform or the other, a segment for which the platforms compete. Our model only refers to this segment.

[5]The analysis is very similar (but mathematically significantly more complicated) with positive fixed and marginal costs.

[6]The history of videogames (Hagiu and Halaburda, 2009) illustrates the potential difficulty. Atari's original business model was BRB, but when under the competitive threat it wanted to switch to SRB and charge royalties from the developers, it could not enforce it, for the lack of the restrictions in the systems code.

business model may constrain the set of prices that the platform can charge.[7] Of course, firms choose their business models considering their expectations about the future prices. In this paper, we investigate how the belief advantage (and disadvantage) affects the business model that the platform chooses.

Timing. The platforms choose their business models (simultaneously). Given the business models, they (simultaneously) decide on the access fees to charge to both sides. The buyers and sellers observe the prices, and then based on the prices make their (simultaneous) decision which platform to join.



Dominant firm equilibria. We focus on equilibria in which one platform "wins" the market and the competing platform earns zero profit. We ignore equilibria with two active platforms because such equilibria are instable. This result follows from our assumption that all buyers are identical. Therefore, in an equilibrium with two active platforms, all buyers should be indifferent between joining each platform. However, if a "marginal" buyer switches from one platform to the other, then more sellers will find it optimal to join this platform, which will induce all buyers to switch to this platform as well. Consequently, a "marginal" change eliminates any equilibrium with two active platforms.

In real-life situations, however, most markets for platforms involve more than one active platform. This is because buyers may differ in their preferences for platforms. Consequently, a platform can always focus on attracting buyers that have strong preferences for this specific platform. In order to keep our model tractable, we consider homogenous buyers. We interpret our dominant-platform equilibrium as a reduced form of an equilibrium in which one platform gains all the buyers who do not have strong preferences for a specific platform, while the other platform (that in our model earns zero profit) focuses on serving buyers that have strong preferences for its specific features.

[7] Casadesus-Masanell and Zhu (2010) provide examples from other industries dominated by platforms: TV channels may choose to be ad-sponsored (most of broadcast channels) or subscription-based (like HBO); most newspapers are sold at positive prices to readers, while there also exist newspapers like *Metro*, which is completely ad-sponsored and free to the readers. As Casadesus-Masanell and Zhu (2010) point out, in both industries firms choosing different business models need to develop different distribution channels (*Metro* is given away at subway stations in large cities, as opposed to shop distribution for other newspapers), and different capabilities (HBO does not need to attract advertisers, but needs to provide samples of its programming to potential subscribers).

α-beliefs refinement. We assume that in the last stage the two sides of the market play a pure strategy Nash equilibrium. As there are multiple equilibria, we consider a refinement of *α-beliefs*, explained in following section.

3 The Concept of α-Beliefs

In the last stage of the game the two platforms charge access fees (F_B^A, F_S^A), (F_B^D, F_S^D), and buyers and sellers simultaneously decide to which platform to join. Since both sides aim to join the same platform, there might be multiple equilibria, resulting from the coordination problem between the sides. We therefore turn to offer a refinement that generates a unique outcome in the second stage subgame, for any (F_B^A, F_S^A), (F_B^D, F_S^D).

Suppose that there are two possible pure strategy subgame equilibria, X and Y. We say that the market has α-beliefs about equilibrium X when all players believe that equilibrium X is played with probability α and equilibrium Y is played with probability $1 - \alpha$.

In the context of our environment, suppose that there are two subgame equilibria in the last stage:

***dominant-A*,** where all buyers and some sellers join A, and there is no trade on platform D; and

***dominant-D*,** where all buyers and some sellers join D, and there is no trade on platform A.

When for some access fees both equilibria exist, we apply the concept of α-beliefs towards platform A:

Definition 1 *The market has α-beliefs about platform A. That is, when both subgame equilibria are possible, agents believe that* dominant-A *is played with probability α, and* dominant-D *is played with probability $1 - \alpha$.*

Motivating example. As a motivating example for this concept, consider the battle between BluRay and HD DVD. There was a common agreement that only one of the two formats would survive. Hence, there were two possible equilibria, in one equilibrium everyone adopts BluRay and in the other everyone adopts HD DVD. But it was not clear which equilibrium would, in fact, be played in the market. Exactly because neither equilibrium was eliminated, either had a positive probability of begin played by the market. We can say that people believed that the market will settle on BluRay with probability α, and on HD DVD with probability $1 - \alpha$.

Discussion of the concept. The α-belief concept is the main focus of our paper, as we want to investigate how a platform's belief advantage or disadvantage affects the platform's business model and competitive advantage. Notice that α-beliefs concept is a generalized form of the "favorable beliefs" refinement, first introduced by Caillaud and Jullien (2001) and Caillaud and Jullien (2003). In particular, for $\alpha = 1$, platform A has a complete beliefs advantage, exactly as in Caillaud and Jullien.[8] Now, however, we can also consider cases where beliefs are not deterministic towards one of the platforms.

In our model we distinguish platform A as the platform with beliefs advantage, that is the platform with α-beliefs, where $\alpha \geq \frac{1}{2}$. Platform D is the platform with belief disadvantage, as $1 - \alpha \leq \frac{1}{2}$. The platforms differ vertically by Δ. But Δ can be positive or negative. Therefore, the only attribute that distinguishes the advantaged platform A from the disadvantaged platform D are more favorable beliefs, i.e., $\alpha > \frac{1}{2}$. Therefore, if $\alpha \leq \frac{1}{2}$, platform D becomes platform A and vice versa.

Higher quality (captured by Δ) and more favorable position in the market (captured by α) constitute two sources of competitive advantage. We can think of the favorable market beliefs as a better brand name, for example. Better, more recognizable brand name may, but does not need to, relate to a higher quality. In most of our analysis, we assume that α and Δ are independent of each other. In Section 5 , we investigate how higher quality may lead to larger belief advantage.

Notice that our concept of α-beliefs differs from a mixed strategy equilibrium. For comparison, consider a typical coordination game, where two players, B and S, can either play A or D. They both get a positive payoff if they coordinate on the same decision, and get 0 if they play different strategies. In such a game, there are two pure strategy equilibria, and one mixed strategy equilibrium. In the mixed strategy equilibrium, B plays A with probability β_B, and with the remaining probability, he plays D. Similarly S plays A with probability β_S, and D with the remaining probability. Probabilities β_B and β_S are unique, and such that the other player is indifferent between playing A or D (a necessary condition for a mixed strategy equilibrium). In such a case, the agents coordinate on A with probability $\beta_B \beta_S$, and coordinate on D with probability $(1 - \beta_B)(1 - \beta_S)$. Under α-beliefs, in contrast, with (exogenous) probability α equilibrium A is played (i.e., both B and S play A), and with probability $(1 - \alpha)$ equilibrium D is played (i.e., both play D). Therefore, this is not equivalent to a mixed strategy equilibrium.

4 Equilibrium

The objective of each platform is to choose the most profitable business model, given the behavior of the other platform, and the agents in the market. To establish which business model is more profitable, we need to analyze subsequent actions of buyers and sellers. Hence, to solve for the

[8] Caillaud and Jullien would call platform A an "incumbent."

equilibrium outcome, we use standard backward induction. We start by solving for the agents' optimal choice of platforms given (F_B^A, F_S^A), (F_B^D, F_S^D). Then, we solve for the equilibrium access fees and business models that the platforms choose, knowing how (F_B^A, F_S^A), (F_B^D, F_S^D) affects the agents' choices.

4.1 Decisions of Buyers and Sellers

In the last stage, buyers and sellers observe the posted fees, (F_B^A, F_S^A), (F_B^D, F_S^D), and simultaneously decide which platform to join. If a buyer or a seller decides not to join any platform, he gets the total payoff of 0.

The decision of buyers depends on the access fees and on the number of sellers they expect to find in each platform. The payoff of every buyer is the same: joining platform A with n sellers yields buyers payoff $U_B(n) - F_B^A + \Delta$, and joining platform D with n sellers yields $U_B(n) - F_B^D$. Hence, they all make the same decision:[9] either they all join platform A, they all join platform D.[10]

This is not so for the sellers. Even though sellers are identical, due to competitive forces, it may be optimal for them to make different decisions. The payoff of a seller decreases with the number of other sellers: joining platform i $(i = A, D)$ with all N_B buyers and n sellers yields each seller a payoff of $N_B(p(n) - k) - F_S^i$. Sellers join a platform only until the payoff is 0. Each additional seller would earn negative payoff. Hence, if $n > 0$ join a platform, this number is uniquely characterized by F_S^i: $N_B(p(n) - k) - F_S^i = 0$.

There exists *dominant-D* equilibrium when following conditions are satisfied:

$$N_B(p(n^D) - k) - F_S^D = 0\,, \tag{1}$$

$$-F_B^D + U_B(n^D) \geq \min\{0, -F_B^A + \Delta\}\,. \tag{2}$$

But for some fees conditions for both *dominant-D* and *dominant-A* are satisfied. Then both equilibria are possible. In such a case, by α-beliefs *dominant-A* is played with probability $\alpha > \frac{1}{2}$, and *dominant-D* is played with probability $1 - \alpha$.

4.2 Choice of Business Models and Access Fees

Consider now the stage where the two platforms choose their access fees, (F_B^A, F_S^A), (F_B^D, F_S^D), taking into account that agents have α-beliefs. To derive the equilibrium, we first focus on the

[9]The only exception may be when the buyers are indifferent. We consider this case in solution, but platforms always want to avoid the case when the buyers are indifferent. Hence we abstract from it for clarity of exposition.

[10]There is also a third potential equilibrium, in which both sides do not join either platform. We assume that if there is such an equilibrium in addition to the two equilibria above, then agents will not play this equilibrium. Intuitively, we focus on a market in which agents believe that the market eventually will succeed in attracting the two sides, so the only question is which platform is going to be successful.

platform A's best response to $(F_B^D,\, F_S^D)$. Setting $(F_B^A,\, F_S^A)$ such that *dominant-A* is an equilibrium is a necessary condition for platform A to win the market, but it is not sufficient. If *dominant-D* is also a subgame equilibrium, *dominant-A* is played only with probability α. Instead, at the cost of marginal loss of profit, platform A may assure that under α-beliefs *dominant-D* is never played. Hence, platform A always prefers to "eliminate *dominant-D* in this way.

Consider first the situation when both *dominant-A* and *dominant-D* are possible. Given α-beliefs, a buyer believes that with probability α all other agents play *dominant-A*, and with probability $1-\alpha$ they play *dominant-D*. Let $U_B^D(F_B^D, F_S^D, \alpha)$ denote the expected payoff of a buyer from joining platform D when both equilibria are possible, and *dominant-D* is played with probability $1-\alpha$. Notice that by our analysis in Section 4.1 this expected payoff depends on the access fees charged by platform D and on α, but it does not depend on the access fees charged by platform A.

In order to eliminate *dominant-D*, platform A needs to charge such access fees (which attract sufficient number of sellers) that the buyers strictly prefer to join platform A even if there is probability $1-\alpha$ that all other agents play *dominant-D*. Given $U_B^D(F_B^D, F_S^D, \alpha)$, platform A has two options—SRB and BRB business models—which we analyze in turn.

Sellers-Revenue-Based (SRB) business model. The first option for platform$A$ is to set $F_S^A > 0$. If the platform charges positive access fee to the sellers, they find it worthwhile to join the platform only if the buyers are joining as well. Then, in a *dominant-A* equilibrium, n^A sellers join platform A, where n^A is determined by

$$N_B\, p(n^A) - K - F_S^A = 0. \tag{3}$$

Given α-beliefs, a buyer believes that those sellers, and all other buyers join platform A with probability α. Consequently, the buyer's expected utility from joining platform A is $\Delta + \alpha U_B(n^A) + (1-\alpha)U_B(0) - F_B^A$. As all buyers are the same, platform A can attract all buyers by setting

$$\Delta + \alpha U_B(n^A) - F_B^A > U_B^D(F_B^D, F_S^D, \alpha)\,. \tag{4}$$

Condition (4) eliminates *dominant-D* as a subgame equilibrium, because it ensures that a buyer strictly prefers to join platform A even if he believes that *dominant-D* will be played with probability $1-\alpha$. When condition (4) is satisfied, all buyers play *dominant-A* with probability 1. Sellers know that, and they will also play *dominant-A*. Notice that if (4) holds in equality, then the α-belief advantage does not eliminate the possibility that agents will play *dominant-D* with some probability. Hence, platform A wants to assure that the condition holds with inequality. At the same time, larger inequality in (4) forces lower F_B^A. In the interest of its profit, the platform wants to keep

F_B^A as high as possible. Hence, the platform sets F_B^A such that condition (4) holds with slight inequality.

With F_S^A and F_B^A identified by (3) and (4), platform As profit under SRB business model, $\Pi_{SRB}^A = n^A F_S^A + N_B F_B^A$, can be expressed as a function of n^A:

$$\Pi_{SRB}^A(n^A) = N_B \left(\pi_{SRB}^A(n^A) + \Delta - U_B^D(F_B^D, F_S^D, \alpha)\right) , \tag{5}$$

where

$$\pi_{SRB}^A(n^A) = n^A(p(n^A) - k) + \alpha U_B(n^A). \tag{6}$$

Let n^{A^*} denote the number of sellers that maximizes (6) (and therefore (5)). While we assume that platforms compete by setting access prices, it is more convenient to solve directly for the optimal number of sellers that platform A wishes to attract, n^A.

Equation (6) reveals that when choosing n^A to maximizing its profit, platform A internalizes all the sellers' gains from trade, but only a fraction α of the buyers' gains from trade. In this sense, platform A is oriented towards capturing the revenues from the sellers' side, and as we show below, use F_B^A as the only tool for competing with platform D. Also, because platform A internalizes only fraction α of the benefits that sellers provide to buyers, SRB involves attracting fewer sellers than the first-best (trade-maximizing level).

Buyers'-Revenue-Based (BRB) business model. Next, we turn to platform A's optimal best response given that it chooses any $F_S^A \leq 0$. Notice first that it is never optimal to set $F_S^A < -K$, because sellers will join platform A even when the platform is saturated: even if more than already joined and $p(n) = 0$, just for benefiting from the subsidy that accedes their development costs. Moreover, notice that it is never optimal for platform A to set $0 \geq F_S^A > -K$. This is because for any $F_S^A > -K$, sellers do not cover their entry costs unless buyers join platform A, which forces platform A to compete in attracting buyers by setting a low F_B^A. Given that it does so, platform A might as well charge a high F_S^A to capture the sellers' profit, by using a SBR. We therefore focus on the case where platform A sets $F_S^A = -K$.

In a *dominant-A* equilibrium where all buyers join platform A, setting $F_S^A = -K$ attracts $\widehat{n}$ sellers join platform A, because for any $n^A < \widehat{n}$, $p(n^A) > 0$ and therefore more sellers would like to join.[11] If both *dominant-A* and *dominant-D* are possible, then by the α-belief advantage sellers expect all buyers to join platform A with probability α. And since sellers are fully compensated for their fixed costs, $F_S^A = -K$ ensures that $n^A = \widehat{n}$ sellers will still find it optimal to join platform A.

[11] If more sellers would like to join platform A, the additional sellers would only enter into a "saturated" platform that includes a price $p = 0$, and would not make positive sales. By our tie-braking assumption, these sellers will not enter.

Intuitively, as sellers are fully compensated for their entry costs, they bear no risk of loosing money from joining into an "empty" platform. As there is some probability that the platform will not be empty, sellers will prefer to join it.

Let us turn now to the buyers side under BRB business model. A buyer knows that when both *dominant-A* and *dominant-D* are possible, sellers join platform A. Therefore, platform A attracts the buyer as long as

$$\Delta + U_B(\widehat{n}) - F_B^A > U_B^D(F_B^D, F_S^D, \alpha)\,. \tag{7}$$

Notice that here α does not appear on the left-hand-side, because the buyer knows that by subsidizing sellers, platform A guarantees sellers participation. Given (7), all buyers join platform A and therefore *dominant-D* is eliminated. As before, in the interest of its profits, platform A sets F_S^A so that (7) holds with slight inequality. Platform A's profit is $\Pi_{BRB}^A = \widehat{n}F_S^A + N_B F_B^A$, or

$$\Pi_{BRB}^A = N_B(\pi_{BRB}^A + \Delta - U_B^D(F_B^D, F_S^D, \alpha))\,, \tag{8}$$

where

$$\pi_{BRB}^A = U_B(\widehat{n}) - \widehat{n}k\,. \tag{9}$$

Equation (9) reveals that now platform A fully internalizes the buyers' gains from trade, but does not provide any gains from trade to the sellers' side (because $p(\widehat{n}) = 0$). In this sense, platform A is oriented towards creating maximal value to the buyers side,[12] and capturing all of it. From now onwards we refer to this business model as *Buyers-Revenue-Based (SRB).*

Optimal business model for platform A. By comparing (5) and (8)—maximal profit under each business model—we find that platform A prefers to adopt the SRB business model if $\pi_{SRB}^A(n^{A*}) > \pi_{BRB}^A$, and prefers to adopt the BRB business model if $\pi_{SRB}^A(n^{A*}) < \pi_{BRB}^A$. Notice that both n^{A*} and $\pi_{SRB}^A(n^{A*})$ depend on α, while π_{BRB}^A does not.

Optimal business model for platform D. Platform D's best response is similar to the discussion above, with the exceptions that platform D's belief advantage is $1-\alpha$ instead of α, and that platform D's quality is 0 instead of Δ. Platform D therefore adopts a SRB if $\pi_{SRB}^D(n^{D*}) > \pi_{BRB}^D$, and adopts the BRB business model if $\pi_{SRB}^D(n^{D*}) < \pi_{BRB}^D$, where:

$$\pi_{SRB}^D(n^{D*}) = (1-\alpha)U_B(n^{D*}) + n^{D*}(p(n^{D*}) - k)\,, \tag{10}$$

$$\pi_{BRB}^D = U_B(\widehat{n}) - \widehat{n}k, \tag{11}$$

[12]Within the bounds of controlling the sellers side.

and n^{D*} maximizes (10). It is straightforward to show that if platform D uses a SRB, the optimal number of sellers that platform D attracts, n^{D*}, is lower than n^{A*}, and is decreasing with α. Intuitively, whenever platform D chooses SRB, it has a lower ability to internalize the buyers' utility than platform A, because $\alpha < 1$. Clearly, $\pi^D_{SRB}(n^{D*})$ depends on α. Notice also that if platform D adopts a BRB, than it attracts the same number of sellers, $\widehat{n}$, as platform A's BRB business model. From now onwards we can define $\pi_{BRB} \equiv \pi^D_{BRB} = \pi^A_{BRB}$.

Lemmas 1 and 2 summarize the properties of each business model.

Lemma 1 (features of SRB business model) *If platform $i = A, D$ chooses a Sellers-Revenue-Based (SRB) business model, then it charges a positive access fees from the sellers, $F^i_S > 0$, and uses the buyers' access fees as the exclusive tool for competing with the other platform. Platform i attracts fewer sellers than the trade-maximizing level, $n^{i*}(\alpha) < n^*$ for all $\alpha < 1$. Moreover,*

(i) for platform A, $n^{A*}(\alpha)$ is increasing with α, and $n^{A*} = n^*$ for $\alpha = 1$;

(ii) for platform D, $n^{D*}(\alpha) < n^{A*}(\alpha)$ for any α, and $n^{D*}(\alpha)$ is decreasing with α.

Proof. See Appendix.

Lemma 2 (features of BRB business model) *If platform $i = A, D$ chooses a Buyers-Revenue-Base (BRB) business model, then it charges a negative access fees from the sellers, $F^i_S = -K < 0$, and a positive access fees form the buyers, $F^i_B > 0$. The platform attracts $\widehat{n}$ sellers, which is more than the trade-maximizing level, $\widehat{n} > n^*$ for all α.*

Proof. See Appendix.

The lemmas show that under SRB, a platform attracts fewer sellers than the trade-maximizing level, while under BRB, a platform attracts more sellers than the trade-maximizing level. This result differs from Hagiu (2006) that shows that a platform that benefits from a full belief advantage (corresponding to platform A in our model) does not distort its level of trade while the competing entrant (corresponding to platform D) distorts the level of trade downwards. These results also differ from Halaburda and Yehezkel (2011) that shows that both platforms distort the level of trade downward regardless of whether they attract the buyer of the seller. For business strategy, Lemmas 1 and 2 show that a platform's business model also specify whether a platform should "oversell" or "undersell" applications.[13]

[13]For example, Claussen, Kretschmer and Mayrhofer (2011) illustrate this on the example of applications on Facebook: "Facebook encouraged entry of as many developers as possible. The company offered strategic subsidies to

Business models in equilibrium. Having analyzed the best responses of each platform, in terms of the access fees charged, and the business model adopted, we now turn to the business models chosen in equilibrium. Figure 1 shows the business models that platforms A and D choose in the equilibrium, given α and k. The figure reveals that there are three regions: (Ω_{BB}) both platforms adopt a BRB, (Ω_{SS}) both platforms adopt SRB, (Ω_{SB}) platform A adopts SRB and platform D adopts BRB.

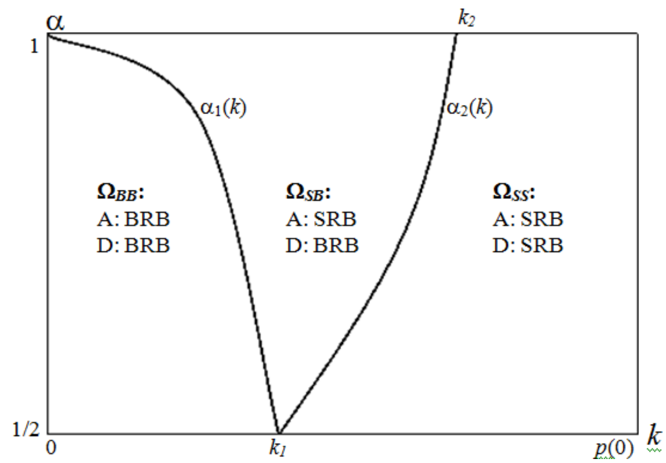


Figure 1: The three subsets: Ω_{BB}, Ω_{SB} and Ω_{SS}

The optimal business model depends on the size of the belief advantage. For a platform i ($i = A, D$) it is optimal to adopt SRB business model when $\pi^i_{SRB}(n^{i*}) > \pi_{BRB}$, and otherwise it is optimal to adopt the BRB business model. In each case, the direction of inequality depends on α and k. As k increases, π_{BRB} decreases. That is, subsidizing sellers becomes increasingly expensive, and brings lower profits. Therefore it is more likely that a platform Adopts SRB business model, where it collects most of the revenue from the sellers while lowering the buyers access fee to make the platform more attractive.

The cost of subsidizing the sellers changes along k in the same way for both platforms. Therefore, both platforms are likely to switch from BRB to SRB as k increases. The cost of subsidizing the buyers, however, changes along α differently for the two platforms. Hence, the platforms have different response to increasing α. As α increases, attracting the buyers (in order to collect the revenue from the sellers) is increasingly cheaper for platform A, and increasingly more expensive

third-party developers by providing open and well-documented application programming interfaces, multiple development languages, free test facilities, as well as support for developers through developer forums and conferences" (p. 5). In result more than 30,000 applications had been developed within a year of lunching the service. However, not all applications have been adopted by users, and of those that have been installed, not all had been actively used. This suggests that the platform was oversupplying applications.

for platform D. Hence, increasing α makes SRB business model more appealing to platform A, while it makes BRB business model more appealing to platform D.

The two platforms may choose the same business model. If they choose different business models, it is possible that platform A adopts the SRB business model, while platform D adopts BRB, but never the other way around. This is because it is always cheaper for platform A than for platform D to attract the buyers and collect the revenue on the sellers side.

These results are summarized in the following proposition.

Proposition 1 (equilibrium choice of business model) *For any $\alpha \in [\frac{1}{2}, 1]$, and any $k \in [0, u_B(0)]$, a pair (α, k) belongs to exactly one of three regions:*

(i) $\Omega_{SS} \equiv \{(\alpha, k) | \pi^A_{SRB}(n^{A^*}) > \pi^D_{SRB}(n^{D^*}) > \pi_{BRB}\}$, *where both platforms adopt SRB business model;*

(ii) $\Omega_{BB} \equiv \{(\alpha, k) | \pi_{BRB} > \pi^A_{SRB}(n^{A^*}) > \pi^D_{SRB}(n^{D^*})\}$, *where both platforms adopt BRB business model; or*

(iii) $\Omega_{SB} \equiv \{(\alpha, k) | \pi^A_{SRB}(n^{A^*}) > \pi_{BRB} > \pi^D_{SRB}(n^{D^*})\}$, *where platform A adopts SRB and platform D adopts BRB business model.*

The thresholds between those regions are characterized by $\alpha_1(k)$ and $\alpha_2(k)$, where

$\alpha_1(k)$ *is uniquely defined by $\pi^A_{SRB}(n^{A^*}) = \pi_{BRB}$; moreover, $\alpha_1(0) = 1$, $\alpha_1' < 0$ and $\alpha_1'' > 0$; also, there is a point k_1 such that $\alpha_1(k_1) = \frac{1}{2}$, where $0 < k_1 < u_B'(0)$.*

$\alpha_2(k)$ *is uniquely defined by $\pi^D_{SRB}(n^{D^*}) = \pi_{BRB}$; moreover, $\alpha_2(k_1) = \frac{1}{2}$, $\alpha_2' > 0$ and $\alpha_2'' > 0$; also, there is a point, k_2, such that $\alpha_2(k_2) = 1$, where $k_1 < k_2 < u_B'(0)$.*

Proof. See Appendix.

In Figure 1, we can clearly see that increase in motivates platform A to switch from BRB business model to SRB. Platform D may be motivated to switch in the opposite direction when α increases. As k increases, both platforms are motivated to switch from BRB to SRB business model.

When $\alpha = \frac{1}{2}$, the platforms are symmetric in the market beliefs, and none has an advantage over the other in attracting the buyers. Hence, both platforms choose the same business model. For k lower than k_1, it is cheaper to subsidize the sellers, and adopt BRB business model. For k higher than k_1, substituting the sellers becomes more expensive, and both platforms prefer to adopt SRB business model. As α increases and the platforms become more asymmetric in the market beliefs they are facing, they are more likely to adopt different business models.

For higher α, it becomes cheaper for platform A to attract the buyers. This is because the buyers believe that there is better chance that sellers will also join platform A, even though platform A charges sellers a positive access fee. When α increases above α_1 (for $k < k_1$), the effect is strong enough that platform A finds it optimal to switch its business model from BRB to SRB.

For platform D it is the exact opposite: As α increases, it becomes more expensive for platform D to attract the buyers. Hence, if for $k < k_1$ platform D subsidized the sellers in BRB business model for $\alpha = \frac{1}{2}$, it continues to do so for any larger α. However, for $k \in (k_1, k_2)$ when α increases above α_2, attracting buyers becomes more expensive for platform D then subsidizing sellers, and the platform finds it optimal to switch its business model from SRB to BRB. However, for $k > k_2$ subsidizing sellers is too expensive for both platforms. Even the disadvantaged platform, D, for such high k prefers to bear the cost of attracting the buyers and sticks to SRB for any α. The corollary below summarizes this discussion.

Corollary 1 *Small change in α may lead a platform to choose a different business model:*

(i) For $k < k_1$, an increase in α motivates platform A to switch from BRB to SRB business model.

(ii) For $k_1 < k < k_2$, an increase in α motivates platform D to switch from SRB to BRB business model.

(iii) For $k > k_2$, an increase in α has no effect on platforms optimal business models.

The results above show that the relative position in the marketas measured by the strength of the belief advantagehas a significant effect on the choice of a business model. The respective choices of the business models in turn affect the platforms' pricing decisions.

Winning platform. We now turn to showing which platform wins the market. Again, it is important to emphasize that in real-life situation, more than one platform can gain positive market share because of horizontal product differentiation that we didn't incorporate into our model. We therefore interpret the question of who wins the market as who wins the indifferent consumers: the consumers who do not have a strong preference towards one of the platforms. In real-life situation, this "wining" platform is the one to gain the higher, though not exclusive, market share.

In competing with one another, each platform reduces its access fees to the buyers in order to attract them. Eventually, one platform cannot reduce its access fees any longer without making negative profits, while the competing platform still makes positive profit and therefore can win the market.

From the discussion above, we know that platform D wins the market only if its quality is sufficiently higher than the quality of platform A. To identify the winning platform, in Proposition 2 we define the cutoff level $\overline{\Delta}$, such that in equilibrium, platform A wins the market for $\Delta > \overline{\Delta}$, and platform D wins the market otherwise. The threshold indicates extend to which higher quality relates to winning the market.

Proposition 2 (winning platform) *Let*

$$\overline{\Delta} = \begin{cases} \pi^D_{SRB}(n^{D*}) - \pi^A_{SRB}(n^{A*}) & \textit{when } (\alpha, k) \in \Omega_{SS}\,; \\ \pi_{BRB} - \pi^A_{SRB}(n^{A*}) & \textit{when } (\alpha, k) \in \Omega_{SB}\,; \\ 0 & \textit{when } (\alpha, k) \in \Omega_{BB}\,. \end{cases}$$

Then, platform A wins the market if and only if $\Delta > \overline{\Delta}$, and earns $\Pi^A = (\Delta - \overline{\Delta})N_B$. Platform D wins the market if and only if $\Delta < \overline{\Delta}$, and earns $\Pi^D = (\overline{\Delta} - \Delta)N_B$.

Notice that the threshold $\overline{\Delta}$ depends on α and k. Moreover, $\overline{\Delta} \leq 0$, which means that platform A can win the market even when it is of lower quality than platform D.[14] This is because platform A has the beliefs advantage. The following corollary describes how $\overline{\Delta}$ depends on α and k.

Corollary 2 (comparative statics on $\overline{\Delta}$)

(i) ***The effect of the belief advantage,*** α***:*** *For all regions, if $\alpha = \frac{1}{2}$ then $\overline{\Delta} = 0$. For regions Ω_{SS} and Ω_{SB}, $\overline{\Delta} < 0$, and Π^D are decreasing with α and Π^A is increasing with α. For Ω_{BB}, $\overline{\Delta} = 0$ and Π^A and Π^D are independent of α.*

(ii) ***The effect of the seller's fixed costs,*** k***:*** *For regions $\Omega_{SS}(\Omega_{SB})$, $\overline{\Delta}$ and Π^D are increasing (decreasing) with k, while Π^A is decreasing (increasing) with k. For region Ω_{BB}, $\overline{\Delta} = 0$ and Π^A and Π^D are independent of k.*

Proof. See Appendix.

The first part of Corollary 2 shows that when platforms have no belief advantage ($\alpha = \frac{1}{2}$), then the platform with higher quality wins the market, regardless of the business model that each platform adopted. Intuitively, without belief advantage, the platforms are symmetric except for their qualities. Hence the quality is the only source of competitive advantage, and it determines the identity of the winning platform.

[14] Recall that we treat α and Δ as independent of each other. In Section 5 we investigate how higher quality may lead to larger belief advantage.

For region Ω_{BB}, quality alone determines the identity of the winning platform for all $\alpha > \frac{1}{2}$. Both platforms adopt BRB business model and subsidize sellers to collect highest possible revenue from the buyers. Given the subsidy to the sellers, α-beliefs do not play a role, as each platform assures $\widehat{n}$ sellers. Consequently, the platform with the highest quality wins the market. In this region the profits are also determined solely by Δ.

In the other two regions, i.e., when at least one platform adopts SRB business model, the beliefs play a role in determining which platform wins, and what are the profits of the winning platform. Larger belief advantage gives larger competitive advantage to platform A. Therefore, this platform can win the market even if it offers lower quality ($\overline{\Delta} < 0$). This is because with larger α it can command higher access fee (or lower subsidy) from the buyers. However, if it offers lower quality, it limits the access fee it can command from the buyers. Therefore, platform D can profitably win the market if its quality advantage is sufficiently large. The higher is α, the larger quality difference platform D needs to win the market, in order to compensate for platform A's belief advantage ($\overline{\Delta}$ is decreasing in α). We also find that the negative effect of α on $\overline{\Delta}$ is stronger when both platforms adopt SRB business models, than when only platform A does so. Intuitively, in Ω_{SS}, both platforms need to offer a discount on buyers to collect the revenue from the sellers; and an increase in α increases platform A's ability to attract the buyers without too large of a discount, while the same increase forces platform D to offer a larger discount to attract the buyers. Thus, a larger quality advantage is needed to overcome both effects. In Ω_{SB}, α only has effect on platform A's discount, since platform D subsidizes sellers, and that subsidy does not depend on α.

Next consider the effect of k. As $k = \frac{K}{N_B}$, k is increasing with the sellers' fixed costs and decreasing with the number of buyers. An increase in k makes it more costly for both platforms to attract sellers into the platform, either because the fixed entry costs increased, or because there are fewer potential buyers to buy from each seller. Now, as Lemma 1 showed, in Ω_{SS}, platform A's business model involves attracting more sellers than platform D ($n^{A*} > n^{D*}$). Consequently, as k increases, platform A's ability to win the market decreases, i.e., $\overline{\Delta}$ decreases. Moreover, platform A"s profit in case it does win the market decreases, while platform D's profit increases. In contrast, in Ω_{SB}, platform A's business model involves attracting fewer sellers than platform D, because $n^{A*} > \widehat{n}$. Therefore, as k increases, platform A's ability to win the market increases, i.e., $\overline{\Delta}$ decreases; and platform A's profit in case it does win increases, while platform D's profit decreases. Finally, in Ω_{BB} both platforms attract the same amount of sellers, $\widehat{n}$, and therefore an increase in k does not change their comparative competitive advantage; hence in this case $\overline{\Delta}$, Π^A and Π^D are independent of k.

4.3 Example

Suppose for simplicity that $N_B = 1$, and that the buyer's utility is:

$$u_B(n) = \Lambda\, n - \frac{n^2}{2}$$

where Λ is a demand parameter, with $K = k < \Lambda$. Solving the above model given this functional form we obtain that $n^* = \Lambda - k$, $n^{D^*} = (\Lambda - k)/(1+\alpha)$, $n^{A^*} = (\Lambda - k)/(2-\alpha)$, and $\widehat{n} = \Lambda$. Notice that n^*, n^{D^*}, n^{A^*}, and $\widehat{n}$ indeed satisfy all of the finding of Proposition 1. In this case, $\alpha_1(k)$ and $\alpha_2(k)$ are:

$$\alpha_1(k) = \begin{cases} \frac{k^2}{\Lambda(\Lambda-2k)} & if \quad k < k_1 = \frac{1}{2}(\sqrt{3}-1)\Lambda; \\ \frac{1}{2}; & if \quad k > k_1; \end{cases}$$

$$\alpha_2(k) = \begin{cases} \frac{1}{2}; & if \quad k < k_1 = \frac{1}{2}(\sqrt{3}-1)\Lambda; \\ \frac{\Lambda^2-2\Lambda k-k^2}{\Lambda(\Lambda-2k)}; & if \quad k_1 < k < k_2 = (\sqrt{2}-1)\Lambda; \\ 1 & if \quad k_2 < k. \end{cases}$$

Moreover, drawing $\alpha_1(k)$ and $\alpha_2(k)$ obtains Figure 1.

5 Investing in Beliefs (α)

Previous section analyzed equilibria in a competition game where α is exogenously given and cannot be changed. In this section, we assume that before the competition game, the platforms can influence α. In an investment stage—before choosing busienss models and competing for buyers and sellers—exogenously given initial beliefs, α_0, can be affected by platforms' costly actions. The advantage in the initial beliefs can be interpreted as brand advantage of one platform over the other, while advertising is an example of platforms' costly action to influence α. We show that under such interpretation, advertising can be sometimes strategic complement and sometimes strategic substitute.

5.1 Setup

The initial beliefs, α_0, are exogenously given. However, the platforms can take costly action to affect the beliefs in their favor, s^A and s^D. The beliefs that result from those investments, $\alpha(s^A, s^D)$, are the market beliefs of the game described in Section 4. Since the actions are affecting the beliefs in respective platform's favor, α is increasing with s^A and decreasing with s^D. Moreover, if the investments are equal, the beliefs do not change, $\alpha(s, s) = \alpha_0$, for all s. The cost of the investment

s for each platform is $c(s)$, where $c'(s) > 0$ and $c''(s) < 0$.

The timing of the game with the additional investment stage is following: In the investment stage, the two platforms do not know Δ, but they know that Δ is distributed between $[\Delta_0, \Delta_1]$ according to the distribution $g(\Delta)$, with a cumulative distribution $G(\Delta)$. In this stage, the platforms invest by setting s^A and s^D simultaneously. At the beginning of the next stage—just before deciding on the business model—all players observe the actual value of Δ, and the investments s^A and s^D. Based on the investments, the market updates α. With this information, the two platforms compete by choosing business models and setting the fees (F_B^A, F_S^A) and (F_B^D, F_S^D). In the last stage the buyers and the sellers observe the fees and decide which platform to join.

We assume that the support of Δ in the investment stage is wide enough, i.e., $\Delta_0 < \pi_{SRB}^D(n^{D*}) - \pi_{SRB}^A(n^{A*}) < \Delta_1$ for all $\alpha \in [0, 1]$. This condition ensures that each platform has a positive probability of winning the market. Therefore, both platform have an incentive to invest in α. Otherwise, the platform that can never win, never invests in α.

The continuation game, starting with the decision about business model, is the same as the game analyzed in Section 4. In what follows, we focus on the case where both platforms choose SRB business model, i.e., where $(\alpha, k) \in \Omega_{SS}$ for all α. This will be the case if $k > k_2$. We first identify the best responses of the two platforms in the investment stage. That is, how each platform adjusts its own level of investment in advertising, given the investment of the competing platforms. Then, we move to characterize the equilibrium levels of investment.

5.2 Platforms' Best Responses

This section identifies the characteristics of each platform's optimal investment in advertising. The ex-ante expected profits are therefore:

$$E\Pi^A(s^A|s^D) = \int_{\pi_{SRB}^D - \pi_{SRB}^A}^{\Delta_1} \left(\pi_{SRB}^A - \pi_{SRB}^D + \Delta\right) g(\Delta) d\Delta - c(s^A),$$

$$E\Pi^D(s^D|s^A) = \int_{\Delta_0}^{\pi_{SRB}^D - \pi_{SRB}^A} \left(\pi_{SRB}^D - \pi_{SRB}^A - \Delta\right) g(\Delta) d\Delta - c(s^D),$$

where $\alpha \equiv \alpha(s^A, s^D)$.

The first order conditions for the platforms to maximize their profits, by choosing s^A and s^D

respectively, are

$$\overbrace{\left[1 - G\left(\pi^D_{SRB} - \pi^A_{SRB}\right)\right]}^{\text{probability of winning}} \overbrace{\left(\frac{d\pi^A_{SRB}}{d\alpha} - \frac{d\pi^D_{SRB}}{d\alpha}\right)}^{\text{profit change}} \frac{\partial \alpha}{\partial s^A} = c'(s^A) \tag{12}$$

$$\left[G\left(\pi^D_{SRB} - \pi^A_{SRB}\right)\right] \left(\frac{d\pi^D_{SRB}}{d\alpha} - \frac{d\pi^A_{SRB}}{d\alpha}\right) \frac{\partial \alpha}{\partial s^D} = c'(s^D)\,. \tag{13}$$

The interpretation of the above equations is following. Consider the platform A's first order condition, (12). The right-hand side is the marginal cost of increasing s^A. The left-hand side is the marginal benefit to the platform A. Increasing s^A increases α. The beliefs affect the probability of winning the market, and also the profit if the platform A indeed wins the market, $\pi^A_{SRB} - \pi^D_{SRB} + \Delta$. It is straightforward to show that the marginal effect of an increase in α on the platform A's profit is

$$\frac{d\pi^A_{SRB}}{d\alpha} - \frac{d\pi^D_{SRB}}{d\alpha} = U_B(n^{A^*}) + U_B(n^{D^*})\,.$$

This is because as α increases, buyers assign a higher probability to the subgame equilibrium where sellers join the platform A, and therefore the platform A benefits twice: First, increasing α allows the platform A to extract more payoff from the buyer through $U_B(n^{A^*})$. Second, the platform A also benefits from platform D's reduced ability to extract payoff from the buyer, $U_B(n^{D^*})$. The last term in the platform A's marginal benefit is the effect of s^A on α. All three terms are positive, so the left-hand side of the platform A's first order condition is positive. The intuition behind the platform D's first order condition is similar.

Since α depends on both s^A and s^D, equations (12) and (13) define the best responses of the two platforms, $s^A(s^D)$ and $s^D(s^D)$. The equilibrium strategies, s^{A^*} and s^{D^*}, satisfy $s^{A^*} = s^A(s^{D^*})$ and $s^{D^*} = s^D(s^{A^*})$. To illustrate some of the features of the best response functions, the following lemma is crucial for the analysis:

Lemma 3 *Suppose that* $\frac{d^3\pi^A_{SRB}(n)}{dn^3}$ *and* $\frac{d^3\pi^D_{SRB}(n)}{dn^3}$ *are sufficiently close to zero. Then,*

(i) if $\alpha > \frac{1}{2}$, *then* $U_B(n^{A^*}) + U_B(n^{D^*})$ *is increasing with* α*;*

(ii) if $\alpha < \frac{1}{2}$, *then* $U_B(n^{A^*}) + U_B(n^{D^*})$ *is decreasing with* α*;*

(iii) if $\alpha = \frac{1}{2}$, *then* $U_B(n^{A^*}) + U_B(n^{D^*})$ *is independent of* α*.*

Proof. See Appendix.

Analyzing the first order conditions (12) and (13) with the help of the lemma above, we can obtain the following results concerning the shape of the best response functions.

Proposition 3 (platform A's best response) *Suppose that* $\frac{d^2\alpha(s^A,s^D)}{ds^A ds^D}$ *is sufficiently close to zero. Then, or all* $\alpha(s^A, s^D) > \frac{1}{2}$ *and for* $\alpha(s^A, s^D) < \frac{1}{2}$ *but sufficiently close to* $\frac{1}{2}$*, the platform A's best response is downward sloping. Moreover, as* α_0 *increases, the platform A's best response shifts upwards.*

Proof. Follows directly from Lemma 3.

The sign of the slope of the best response is identical to the sign of the derivative of the left-hand side of the platform A's first order condition (12). In other words, the slope is affected in the same way as s^D affects the marginal benefit from increasing s^A. As s^D increases, α decreases. This has two effects. First, the platform A has a lower probability of winning (the term in the first brackets decreases). Second, from the Lemma 3 if $\alpha(s^A, s^D) > \frac{1}{2}$, then the marginal profit—given that the platform A indeed wins the market—also decreases (the second brackets). Therefore, as s^D increases, the platform A's marginal benefit of investing in α decreases and the platform A will reduce s^A. (Notice that there is a third effect because s^D may also affect $\frac{d\alpha}{ds^A}$, but we assume that this effect is negligible.) If $\alpha(s^A, s^D) < \frac{1}{2}$ but close to $\frac{1}{2}$, then the second effect changes as now the marginal profit given that the platform A wins increases (from the lemma), but if $\alpha s^A, s^D$ is sufficiently close to $\frac{1}{2}$, the first effect will still dominate. Notice that the same logic follows to an increase in α_0, which directly affects $\alpha(s^A, s^D)$.

Proposition 4 (platform D's best response) *Suppose that* $\frac{d^2\alpha(s^A,s^D)}{ds^A ds^D}$ *is sufficiently close to zero. Then, if* $\alpha(s^A, s^D)$ *is close to* $\frac{1}{2}$ *(either from above or from below), then the platform D's best response is downward sloping, and is decreasing in* α_0*. However, if* $\alpha(s^A, s^D)$ *is sufficiently high, the platform D's best response might be upward sloping and increasing in* α_0*.*

Proof. Follows directly from Lemma 3.

As s^A increases, $\alpha(s^A, s^D)$ increases and again there are the two effects to the platform D's best response. First, the platform D's probability of winning decreases, which reduces the marginal benefit from investing in s^D (the first brackets in the platform D's first order condition). Second, if $\alpha(s^A, s^D) < \frac{1}{2}$, then the marginal profit given that the platform D wins also decreases, so the platform D's incentive to invest in s^D decrease. If however $\alpha(s^A, s^D) > \frac{1}{2}$, the second effect goes in the opposite direction. If $\alpha(s^A, s^D)$ is sufficiently high, the platform D's best response may shift the sign from negative to positive. The effect of α_0 follows the same logic.

5.3 Equilibrium Investments

After establishing the platforms' best response functions, we turn to identifying the equilibrium investment levels. In particular, we are interested in showing how an increase in the base beliefs α_0, affects the equilibrium levels of investment.

From the analysis above, it follows that if $\alpha_0 = \frac{1}{2}$, the two best responses are downwards sloping, and there is a symmetric equilibrium with $s^{A^*} = s^{D^*}$ and $\alpha(s^{A^*}, s^{D^*}) = \frac{1}{2}$. As α_0 increases, the platform A's best response shifts upwards, the platform D's best response shifts downwards, and therefore s^{A^*} increases, s^{D^*} decreases and $\alpha(s^{A^*}, s^{D^*})$ increases. As α_0 further increases, the platform D's best response can be upwards sloping, and then both s^{A^*} and s^{D^*} will increase.

Corollary 3 *If $\alpha_0 = \frac{1}{2}$, both platforms invest the same positive amount of money for affecting beliefs, so beliefs remains the same and the investment is wasteful. As α_0 increases, s^{A^*} increases while s^{D^*} is first decreasing and then increasing.*

Notice also that for $\alpha_0 > \frac{1}{2}$, s^{A^*} is always higher than s^{D^*}. Hence, $\alpha(s^{A^*}, s^{D^*}) > \alpha_0$. Moreover, as α_0 increases, while both s^{A^*} and s^{D^*} change, in total $\alpha(s^{A^*}, s^{D^*})$ is always increasing. This implies that platforms' ability to affect beliefs amplifies the gap in the degree of "favorableness" of beliefs, instead of reducing it.

Corollary 4 *For all values of $\alpha_0 > \frac{1}{2}$, $\alpha(s^{A^*}, s^{D^*}) > \alpha_0$, and $\alpha(s^{A^*}, s^{D^*})$ is increasing with α_0.*

As α_0 increases, the marginal benefit for the platform A of investing increases quicker than the marginal cost. This is because not only the starting probability of winning the market increases, but also the increase in profit conditional on winning the market is larger for larger α_0. Hence, the platform A has incentive to invest more when alpha0 is large, i.e., to be more aggressive with its advertising. Thus, the platform A uses its investment to strengthen its already dominant position.

If the platform D invested a lot, it could sway the market in its favor, i.e., achieve $\alpha_0 < \frac{1}{2}$. However, it is never profitable for the platform D to do so. This is because any level of investment brings higher returns to the platform A than to the platform D. Therefore, the platform A invests more than the platform D (except for the special case of $\alpha_0 = \frac{1}{2}$). Thus, the equilibrium alpha not only will be above $\frac{1}{2}$, but it also will be above α_0. Higher α means lower profits for the platform D. The platform D uses the investment to keep α from increasing too much. And so the investment is used defensively do decrease the level of "losses."

For α_0 at $\frac{1}{2}$, the platform D has a good chance of winning the market, and invests to take advantage of this chance. In result, the competition is fierce and the investment is high. As α_0 moves away from $\frac{1}{2}$, the platform D has smaller chances of winning the market. It is not worthwhile to invest in trying to win it. And if the platform A is not too aggressive (i.e., for smaller α_0), the platform D does not need to invest too much in the defense. In fact, it may be cheaper to give up a little bit of α, than to invest in keeping it down. Then, the optimal investment of the platform D decreases as α_0 increases.

As α_0 increases further, not only the initial situation of the platform D is worse, but the platform A is also more aggressive in his investment. Now, it may be too costly to cede the remaining α. Then, it is optimal for the platform D to increase the investment so that α does not increase too far.

5.4 Example

We illustrate this result with the following numerical example. Consider the example in Section 4.3. Suppose in addition that $\alpha = \alpha_0 + s^A - s^D$, $\Lambda = 2$, $k = 1$, $g(\Delta) = \frac{1}{2}$, and $c(s) = s^2$. We solve the model for values of α_0 on the interval [0.5, 0.85]. We didn't solve for any $\alpha_0 > 0.85$ to ensure that the total equilibrium probability, $\alpha_0 + s^A - s^D$, is below one. We then numerically solved for the best responses and the equilibrium s^A and s^D by differentiating the above two functions with respect to s^A, s^D (we find that second order conditions are always satisfied).

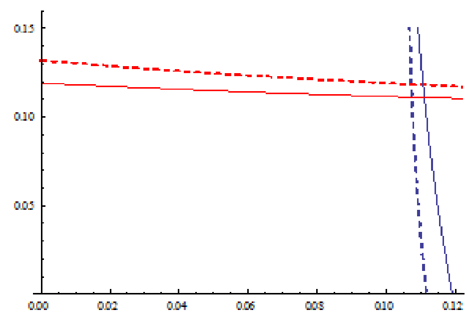


Figure 2: The best responses for $\alpha_0 = 0.5$ (bold lines) and $\alpha_0 = 0.6$ (dashed lines)

Figure 2 shows the best responses of the two platforms for $\alpha_0 = 0.5$ (the bold line) and $\alpha_0 = 0.6$ (the dashed line). Notice that for α_0= 0.5, the two best responses are downward sloping, and that there is a unique and stable equilibrium. As α_0 increases to $\alpha_0 = 0.6$ (moving to the dashed lines), the platform A's best response shifts upwards, the platform D's best response shifts downwards, and there is a new unique and stable equilibrium with a higher s^{A*} and a lower s^{D*}.

The case of asymmetric base utility (α_0 is high). Suppose now that α_0 is high. Now, as α_0 further increases, the platform D's best response can be upwards sloping, and then both s^{A*} and s^{D*} might increase.

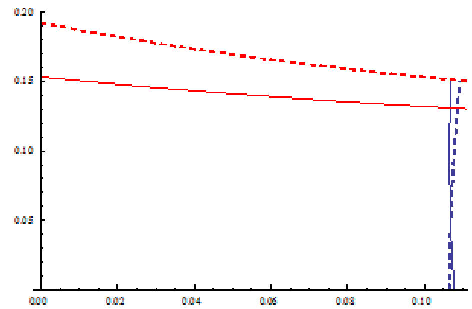


Figure 3: The best responses for $\alpha_0 = 0.7$ (bold lines) and $\alpha_0 = 0.8$ (dashed lines)

To illustrate this point, Figure 3 shows the best responses of the two platforms for $\alpha_0 = 0.7$ (the bold line) and $\alpha_0 = 0.8$ (the dashed line). Notice that for α_0= 0.7, the platform A's best response is downward sloping, but the platform D's best response is now upwards sloping. The equilibrium is still unique and stable. As α_0 increases (moving to the dashed lines), the platform A's best response shifts upwards, the platform D's best response also shifts upwards, and there is a new unique and stable equilibrium with a higher s^{A*} and a higher s^{D*}.

The non-monotonic effect of α_0 on s^{D*}. Figure 4 shows the effect of α_0 on s^{A*} and s^{D*}. given the numerical example considered above. We should note that we were unable to solve this part the model analytically, because the mathematical expression involves a high-order polynomial expressions.

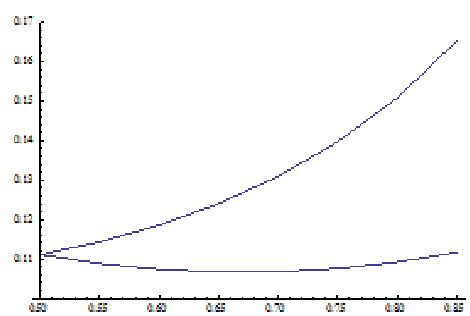


Figure 4: The equilibrium s^A and s^D as a function of α_0

Figure 4 shows that for $\alpha_0 = 1/2$, the equilibrium values of s^{A*} and s^{D*} are indeed identical. As α_0 increases, s^{A*} always increases, while s^{D*} first decreases and only afterwards increases. This is a direct consequence of the general results above that the platform A's best response is always downwards sloping while the platform D's best response is downwards sloping for low values of α_0, but might be upwards sloping for high values of α_0.

Notice that s^{A*} is always higher than s^{D*}, implying that even though the effect of α_0 on s^{D*} is non-monotonic, $\alpha(s^{A*},\, s^{E}*)$ is always above α_0.

Appendix: Proofs

Proof of Lemma 1. Since $U_B(n) = u_B(n) - n\,u'_B(n)$ and $p(n) = u'_B(n)$, we can write the first-order conditions that determine n^*, n^{A*} and n^{D*}, respectively, as

$$u'_B(n^*) - k = 0\,, \tag{14}$$

$$u'_B(n^{A*}) - k + (1-\alpha)[n^{A*}\,u''_B(n^{A*})] = 0\,, \tag{15}$$

$$u'_B(n^{D*}) - k + \alpha[n^{D*}\,u''_B(n^{D*})] = 0\,. \tag{16}$$

(14) follows directly from maximization problem on page 7. (15) follows from equation (6), and (16) follows from (10).

Since by assumption, $u''_B < 0$, the terms in the squared brackets in (15) and (16) are negative. Since $\frac{1}{2} < \alpha < 1$, it follows from the above equations that $n^* > n^{A*} > n^{D*}$. Moreover, (15) and (16) implies that as α increases, n^{A*} increases and n^{D*} decreases, with $n^{A*} = n^*$ for $\alpha = 1$. The second order conditions are:

$$u''_B(n*) < 0\,, \tag{17}$$

$$u''_B(n^{A*}) + (1-\alpha)[n^{A*}u'''_B(n^{A*}) + u''_B(n^{A*})] < 0\,, \tag{18}$$

$$u''_B(n^{D*}) + \alpha[n^{D*}u'''_B(n^{D*}) + u''_B(n^{D*})] < 0\,, \tag{19}$$

which are satisfied by assumptions of $u''_B(n) < 0$ and $u'''_B(n) < -u''_B(n)/n$.

Proof of Lemma 2. Recall that $\widehat{n}$ is the solution to $u'_B(\widehat{n}) = 0$. Comparing $u'_B(\widehat{n}) = 0$ with (14) yields that $\widehat{n} > n^*$ for $k > 0$ (if we allowed for $k = 0$, then $\widehat{n} = n^*$).

Proof of Proposition 1. To prove the characteristics of $\alpha_1(k)$ and $\alpha_2(k)$, we use the following claims.

Claim. There is at most one α, $\alpha_1(k)$, that solves $\pi^A_{SRB}(n^{A*}) = \pi_{BRB}$, such that if $1/2 \leq \alpha_1(k) \leq 1$. Moreover, $\pi^A_{SRB}(n^{A*}) > (<)\pi_{BRB}$ for $\alpha > (<)\alpha_1(k)$.
Proof. $\pi^A_{SRB}(n^{A*})$ is strictly decreasing with α while π_{BRB} is independent of α, thus $\pi^A_{SRB}(n^{A*})$ can intersect π_{BRB} only once, and $\pi^A_{SRB}(n^{A*}) > (<)\pi_{BRB}$ for $\alpha > (<)\alpha_1(k)$.

Claim. $\alpha_1(0) = 1$.

Proof. To prove the claim we need to show that evaluated at $(\alpha, k) = (1, 0)$, $\pi^A_{SRB}(n^{A^*}) = \pi_{BRB}$. From Lemmas 1 and 2 show that at $(\alpha, k) = (1, 0)$, $n^{A^*} = \widehat{n} = n^*$. Substituting $n^{A^*} = \widehat{n}$ and $\alpha = 1$ into $\pi^A_{SRB}(n^{A^*})$ yields:

$$\pi^A_{SRB} = U_B(\widehat{n}) + \widehat{n}(p(\widehat{n}) - k) = U_B(\widehat{n}) - k\widehat{n} = \pi_{BRB}\,, \tag{20}$$

where the second equality follows because, $p(\widehat{n}) = u'_B(\widehat{n}) = 0$, and the last equality follows from the definition of π_{BRB}.

Claim. $\alpha'_1(0) = 0$, $\alpha'_1(k) < 0$ and $\alpha''_1(k) > 0$.

Proof. Since $\alpha_1(k)$ is the solution to $\pi^A_{SRB}(n^{A^*}) = \pi_{BRB}$, we have

$$\frac{d\,\alpha_1(k)}{d\,k} = -\frac{\frac{d\,(\pi^A_{SRB}(n^{A^*}) - \pi_{BRB})}{d\,k}}{\frac{d\,(\pi^A_{SRB}(n^{A^*}) - \pi_{BRB})}{d\,\alpha}}\,. \tag{21}$$

The nominator of (21) is

$$\frac{d\,(\pi^A_{SRB}(n^{A^*}) - \pi_{BRB})}{d\,k} = \frac{\partial\pi^A_{SRB}(n^{A^*})}{d\,k} + \frac{\partial\pi^A_{SRB}(n^{A^*})}{\partial\,n}\,\frac{\partial n^{A^*}}{\partial k} - \frac{d\,\pi_{BRB}}{d\,k} = -(n^{A^*} - \widehat{n})\,, \tag{22}$$

where the last equality follows from the envelope theorem and from the definitions of $\pi^A_{SRB}(n^{A^*})$ and π_{BRB}. The denominator of (21) is

$$\frac{d\,(\pi^A_{SRB}(n^{A^*}) - \pi_{BRB})}{d\,\alpha} = \frac{\partial\pi^A_{SRB}(n^{A^*})}{d\,\alpha} + \frac{\partial\pi^A_{SRB}(n^{A^*})}{\partial\,n}\,\frac{\partial n^{A^*}}{\partial\alpha} - \frac{d\,\pi_{BRB}}{d\,\alpha} = U_B(n^{A^*})\,, \tag{23}$$

where the equality follows from the envelope theorem and from the definitions of $\pi^A_{SRB}(n^{A^*})$ and π_{BRB}. Substituting (22) and (23) back into (21) yields:

$$\frac{d\,\alpha_1(k)}{d\,k} = -\frac{\widehat{n} - n^{A^*}}{U_B(n^{A^*})}\,.$$

Now, for $(\alpha, k) = (1, 0)$, $n^{A^*} = \widehat{n}$, implying that $\alpha'_1(0) = 0$. As k increases, $\widehat{n}$ remains constant but n^{A^*} decreases, implying that $\alpha'_1(k) < 0$ and $\alpha''_1(k) > 0$.

Remark. Since $\alpha'_1(k) < 0$ and $\alpha''(k) > 0$, it has be that there is a k such that $\alpha_1(k) = 1/2$. We define the solution to $\alpha_1(k) = 1/2$ as k_1. As $\alpha_1(0) = 1$, it has to be that $k_1 > 0$, but we still need to prove that $k_1 < u'_B(0)$. It would be convenient for us to do this for the subsequent proof of characteristics of $\alpha_2(k)$.

Claim. There is at most one α, $\alpha_2(k)$, that solves $\pi^D_{SRB}(n^{D*}) = \pi_{BRB}$, such that if $1/2 \leq \alpha_2(k) \leq 1$. Moreover, $\pi^D_{SRB}(nDs) > (<)\pi_{BRB}$ for $\alpha < (>)\alpha_2(k)$.
Proof. $\pi^D_{SRB}(n^{D*})$ is strictly decreasing with α while π_{BRB} is independent of α, thus $\pi^D_{SRB}(n^{D*})$ can intersect π_{BRB} only once, with $\pi^D_{SRB}(nDs) > (<)\pi_{BRB}$ for $\alpha < (>)\alpha_2(k)$.

Claim. $\alpha_2(k_1) = \frac{1}{2}$.
Proof. Recall that k_1 is the solution to $\alpha_1(k) = 1/2$, thus evaluated at $(\alpha, k) = (1/2, k_1)$, $\pi^A_{SRB}(n^{A*}) = \pi_{BRB}$. To prove that it is also the solution to $\alpha_2(k_1) = 1/2$, we need to show that evaluated at $(\alpha, k) = (1/2, k_1)$, $\pi^D_{SRB}(n^{D*}) = \pi_{BRB}$, which holds if $\pi^D_{SRB}(n^{D*}) = \pi^A_{SRB}(n^{A*})$. To see that, notice that (15) and (16) imply that at $\alpha = 1/2$, $n^{D*} = n^{A*}$. Using the definitions of $\pi^D_{SRB}(n^{D*})$ and $\pi^A_{SRB}(n^{A*})$, we then have $\pi^D_{SRB}(n^{D*}) = \pi^A_{SRB}(n^{A*})$.

Claim. $\alpha_2'(k) > 0$ and $\alpha_2''(k) > 0$.
Proof. Using the envelope theorem, and applying similar calculations as in (21), (22) and (23) yields

$$\frac{d\,\alpha_2(k)}{d\,k} = \frac{\widehat{n} - n^{D*}}{U_B(n^{D*})}\,. \tag{24}$$

From Lemmas 1 and 2, $n^{D*} < \widehat{n}$, hence $\alpha_2'(k) > 0$. Moreover, as α increases, $\widehat{n}$ remains constant, while n^{D*} decreases, thus (24) increases.

Claim. There is a point, k_2, such that $\alpha_2(k_2) = 1$, where $0 < k_1 < k_2$.
Proof. Since $\alpha_2'(k) > 0$ and $\alpha_2''(k) > 0$, there is a k such that $\alpha_2(k) = 1$. Also, as $\alpha_1(0) = 1$ and $\alpha_1'(k) < 0$, it has to be that $0 < k_1$, while as $\alpha_2'(k) > 0$, it has to be that $k_1 < k_2$.

Claim. $k_2 < u_B'(0)$.
Proof. To show that $k_2 < u_B'(0)$, it is sufficient to show that evaluated at $k = u_B'(0)$, it is always the case that $\pi^D_{SRB}(n^{D*}) > \pi_{BRB}$. This is because if at $k = u_B'(0)$, $\pi^D_{SRB}(n^{D*}) > \pi_{BRB}$ for all α, it has to be that $\alpha_2(k)$ is always to the left-hand side of the vertical line defined by $k = u_B'(0)$. To show that, notice that (16) implies that if $k = u_B'(0)$, then $n^{D*} = 0$. This in turn implies that at $k = u_B'(0)$, $\pi^D_{SRB}(n^{D*}) = 0$. Turning to π_{BRB}, evaluating π_{BRB} at $k = u_B'(0)$ yields

$$\begin{aligned}
\pi_{BRB} &= u_B(\widehat{n}) - \widehat{n}\,u_B'(\widehat{n}) - \widehat{n}\,k \\
&= u_B(\widehat{n}) - \widehat{n}\,0 - \widehat{n}\,u_B'(0) \\
&= \int_0^{\widehat{n}} \left(u_B'(n) - u_B'(0)\right)\,d\,n < 0\,,
\end{aligned}$$

where the first equality follows because $u'_B(\widehat{n}) = 0$ and $k = u'_B(0)$, the second equality follows because $u_B(0) = 0$, and the last inequality follows because $u'_B(n)$ is decreasing in n. We therefore have that evaluated at$k = u'_B(0)$, $\pi^D_{SRB}(n^{D*}) = 0 < \pi_{BRB}$.

Proof of Corollary 2. Consider first the effects of α. From Lemma 1, if $\alpha = 1/2$, then $n^{D*} = n^{A*}$, implying that $\pi^D_{SRB}(n^{D*}) = \pi^A_{SRB}(n^{A*})$ and therefore $\overline{\Delta} = 0$. Using the envelope theorem, the derivatives of Π^A, Π^D and $\overline{\Delta}$ with respect to α, in region Ω_{SS}, are

$$\frac{d\,\Pi^A}{d\,\alpha} = U_B(n^{A^*}) + U_B(n^{D^*}) > 0, \qquad \frac{d\,\Pi^D}{d\,\alpha} = \frac{d\,\overline{\Delta}}{d\,\alpha} = -(U_B(n^{A^*}) + U_B(n^{D^*})) < 0\,.$$

In regions Ω_{SB}, the derivatives are

$$\frac{d\,\Pi^A}{d\,\alpha} = U_B(n^{A^*}) > 0, \qquad \frac{d\,\Pi^D}{d\,\alpha} = \frac{d\,\overline{\Delta}}{d\,\alpha} = -U_B(n^{D^*}) < 0\,.$$

In region Ω_{BB}, it is straightforward to see that α does not affect Π^A, Π^D and $\overline{\Delta}$.

Next, we turn to the effects of k. Using the envelope theorem, the derivatives of Π^A, Π^D and $\overline{\Delta}$ with respect to$k$, in region Ω_{SS} are

$$\frac{d\,\Pi^A}{d\,k} = -n^{A^*} + n^{D^*} < 0 \qquad \frac{d\,\Pi^D}{d\,k} = \frac{d\,\overline{\Delta}}{d\,k} = n^{A^*} - n^{D^*} > 0\,,$$

where the inequalities follow because $n^{D*} < n^{A*}$.

In region Ω_{SB}, the derivatives are

$$\frac{d\,\Pi^A}{d\,k} = -n^{A^*} + \widehat{n} > 0 \qquad \frac{d\,\Pi^D}{d\,k} = \frac{d\,\overline{\Delta}}{d\,k} = n^{D^*} - \widehat{n} < 0\,,$$

where the inequalities follow because $\widehat{n} > n^{A*}$. Finally, it is straightforward to see that in region Ω_{BB}, k does not affect Π^A, Π^D and $\overline{\Delta}$.

Proof of Lemma 3. We first solve for $\frac{d\,U_B(n^{A^*})}{d\,\alpha}$. Notice that

$$\frac{d\,U_B(n^{A^*})}{d\,\alpha} = \frac{d\,U_B(n^{D^*})}{d\,n}\,\frac{d\,n^{A^*}}{d\,\alpha}\,.$$

Since n^{A*} is the solution to $d\,\pi^A_{SRB}/d\,n = 0$, we have that

$$\frac{d\,n^{A^*}}{d\,\alpha} = -\frac{\frac{d^2\,\pi^A_{SRB}(n^{A^*})}{d\,n\,d\,\alpha}}{\frac{d^2\,\pi^A_{SRB}(n^{A^*})}{d^2\,n}} = \frac{\frac{d\,U_B(n^{A^*})}{d\,n}}{\left|\frac{d^2\,\pi^A_{SRB}(n^{A^*})}{d^2\,n}\right|}\,.$$

Substituting it back into the above equation yields

$$\frac{d\,U_B(n^{A*})}{d\,\alpha} = \frac{\left(\frac{d\,U_B(n^{A*})}{d\,n}\right)^2}{\left|\frac{d^2\,\pi_{SRB}^A(n^{A*})}{d^2\,n}\right|}.$$

Applying the same calculations for $U_B(n^{D*})$ yields:

$$\frac{d\,U_B(n^{A*})}{d\,\alpha} + \frac{d\,U_B(n^{D*})}{d\,\alpha} = \frac{\left(\frac{d\,U_B(n^{A*})}{d\,n}\right)^2}{\left|\frac{d^2\,\pi_{SRB}^A(n^{A*})}{d^2\,n}\right|} - \frac{\left(\frac{d\,U_B(n^{D*})}{d\,n}\right)^2}{\left|\frac{d^2\,\pi_{SRB}^D(n^{D*})}{d^2\,n}\right|}. \tag{25}$$

Next we turn to show that this term is positive (negative) for $\alpha > (<)1/2$. We first show that if $\alpha > (<)1/2$, then the nominator of the first term in (25) is higher (lower) than the nominator of the second in (25). For $\alpha > 1/2$, $n^{A*} > n^{D*}$, and since $d^2U_B(n)/d^2n > 0$, we have that $d\,U_B(n^{A*})/d\,n > d\,U_B(n^{D*})/d\,n$, implying that the nominator in the first term is higher than the second term. Next we turn to show that the denominator of the first term in (25) is lower (higher) than the second term for $\alpha > (<)1/2$. We can write the two second order conditions as:

$$\frac{d^2\,\pi_{SRB}^A(n^{A*})}{d^2\,n} = 2u_B''(n^{A*}) + n^{A*}u_B'''(n^{A*}) - \alpha\left[u_B''(n^{A*}) + n^{A*}u_B'''(n^{A*})\right],$$

$$\frac{d^2\,\pi_{SRB}^D(n^{D*})}{d^2\,n} = 2u_B''(n^{D*}) + n^{D*}u_B'''(n^{D*}) - (1-\alpha)\left[u_B''(n^{D*}) + n^{D*}u_B'''(n^{D*})\right].$$

We first compare between the two terms for a given identical n. Since by assumption, $u_B''(n) < -n\,u_B'''(n)$, the terms in the squared brackets are negative. Therefore, evaluated at the same n, $d^2\pi_{SRB}^A/d^2n > (<)d^2\pi_{SRB}^D/d^2n$ if $\alpha > (<)1/2$. Since these two second order conditions are negative, this implies that evaluated at the same n, the denominator of the first term of (25) is lower (higher) than the denominator of the second term in (25). Now, if $d^3\pi_{SRB}^A(n)/d^3n = (3-2\alpha)u_B'''(n) + (1-\alpha)n\,u_B''''(n)$ and $d^3\pi_{SRB}^D(n)/d^3n = (1+2\alpha)u_B'''(n) + \alpha\,n\,u_B''''(n)$ are sufficiently close to zero, then substituting n with $n^A(\alpha)$ and $n^D(\alpha)$ in the analysis above will not change the results, and the denominator of the first term in(25) is still lower (higher) then the denominator in the second term for $\alpha > (<)1/2$. Notice whether $d^3\pi_{SRB}^A(n)/d^3n$ and $d^3\pi_{SRB}^D(n)/d^3n$ are sufficiently close to zero depends only on the third and forth derivatives of $u_B(n)$, thus the lemma always holds if $u_B'''(n) = 0$.

References

[1] Caillaud, Bernard and Bruno Jullien. "Competing Cybermediaries," European Economic Review, Vol. 45 (2001), pp. 797–808.

[2] Caillaud, Bernard and Bruno Jullien. "Chicken & Egg: Competition among Intermediation Service Providers," RAND Journal of Economics, Vol. 34 (2003), pp. 309–328.

[3] Casadesus-Masanell, Ramon and Joan Enric Ricart. "From Strategy to Business Models and onto Tactics," Long Range Planning, Vol. 43 (2010), pp. 195–215

[4] Casadesus-Masanell, Ramon and Feng Zhu, "Strategies to Fight Ad-Sponsored Rivals." Management Science, Vol. 56 (2010), pp. 1484–1499

[5] Claussen, Jorg, Tobias Kretschmer and Philip Mayrhofer, "Incentives for Quality over TimeThe Case of Facebook Applications." Mimeo, December 2011.

[6] Ghemawat, Pankaj, Commitment: The Dynamic of Strategy. Free Press, New York 1991.

[7] Hagiu, Andrei. "Pricing and Commitment by Two-Sided Platforms," RAND Journal of Economics, Vol. 37 (2006), pp. 720–737.

[8] Halabruda, Hanna and Andrei Hagiu, "Responding to the Wii?" Harvard Business School Case, No. 709-448, 2009

[9] Halaburda, Hanna and Yaron Yehezkel "Platform Competition under Asymmetric Information," Harvard Business School Working Paper, No. 11-080, February 2011

[10] Jullien, Bruno "Competition in Multi-Sided Markets: Divide-and-Conquer", American Economic Journal: Microeconomics, Vol. 3, (2011), p. 1–35.

[11] Katz, Michael and Karl Shapiro. "Network Externalities, Competition and Compatibility," American Economic Review, Vol. 75 (1985), pp. 424–440.

[12] Lopez, Angel and Patrick Rey, "Foreclosing Competition through Access Charges and Price Discrimination." IDEI Working Paper, n. 570, June 2009.

[13] Rochet, Jean-Charles and Jean Tirole. "Platform Competition in Two-sided Markets." Journal of the European Economic Association, Vol. 1 (2003), pp. 990–1029

[14] Spiegler, Ran. "Extracting Interaction-Created Surplus." Games and Economic Behavior, Vol. 30 (2000), pp. 142–162