Innovation, Fast Seconds, and Patent Policy

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Abstract

We develop a model of innovation in which entrepreneurs develop a new (differentiated) product market that is subsequently exploited by a well-established firm that “stretches” its brand to enter the new market as a “fast second”. In this setting, there is a positive externality to the pioneering efforts of the initial entrants that may well increase with the number of such entrants. We develop a model that exhibits this externality and use it to analyze the effectiveness of patent policy—specifically patent breadth—in encouraging the socially optimal amount of initial entry.

Key Words: fast second, product differentiation, patent breadth

JEL Classification Codes: L5, O25

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1 This research benefited from a Tilburg Law and Economics Center (TILEC) IIPC grant. We are grateful to participants at the EARIE Conference in Istanbul, 2010 for comments on an earlier draft.
1. Innovative Entrepreneurs and “Fast Second” Entrants

This paper explores the dynamics of product innovation and market evolution, and the role of patent policy in insuring that this process yields an efficient outcome. Economists have been concerned about the processes underlying the evolution of industrial structure at least since the work of Gibrat (1931) and Schumpeter (1942). Over time, this has led to the recognition of the critical role played by entrepreneurial startups in bringing basic inventions to the market in the form of new, innovative applications: see, for example, Kortum and Lerner (2000).

Yet if established firms are not always the originators of new product markets, they often play an equally critical role in building upon the experiences of the initial startups and entering with a branded version of the product that consumers prefer. By playing this “fast second” role, emphasized by Markides and Geroski (2005), the commercial advantages of large, established firms frequently allow them to emerge as the dominant player in the industry despite their late entry. Procter & Gamble for instance, was more than a decade behind the initial introduction of disposable diapers, with a number of firms marketing such a product by the time that P&G finally launched their “Pampers” diaper in 1964. Yet by 1968, P&G’s market share was well over 90 percent. Similarly, it was Microsoft that built on the early spreadsheet programs of Visi-calc and Lotus to develop the ultimate market winner, Excel.

Markides and Geroski (2005) show that these examples can be multiplied many times over. In fact, it is now clear that such “fast second” moves are an explicit part of corporate strategy. Schnars (1994) quotes a Coca-Cola manager giving dramatic testimony in support of this strategy: “We let others come out, stand back and watch, and then see what it takes to take the category over.” Baumol (2010) also describes this division of labor between small, entrepreneurial firms that make initial, innovative breakthroughs and large, bureaucratic firms...
that build on these breakthroughs by further *replicative* enhancements as a fundamental “David and Goliath” feature of the innovation process.

The later entry of large, established firms as “fast seconds” is likely an important reason that a large proportion of the many small new firms that start each year typically exit within a short time and earn little economic profit. The well-known manufacturing study of Dunne, Roberts, and Samuelson (1988) and the more recent survey of retailing entry by Jarmin, Klimek, and Miranda (2004) both find that over 60 percent of new firms exit within five years of entry and 80 percent leave within ten years. Of course, such exit can be accomplished by selling out to another firm rather than simple failure. Leading textbooks in entrepreneurship, such as Barringer and Ireland (2008), explicitly identify selling to a major established firm as an entrepreneurial exit strategy. However, Cummings and Macintosh (2003) and Cochrane (2005) present evidence that from 1987 to 2000 only about 20% of venture backed firms were acquired. Moreover, even when a small entrepreneurial firm is acquired by a large, established one the terms of acquisition do not necessarily favor the acquired firm. The evidence from Astebro (2003) and Nordhaus (2004) is that entrepreneurs typically earn close to zero economic profit and capture only a small fraction of the surplus that their inventions generate.

In short, a common pattern of industrial evolution is one in which a first generation of small, pioneering firms create a market that is later targeted for entry by a large, established firm. It follows that the fact of such fast second entry and its anticipation by the pioneering firms will have a major impact on the market environment. Viewed in this light, the innovation process is analogous to an ecological food chain in which a thriving population of little fish provides important nutrients for the larger “big fish”. In turn, these fast second big fish provide nontrivial product enhancements to final consumers. In both systems, there is a danger that the big fish or
fast second firms will over-harvest and leave too few little fish, thereby threatening the health of the entire system.

In this paper, we build on our earlier work (Norman, Pepall and Richards, 2008; referred to hereafter as NPR) to develop a model of market evolution that incorporates this “big fish-little fish” analogy. The central focus of our analysis is to explore the role of patents as a mechanism for enhancing the performance and sustainability of this dynamic market environment. Specifically, we explore the market evolution and the associated welfare outcomes when initial startups create a new market but do so anticipating the later entry of a fast second offering a competing product that consumers value more highly. To some extent, our model incorporates the sequential aspect of innovation emphasized by Bessen and Maskin (2009), but does so in the specific “big fish-little fish” scenario that may well be the most common example of such sequentiality.

In our framework, the externality between the initial entrants and the gains provided by the fast second is a central reason that unconstrained market evolution fails to maximize total surplus. In the absence of patents, rational anticipation by the initial startups of the fast second’s later entry and the likelihood of survival leads to too little initial entry. Intuitively, the externality between the initial startups and fast second is that the added value from the “fast second’s” later entry increases with the number of initial entrants.

Our primary concern is to investigate whether, and under what circumstances patent policy, particularly patent breadth, can correct this market failure. We find that somewhat broad patents covering a wide product spectrum may actually induce more initial, innovative entry than would otherwise occur and thereby raise social welfare. Viewed in this light, our work reveals an important qualification to Bessen and Maskin (2009) result that broad patents make it difficult for further product enhancement by subsequent producers. Because broad patents encourage
early entry, patents increase the value added by a fast second and hence the incentive for late entry. However, we also find that achieving a first-best solution via patent policy is not always possible. A novel feature of our paper is that we find patent policy is more likely to achieve the social optimum in markets where consumers have particularly strong attachments to their most preferred versions of the product.

Our basic model of initial entry and survival after the emergence of a fast-second is presented in the next section. Optimal patent breadth policy is analyzed in Section 3. A summary with concluding remarks follows in Section 4.

2. A Big Fish, Little Fish Model of Innovation and Market Evolution

We consider a technological breakthrough whose commercialization leads a number of startup firms to create a new, horizontally differentiated, product market. The assumption of horizontal product differentiation makes sense for product innovation in “high tech” sectors such as information technology, biotechnology, aerospace, nanotechnology and robotics.

Product differentiation is represented by the “Salop circle”, a one-dimensional circular space $\Lambda$ whose length is normalized to unity. Consumer preferences for these new products are assumed to be uniformly distributed over the circle at density $d$, which we can also normalize to unity. The “location” of consumer $s$ defines that consumer’s most preferred specification for a new product in this market. For each consumer $s$, the surplus obtained from buying the new product with specification $x$ at price $p(x)$ is:

$$U(x,s) = V - p(x) - t|x - s| \quad (1)$$

Here, $V$ is the consumer’s reservation price and $t$ is the loss of utility per unit “distance” that the consumer incurs in buying other than her most preferred product. In other words, $t$ is a direct measure of the intensity of consumer preferences. Each consumer buys exactly one unit of the
product that offers her the greatest consumer surplus (we assume that \( V \) is great enough to ensure that the market is covered).

When an innovative entrant commercializes the new technology it incurs a sunk cost \( f \) and locates on the circle.\(^2\) We assume that marginal costs for all firms are constant and so can be normalized to zero without loss of generality. The “location” \( x_i \) of innovative firm \( i \) is the specification of its basic new technology. Relocation of the basic technology is assumed to be prohibitively costly, but the innovator can adapt its basic technology incurring versioning costs to offer a line of products designed specifically to meet the most preferred specifications of its potential consumers. Versioning by innovator \( i \) of the basic technology \( x_i \) to the product specification \( x \) incurs the cost:

\[
MC_i(x_i, x) = r|x_i - x| \tag{2}
\]

The versioning technology has two essential features (Eaton and Schmitt, 1994; Norman and Thisse, 1999). First, it is perfect in the sense that a consumer is indifferent between the product \( s \) and a basic product \( x_i \) versioned to \( s \) if these two products are offered at the same price. Second, we assume that the versioning cost \( r < t \). As a result, the early innovators have a profit incentive to version their products to the preferences of each consumer they serve in the new market.\(^3\)

An important property of versioning (Norman and Thisse, 1999) is that it forces our innovative firms to adopt discriminatory prices. To see why, suppose, by contrast, that firm \( i \) adopts non-discriminatory (fob) pricing. Versioning gives a firm control of “delivery” of its product to consumers. As a result, a neighboring firm \( j \) can offer a versioned product that is priced to undercut firm \( i \) at any of firm \( i \)’s consumer locations such that firm \( i \)’s delivered price is greater than firm \( j \)’s production plus versioning costs. Hence, no commitment to adopt non-

\(^2\) Since profit is homogeneous of degree one in consumer density \( d \), we could alternatively consider sunk costs per capita: \( f = F/d \).

\(^3\) Without this assumption, no firm would offer a versioned product.
discriminatory pricing is sustainable. Rather, offering versioned products at discriminatory prices is a dominant strategy for all firms.4

Competition in the new market is modeled as a four-stage, two-period game. In the first stage and first period a set of risk-neutral innovative firms choose whether or not to open the market by spending on research and development. Each innovative pioneer $i$ expects to secure a product patent that covers part or all of its market area. Formally, if there are $n$ initial entrants we assume that the patent gives each firm $i$ the exclusive right to produce versioned products within an arc of length $w/n$ centered on its basic product $x_i$ with $0 < w \leq 1$ to be determined by the patent office.5 In the second stage after the patents have been awarded, these firms compete in prices.

In the third stage and second period a fast-second chooses whether or not to enter the market. It will do so if, because of its brand name or other advantages, it can offer a product considered by consumers to be superior to the substitutes offered by the early entrant firms. Such entry results in the exit of a number of the initial entrants, the extent of exit being determined by the relative brand strengths of the initial entrants and the fast-second. In the fourth stage all firms remaining in the market compete in prices.

We solve for the perfect equilibrium to this 4-stage game. This implies that the innovative firms, in making their entry decisions in stage 1, correctly anticipate the price competition outcome in stage 2, the possibility of fast-second entry in stage 3, and the post-entry price competition and equilibrium of stage 4. All firms have a unit discount factor.

4 Baumol (2010) also presents a model of innovation in which discriminatory prices are a necessary part of any equilibrium.

5 This assumption has much in common with Klemperer’s (1990) analysis of patent breadth. In common with other Salop-style models, we assume that the innovative entrants are uniformly distributed over the consumer preference space. As a result, we need not worry about the possibility of patent infringement by the initial entrants.
2.1 Opening the Market

Consider the price equilibrium outcome for stage 2 of the game. Assume that there are \( n \) initial entrants in stage 1 uniformly distributed over the market space. As noted above, the patent authority awards each original innovative entrant a patent of arc length \( w/n \) \((0 < w \leq 1)\) centered on its basic technology. The patent regime changes the equilibrium Bertrand-Nash prices as compared to the no-patent price equilibrium outcome as illustrated in Figure 1.

Consider a representative consumer whose “location” is \( s_i \in \left[ \frac{w}{2n}, \frac{1}{2n} \right] \) “to the left” of innovative firm \( i \). Firm \( i - 1 \) can offer this consumer a product perfectly customized to \( s \) without infringing upon firm \( i \)’s patent. Standard analysis (Norman and Thisse op. cit.) gives the Bertrand-Nash equilibrium price to this consumer as

\[
p_i(s_i) = r \left( \frac{1}{n} - s_i \right)
\]  

(3)

Now consider a representative consumer with location \( s_0 \in \left[ 0, \frac{w}{2n} \right] \) “to the left” of innovative firm \( i \). The best that firm \( i - 1 \) can offer this consumer is the product customized to the boundary of firm \( i \)’s patent region.\(^6\) In order to purchase this product a consumer at \( s_0 \) has to “travel” a distance \( w/2n - s_0 \), incurring costs \( t(w/2n - s_0) \). As a result, the Bertrand-Nash equilibrium price to this consumer is

\[
p_i(s_0) = r \left( \frac{1}{n} - \frac{w}{2n} \right) + t \left( \frac{w}{2n} - s_0 \right)
\]  

(4)

The resulting Bertrand-Nash equilibrium prices are illustrated by the bold line in Figure 1.

(Figure 1 hear here)

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\(^6\) As noted above, our approach is similar to Klemperer (1990). Important differences are: 1) Klemperer’s consumers all agree on the most preferred product whereas our consumers have heterogeneous preferences over product types; 2) Klemperer has one patent whereas we allow for multiple patents; and 3) Klemperer’s consumers have heterogeneous taste parameters \( t \) whereas our consumers have identical taste parameters.
The price at each consumer location equals the cost incurred by the innovator owning the second-nearest basic technology to provide the closest versioned product that the patents allow, plus any utility cost to the consumer if the best alternative involves purchasing a product that is not that consumer’s most preferred product.

The first-period profit of an innovative entrant, ignoring the sunk costs of commercializing the technology, is:

\[
\pi_t = \frac{r}{2n^2} + \frac{(t-r)w^2}{4n^2}
\]  

(5)

The second term in (5) is the additional profit generated by the patent regime. As can be seen, a firm’s ability to compete beyond its patent boundary now depends on the consumer taste parameter \( t \). The patent regime increases the first-period profits of the innovative entrants because the patents restrain price competition. Moreover the additional profits generated by the patent regime increase as the taste parameter \( t \) increases. In other words, the protection afforded by the patent regime is more valuable to firms when consumer tastes are more intense. This link between the profit-impact of patents and the taste parameter \( t \) is a unique dimension of our model.

2.2 Fast Second Entry and Innovative Survival

Innovative entry reveals the new market to potentially bigger predators. From the perspective of a potential fast second, this is an important and valuable role played by start-up firms. A fast second firm \( b \) is by assumption an established firm in another market that is able either to make a product or service improvement or leverage its brand strength in a way that gives it a brand or quality advantage relative to the pioneer innovators. The result is that consumers are willing to pay more for firm \( b \)'s new products than for those developed by the original innovative firms.
As in NPR, we assume that there is a population of established firms, each characterized by a parameter \( \alpha \) uniformly distributed between 0 and 1. Nature randomly selects one of these firms and that firm, having a specific value of \( \alpha \), decides whether to enter the new market. If the fast second enters we assume that it operates the same customizing technology as the innovative entrants. A technical assumption that we make throughout the analysis is:

Assumption r: \( r \leq 1 \).

The effect of this assumption is that if the fast-second has maximal brand strength \( \alpha = 1 \) it is able to monopolize the market in the absence of patent protection.

At the time of the fast second’s entry or stage 3 there are \( n \) innovative entrants, symmetrically located on the circle, each with a market share of \( 1/n \). We follow NPR and assume that:

“the brand advantage that the fast second enjoys relative to each of the entrepreneurial entrants is measured by the difference between its own brand strength \( \alpha \) and the pioneer’s brand strength \( 1/n \). Accordingly, if the fast second enters, this advantage has the effect of increasing the consumer willingness to pay for the brand-stretcher’s product above that for early entrant firms’ products by its relative brand strength.”

(NPR, p. 6)

In other words, the expertise and advantage that the fast second firm brings to the new market is captured by assuming that consumers are willing to pay \( (\alpha - 1/n) \) more for its product than they are willing to pay for a substitute product offered by a pioneering firm.\(^7\) This assumption captures the idea that the product enhancement brought by the fast second is greater the greater is the number of initial entrepreneurial entrants, but then precisely for that reason,

\(^7\) Pepall and Richards (2002) and NPR provide an extensive discussion in support of the reasonableness of this assumption.
each entrepreneurial first entrant is more vulnerable to fast-second entry the more such initial entrants there are.

We know that uncertain survival is a fact of life for startups. That uncertainty is reflected in our model in two ways. First, whether the fast second will enter is probabilistic owing to Nature’s random selection of $\alpha$. Second, if the fast second enters, its location along the circle is also uncertain. To be precise, an innovative firm $j$ faces three possible outcomes in the wake of $b$’s entry in stage 3. One is that nothing changes, which happens if there is at least one surviving innovator in both directions between $j$ and $b$. Price discrimination via versioning localizes all competition with the result that the two firms $j$ and $b$ do not compete for customers in this case. The second possible outcome is that competition with $b$ is so intense that firm $j$ exits. The final possibility is that all the early innovators between $j$ and $b$ exit so that $j$ becomes either the left-or right-hand survivor that directly competes with $b$. In this case, for the innovator $j$ to survive it must be able to undercut profitably the big firm’s superior brand and retain market share.

At the beginning of stage 3 there is a patent regime such that each initial entrant has a patent to supply customers within the arc length $w/2n$ on each side of its basic technology. We distinguish two cases. In Case 1, we have $w < 1$ and the fast second can enter mid-way between any two initial entrants without infringing their patents. In Case 2, $w = 1$ and patent infringement becomes an issue. The fast second, in order to be able to enter, must purchase the patents of some subset of the innovative entrants.

2.1.1 Case 1: Partial Patents ($w < 1$)

Assume that the fast-second firm $b$ enters mid-way between two early entrants or incumbents. This location ensures that $b$ does not infringe on any existing patents.\(^8\) We can then calculate the number, $n_s$, of innovative entrants that can be expected to survive $b$’s entry.

\(^8\) NPR identify the conditions under which this is an optimal location choice for the fast second in the absence of patent protection.
Since \( n \) is independent of \( b \)'s actual location, we can number the early entrants such that \( b \) enters between firms 1 to the right and \(-1\) to the left of \( b \) (or 1 and \( n \)). It follows that the probability that any initial entrant survives is \( n/n \), again independent of \( b \)'s actual location. What determines \( n \)?

To eliminate the two early entrants that are the \( j \)th nearest neighbors to \( b \) it is not sufficient for \( b \) to have relative brand strength \( r((j - 1)/n + 1/2n) \) as in NPR. This will capture all of \( j \)'s consumers outside \( j \)'s patent region but not those within \( j \)'s patent region. What is necessary is that \( b \) can capture a critical consumer \( s_0 \) located within the patent region “to the right” of \( j \) (or “to the left” of \(-j\)). Consider a consumer distance \( s \) “to the right” of \( j \) and within \( j \)'s patent region. If this consumer travels “to the right” to purchase from \( b \) the best offer that \( b \) can make is

\[
p_r(s,j) = r\left(\frac{j-1}{n} + \frac{1}{2n} + \frac{w}{2n}\right) + t\left(\frac{w}{2n} - s\right)
\]  \hspace{1cm} (6)

The first term in (6) is \( b \)'s cost of providing a product versioned to the right-hand boundary of \( j \)'s patent region and the second term is the additional cost that a consumer at \( s \) incurs in purchasing this product. By contrast, if this consumer purchases from \( b \) by travelling “to the left”, the best offer that \( b \) can make is

\[
p_l(s,j) = r\left(\frac{j-1}{n} + \frac{1}{2n} - \frac{w}{2n}\right) + t\left(\frac{w}{2n} + s\right)
\]  \hspace{1cm} (7)

with an interpretation similar to (6). When the prices in equations (6) and (7) are equal, a consumer at \( s \) is indifferent between purchasing firm \( b \)'s product either by traveling to the left or right boundary of \( j \)'s patent region. This implies that the critical value of \( s \) is:

\[
s_0 = \frac{rw}{2nt}
\]  \hspace{1cm} (8)

Note that \( s_0 \) is independent of \( j \). Note also that if the consumer at \( s_0 \) buys from the brand stretcher \( b \), then all of \( j \)'s consumers both to the right of this point and to the left will also buy
from $b$. In other words, if $b$ can capture the consumer distance $s_0$ “to the right” of incumbent $j$ then $b$ captures $j$’s entire market. Substitution of (8) into either (6) or (7) implies that the best offer that $b$ can make to a consumer at $s_0$ is

$$p_r(s_0, j) = \frac{(2j - 1)r + tw}{2n}$$

(9)

In contrast, the best offer that the incumbent firm $j$ can make is

$$p_j(s_0) = rs_0 = \frac{r^2w}{2nt}$$

(10)

The difference between these two prices is the minimum relative brand-strength that will allow $b$ to undercut the two incumbent firms $j$ and $-j$ and steal all their customers.

We can put this another way. Incumbent firms $j$ and $-j$ survive $b$’s entry so long as $b$’s relative brand strength $\alpha - \frac{1}{n} < p_r(s_0, j) - p_j(s_0)$, which gives the critical value of $\alpha$

$$\alpha_s(j, n) = \frac{1}{n} + \frac{(2j - 1)rt + (t^2 - r^2)w}{2nt}$$

(11)

An incumbent firm is less likely to be forced to exit if patent protection ($w$) is broader and consumer tastes ($t$) are more intense.

To ease calculations, assume that $n$ is even and equal to $2m$. (This loses no generality since, in the analysis, we smooth out what would otherwise be step-functions by treating $n$ as a continuous variable.) The number of incumbents that can be expected to survive $b$’s entry is:

$$n_s(n, r, t, w) = \int_0^{\alpha_s(1,2m)} 2md\alpha + \sum_{j=1}^{m} (2m - 2j) \int_{\alpha_s(j,2m)}^{\alpha_s(1,2m)} 1d\alpha = 1 + \frac{nr}{4} + \frac{(t^2 - r^2)w}{2t}$$

(12)

The first term is the range of $\alpha$ over which either $b$ does not enter or all the initial entrants survive $b$’s entry, while each term in the summation is the range of $\alpha$ over which the $j$th nearest neighbors to $b$ are undercut and so exit the market. As we would expect, a greater number of
initial entrants survive if patents are broader or consumer tastes are more intense. The probability of an innovative entrant surviving fast-second entry is therefore:

\[ \rho(n,r,t,w) = \frac{n_s(n,r,t,w)}{n} \]  \hspace{1cm} (13)

Conditional on survival, a pioneering entrant will earn a total profit over both stage 2 and 4 equal to

\[ \pi_1 + \left( \pi_1 - k(\alpha) \right) \frac{2}{n_s} + \pi_1 \left( \frac{n_s(n,r,t,w) - 2}{n_s} \right) \]

where \( \pi_1 - k(\alpha) \) is the profit that the pioneer expects to earn in stage 4 given that it is one of the two nearest surviving neighbors to \( b \), which occurs with probability \( 2/n_s(n,r,t,w) \). In this equation \( k(\alpha) \) is the profit that such a nearest neighbor loses as a result of the encroachment by \( b \) on its market. Rather than calculate \( k(\alpha) \) explicitly\(^9\) we consider the boundary values of \( k(\alpha) \), which are \( \pi_1 \) and 0. As a result, for a pioneering entrant, the total expected profit over both is bounded by:

\[ \pi_{eu}(f,n,r,t,w) = \pi_1 + \rho(n,r,t,w)\pi_1 - f \]

\[ = \frac{1}{16tn^3} \left( 2r + (t-r)w^2 \right) \left( 4t(n+1) + ntr + 2w(t^2 - r^2) \right) - f \]

\[ \pi_{el}(f,n,r,t,w) = \pi_1 + \rho(n,r,t,w) \left( \frac{n_s(n,r,t,w) - 2}{n_s} \right) \pi_1 - f \]

\[ = \frac{1}{16tn^3} \left( 2r + (t-r)w^2 \right) \left( 4t(n-1) + ntr + 2w(t^2 - r^2) \right) - f \]  \hspace{1cm} (14)

The equilibrium number of innovative entrants is, therefore, bounded by

\[ n_e(f,r,t,w) \in \left[ n_i : \pi_{ei}(f,n,r,t,w) = 0, n_u : \pi_{eu}(f,n,r,t,w) = 0 \right] \]  \hspace{1cm} (15)

While (14) and (15) do not permit of a simple, “clean” solution for \( n_e \), we can show that the upper and lower bounds on \( n_e \) are increasing in \( w, t \) and \( r \) and decreasing in \( f \).

\(^9\) Calculation is both complex and tedious.
The patent regime protects each innovative entrant from competing directly with the products of the fast-second late entrant. Protection is stronger in markets where consumer tastes, as measured by the parameter $t$, are more intense and when the patent breadth is greater. As a result, we expect there to be more innovative entry when consumer tastes are more intense and patents are broader. This is illustrated in Figure 2, which shows how the equilibrium number of innovative entrants is affected by various parameters in the model: the solid contours are for $r = 0.2$ and the dashed contours for $r = 0.3$. In each case, the double contours show the upper and lower bounds on the number of innovative entrants for $k(\alpha)$ equal to 0 and $\pi_1$, respectively.

(Figure 2 near here)

Of course, what is important to any pioneering entrepreneur is its expected profitability over the entire game which, in turn, reflects its probability of survival. In this respect, the results above carry a mixed message. On the one hand, there is a *protection effect* in that broader patent width and more intense consumer preferences raise the likelihood of survival for any entrepreneur given the number of such initial entrants. Yet, as also just shown, increases in $w$ and $t$ also have a *brand weakening effect* in that they raise the number of initial entrants. More initial entrants means that each such entrant has a smaller market share and therefore a weaker relative brand identity, making them more vulnerable to fast-second entry.

We illustrate two typical cases in Figure 3 given that $n_e() = n_u$.\(^{10}\)

(Figure 3 near here)

In general, the probability of survival of an innovative early entrant is greater when consumer tastes $t$ are stronger. By contrast, increased patent width generally decreases the probability of survival when versioning costs are low but increases it when versioning costs are high. Intuitively, when versioning costs are low there is less innovative entry, with the result that

\(^{10}\) The comparative statics are qualitatively identical for $n_e = n_u$. 
each innovative entrant has relatively high brand strength. But then an increase in patent breadth will have a strong brand weakening effect.

Note also that, while an increase in patent width $w$ increases the proportion of the total differentiated product market that is covered by patent protection, the actual span of patent protection $w/n_e$ for an individual innovative entrant may be greater or smaller. Which is the case depends upon the elasticity of $n_e$ with respect to $w$.

**2.1.2 Case 2: Full Patent ($width = w_p$)**

In Case 2 each innovative entrant expects to be awarded a patent that covers its full market area. Note, however, that this does not imply that the Patent Office sets $w = 1$, in which case there would be $n_e(f, r, t, 1)$ innovative entrants. Rather, the full patent case is one in which the Patent Office awards each innovative entrant a patent of width $w_p$, and as a result, patent width determines the number of innovative entrants. It is convenient for the full patent case to change our notation slightly. Instead of identifying the arc length of patent width as $w/n$ as in Case 1, we now define patent width simply as $w_p$ and impose the constraint that $n_p = 1/w_p \leq 1/n_e(f, r, t, 1)$. This approach anticipates our discussion of optimal patent breadth in which we consider three possibilities: 1) the social optimum can be obtained by awarding a partial patent; 2) the social optimum can be obtained by awarding a full patent of width $w_0$; 3) the Patent Office has to settle for second-best, awarding a full patent that maximizes total surplus subject to the constraint that the innovative entrants just break even.

With a full patent, the first-period profit of an innovative entrant, again ignoring the sunk costs of commercializing the technology, is from (5):

$$\pi_{1p} = (r + t)w_p^2 / 4 = (r + t)/4n_p^2$$

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In contrast to the partial patent system, for entry of the fast-second to be feasible while avoiding patent infringement, firm $b$ must purchase the patent of at least one early entrant. We
assume non-cooperative behavior among the early entrants so that it is not possible for them to act as a coalition and jointly refuse to sell their patent rights to firm $b$.

We also assume that if firm $b$ purchases early entrants’ patents, then $b$ markets these products by versioning its own basic technology. In other words, the fast second purchases the right to offer the versioned products but does so using its basic technology and leveraging its relative brand strength of $\alpha - 1/n_p$. One possible justification for assuming a single basic technology is that it is a better “branding” strategy and minimizes risk to $b$’s brand name in this market.\(^{11}\)

The remaining question is how much the fast second pays to acquire an early entrant’s patent. We assume that each early entrant establishes a reservation price at which it is willing to sell its patent based on its expected profit in stage 4, taking into account the probabilistic nature of the fast second’s brand advantage and of its entry location in stage 3. We also assume that an early entrant, in calculating its probability of survival given that it is the $j$th nearest neighbor to the fast-second reckons that the fast-second has purchased the patents of its $(j - 1)$ nearest neighbors. We can then work out the number of early entrants that are expected to survive entry of the fast second, and from that calculate the probability that each early innovative entrant will survive in stage 4.

Assume that the fast second enters by buying the patent of firm 0.\(^{12}\) To be profitable the fast second late entrant must have a brand strength or advantage of at least $\alpha = w_p = 1/n_p$. For the two nearest neighbors, 1 and $n - 1$, to survive it must be that the fast-second is unable to undercut its nearest neighbor 1 in supplying that nearest neighbor's right-most consumer, given that this consumer has to “travel” in order to purchase the fast-second's customized product. The lowest

\(^{11}\) Note that if it were sensible for the fast second to produce at multiple locations, the problem would become trivial as it would simply buy all initial entrants’ patents.

\(^{12}\) We can always number the innovative entrants such that this is the case.
price that the consumer faces in buying from the fast-second is $rw_p/2 + tw_p$ while the price paid to firm 1 is $rw_p/2$. So we require that $\alpha - w_p < tw_p$. Similarly, for the second-nearest neighbors 2 and $n - 2$ to survive, now given that these firms assume that the fast-second has acquired the patents of 1 and -1, the lowest price that the consumer faces in buying from the fast second is $3rw_p/2 + tw_p$ while the price paid to firm 2 is $rw_p/2$. So we require $\alpha - w_p < tw_p + rw_p$. For the third-nearest neighbors 3 and $n - 3$ to survive we have the same argument again. The lowest price that the consumer incurs in buying from the fast second is $5rw_p/2 + tw_p$ while the price paid to firm 3 is $rw_p/2$. So we require $\alpha - w_p < tw_p + 2rw_p$.

We can extend the above argument repeatedly for $m_p$ early entrants on either side of the fast second’s point of entry on the circle under the assumption that the initial number of early entrants $n_p = 1/w_p$ is odd and equal to $2m_p + 1$.\(^{13}\) Given the distribution of the fast-second’s brand advantage $\alpha$, this allows each early entrant to determine the likely number of survivors in the wake of a fast second’s entry. This is:

$$s_p(n_p, r, t) = \int_{0}^{n_p} n_p d\alpha + \int_{n_p}^{(1+n_p)/2} 2m_p d\alpha + \sum_{j=2}^{n_p} \left(2m_p + 2 - 2j\right) \int_{(1+r* (j-2))/n_p}^{(1+r* j)/n_p} 1 d\alpha$$

$$= 1 + \left(\frac{n_p-1}{4n_p}\right)\left((n_p-3)r + 4t\right)$$

The first term in (17) is the range of $\alpha$ such that firm $b$ does not enter, the second term the range of $\alpha$ such that all firms other than firm $0$ survive, and each term in the summation the range of $\alpha$ such that the $j$th nearest neighbors survive. It follows that the probability of an innovative entrant surviving fast-second entry is:

$$\rho_p(n_p, r, t) = \frac{s_p(n_p, r, t)}{n_p}$$

\(^{13}\) As in Case 1, since we ignore integer constraints on $n$ in our formal analysis no generality is lost in Case 2 by assuming $n$ is odd in calculating the probability of survival.
As in the partial patent case, the probability of survival of an entrepreneurial entrant is increasing in the intensity of consumer preferences.

Given equations (18) and (16), each early entrant can work out its expected profit for stage 4 of the game $E(\pi^2_p)$ As in Case 1 this takes the form

$$E(\pi^2_p) = \rho_p(n_p, r, t) \left[ (\pi_{1p} - k_p(\alpha)) \frac{2}{s_p(\cdot)} + \pi_{1p} \frac{s_p(\cdot) - 2}{s_p(\cdot)} \right]$$

(19)

where $k_p(\alpha)$ is bounded between 0 and $\pi_{1p}$. As in Case 1 we identify bounds on the innovative entrants’ expected profits rather than calculate $k_p(\alpha)$ explicitly.

The innovative entrants expect that if the fast second enters in stage 3 it does so by announcing an auction for the patent rights of early entrants. The innovative entrants expect this auction to clear at price $E(\pi^2_p)$. Each innovative entrant expects to earn this profit, on average, whether or not it actually sells its patent. It is then straightforward to show that for an innovator or early entrant, the total expected profit for the game is bounded by:

$$\pi_{pe}(f, n_p, r, t) = \frac{(r + t)}{4n_p^2} \left( 1 + \rho_p(n_p, r, t) \right) - f$$

$$= \frac{(r + t)}{16n_p^2} \left( n_p^2 \left( 4 + r \right) + 4n_p \left( 1 + t - r \right) - (4t - 3r) \right) - f$$

$$\pi_{pl}(f, n_p, r, t) = \frac{(r + t)}{4n_p^2} \left( 1 + \frac{s_p(\cdot) - 2}{s_p(\cdot)} \rho_p(n_p, r, t) \right) - f$$

$$= \frac{(r + t)(n - 1)}{16n_p^4} \left( n_p (4 + r) + 4t - 3r \right) - f$$

(20)

As we noted at the beginning of this subsection, the number of early entrants with patent rights is fully determined by the Patent Office. From this, it is easy to confirm that the probability of survival and the expected profit of an early entrant are both increasing in $w_p$, where $w_p = 1/n_p$, as well as increasing in $r$ and in $t$. 

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The question to which we now turn is whether the patent regime can be designed to secure the socially optimal level of innovative entry. In calculating the social optimum we must allow for the fact that the fast-second’s improved products do bring consumers additional value. The socially optimal number of innovative entrants minimizes total versioning plus set-up costs – including those of the fast-second – minus the additional expected surplus created by the fast second. As in NPR, we refer to this as expected net cost, which we denote ENC(n, f, r), where n is the number of innovative entrants.

Since the social planner’s objective is to minimize ENC, the efficient allocation by the social planner is unaffected by whether or not there is a patent regime in place. Rather, the efficient allocation is precisely the allocation that would be determined by competition between the fast-second and the innovative first entrants in the absence of the patent regime. So an incumbent should be allowed to survive only if it can sell at a positive price given the fast-second’s brand strength. Assume that, in common with consumers, the social planner does not know the brand-strength of the fast-second. Suppose also, as in Case 1 above, that the fast-second enters midway between incumbents 1 and -1. The incumbents that are the jth nearest neighbors to the fast-second survive if the fast-second’s brand strength lies in the interval

$$\alpha \in \left[ \frac{1}{n} + \frac{r}{2n} + \frac{(j-1)r}{n}, \frac{1}{n} + \frac{r}{2n} + \frac{jr}{n} \right]$$

The market arc length of the fast-second, given that the nearest incumbents to survive are incumbents j and -j is then:

$$s_b(j) = s |rs - (\alpha - \frac{1}{n}) = r \left( \frac{1}{2n} + \frac{j-1}{n} - s \right) \Rightarrow s_b(j) = \frac{1}{2r} \left( \alpha - \frac{1}{n} \right) + \frac{1}{4n} (2j - 1)$$

and the market arc length for the jth nearest neighbors on the side nearest the fast-second is
\[ s_c(j) = \frac{1}{2n} + \frac{j-1}{n} - s_b \]  

(22)

Assume that the number of innovative entrants is even, given by \( n = 2m \). The first element of expected net cost is then

\[
NC_1 = \sum_{j=1}^{m} \left[ \frac{1}{n} \int_{ \frac{1}{n} }^{ \frac{j}{2n} } \left( r s_j(j)^2 + s_b(j)^2 \right) + \frac{1}{n} \int_{ \frac{1}{n} }^{ \frac{j-1}{n} } r s_j(j)^2 - 2 \left( \alpha - \frac{1}{n} \right) d\alpha \right] 
\]

The first two terms are versioning costs for the fast-second and for the two surviving \( j \)th nearest neighbors on the side nearest the fast-second; the third term is versioning costs for the remaining surviving firms plus those for the \( j \)th nearest neighbors on the opposite side to the fast second; the fourth term is the additional surplus that the fast-second creates.

To this we add the following:

\[
NC_2 = \left( \frac{1}{n} \int_{ \frac{1}{n} }^{ \frac{1}{2n} } \frac{r}{4} \left( \alpha - \frac{1}{n} \right) d\alpha + \frac{n+1}{2n} \int_{ \frac{1}{n} }^{ \frac{1}{2n} } \frac{r}{4n} d\alpha \right) + \frac{r}{4n} + nf 
\]

The first term in the bracket is the expected net cost when the \( m \)th nearest incumbents are forced to exit and the fast-second monopolizes the market while the second is the versioning costs of the incumbent in stage 4 when there is no fast-second entry. The third term is versioning costs of the innovative entrants in stage 2 and the final term is aggregate sunk costs of the innovative entrants. The social planner chooses \( n \) to minimize expected net costs \( ENC(n, f, r) = NC_1 + NC_2 \). The socially optimal number of innovative entrants \( n_0 \) solves:

\[
n_0(f, r) = n : \frac{n \left( 48 + 6r - 3r^2 \right) - 4 \left( 9 - 9r + r^2 \right)}{48n^3} - f = 0 
\]

(23)

\[ ^{14} \text{As in the previous section this eases computation but loses no generality given that we treat } n \text{ as continuous in the remaining analysis.} \]

\[ ^{15} \text{We can ignore any sunk costs of the fast-second since these are independent of the number of innovative entrants.} \]
The question to which we now turn is whether the Patent Office can set patent width to replicate the degree of innovative entry that would be chosen by the social planner, given that the patent system has to be designed such that the innovative entrants expect to at least break even: otherwise the market would never be opened. In other words, the constrained optimization problem confronting the patent office is

$$\min_n \ ENC(n, f, r) \text{ subject to } \pi_i \geq 0$$

(24)

where $$\pi_i = \pi_e(f, n, r, t, w)$$ with a partial patent and $$\pi_p(f, n_p, r, t)$$ with a full patent. In the remainder of the analysis we confine attention to the upper bounds on the expected profits of the innovative entrants (see equations 14 and 20). As a result, we identify necessary conditions for the patent system to generate the socially optimum degree of entry. Note that our qualitative conclusions, and the comparative static analysis would be unaffected if we were to take the lower bounds.

There are three possible outcomes: (i) $$f, r$$ and $$t$$ are such that the social optimum can be attained with a partial patent; (ii) $$f, r$$ and $$t$$ are such that the social optimum can be attained with a full patent of width $$w_p = 1/w_0$$; (iii) $$f, r$$ and $$t$$ are such that the social optimum cannot be attained.

Consider (i). This implies that there is a critical value of the consumer taste parameter $$t$$, denoted $$t_1(f, r)$$, above which a partial patent achieves the social optimum. Formally:

$$t_1(f, r) = t : \pi_{eu}(f, n_0(f, r), r, t, w) \bigg|_{w=1} = 0$$

(25)

where $$\pi_{eu}$$ is given by (14). By contrast, (ii) implies that there is a critical value of the consumer taste parameter $$t$$, denoted $$t_2(f, r)$$, above which a full patent achieves the social optimum. Formally:

$$t_2(f, r) = t : \pi_{eu}(f, n_0(f, r), r, t) = 0$$

(26)
where $\pi_{pu}$ is given by (20). The functions $t_1(f, r)$ and $t_2(f, r)$ are illustrated in Figure 4. Figure 5 illustrates two typical “slices” through the surfaces in Figure 4, for $r = 0.3$ and $r = 0.6$. As can be seen, both functions are decreasing in $f$ and $r$.

(Figures 4 and 5 near here)

We can conclude:

**Result 1:** Suppose that there is the possibility of fast-second entry. The Patent Office can set patent breadth to generate the first-best degree of innovative entry $n_0(f, r)$:

(i) by awarding a partial patent if $t > t_1(f, r)$;

(ii) by awarding a full patent of width $1/n_0(f, r)$ if $t_2(f, r) < t \leq t_1(f, r)$.

If $t < t_2(f, r)$ the Patent Office can achieve only second best, setting patent breadth to minimize expected net cost subject to the constraint that the innovative entrants just break even.

There is also the following:

**Corollary:** Suppose that there is the possibility of fast-second entry and that $t > t_1(f, r)$. Optimal patent breadth is decreasing in set-up costs $f$; decreasing in the consumer taste parameter $t$ and decreasing in versioning costs $r$.

The intuition underlying these results is relatively easily explained. We have seen that the patent system increases the profitability of the innovative entrants and so encourages entry. The question is whether this additional entry is sufficient to give the social optimum. In the absence of a patent regime, NPR demonstrated that “The equilibrium number of early entrants is always less than the socially optimal number…. (T)he degree to which there is insufficient early entry decreases as versioning costs $r$ rise (and) as set-up costs $f$ rise” (NPR, p. 20) As a result, when $f$ and/or $r$ are “high” patents can be narrower while still being able to generate efficient innovative entry. By a similar argument, in the presence of a patent regime, profits of the innovative
entrants are greater when consumer tastes are more intense. This allows patent breadth to be narrower while generating efficient innovative entry.

4. Conclusion

Recent research on innovation and patent policy, e.g., O'Donoghue, Scotchmer, and Thisse (1998), Hunt (2004), and Bessen and Maskin (2009) has emphasized the sequential nature of the innovative process in which later innovations build on earlier ones. At the same time, there is a somewhat parallel literature, e.g., Baumol (2010) that recognizes that such sequentiality occurs when the initial breakthroughs are made by small, entrepreneurial firms. For these firms competitive price discrimination is often a fact of life, while the later improvements on those breakthroughs are realized by well-established firms that have some brand advantage over these initial upstarts.

In this paper, we have attempted to build a model that captures these dimensions of innovation and market evolution. Our model is based on the familiar Salop model of horizontal product differentiation, in which a market is initially opened by a number of symmetric but differentiated entrepreneurs. Subsequently, an advantaged fast second enters with the ability to improve the product (at least in the eyes of consumers) and thereby charge a higher price for its products. In this setting, we investigate how optimal patent design varies with the structural features of markets, such as the extent of sunk costs for product development, the intensity of competition, and the intensity of consumer tastes.

A key feature of our model that underpins the analysis is the externality that early entrepreneurs provide to a late-entering, brand advantaged firm. It is this externality that lies at the heart of the sequential innovation process that has been the focus of much recent literature. A further attractive feature of our model is that it also captures the complementarity factor emphasized by Bessen and Maskin (2009) that the richer the variety of early-patented products
created by pioneering entrepreneurs, the greater the gains from subsequent innovation will be. As they note, such richness “increases the bio-diversity of the technology.” In turn, this facilitates the possible lines of future development, making further advances more possible.

Of course, as in most ecosystems, our model exhibits a delicate balance. On the one hand, both the profit and the value that fast second generates grow with the number of innovative entrants and so technological varieties that open the market. On the other hand, when the fast seconds realize that profit they often do so by eating the innovative entrants or otherwise forcing them to exit. In turn, this discourages the innovators from playing their market-opening role in the first place.

Patent policy can play an important role here. We know that this market evolution will, in the absence of patent rights, result in less than efficient entry by the initial entrants. More importantly, we have shown that contrary to the fears expressed in Bessen and Maskin (2010), a regime of fairly broad patents may well enhance the dynamic gains from sequential innovation. Patent protection does, indeed, protect the initial entrants by making it harder for the fast second to compete with them. As a result, patent protection can actually increase the number of initial entrepreneurs and thereby also raise the value of the later product enhancement that the fast second brings to the market. In fact, we show that a patent regime characterized by fairly broad patents offers a means of achieving the first-best social optimum. We are also able to identify the influence of key market characteristics on the optimal patent breadth. Specifically, we find that patents should be narrower in markets with high set-up costs or low versioning costs.

We have also demonstrated an important link between the intensity of consumer preferences and the optimal design of the patent system. For a well-designed patent regime to achieve the first-best social outcome it is necessary that consumers be intensely committed to their most preferred product variety. If, in contrast, consumers are only weakly attached to their most
preferred product variety, the patent system may have to settle for a second-best outcome that maximizes social surplus subject to the constraint that innovative entrants at least break even. Intuitively, with the introduction of a patent regime, the profitability of an innovative entrant and the probability that such an early entrant survives fast-second entry are both increasing in the intensity of consumer preferences for the differentiated products. As a result, if the intensity of consumer preferences is “strong enough” the patent regime can be designed to offset the inefficiently low level of innovative entry that would otherwise result.
References


Figure 1: Price Equilibrium with $n$ Innovative Entrants – Patent Regime

Figure 2: Equilibrium Number of Innovative Entrants – $f = .001$
Figure 3: Probability of Innovative Firm Survival
Figure 4: Critical Values of the Taste Parameter
Figure 5: Critical Value of Consumer Taste Parameter – $r = 0.3; r = 0.6$