Patent Pools and Dynamic R&D Incentives[1]

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Abstract

Patent pools are cooperative agreements between two or more firms to license their related patents as a bundle. In a continuous-time model of multi-stage innovations we characterize firms' incentives to perform R&D when they anticipate the possibility of starting a pool of complementary patents, which can be essential or nonessential. A coalition formation protocol leads the first innovators to start the pool immediately after they patent the essential technologies. The firms invest more than in the no-pool case and increase the speed of R&D for essential technologies as the number of patents progresses to the anticipated endogenous pool size, to the benefit of consumers. There is overinvestment in R&D compared to a joint profit-maximization benchmark. If firms anticipate the addition of nonessential patents to the pool they reduce their R&D efforts for the essential patents at each point in time, resulting in a slower time to market for the pooled technologies.

Keywords: R&D races, Innovation, Licensing, Competition policy.

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1 Introduction

Patent pools are cooperative agreements among several firms to license as a bundle their respective patents to third parties. Although patent pools have long been suspected of facilitating the implementation of anti-competitive behavior, regulatory authorities do recognize the potential virtues of patent pools, including "integrating complementary technologies, reducing transaction costs, clearing blocking positions, decreasing infringement litigation and the uncertainties related to it, and promoting the dissemination of technology" (US Department of Justice and the Federal Trade Commission, 2007, pp. 84-85).[1] By allowing one-stop shopping, the pool gives access to more efficient licensing. Thus, the pool can increase the private value of the constitutive patents and also social welfare by facilitating the diffusion of innovations. As a consequence of this more favorable position, patent pools re-emerged in the recent years, mainly in high-technology sectors.[2] Examples include MPEG-2 (1997), MPEG-4 (1998), Bluetooth (1998), DVD-ROM (1998), DVD-Video (1999), 3G-Mobile Communications (2001), One-Blue (2009). Pools have also formed in the pharma/biotech industry. Examples include the POINT (Pool for Open Innovation against Neglected Tropical diseases) and the Medicines Patent Pool, established in 2009 and 2010, respectively.[3]

The objective of this paper is to characterize the dynamic incentives to perform research and development (R&D) when firms anticipate from the outset participation in a patent pool. We construct a continuous-time model where R&D programs are described as a series of successive patent races in which firms interact strategically. The pool size is the outcome of a coalition formation protocol in which only successful innovators can participate. The pooling of complementary patents, which can be technically essential or nonessential, allows firms to coordinate their licensing behavior and thus increases the return on their investment. After the foundation of the pool, a late innovator can benefit from the bundled technologies if the firm contracts as a licensee with the pool initiators.

As a result, in this setup the pool is started immediately after all essential technologies have been patented. The perspective of being among the pool initiators works as a prize and enhances the speed of R&D. Specifically, the equilibrium pattern of innovative efforts increases stepwise

[1] One reads a very similar statement in the Antitrust Guidelines for the Licensing of Intellectual Property (US Department of Justice and the Federal Trade Commission, 1995, p. 28).

[2] See Newberg (2000) and Merges (2001) for detailed descriptions of organizational forms and contractual provisions of past and current pooling arrangements.

[3] For a discussion on patent pools as a mechanism for more accessible genetic inventions, see Matthijs et al. (2006). For more information on recent patent pools in the biopharmaceutical industry, see Matthijs et al. (2011).

over time before the formation of the pool and falls to the no-pool level afterwards. The final race to the pool exhibits an overinvestment in R&D compared to a joint profit-maximizion benchmark. We also find that firms' incentives to obtain the essential patents are diluted if a distortion occurs in the endogenous determination mechanism of the pool size so that the addition of nonessential patents is anticipated at the outset. The distortion implies strictly lower equilibrium R&D investments and a delayed expected time to market of the pooled technologies.

Most contributions to the economics literature on patent pools adopt an *ex post* perspective which follows antitrust practices for reviewing the impact on welfare of a pool after it was founded. The objective then is to identify what kind of pools should be authorized by the regulator. Shapiro (2001) examines this question in a pioneering contribution, where a simple model lends theoretical support to the idea that welfare is reduced when patents are substitutes (as in the case of goods coordinated by a cartel), and enhanced when patents are complements. In the latter case, royalty rates are reduced because the pool participants internalize the effect of their pricing on the demand for complementary patents. However, when all patents are substitutes, if firms determine the number of licensees in addition to the license Kato (2004) identifies circumstances in which a pool enhances social welfare. When all patents are complements, Aoki and Nagaoka (2005) show that a pool with all complementary technologies does not form when the number of patent holders is large. In an oligopoly model with an upstream patent licensing stage and a downstream production stage, Kim (2004) shows that vertical integration always lowers the price of the final product if there is a pool and all patents are complements. In a more general setup, Schmidt (2009) establishes that although vertical integration partially solves a double marginalization problem, it might also result in higher royalties and less output for the downstream market. Moreover, a merger at the upstream stage implies a reduction in total royalties and an increase in the output of the downstream market.

Whether given patents are substitutes or complements is not always obvious, so that an objective for research is to provide the regulator with some means to discriminate among pool candidates. Lerner and Tirole (2004) address this problem in a model that describes the full range between the extreme cases of perfectly substitutable and perfectly complementary patents. Compulsory individual licensing (i.e., the requirement that independent licenses be offered by pool members to third parties) performs as a screening device. The latter is innocuous when patents are complements, but destabilizes a pool of substitutes by reducing its profits. In a related model for the formation mechanism of pools, Brenner (2009) shows that exclusive pool membership (i.e., a firm participates in the startup of a pool only if all other pool initiators agree) must be added to compulsory individual licensing for a pool to be welfare enhancing. However, in another setup where complementary patents can be either essential or nonessential, Quint (2012) finds that a pool containing only nonessential patents can reduce social welfare, although the pool is stable to compulsory individual licensing.

The present paper contributes to a more recent stream of literature that adopts an *ex ante* viewpoint to characterize endogenous R&D efforts toward the startup of a pool. As Scotchmer (2004, p. 178) describes, "[p]rospective inventors face different rewards if their intellectual property goes into a patent pool than if they license individually". In this perspective Lerner and Tirole (2008, Appendix B) offer simple theoretical foundations to the conjecture that the perspective to form a pool with independent licensing is preferable to no pool because its lead to more innovation and make users better off. This strengthens the case in favour of pool agreements that allow firms to license their R&D outputs separately. Our starting point is different in that we rule out substitutable patents by assumption in order to eliminate the antitrust concern in patent pooling. This assumption follows a recent paper by Gilbert and Katz (2011), who study formally how alternative reward schemes for two firms impact the choice of R&D levels. Similarly, we also have the specification that technologies are invented sequentially, and are more valuable when used together than separately, although the technologies result from independent and uncertain R&D processes. An important difference is that Gilbert and Katz (2011) identify the properties that an innovation reward scheme must satisfy to support efficient R&D efforts, while we specify the simplest possible reward structure before examining its effect on firms' R&D choices.[4]

A growing empirical literature exists on patent pools. By examining sixty-three such agreements, Lerner, Strojwas, and Tirole (2007) confirm the theoretical prediction that pools of complementary patents are more likely to authorize independent licensing by member firms. Lampe and Moser (2010, 2011) use data on patent grants in the nineteenth century sewing machine industry. They find that the creation of a pool discourages subsequent innovation in complementary technologies, and also strongly encourages innovation in technologically inferior substitutes by outsiders. Layne-Farrar and Lerner (2011) examine nine modern patent pools to identify factors that drive the decision to join an existing pool. The likelihood of joining is shown to be reduced in case of a large group of pool initiators, and if license revenues are shared according to each firm's share of the total number of patents in the pool. The analysis of patents relating to information and telecommunication technologies leads Baron and Delcamp (2010) to suggest that pool initiators have strong bargaining power vis-à-vis other firms, as they are able to introduce lower quality patents than outsiders. Baron and Pohlman (2011) exploit a large data set to show

[4]See Gallini (2011) and Schmidt (2011) for thorough discussions on the theoretical economics literature on the efficiencies and potential anti-competitive effects of patent pools.

that pools have a positive effect on the number of patent declarations relating to major standards both before and after their startup. Delcamp (2011) finds that pools generally select patents with a higher number of citations – which represent patent value – than other patents with similar characteristics in a control sample.

The paper is organized as follows. Section 2 builds a model of pool formation in the terms of a differential game. Section 3 offers a characterization of the symmetric Markov Perfect Equilibria of the game. Section 4 characterizes the pool size as the outcome of a coalition formation protocol. Section 5 focuses on the pattern of equilibrium R&D efforts toward the pool foundation. Section 6 investigates the effects of firms anticipating the addition of nonessential patents in the pool. Section 7 concludes with policy implications. The Appendix contains all proofs.

2 A Model for Patent Pool Startup

In this section, we construct a N-firm industry model. Each firm invests in a specific R&D program, which is risky, to discover a new technology. At $t = 0$, no patent pool exists. The firms anticipate that, if they patent an innovation, they might participate in a pool startup process.

R&D investments. Firms indexed on a set $\{1, \ldots, N\}$ invest in a specific R&D program to discover a new technology. The investment process is formalized as a continuous time non-cooperative profit-maximization problem with discounting. The parameter r denotes the interest rate common to all firms, which are risk neutral. At each point in time, each firm i can decide independently to exert an R&D effort, measured by the non-negative continuous variable x^i. The time taken for an innovation to occur follows a continuous exponential probability distribution with intensity x^i, so that the probability that firm i succeeds in R&D before τ is $1 - e^{-x^i \tau}$. The flow cost of R&D is $c(x^i)$, with $c(0) = 0$, $c'(x^i) \geq 0$, and $c''(x^i) > 0$. We also assume that $c'(0) = 0$ and $c'(+\infty) = +\infty$, for each firm's optimal strategy to be an interior solution of its expected profit-maximization program.[5] The solution concept is the symmetric Markov perfect equilibrium (see Subsection 3.3). When a firm succeeds in R&D, it receives a patent of infinite length and can secure a given profit flow $\underline{v} \geq 0$ by licensing the technology independently. This licensing is the outside option of a patent holder.

Essential and nonessential patents. As in Gilbert and Katz (2011), all patents are complementary. The technological complementarities result in more than $\underline{v}$, the reservation value, only if a subset

[5] The specification that firms incur a variable cost in the R&D technology is as in Lee and Wilde (1980). However, here each firm i can instantaneously adjusts its rate of effort x_i at each point in time.

of $K < N$ patents, which we refer to as technically "essential", are used together. If fewer patents are granted, they are not worth more when combined together than when used separately. In Gilbert's (2010a) terminology, the essential patents are two-way blocking because each of them can block the use of any other patent. The other $N - K$ complementary patents are "nonessential". They are one-way blocking patents because none results in more than $\underline{v}$ unless they are used with all essential patents, but they cannot block an essential patent.[6,7]

Patent values. When included in a pool, each essential patent generates a profit flow $\bar{v}$, while nonessential patents generate a profit flow $\hat{v}$, with

$$\underline{v} < \hat{v} \leq \bar{v}. \tag{1}$$

The first (strict) inequality sign formalizes the fact that a pool increases the private value of all patents (as the pool comprises the essential patents, it results in more value for the nonessential participating patents as well). The second (weak) inequality sign reflects the fact that the non-essential patents relate to improvement technologies (they are generally less valuable than essential technologies).[8]

The pool size. The pool size S, defined as the number of patents the pool initiators hold, is endogenous. A pool has no *raison d'être* if it does not include K essential patents, although nonessential patents are not *a priori* excluded from the formation process of the pool. It follows that $K \leq S \leq N$. We assume that any subset of S patents is admissible.[9,10]

[6] Gilbert (2010a) also qualifies two-way blocking (or technically essential) patents as "dominant", and one-way blocking (nonessential) or improvement patents as "subservient". See also Gilbert (2010b).

[7] This specification closely corresponds to the MPEG-2 pool, which "separates its patents into different categories. The most fundamental are the "essential" patents: the basic complementary technologies that in effect make up the MPEG-2 standard. The charter also recognizes "related patents," which are patents that represent improvements upon the existing essential patents in the pool. If the related patents were left unlicensed, they would infringe upon the MPEG-2 standard" (Lind *et al.*, 2003, p. 15).

[8] If $\underline{v} = 0$ we obtain the case of *perfect* complements examined by Gilbert and Katz (2011), where the social and private benefits of patents is set equal to zero when less than a finite number of technologies is included in the pool.

[9] This follows Schmalensee (2009), where a standard can be formed by any combination of M components, resulting in 2^M possibilities. In the present model, in a set of N patents there are $\frac{N!}{K!(N-K)!}$ combinations of K essential patents. Each combination can be augmented by one of the 2^{N-K} distinct possible subsets of non-essential patents (including the empty set).

[10] In an alternative setup, all firms' R&D projects can be assumed to be initially directed to the K pre-determined essential technologies, with several firms having identical R&D programs whenever $K < N$. In that case, if a firm succeeds in patenting the laggards must be allocated one of the remaining R&D programs. This specification does not impose to modify the formalization of investments because the exponential process implies no accumulation of knowledge (Reinganum, 1981, 1989).

The pool's startup process. As in Brenner (2009), the pool's startup process is modeled as a coalition formation protocol that can result in only one pool. Immediately after K firms have been granted a patent, one of them, firm i, proposes the startup of a pool to a subset of size S_i of patent holders (including itself), while the non-patent holders continue their R&D investment. The firms that receive the proposal are asked sequentially, with arbitrary ordering, for their approval.[11] If all these firms accept the proposal, the pool is formed, so that $S = S_i$. Otherwise, the next firm in the ordering makes a new proposal, and so on. The resulting pool consists of those patents held by the initiator and all the addressees of the successful pool proposal. Each firm can make only one proposal, and no coalition can start if none of the proposals is successful. In that case, the coalition formation protocol restarts with the $K+1$-th initiator, immediately after it receives a patent, and so on. The iterative process ends when one pool exists.[12]

The profit-sharing mechanism. After the pool's startup, the S initiators license their IP rights as a bundle to the successful "outsiders" (i.e., any of the $N-S$ non-pool members that discover a complementary technology). Each of the latter firms holds a one-way blocking patent that has the value $\hat{v}$ only if used with the two-way blocking patents. The pool extracts the monopoly revenue by charging the highest possible royalty $\hat{v} - \underline{v} - \varepsilon$, where ε is arbitrarily small. (Hereafter ε is set equal to zero.) The total pool profits are shared equally among the pool initiators.[13] More formally, at each point in time a pool initiator, as a residual claimant, receives a share of the profits generated by the S patents, that is $\frac{1}{S}\left(K\bar{v} + (S-K)\,\hat{v}\right)$. In addition, each pool initiator receives a share $\frac{1}{S}\left(\hat{v} - \underline{v}\right)$ from any nonessential technology discovered by the $N-S$ outsiders after the pool's startup. As each firm holds at most one patent, this profit-sharing mechanism is the simplest possible example of numeric proportional rule, whereby members get a share of total earnings which is a function of the number of patents in the pool. This reflects a majority of recent real-world cases, including the MPEG-2 pool that Layne-Farrar and Lerner (2011, p. 296) qualify as "typical".[14]

[11] Layne-Farrar and Lerner (2011) note that, in most models in the economic literature on patent pools, all firms with a relevant patent are assumed to join the pool. Here participation is voluntary and occurs as an endogenous outcome.

[12] The coalition formation protocol is a simplified finite version of the infinite-horizon unanimity game by Ray and Vohra (1999). The only difference with Brenner (2009) is that, in the present model, the number of patent holders increases over time. This implies that the protocol can *a priori* start up to $N-K+1$ times.

[13] As an alternative specification, the $N-S$ outsiders, when successful in R&D, can integrate the pool by paying the access price $\hat{v} - \underline{v} - \varepsilon$. However, in a detailed empirical analysis of a large dataset of 1337 U.S. patents included in seven pools, Baron and Delcamp (2010) find that only a minority of late patent introductions can be explained by new patent holders joining the pool.

[14] Layne-Farrar and Lerner (2011) find that five out of nine modern patent pools (1394, AVC, DVB-T, MPEG-2, MPEG-4) share earnings by applying a numeric proportional rule, whereby members get a share of total earnings

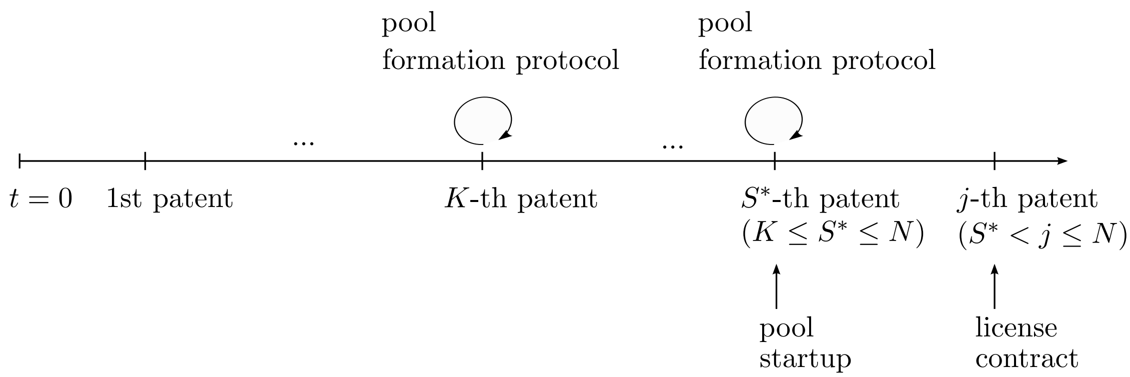

Figure 1: Timing.

The timing. Given that $s - 1$ firms have already patented a technology (with $s = 1$ at $t = 0$, and $0 \leq s - 1 \leq N$ afterwards), the specific timing of the model is as follows: 1) the patent holders stay put, while each other firm continuously and non-cooperatively invests in R&D until it discovers the s-th technology; 2) immediately after receiving a patent the last innovator initiates the coalition formation protocol, which is instantaneous, and all firms return to the first stage; and 3) as soon as a pool forms it licenses the IP rights as a bundle to each outsider that succeeds in R&D, and the total benefits are continuously shared among the pool initiators.

3 Computation of Patent Values

In this section, we first examine the no-pool benchmark. Next, we compute the value of starting a pool of given size S. Then, we describe a recursive formulation of the race to the pool.

3.1 The No-Pool Benchmark

As a benchmark, suppose that no patent pool can form. In that case, because R&D programs are independent of one another, the environment is non-strategic and we can analyze any firm's decision by studying the optimal pattern of effort x for a single representative firm. The maximum value function V of the firm's program depends on the innovation's state, which can take values of zero (failure) or one (success).

that is function of the number of patents they pour in the pool. Three other pools (DVD-1, DVD-2, PRK) use value-added rules, so that shares are defined in proportion to the value of patents. Only one pool (Bluetooth) operates under a royalty-free licensing policy.

For success (state 1), the value of a patent is $V(1) = \int_0^\infty e^{-rt}\underline{v}dt = \frac{\underline{v}}{r}$, where r is the common interest rate. In the absence of innovation (state 0), the value $V(0)$ of the firm's R&D program verifies that

$$rV(0) = \max_x \left[x\left(V(1) - V(0)\right) - c(x)\right]. \tag{2}$$

In the usual terminology, this Bellman equation formalizes that the return r on the "asset" $V(0)$ is equal to the expected "capital gains" $x(V(1) - V(0))$ minus the flow of the "dividends" $c(x)$.[15] The first-order condition for an optimal level of R&D effort implies that

$$V(0) = \frac{\underline{v}}{r} - c'(\underline{x}), \tag{3}$$

at all points in time, where $\underline{x}$ solves

$$\underline{v} - (r + \underline{x})\, c'(\underline{x}) + c(\underline{x}) = 0, \tag{4}$$

which is obtained by plugging (3) into (2).

3.2 The Value of Starting the Pool

Consider first the firms outside the pool. After the formation of the S-patent pool, the R&D decisions of all outsiders – the firms with no patent yet and thus do not contribute to the pool – are the same as in the no-pool configuration (Subsection 3.1). Each of them invests $\underline{x}$, that is the solution to (4), and the value of each R&D program is as displayed in (3), that is

$$V_{S+1}(0) = \frac{\underline{v}}{r} - c'(\underline{x}). \tag{5}$$

In other words, $V_{S+1}(0)$ is the value of not being among the pool initiators (if $S = N$, no outsider exists).

Consider now the pool members. These firms gain two kinds of benefits. First, they make a higher return on their own patent, that is $\bar{v}$ or $\hat{v}$, than in the no-pool case. In addition, they extract a rent, that is a flow profit $\hat{v} - \underline{v}$ from each outsider that, if successful in R&D, contracts with the pool as a licensee. Formally, the value of participating in the startup of a pool of given size S is

$$V_S(1) = \frac{1}{r}\left[\frac{1}{S}\left(K\bar{v} + (S-K)\,\hat{v}\right) + \frac{N-S}{S}\frac{\underline{x}}{r+\underline{x}}\left(\hat{v} - \underline{v}\right)\right]. \tag{6}$$

[15]The value of dynamic R&D programs, as formalized in this section, results from the optimal control approach described in Grossman and Shapiro (1986, pp. 584-586).

This expression describes the value of holding a patent and being the initiator, or recipient, of a successful proposal in the coalition formation protocol. The first term between the brackets is the share, as received by each pool founder, of the value of the K essential patents (flow profit $\bar{v}$) and of $S-K$ nonessential patents (flow profit $\hat{v}$) at the time of the pool's startup. The second term reflects the fact that each of the potential $N-S$ subsequent innovators contributes to the pool value by an amount $\hat{v}-\underline{v}$. This profit flow is equally divided among the S pool initiators and discounted by the "adjusted probability" of the success of the outsiders' R&D programs, that is $\frac{\underline{x}}{r+\underline{x}}$.[16]

Note that $V_S(1)$ in (6) increases with the optimal level of effort by the firms that remain outside the pool, and thus with their adjusted probability of success. $V_S(1)$ also increases with the number of firms outside the pool. In other words, from the group of initiators' viewpoint, the larger the number of potential contributors – i.e., the size of the subset of outsiders – to the existing pool, the more profitable. Or, for a given total number of firms N, the value of participating in the startup of the pool decreases in the number of initiators S, because more parties share in the rent extraction process afterwards. This property has been the object of an empirical test by Layne-Farrar and Lerner (2011) which lends support to the present theoretical formalization.

3.3 A Recursive Formulation of the Race to the Pool

We now concentrate on the period starting at date zero and finishing with the startup of the pool, for a given size S.

The dynamic process to the pool is analogous to a race in which the prize consists in being among the S first innovators, because this gives access to a portion of the pool value as a residual claimant. Equivalently, we choose to describe this process hereafter as a series of S successive patent races. The environment is now strategic because each firm's expected return from a patent depends on the achievement of other firms to be among the S first patentees, and thus on their respective strategies x^i, $i=1,\ldots,N$, at any t. It follows that the maximum value of a firm's

[16] The "adjusted probability" expression follows Denicolò (2000). Recall that the stationary investment is $\underline{x}$ for each outsider. As $\underline{x}$ is the intensity of a continuous exponential probability distribution, for any flow profit v the present value of an outsider's expected payoff is

$$\int_0^{+\infty} \underline{x}e^{-(r+\underline{x})t} v dt = \frac{\underline{x}}{r+\underline{x}} v.$$

In other words, everything happens as if by continuously investing $\underline{x}$ the firm were instantaneously successful with probability $\frac{\underline{x}}{r+\underline{x}}$.

R&D program is a function not only of the innovation's state (i.e., to be successful or not), but also of the number of firms that have already patented.

The solution concept. When exactly $s-1$ firms have patented a technology, $N-s+1$ other firms keep investing in an R&D program. We compute each firm i's equilibrium strategy x_s^i toward the s-th technology, with $s \leq N$. As firms are symmetric, and for any given number of patented technologies the strategies x^i depend only on the innovation's current state (0 for failure, 1 for success), we restrict attention to symmetric Markov perfect equilibrium (MPE) solutions.

Value functions can thus be indexed only by the number of firms that have already patented an innovation or, equivalently, by the rank s of the patent race in which a firm participates. By generalizing the previous notation, $V_s^i(0) > 0$ is the value of the research program that aims at discovering the s-th technology. Firm i's R&D program can either be successful before others, and thus lead to an innovation of value $V_s^i(1)$, or fail in patenting the s-th innovation and be valued at $V_{s+1}^i(0)$.[17] Therefore, the value $V_s^i(0)$ of firm i's R&D program of rank s verifies

$$rV_s^i(0) = \max_{x^i} \left[x^i(V_s^i(1) - V_s^i(0)) + X_t^{-i}(V_{s+1}^i(0) - V_s^i(0)) - c(x^i)\right], \quad (7)$$

where X_t^{-i} is the sum of the instantaneous R&D efforts made by all the other firms that have not innovated yet. In other words, the Bellman equation in (7) describes the return on the "asset" $V_s^i(0)$ as the expected "capital gains", which depend on all R&D investors' strategies, net of the "dividends", which are firm i's flow cost. In contrast to the no-pool case, capital gains can take two forms that depend on whether firm i innovates first or another firm does. The first-order condition leads to firm i's optimal effort $x_s^i = (c')^{-1}\left(V_s^i(1) - V_s^i(0)\right)$ for any given s at each point in time. Now the symmetry implies that $V_s^i(1) = V_s(1)$, $V_s^i(0) = V_s(0)$, and consequently $x_s^i = x_s$, for all i, with

$$x_s = \left(c'\right)^{-1}\left(V_s(1) - V_s(0)\right), \quad (8)$$

which is a symmetric MPE of the R&D program of rank s. Using (8), equation (7) can now be rewritten as

$$rV_s(0) = x_s c'(x_s) + (N-s)x_s(V_{s+1}(0) - V_s(0)) - c(x_s). \quad (9)$$

Further, to completely characterize the value function associated with this game, we must compute the value holding a patent in the s-th race, that is $V_s(1)$. This computation is done by observing that during the $s+1$-th race, that is in the period that follows the discovery of the s-th

[17] When a firm participates in a patent race of rank s and does not succeed in being the first to innovate, it initiates a new R&D program of rank $s+1$. Accordingly, the value of a program that fails to patent the s-th innovation can be denoted by $V_{s+1}^i(0)$. For $s = N$, we set $V_{N+1}^i(0) = 0$.

innovation, and before the discovery of another innovation, the event that one of the "remaining" $N-s$ firms succeeds in patenting an innovation can occur with an "adjusted probability" of success of $\frac{(N-s)x_{s+1}}{r+(N-s)x_{s+1}}$, in which case the value of all patents at the issue of the race is equal to the value of innovating at rank $s+1$, that is $V_{s+1}(1)$. Otherwise, all R&D programs fail with probability $\frac{r}{r+(N-s)x_{s+1}}$, and the actualized value of each of the existing s patents remains equal to $\frac{v}{r}$, as in the no-pool benchmark. This leads to

$$V_s(1) = \frac{r\left(\frac{v}{r}\right) + (N-s)\, x_{s+1} V_{s+1}(1)}{r + (N-s)\, x_{s+1}}. \tag{10}$$

In other words, $V_s(1)$ is the value of a lottery, in which a firm can gain $\frac{v}{r}$ with probability $\frac{r}{r+(N-s)x_{s+1}}$, or $V_{s+1}(1)$ with probability $\frac{(N-s)x_{s+1}}{r+(N-s)x_{s+1}}$.

4 The Endogenous Pool Size

In this section, we compute the equilibrium pool size as endogenously obtained from the coalition formation protocol.

Note from (6) in section 3.2 that $V_S(1)$ is monotone decreases monotonically in S. This monotonicity is a consequence of the fact that, when the pool size increases, the revenues accruing to a pool initiator are not only reduced – there are less potential entrants from which to extract a rent – but also shared by a larger number of residual claimants.

The first technical result exploits this monotonicity property.

Lemma 1 *For all $S \leq N$:* $\quad V_S(1) > \frac{v}{r}$.

This lemma is useful for determining the endogenous pool size as a result of the coalition formation protocol.

Proposition 1 *The pool is started as soon as the essential technologies are patented: $S^* = K$.*

This proposition establishes that the first K innovators maximize the private value of holding a patent by forming the pool as soon as technologically possible, so that there are only essential patents in the pool. This outcome is consistent with the empirical evidence by Delcamp (2011) that patents in a pool have a higher value than technologically similar patents that are not included in a pool.

A consequence of Proposition 1 is that the value of participating in the pool's startup, as introduced in (6), becomes

$$V_{S^*}(1) = \frac{1}{r}\left[\bar{v} + \frac{N-K}{K}\frac{\underline{x}}{r+\underline{x}}\left(\hat{v} - \underline{v}\right)\right]. \tag{11}$$

Therefore, in (11) $V_{S^*}(1)$ decreases (weakly) in K, and increases monotonically with N, and hence increases with the number of outsiders. If $K = N$, so that all patents are essential, we get $V_{S^*}(1) = \frac{\bar{v}}{r}$.

The first proposition qualifies the regulatory pressure put on firms by the legislation that emphasizes the essentiality of patents as an enforcement principle of antitrust policy, both in the U.S. and European contexts: "The European Commission shares the DOJ's concerns about situations when patents that are substitutes are included in the pools, as well as the inclusion of non-essential (and particularly invalid) patents in these arrangements" (Lerner and Tirole, 2008, p. 161). For example, in the MPEG-2 case, the DOJ stipulated in its 1997 Business Review Letter that the proposed pool is limited to "technically essential patents, as opposed to merely advantageous ones."[18] The European Commission also approved the pool in 1998, and expressed that each of the parties "is a holder of patents essential to the implementation of the standard, i.e. patents that must be used to produce equipment or recordings that conform to the standard."[19]

In our formal setup, the regulatory principle that pools should be limited to essential patents is captured by the simple constraint $S \leq K$.[20] In the *ex ante* perspective we adopt, this constraint is exactly satisfied by the firms' choice of S^* in Proposition 1. This result thus offers an additional economic foundations to the regulator's position not to enlarge pools for one-way blocking patents (i.e., nonessential though complementary technologies).

5 The Pattern of R&D Efforts

In this section, we characterize the symmetric MPE of the $S^* = K$ races to the pool, compare them with the stationary investment level $\underline{x}$ of the no-pool benchmark, and with a joint profit-maximizing R&D level $\overline{x}$. Toward this aim, we first derive a series of technical results.

[18] Business Review Letter from Joel I. Klein, Assistant Attorney general, U.S. Department of Justice, to Garrard R. Beeney, June 26, 1997, available at http://www.usdoj.gov/atr/public/busreview/215742.pdf.

[19] Case No IV/C-3/36.849 MPEG-2 Licensing Programme, Official Journal of the European Communities (available at http://eur-lex.europa.eu/).

[20] From an *ex post* viewpoint this constraint plays no role because if all N technologies were discovered in a previous period, the same total value $K\bar{v} + (N-K)\hat{v}$ is generated for any pool size $S \geq K$.

Lemma 2 *For all* $s \leq S^* - 1 : \frac{v}{r} < V_s(1) < V_{s+1}(1)$.

This lemma establishes that the value of a patent increases as the rank s progresses to S^*. Indeed, the reward accruing to a pool initiator is discounted less in race $s+1$ than at rank s. Furthermore, holding one of the essential patents gives access to a share of the pool's profits on top of the flow of profits obtained in the no-pool situation. Hence, we have $V_s(1) > \frac{v}{r}$.

Lemma 3 $x_{S^*} > \underline{x}$.

The reasoning behind this claim becomes intuitive from observing that, in the S^*-th patent race, the firms have a last chance to participate in the startup of the pool. This perspective encourages them to choose a more aggressive strategy than in the absence of hope to benefit from the pool's profits. The next lemma establishes the monotonicity of equilibrium investment efforts.

Lemma 4 *For all* $s \leq S^* - 1 : \quad x_s < x_{s+1} \Leftrightarrow \underline{x} < x_s$.

By combining these claims we can now state the main result of this section.

Proposition 2 *For all* $s \leq S^* - 1 : \quad \underline{x} < x_s < x_{s+1}$.

This proposition characterizes the impact on R&D activity of the possibility for firms to form a patent pool of a given size S^*. Before the formation of the pool, the speed of innovation is higher than in the no-pool configuration, and R&D efforts increase with the rank of the race (see Figure 2). The perspective of possibly participating in the pool startup acts *ex ante* as an additional reward that enhances incentives to perform R&D. After the startup of the pool, the outsiders' R&D efforts drop to the no-pool benchmark level $\underline{x}$.[21]

[21] The empirical evidence confirms some of our findings on the pattern of R&D efforts. In an analysis of patent declarations that relate to several standards Baron and Pohlman (2011, p. 21) show that "prospective patent pool creation induces a race of declarations.". By using data on the 19th-century sewing machine industry Lampe and Moser (2011, p. 22) find that "[p]ool members and other firms produced fewer patents after the pool had formed."

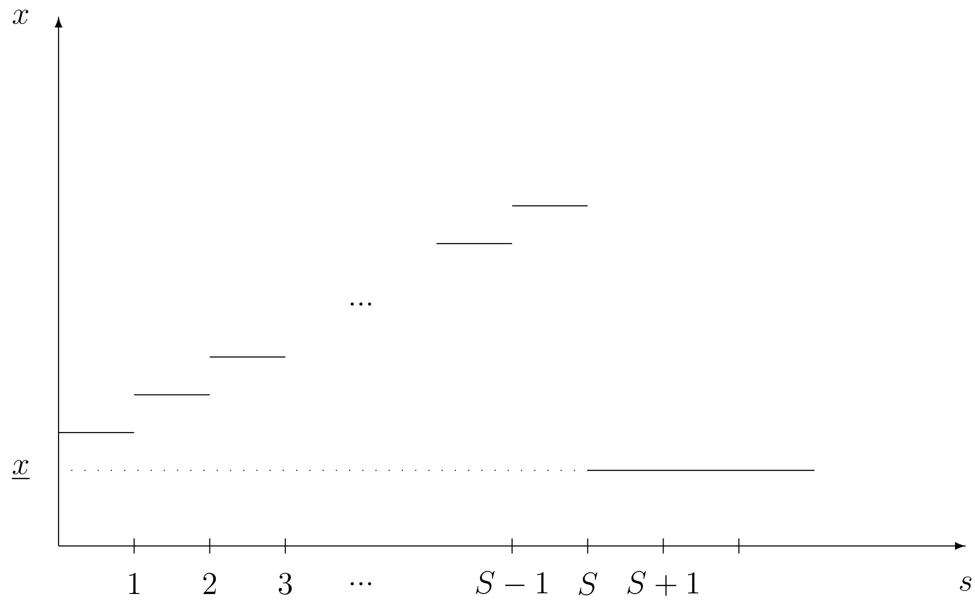

Figure 2: Pattern of equilibrium R&D effort levels (with $S^* = K < N$).

To compare, recall that Lee and Wilde (1980), under some stability conditions, clearly indicate that more competitors increase the equilibrium level of R&D for each firm. The upward pattern of R&D efforts in Proposition 2 is thus surprising, because the number of participants in the $s+1$-th patent race is *smaller* than in the s-th race. However, in our model, the impact of the reduction in the number of participants from one race to another is counterbalanced by the following two effects. First, if a firm wins the s-th race, then it waits longer for the startup of the pool than if it succeeds in patenting the $s+1$-th innovation. Time being discounted, the reward is thus smaller in the s-th race. Second, the failure to innovate is less damaging in the s-th race than in the $s+1$-th race. The reason is that, at the lower rank, there is one more race to run before the pool startup, and thus one additional chance to be among the S^* first patentees. The latter two effects dominate the first one, leading to $x_s < x_{s+1}$. This inequality, together with $\underline{x} < x_s$, establishes that all equilibrium R&D levels in all S^* races are above the "floor" level $\underline{x}$ of the no-pool benchmark.

The time to market implication of Proposition 2 is straightforward. In this model, the probability that any given firm i innovates before time t increases monotonically with its R&D effort x^i. Therefore, in expectation, the pool makes the essential technologies available earlier to consumers.

This result does not hold for the other technologies, because the R&D efforts of the outsiders for the remaining $N-K$ patents are the same as in the no-pool case.

The final step in this section is to compare the highest equilibrium strategy x_{S^*} with a reference measure. To do that, we first establish as a technical result that the prospect of starting a pool increases the value of all research programs of rank $s = 1, \ldots, S^*$ in comparison to the no-pool case.

Lemma 5 *For all* $s \leq S^* : V_s(0) > V_{S^*+1}(0) = \frac{v}{r} - c'(\underline{x})$.

Consider now the industry acting as a single player, as if all firms cooperate in R&D. The objective of this fictitious player is to maximize joint profits by choosing $x_1, \ldots, x_N$. Assume the following technological conditions: (1) only one patent is essential (i.e., $K = 1$, so that a pool can be created as soon as a single firm innovates), and (2) the value of improvement and essential patents is the same (i.e., $\hat{v} = \bar{v}$, so that complementarities result in the highest possible total pool value).

This cooperative situation parallels the similarly non-strategic case of the no-pool benchmark in subsection 3.1. Therefore, the joint profit-maximization strategy of the industry for each R&D project is $\overline{x}$, as implicitly defined by the first-order condition $\overline{v} - (\overline{x} + r)c'(\overline{x}) + c(\overline{x}) = 0$. As a result:

Proposition 3 $x_{S^*} > \overline{x} > \underline{x}$.

This proposition establishes that the final race to the pool exhibits an overinvestment in innovation compared to the joint profit-maximizing R&D level. This overinvestment occurs because, in our model, being among the pool initiators is over-rewarded through the extraction of a rent from the outsiders. There is no such transfer in the cooperative case. The consequence for consumers is that the K-th patent, which triggers the startup of the pool (Proposition 1), occurs earlier in expectation than in the most favorable case of a cooperative agreement that comprises all firms.

Linking this new proposition to the research stream on multi-stage patent races that concentrates on the cumulative nature of innovation might be interesting. This literature is mainly concerned with the problem of rewarding the early inventors for opening the way to subsequent improvements, and generally insists on the insufficient incentives given by patent protection. (See Green and Scotchmer (1995), Chang (1995) or Matutes, Régibeau and Rockett (1996) for representative contributions.) We find that a patent pool can be conceived as a means to increase the rewards to early inventors so long as it enhances their bargaining power when the time comes to negotiate a license contract with external patent holders. The *ex ante* viewpoint thus shows that patent pools are a policy instrument for stimulating early investments in R&D toward the generation of a sequence of complementary patents. This conclusion contrasts with the *ex post* antitrust concern that "combining patent rights in a pool could discourage R&D" (IP Report, 2007, p. 6) when a pool agreement stipulates the cross-licensing of present as well as future patents, because firms might get a free ride from the other pool participants' discoveries.

Note that so far we do not address the question of the uniqueness of the solution to the recursive system. Actually, it is not clear that the Bellman equation (7) has only one symmetric solution, meaning that the differential game we consider might have several symmetric MPEs. However, Proposition 2 and 3 are proved without relying on uniqueness. Therefore, our results are valid for all possible equilibria. In all equilibrium paths, race participants' R&D levels x_s, which are always above the no-pool level $\underline{x}$, increase over time until the pool is started, with x_{S^*} greater than the joint profit-maximizing mark $\overline{x}$.

6 Inclusion of Nonessential Patents

In this section we investigate the effects on equilibrium R&D efforts of firms' anticipation that improvement patents will be added to the pool.

In practice the distinction between essential and non-essential technologies is difficult to ascertain (see, e.g., Carlson, 1999). "In many cases, patents in a pool are not pure complements or pure substitutes, but display characteristics of both" (US Department of Justice and Federal Trade Commission, 2007, p. 4). Recent US cases show that pool candidates rely on the assessment of patents by an independent expert in advance of soliciting a decision by the regulator.[22] "The use of an independent patent expert to assess essentiality provides some comfort, but essentiality is often difficult to determine even for an unbiased expert" (Gilbert, 2010a, p. 338). Indeed these pools typically relate to a standard, to which additional technological features can be added that improve it, although the associated patents are only one-way blocking: "If additional features are

[22] On this see the four U.S. Department of Justice Business Review Letters (1997, 1998, 1999 and 2002, available at: *www.usdoj.gov/atr/public/busreview/1170.htm*) regarding MPEG-2, the two DVD pools, and the 3G platform, respectively. In the European Union, the regulatory treatment of patent pools is presented in the European Commission's Guidelines on the application of Article 81 of the EC Treaty to technology transfer agreements (2004/C 101/02), April 27, 2004 (available at: *http://eur-lex.europa.eu/en/index.htm*). The European guidelines do not require firms to solicit a review process.

added to a standard, does the definition of an essential patent expand to include patents that are necessary to implement these new features? (...) The agencies recognize that it is difficult to assure that a pool includes only essential patents" (Gilbert, 2010a, p. 337-8). As in the case of the related standard, the determination of the pool boundaries can also involve some politics that distort a purely economic rationale.[23] The principle that only essential patents should be included in a pool is thus unlikely to be enforced strictly.[24]

In the formal context of our model, any departure from the endogenous pool size, as obtained when firms are accurately informed (section 4), comes as a distortion. The impact of this distortion on the dynamics of the innovation process can be measured by the comparison of R&D levels, when firms anticipate from the outset the formation of a L-patent pool (for some reasons which are exogenous to our model), with the equilibrium efforts of the S^*-patent case (as in section 5), with $L > K$.

In order to compare the symmetric MPE in all ranks $s = 1, \ldots, K$ for different pool sizes, we extend the notation by incorporating S as an argument of strategies and of value functions. For any given pool size S, we denote by $x_s(S)$ the strategies of the firms involved in the s-th patent race, and by $V_s(1, S)$ (resp. $V_s(0, S)$) the value of a patent (resp. a research program) when exactly s innovations (resp. exactly $s - 1$ innovations) have already been patented.

The effect on the first K races of firms' anticipation of a L-patent pool is not obvious, so an intermediate result is needed. It introduces a mildly sufficient condition for the uniqueness of the symmetric MPE in each patent race of rank s.

Lemma 6 *If $c'''(x) \geq 0$, there exists a unique solution x_s to (9), for any pool size S.*

The proof of the next proposition (in Appendix 4) uses the assumption that $c''' \geq 0$, implying from Lemma 6 that there exists a unique equilibrium path $x_1, \ldots, x_s, \ldots, x_S$ to a S-patent pool.

As for the final races of ranks $K + 1$ to L, we know from Proposition 2 that an increase in the pool size should raise the R&D efforts above the no-pool level $\underline{x}$. However, if the firms take L as the pool size from $t = 0$ onward, and still accurately evaluate the non-essential patents at

[23] Such distortions are documented in the literature. In an empirical contribution, Simcoe (2010, p. 17) mentions the "the difficulty of identifying the handful of patents essential to a particular standard". In a discussion on patent thickets, Regibeau and Rockett (2011, p. 14) note that "the difficulties encountered within standard-setting organisations themselves strongly suggest that real life bargaining is unlikely to reliably produce the most efficient outcome".

[24] Recent contributions to the law literature (e.g., Nelson, 2007) give *prima facie* evidence that pools were cleared that include more than technically essential patents.

$\hat{v} \leq \bar{v}$, the equilibrium R&D efforts in all races of rank $s = K+1, \ldots, L$ can only be weakly lower than the equilibrium levels that would be chosen with essential patents only. In what follows, we set $\hat{v} = \bar{v}$. Accordingly, the R&D efforts for the additional improvement patents in the L-patent pool, as characterized in the following proposition, describe an upper bound to the range of firms' equilibrium choices. In other words, the distortion is at a minimum.

Proposition 4 *For any pool size $L > S^* = K$, the equilibrium R&D efforts are such that:*

(i) $x_s(L) < x_s(S^*)$, *all* $s \leq S^*$;

(ii) $x_s(L) = x_s(S^*) = \underline{x}$, *all* $s > L$;

(iii) $x_s(L) > x_s(S^*) = \underline{x}$, *all* $L \geq s > S^*$;

(iv) $x_L(L) < x_{S^*}(S^*)$.

This result offers a complete comparison of equilibrium R&D effort across the two pool cases: (*i*) establishes that race participants' R&D efforts at each rank s in the non-distorted S^*-size case are higher than in the large size case; (*ii*) states that the post pool-formation R&D efforts of outsiders fall at the no-pool optimal level $\underline{x}$ for all pool sizes (whereas pool initiators do not invest anymore); eventually, an increase in pool size implies a prolongation of race participants' equilibrium R&D efforts, which according to (*iii*) are above the no-pool optimal level, and from (*iv*) are below the final race level of the S^*-patent pool. Figure 3 illustrates these points.

In this framework, the introduction of additional improvement patents dilutes the incentives to obtain the essential technologies, and thus decreases the equilibrium R&D efforts. This effect is the consequence of three distinct factors. First, patenting in the s-th race is more valuable if the pool size is smaller (i.e., only $S^* = K$ patents) because the discount applied on the reward is smaller (the waiting time is shorter). Second, the reward itself is higher for a smaller pool size because the pool's profit pie is divided among a smaller set of residual claimants. Further, the larger the pool size, the less damaging a failure in the s-th race, because the number of opportunities to belong to the set of pool initiators (that is, the number of subsequent races $S^* - s$), as faced by each competing firm, increases with the pool size.

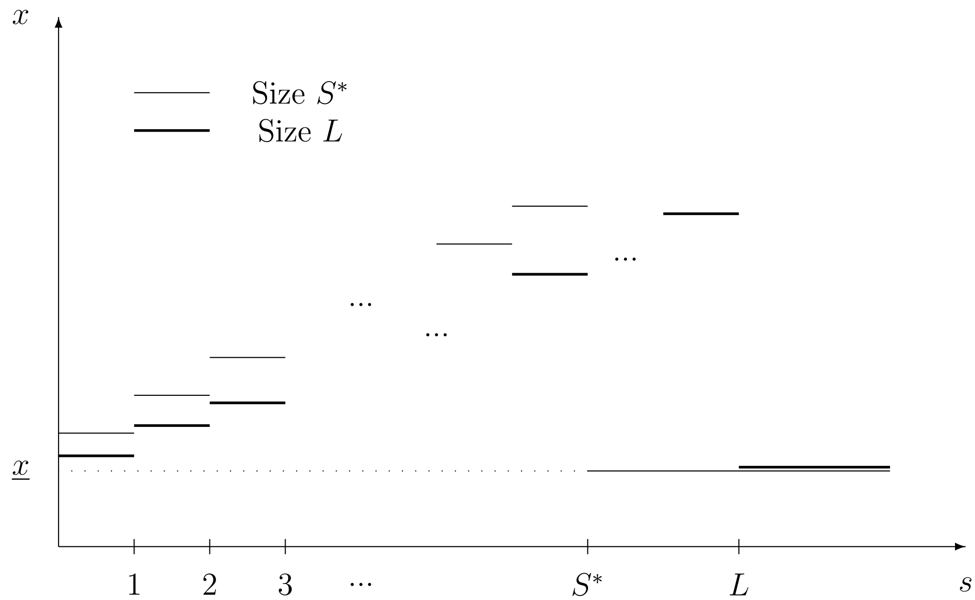

Figure 3: Comparing equilibrium R&D efforts for two pool sizes (with $L > S^* = K$).

In our *ex ante* perspective, Proposition 4 establishes that R&D efforts for the discovery of essential technologies are lower as a consequence of firms anticipating the inclusion of additional patents at the pool startup date. The expected duration of the K first research programs is thus longer (before the $L - K$ improvement innovations must be discovered). Therefore, consumers can only benefit later from the use of the pooled technologies.

7 Conclusion

Our main result contrasts sharply with the antitrust concern that, after the startup of the pool, the cross-licensing of future patents implies that firms are likely to free ride on other pool members' R&D. In the *ex ante* perspective of our model, we find that firms increase their R&D effort for essential technologies, as the number of patents grows to the anticipated pool size, and then up to a final race level that exceeds a joint profit-maximization benchmark. This main result gives theoretical support to patent pools as a policy lever for stimulating the generation of the first patents in a sequence of R&D programs, and thus for accelerating time to market of essential technologies, to the benefit of consumers. In the theoretical context of our model, we also find that the endogenous pool startup mechanism is not constrained by the regulatory principle that only essential patents should be included. Moreover, any departure from that principle, as anticipated by firms, can only imply lower R&D levels, thus delaying access for consumers to essential technologies. The latter two results thus reinforce a policy consensus derived in the *ex post* perspective of the economic literature, because they lend additional support to a strict enforcement of the essentiality criterion currently used in the United States and the European Union.

Appendix

A1. Proof of Lemma 1 From (6) $V_{S+1}(1) - V_S(1) = \frac{-1}{r(S+1)S}\left[K(\bar{v}-\hat{v}) + \frac{x}{r+x}N(\hat{v}-\underline{v})\right]$ is negative because $\underline{v} < \hat{v} \leq \bar{v}$, so that $V_S(1)$ is minimized if $S = N$. Then it is sufficient to check that $V_N(1) - \frac{\underline{v}}{r} = \frac{K(\bar{v}-\hat{v})+N(\hat{v}-\underline{v})}{rN} > 0$. $\square$

A2. Proof of Proposition 1 We know from the proof of the previous result that $V_{S+1}(1) - V_S(1) < 0$, which implies that $\arg\max_{S\geq K} V_S(1) = K$. Suppose that all N firms hold a patent, and rank them in the time order of the patenting process (i.e., firm 1 patented first, etc.). Then proceed backwards, as follows:

- Firm N proposes a pool with $S \geq K$ firms, including itself. In that case the value of firm N's patent, and of the addressees' patents, is $V_S(1)$. Otherwise, no pool forms and the value of all firms' patent is $\frac{\underline{v}}{r}$. As $V_S(1) > \frac{\underline{v}}{r}$ from Lemma 1, it is a dominant strategy for any of firm N's addressees to participate in the foundation of a S-patent pool, all S. Then $\arg\max_{S\geq K} V_S(1) = K$ implies that firm N's value maximizing proposal is $S = K$, and all K addressees accept it.

- Firm $N-1$ proposes a pool with $S \geq K$ firms, including itself. In that case, the value of firm $N-1$'s patent, and of the addressees' patents, is $V_S(1)$. Otherwise, no pool forms and the value of all firms' patent is the constant $\tilde{V}_{N-1} = \frac{K-1}{N-1}V_K(1) + \frac{N-K}{N-1}\frac{\underline{v}}{r}$. The latter expression is the expected value of any firm's patent when the proposal is rejected, implying that the protocol restarts with firm N. More specifically:

  (i) with probability $\frac{K-1}{N-1}$, any of the $N-1$ firms holding a patent participates in the K-patent pool that forms as a response to firm N's proposal (see previous case, where firm N founds a pool with $S = K$ patents), in which case this firm's patent is worth $V_K(1)$;[25]

[25] There are C_{N-1}^{K-1} combinations with K firms including firm N, and C_{N-2}^{K-2} combinations with K firms including both firms $N-1$ and N. The ratio of the two scalars is the probability that firm $N-1$ participates in the K-patent pool initiated by firm N.

(ii) with probability $1-\frac{K-1}{N-1}=\frac{N-K}{N-1}$, any of the $N-1$ firms holding a patent is not included in firm N's proposal, in which case this firm's patent is worth $\frac{\underline{v}}{r}$.

Any firm, including firm $N-1$, participates in the foundation of a S-patent pool provided that $V_S(1) > \tilde{V}_{N-1}$. Then a) recall that $\arg\max_{S \geq K} V_S(1) = K$, and b) observe that $V_K(1) > \tilde{V}_{N-1}$ because $\tilde{V}_{N-1}$ is a linear combination of $V_K(1)$ and $\frac{\underline{v}}{r}$ with $V_K(1) > \frac{\underline{v}}{r}$ from Lemma 1. It follows from a) and b) that firm $N-1$'s value maximizing proposal is $S = K$, and all K addressees accept it.

This process works back to firm K, implying that, as soon as K technologies are patented, a pool of size K forms. □

A3. Proof of Lemma 2 For any given pool size $S \geq K$, set $s = S-1$. We know from Lemma 1 that $V_S(1) > \frac{\underline{v}}{r}$. As $V_{S-1}(1)$ is a convex combination of $\frac{\underline{v}}{r}$ and $V_S(1)$ from (10), we obtain directly that $\frac{\underline{v}}{r} < V_{S-1}(1) < V_S(1)$. Then the same reasoning applies by iteration for all $s = S-2, S-3, \ldots, 1$. This holds in particular for $S = S^* = K$. □

A4. Proof of Lemma 3 In (9) set $s = S \geq K$, a given pool size, so that

$$rV_S(0) = x_S c'(x_S) + (N-S)\, x_S \left[V_{S+1}(0) - V_S(0)\right] - c(x_S). \tag{A1}$$

From (8) we have $V_S(0) = V_S(1) - c'(x_S)$, and from (5) we have $V_{S+1}(0) = \frac{\underline{v}}{r} - c'(\underline{x})$. Plugging the latter two expressions in (A1), and reorganizing terms, we find

$$r\left[V_S(1) - \frac{\underline{v}}{r}\right] + \underline{v} - (x_S + r)c'(x_S) + c(x_S) + (N-S)\, x_S \left[V_S(1) - \frac{\underline{v}}{r} + c'(\underline{x}) - c'(x_S)\right] = 0. \tag{A2}$$

Then suppose that $x_S \leq \underline{x}$. Recall that $V_S(1) - \frac{\underline{v}}{r} > 0$ (Lemma 1). Moreover, from (4) and the convexity of $c(.)$ we know that $\underline{v} - (x+r)\, c'(x) + c(x)$ takes the value 0 at $\underline{x}$ and is decreasing in x, implying that $\underline{v} - (x_S + r)c'(x_S) + c(x_S) \leq 0$. The convexity of $c(.)$ also implies that $c'(\underline{x}) - c'(x_S) \geq 0$. Therefore, the expression on the LHS of the equality sign in (A2) is *strictly* positive, a contradiction. Hence $x_S > \underline{x}$, which holds in particular for $S = S^*$. □

A5. Proof of Lemma 4 Consider any $s \leq S-1$, for any size $S \geq K$. From (8) we have $V_s(0) = V_s(1) - c'(x_s)$. By inserting the latter expression in (9), and reorganizing terms, we obtain

$$\underline{v} - (r + x_s)\, c'(x_s) + c(x_s) = \underline{v} - rV_s(1) + (N-s)x_s \left[V_{s+1}(1) - V_s(1) + c'(x_s) - c'(x_{s+1})\right]. \tag{A3}$$

We now prove that $x_{s+1} < x_s \Rightarrow x_s < \underline{x}$. To do that, focus on the RHS of the latter equation, and note from (10) that $\underline{v} - rV_s(1) + (N-s)x_{s+1}\left[V_{s+1}(1) - V_s(1)\right] = 0$, where $V_{s+1}(1) - V_s(1) > 0$

(from Lemma 2). Then $x_{s+1} < x_s$, together with $c'(x_s) - c'(x_{s+1}) > 0$ from the convexity of $c(.)$, imply that the RHS expression in (A3) is positive. It follows directly that

$$\underline{v} - (r + x_s)\, c'\,(x_s) + c\,(x_s) > 0. \tag{A4}$$

As $\partial\left(\underline{v} - (r + x_s)c'(x_s) + c(x_s)\right)/\partial x_s = -(r+x_s)c''(x_s) < 0$ by assumption, and $\underline{v} - (r + \underline{x})\, c'\,(\underline{x}) + c\,(\underline{x}) = 0$ by definition of $\underline{x}$ in (4), we conclude that $x_s < \underline{x}$.

Next, we prove that $x_s < \underline{x} \Rightarrow x_{s+1} < x_s$. Again from the convexity of $c(.)$ and the implicit definition of $\underline{x}$ in (4) we know that $x_s < \underline{x}$ implies $-\,(r + x_s)\, c'\,(x_s) + c\,(x_s) > -\underline{v}$. From (10) we also know that $-\underline{v} = -rV_s(1) + (N - s)x_{s+1}\,[V_{s+1}(1) - V_s(1)]$. Then by transitivity, and a simple reorganization of terms in (A3), we obtain that

$$(N - s)\left\{(x_{s+1} - x_s)\,[V_{s+1}(1) - V_s(1)] + x_s\left[c'(x_{s+1}) - c'(x_s)\right]\right\} < 0. \tag{A5}$$

Given that $V_{s+1}(1) - V_s(1) > 0$ (from Lemma 2) and $c(.)$ is convex, the negative sign in (A5) holds only if $x_{s+1} < x_s$. □

A6. Proof of Lemma 5 For any given pool size $S \geq K$, recall from (5) that $V_{S+1}\,(0) = \frac{\underline{v}}{r} - c'(\underline{x})$, and from Proposition 2 that $\underline{x} < x_s$. Then assume that, for some s, we have $V_{s+1}\,(0) \geq V_{S+1}\,(0) = \frac{\underline{v}}{r} - c'(\underline{x})$. In that case, can we have $V_s\,(0) \leq V_{S+1}\,(0)$? Suppose this holds, so that $\underline{v} - rc'(\underline{x}) \geq rV_s(0)$. The latter inequality, together with (9) according to which $rV_s(0) = x_s c'(x_s) - c(x_s) + (N - s)x_s(V_{s+1}(0) - V_s(0))$, where the last term is non-negative, imply that $\underline{v} - rc'(\underline{x}) \geq x_s c'(x_s) - c(x_s)$. However, the latter inequality can be true only if $x_s \leq \underline{x}$, a contradiction.[26] Therefore, if $V_{s+1}\,(0) \geq V_{S+1}\,(0)$ is verified for some s, we must also have $V_s\,(0) > V_{S+1}\,(0)$. It remains to check that the former weak inequality is indeed verified at rank $s = S$. The proof holds in particular for $S = S^* = K$. □

A7. Proof of Proposition 3 First, recall $\overline{x}$ is implicitly defined by $\overline{v} - (\overline{x} + r)c'(\overline{x}) + c(\overline{x}) = 0$. Together with the implicit definition if $\underline{x}$ in (4), the convexity of $c(.)$ and $\overline{v} > \underline{v}$ imply that $\overline{x} > \underline{x}$. Second, recall that $S^* = K$. Then suppose that $x_K \leq \overline{x}$, and look for a contradiction. Combining $c'(x_K) = V_K(1, K) - V_K(0, K)$ from (8) and $V_K(1, K) \geq \frac{\overline{v}}{r}$ from (11), we obtain that $V_K(0, K) \geq \frac{\overline{v}}{r} - c'(x_K)$ for all $x_K \geq 0$. Then the convexity of $c(.)$ and the supposition that $x_K \leq \overline{x}$ lead to $V_K(0, K) \geq \frac{\overline{v}}{r} - c'(\overline{x})$. Multiplying throughout by r, and observing from the implicit definition of $\overline{x}$ that $\overline{v} - rc'(\overline{x}) = \overline{x}c'(\overline{x}) - c(\overline{x})$, we obtain that $rV_K(0, K) \geq \overline{x}c'(\overline{x}) - c(\overline{x})$.

[26] Recall from (4) that $\underline{v} - rc'(\underline{x}) - \underline{x}c'(\underline{x}) + c(\underline{x}) = 0$, and note that $\partial\left(-xc'(x) + c(x)\right)/\partial x = -xc''(x) < 0$, all $x > 0$ (by assumption). It follows that $\underline{v} - rc'(\underline{x}) - xc'(x) + c(x) \geq 0$ if and only if $x \leq \underline{x}$.

But this is inconsistent with equation (9), which defines the MPE strategy x_K for $s = K$, since $V_{K+1}(0, K) < V_K(0, K)$ from Lemma 5. It follows that $x_K > \overline{x}$. □

A8. Proof of Lemma 6 For any rank $s \leq S - 1$, all $S \geq K$, define

$$\varphi(x_s) = rV_s(0) - x_s c'(x_s) - (N - s)x_s [V_{s+1}(0) - V_s(0)] + c(x_s), \tag{A6}$$

which allows us to rewrite the Bellman expression in (9) simply as $\varphi(x_s) = 0$. From (8) we can substitute $V_s(1) - c'(x_s)$ for $V_s(0)$, and also $V_{s+1}(1) - V_s(1) + c'(x_s) - c'(x_{s+1})$ for $V_{s+1}(0) - V_s(0)$, in (A6). Note from (10) that $V_s(1)$ and $V_{s+1}(1)$ are not functions of x_s. Then $c''(x_s) > 0$ and $c'''(x_s) \geq 0$ imply that $\varphi''(x_s) = -x_s c'''(x_s)(1 + N - s) - c''(x_s)(1 + 2(N - s)) < 0$. As $\varphi(0) = rV_s(0) > 0$, there exists only one x_s such that $\varphi(x_s) = 0$, all s. □

A9. Proof of Proposition 4 We extend the previous notation, so that $V_s(1, S)$ (resp. $V_s(0, S)$) is the value associated with a patent (resp. an R&D program) when exactly s innovations (resp. $s - 1$ innovations) have been patented, and where the second argument is the anticipated pool size. We also incorporate S as an argument of the function introduced in (A6), so that it becomes $\varphi(x_s, S)$. Note that the proofs of Lemmas 2 to 6 remain valid for any $S \geq K$.

The claims in (*ii*) and (*iii*) are direct consequences of Proposition 2. We first prove (*i*). Set $S = S^* < L$, and use (8) to substitute $V_{s+1}(1, K) - V_s(1, K) + c'(x_s) - c'(x_{s+1})$ for $V_{s+1}(0) - V_s(0)$ in the expression of $\varphi(x_s, K)$ as introduced in (A6). As $V_{s+1}(1, K) - V_s(1, K) = \frac{r(V_s(1,K) - \frac{\underline{v}}{r})}{(N-s)x_{s+1}(K)}$ from (10), a reorganization of terms in (A6) leads to

$$\begin{aligned} \varphi(x_s(K), K) = \underline{v} + r\left(1 - \frac{x_s(K)}{x_{s+1}(K)}\right)\left(V_s(1, K) - \frac{\underline{v}}{r}\right) - (x_s(K) + r)\, c'(x_s(K)) \\ - (N - s)x_s(K)\left[c'(x_s(K)) - c'(x_{s+1}(K))\right] + c(x_s(K)) = 0. \end{aligned} \tag{A7}$$

Now we proceed by induction. Suppose that, at rank $s + 1$, we have $V_{s+1}(1, K) > V_{s+1}(1, L)$ and $x_{s+1}(K) > x_{s+1}(L)$, where $L > K$. Then from equation (10) we deduce that $V_s(1, K) > V_s(1, L)$ as in the K-case more weight is given to the most valuable point in the convex combination (that is, $\frac{\underline{v}}{r} < V_{s+1}(1, K)$ from Lemma 2), and this point is more valuable than in the L-case. We also know that $\varphi(x_s(L), L) = 0$ (by definition of $x_s(L)$ as a MPE) and $\varphi(0, L) = rV_s(0, L) > 0$ (by assumption). Next, we check that $\varphi(x_s(K), L) < 0$ because $x_{s+1}(K) > x_s(K)$ (from Proposition 2) and $V_s(1, K) > V_s(1, L)$ (as obtained above). Then, the intermediate value theorem, together with the unicity of the symmetric MPE (Lemma 6), lead to $x_s(L) < x_s(K) = x_s(S^*)$.

For the induction process to be complete it remains to establish that $V_K(1, K) > V_K(1, L)$ and $x_K(K) > x_K(L)$. The first point is obtained directly as $V_K(1, K) > V_L(1, L)$ from the monotonicity of $V_S(1,S)$ in S (see 6), and $V_L(1,L) \geq V_{K+1}(1,L) > V_K(1,L)$ from Lemma 2 (with $L \geq K+1$). For the second point, note from (A6) that the function φ verifies $\varphi(0,S) > 0$, all S, and $\varphi(x_s(K),K) = \varphi(x_s(L),L) = 0$. From $V_K(1,K) > V_K(1,L)$ and $V_{K+1}(0,K) < V_{K+1}(0,L)$ (from Lemma 5) we obtain that $\varphi(x_s(K),L) < 0$. By continuity of φ in its first argument and unicity of the symmetric MPE (Lemma 6), it must be the case that $x_K(K) > x_K(L)$.

Finally, to establish (*iv*) recall that $\varphi(x_S(S),S) = 0$ (by definition of $x_S(S)$ as a MPE) and note again from (A6) that $\varphi(0,S) > 0$, all S. Consider $L > S$, the monotonicity of $V_S(1,S)$ in S (both as a rank subscript and as a pool size argument) implies that $V_S(1,S) > V_L(1,L)$. Moreover $V_{S+1}(0,S) = V_{L+1}(0,L)$ and $V_{S+1}(0,S) - V_S(1,S) + c'(x) < 0$ (from Lemma 5 and $c'(x) \geq 0$). It follows that $\varphi(x_S(S),L) < 0$, and again by continuity of φ in its first argument and unicity of the equilibrium solution (Lemma 6), we conclude that $x_L(L) < x_K(K)$. □

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