Search Advertising

Alexandre de Cornière *

Paris School of Economics and ENSAE-ParisTech

May 27, 2011

Abstract

This article investigates the role of a search engine as an intermediary between firms and consumers. Search engines enable firms to target consumers who have revealed some specific needs through their query. In a framework with horizontal product differentiation, imperfect product information and in which consumers incur search costs, I show that introducing a mechanism which enables firms to target consumers reduces social inefficiencies.

A profit maximizing search engine has incentives to design the matching mechanism so as to soften price competition between firms, in order to extract profit from them. In many cases, this implies lowering the accuracy of the matching mechanism.

Keywords: search engine, targeted advertising, consumer search.

JEL Classification: D43, D83, L13, M37.

*Email: adecorniere@gmail.com. I am grateful to Simon Anderson, Mark Armstrong, Heski Bar-Isaac, Luc Bridet, Bernard Caillaud, Olivier Compte, Gabrielle Demange, Romain de Nijs, Bruno Jullien, Frederic Koessler, Régis Renault and Joana Resende for very useful suggestions. The paper has also benefited from presentations at the Paris School of Economics, Université de Cergy-Pontoise, Northwestern University (2nd Transatlantic Theory Workshop), Stanford University (5th workshop on Ad Auctions), the Toulouse School of Economics, IESE business school (3rd conference on the economics of advertising and marketing) . Previous versions of this paper have circulated under the title "Targeting with consumer search: an economic analysis of keyword advertising".

1 Introduction

Search engines are arguably the most important actors of the digital economy. More than 4 billion search queries are processed every day by search engines such as Google, Yahoo or Bing, to find all sort of information. It is not a surprise that the development of these actors has generated interest from advertisers, to the point that search advertising is nowadays a multi-billion dollars industry.[1]

It turns out that advertising through a search engine is the cheapest way of attracting new consumers: in 2005, the cost of attracting a new customer was $ 8.5 with search advertising, $20 with yellow pages, $50 with banner advertising, $60 with email advertising, and $70 with traditional mail advertising.[2] One may wonder what are the reasons that make it so profitable. Two aspects seem to be of particular importance: (i) advertising is *intent-related* and (ii) costs are paid on a *per click* basis.

Intent-related advertising, as opposed to content-related advertising, exploits the possibility to know what consumers are looking for. Typically, suppose that a hotel located in Paris next to the Eiffel Tower wants to launch an advertising campaign. Using local newspapers may not be a good idea, since people who read them probably do not need a hotel in Paris. Alternatively, advertising through national TV or newspapers is probably too expensive. On the other hand, using a search engine appears like a very natural option, because the firm is able to target consumers who are looking for a hotel in Paris, or, even better, a hotel in Paris with a view on the Eiffel Tower. It might also choose not to target users who are looking for a hotel close to the Charles de Gaulle Airport.

"Per click" pricing, is aimed at ensuring firms that their investments are not wasted, i.e that the consumers for whom they pay are those who actually see the ad *and* were looking for it. The Hilton Hotel in Paris is certainly not willing to pay every time a search engine user enters the query "Paris Hilton".

In this paper I present a model of targeted advertising through a search engine, with differentiated products, which includes the main features mentioned above. Firms are horizontally differentiated *à la* Salop (1979), and consumers do not have prior knowledge of firms' prices or

[1]See Evans (2008) for an interesting presentation of the online advertising industry, with a special emphasis on search engines

[2]See Batelle (2005)

products' characteristics. The search engine is an intermediary between firms and consumers: firms choose which keywords they want to target, and consumers enter keywords and then search sequentially (and costly) at random through the links that appear. I do not study the format of the auction through which slots are allocated.[3] Rather, I shall explore the links between what information is revealed by the search engine and the resulting market outcomes.

In sections 2, 3 and 4, the search engine does not modify the messages which it receives from firms and consumers. The main findings are the following: the targeting technology creates two sorts of efficiency gains, namely better matches for consumers and smaller expenses in search costs. The fact that consumers find products more suited to their tastes is rather in line with the intuition that one may have before going into the details of the model. Indeed, since firms target them, consumers no longer receive non-relevant advertisements and thus choose from a better pool of offers. The model also predicts that, with targeting, consumers do not visit more than one firm, and thus minimize their search costs. These two results combine to improve the efficiency of advertising: the social costs due to imperfect information (bad matches and high search costs) are significantly reduced and thus the presence of a search engine contributes to improving social welfare. In a context in which the search engine competes with another online platform that does not allow targeting through keywords, the search engine is able to monetize the value created by the targeting mechanism. The model predicts that the relation between search costs and search engine's profit is non-monotonic. In fact, for low values of the search costs, I identify two forces that increase profit:(i) the search engine becomes more attractive to consumers with respect to platforms that do not allow for targeting, because consumers expect to search less if they use the search engine; (ii) firms are willing to bid more to access consumers, because they know that consumers will be more willing to accept higher prices. For high values of the search costs, these effects are more than offset by the fact that an increase in the search cost leads to a decrease in consumers participation.

The non strategic matching mechanism is an approximation of how search engines really proceed. For instance, Google sorts firms using a weighted average of the firms' bids and of a "quality score" index. Consumers are also sometimes provided with additional information on the results page, such as a map showing the locations of different vendors. On the other hand, the "Broad match" technology enables search engines to expand the set of keywords

[3]See Edelman, Ostrovsky, and Schwarz (2007), Varian (2007)

corresponding to a given advertisement. Such practices may be regarded as an attempt by the search engine to influence the accuracy of the information transmitted by firms, in one way or another. In section 5 I look at a situation in which a strategic search engine can introduce an arbitrary level of noise (in a sense made precise below) in the information revealed to consumers. The analysis reveals that, even if the search engine could implement the perfect matching costlessly, it would not be optimal to do so. Indeed, implementing an accurate matching mechanism would lead to a hold-up situation (the Diamond paradox) that would dissuade consumers from participating. It can even be optimal for the search engine to implement a matching that is less accurate than the laisser-faire outcome (in which the accuracy is the result of equilibrium behavior by firms and consumers). The reason is that offering a noisy matching mechanism makes consumers more willing to accept high-prices on the product market, because it is more costly to search. Since the search engine can only extract firms' profit, it may then be optimal to use such a strategy. It is not always optimal, because it results in a decrease in the number of active consumers, and so the search engine trades off per-consumer profit and number of consumers.

Related literature

This paper is related to the literature on advertising, search models, as well as to more recent contributions which study internet search engines and two-sided markets.

Targeted advertising has received increased attention in recent years. Esteban, Gil, and Hernandez (2001) show that in a monopoly framework, firms' ability to target consumers reduces both consumers' and total surplus. Roy (2000) or Iyer, Soberman, and Villas-Boas (2005) show how targeted advertising may generate market segmentation in a duopoly, respectively with homogenous and heterogenous products. In this paper I will focus on cases in which there is a large number of firms and market segmentation cannot play any role. Other recent works on targeted advertising include Van Zandt (2004) who suggest ways to avoid information overload, Johnson (2008), who examines ad avoidance behavior, or Bergemann and Bonatti (2010), who study competition between medias with different targeting technologies.

An important paper on advertising in the presence of horizontal differentiation is Grossman and Shapiro (1984). The product space is similar to the one in my paper, and the difference lies in the fact that in their model advertising is perfectly informative and there is no search.

The seminal paper on consumer search is Diamond (1971). In a model with several firms producing an homogenous good, and in which consumers incur a positive cost to obtain price information, in equilibrium firms necessarily charge the monopoly price. The reason for that is that demand is inelastic with respect to price, because a rise in the price inferior to the search cost does not drive consumers away from a firm.

One way to avoid this "Diamond paradox" is to introduce some product differentiation, because it makes the demand price elastic. Wolinsky (1984) and Anderson and Renault (1999) use a framework in which consumers' tastes for different goods are independent, whereas in Wolinsky (1983) competition is "local", meaning that some products are close substitutes and some are not. The latter framework will be the building block of my model.

Another way to circumvent this paradox is to introduce some heterogeneity in consumers' information about the price. In equilibrium firms use mixed strategies, trading off selling at a high price to some uninformed consumers, or setting a low price so as to attract the informed consumers. The informational heterogeneity is either exogenous (as in Varian (1980)), or endogenous due to different search costs (Stahl (1989)).

Robert and Stahl (1993) model the heterogeneity of information as a result of advertising. Advertising conveys price information, but some consumers do not receive advertisements and have to search. Another paper which relates advertising and search is Anderson and Renault (2006). In their model, a monopoly can reveal some information to consumers before they start searching. The nature (price or product characteristics) as well as the accuracy of the information which is transmitted depend on search costs. An important difference between Robert and Stahl (1993) and Anderson and Renault (2006) is that in the former, receiving an ad is a substitute to searching, whereas in the latter consumers always have to pay a search cost, even if they receive information. In my model, receiving an ad does not dispense consumers from searching, but neither does it provide "hard" information regarding the products' characteristics or their price. Rather, advertising acts more like a signal of relevance.

Some papers use consumer search models in the context of a search engine. Athey and Ellison (2007) focus on the design of the auction to allocate advertisement slots, given that consumers search strategically through the slots. However their analysis does not include competition between firms on the product market. Armstrong, Vickers, and Zhou (2009) deal with price competition between firms, in a model in which one firm is made prominent, meaning that although consumers search strategically, they always visit the prominent firm first.

In this paper, rather than modeling the heterogeneity of slots in terms of prominence, I focus on the heterogeneity of firms with respect to their relevance to certain keywords.

Finally, my paper is related to the growing literature on two-sided markets, seminal papers of which include Armstrong (2006), Caillaud and Jullien (2003), Rochet and Tirole (2006). My approach is slightly different from these papers, in the sense that I do not use a reduced-form way of modeling interactions between agents on the platform, in order to account for some important details. Neither do I allow complete flexibility in terms of pricing, focusing instead on the design of the matching process as a way to increase the platform's profit.

Other papers have a similar approach: Baye and Morgan (2001) model an intermediary who acts as an information gatekeeper on a homogenous product market, and look at the optimal two-sided pricing, taking into account subsequent price setting by firms and consumers' search behavior. Hagiu and Jullien (2010) focus on the design of a platform in terms of search diversion, and highlight several reasons why an intermediary does not want to provide the highest quality matching, even when the technology is costless.

White (2008) examines the trade-off faced by a search engine between providing quality organic results (which tend to attract users) and generating clicks on sponsored links (through which the search engine makes money). Gomes (2011) characterizes the optimal mechanism to sell an advertising slot when consumers and advertisers are heterogenous.

To the best of my knowledge, this paper is the first to explicitly model the transmission of information from firms to consumers through a search engine, and how this process may affect prices and welfare. It is also the first to study a model of consumer search with targeted advertising.

2 The model

2.1 Description of the market and of preferences

The framework is based on Wolinsky (1983). Consider a market where a continuum[4] of mass 1 of firms produce a differentiated good at a zero marginal cost. Each product may be described

[4]The continuum assumption makes the derivation of results easier, but is not a necessary condition.

by a single keyword. Keywords are located on a circle, whose perimeter is normalized to one. Thus a firm is characterized by the position of its product's keyword on the circle. The type of a firm, i.e the keyword that describes perfectly its product, will be denoted $\theta \in [0;1]$. θ is private information.

There is a continuum of mass 1 of consumers. Consumers differ along two dimensions: (i) each consumer has a favorite product (or keyword), $\omega \in [0;1]$, uniformly distributed around the circle, and (ii) consumers differ with respect to their willingness to pay for their favorite product. This willingness to pay v is independent of ω, and across consumers. It is distributed on $[0, \overline{v}]$ according to a continuous and increasing cumulative distribution function F , with density f.

Both ω and v are consumers' private information. Consumers have use for at most one unit, and the utility that a consumer located in ω gets from consuming product θ, with $d(\theta, \omega) = d$, is

$$u(v, d, p) = v - \phi(d) - p \tag{1}$$

where p is the price of the good and ϕ is increasing and twice continuously differentiable. $\phi(d)$ is often referred to as a transportation cost in traditional models of spatial competition. Here, I will use the terminology "mismatch cost".

2.2 Advertising technology

Consumers have imperfect information about firms' characteristics: they do not know firms' position on the circle (θ) nor their price, and thus have to search before buying.

Interactions between firms and consumers are only possible through a search engine. The search engine plays the role of an intermediary: on the one hand firms announce the set of keywords that they want to target. This set is assumed to be symmetric around θ and convex: $\mathcal{K}(\theta) = [\theta - D_\theta; \theta + D_\theta]$.[5] On the other hand, consumers announce the keyword they are interested in $\mathcal{K}(\omega) = \{\omega\}$.[6] If a certain keyword ω is entered by a consumer, the search engine randomly selects a firm θ such that $\omega \in \mathcal{K}(\theta)$. The consumer incurs a search cost $s > 0$, which corresponds to the time spent looking for θ's offer, and learns the price and position of this

[5]It is straightforward to see that even if $\mathcal{K}(\theta)$ could be any measurable set, the equilibrium in this game would still be an equilibrium.

[6]I assume away issues of strategic communication by consumers. In Section 6, I look at the case in which $\mathcal{K}(\omega)$ can be of the form $[\omega - L_\omega; \omega + L_\omega]$.

firm. The firm θ pays a fee $a > 0$ to the search engine. At that point, the consumer has three options: (i) he can accept the offer and leave the market, (ii) he can refuse the offer and leave the market, (iii) he can hold the offer and continue searching. In that case, the search engine randomly selects another firm θ' such that $\omega \in \mathcal{K}(\theta')$, and the process starts over.

At any point, consumers can come back at no cost towards a firm they have visited previously (recall is costless). It is the case if for instance consumers open a new window every time they click on a link.

The assumption that consumers do not observe anything before clicking on a link seems appropriate in many contexts. Indeed, firms can provide very little information with the text under their link on a search engine's page. Consumers have to click on the link to get more precise information. In this respect, advertising is not informative in the usual sense: it does not provide information in itself, but in equilibrium consumers infer correctly that a firm which targets them is not farther than a certain distance.The assumption is less relevant when consumers have a previous knowledge of the firms and/or products (if they bought in the past, of if they know the brand). I assume away these kinds of situations, which certainly deserve a proper analysis.

2.3 Strategies and equilibrium concept

Timing and strategies The timing of the game is the following:

1. **Platform pricing:** In the paper I consider three forms of market structure, each one providing a different insight: in subsection 4.1, I assume that the search engine is the only way through which consumers can find a product, and it can choose the per-click fee a. In section 4.2, I assume that consumers can either use the search engine (SE) or another platform (P) which does not allow targeting. Both platforms can choose the price a_{SE} and a_P that firms pay. In section 4.3, the search engine is a monopoly, but a_{SE} is determined through an auction. Irrespective of the market structure, firms and consumers observe a_{SE} and a_P before playing.

2. **Firms pricing and targeting:** Firms can multi-home among platforms, as long as they expect a non-negative profit from doing so. Each firm θ jointly chooses a price $p_{\theta,SE}$ and an advertising strategy $D_{\theta,SE}$ if it joins the search engine. If there is another platform on

the market, firms can charge a different price on each platform (and thus a firm's strategy is $(p_{\theta,SE}, D_{\theta,SE} p_{\theta,P})$, and it is not observed by other firms nor by consumers).

3. **Consumer search:** Consumers decide whether they want to use the search engine or the other platform[7]. If they choose the search engine, they enter the keyword corresponding to their favorite product (ω), and start a sequential search among firms such that $d(\theta,\omega) \leq D_\theta$. Firms are drawn uniformly from $\{\theta \quad s.t. \quad d(\theta,\omega) \leq D_\theta\}$. If they choose the other platform, they search randomly among all the firms.

Stages 2 and 3 constitute a subgame $\Gamma(a)$, where a is the vector (a_{SE}, a_P).

Consumers face a sequential search problem, with an infinite number of firms. We know, from Kohn and Shavell (1974), that the optimal strategy for a consumer is a stationary decision rule. If, at any point, the best available offer comes from a firm located at a distance $\hat{d}$ from ω, with a price of $\hat{p}$, the consumer continues to search if and only if $v - \phi(\hat{d}) - \hat{p} < U_R$. Therefore, the strategy of a consumer consists in the choice of the reservation utility U_R, or, alternatively, in the choice of a reservation distance $R(\hat{p}) \equiv \phi^{-1}(v - \hat{p} - U_R)$. Notice that $R(.)$ will depend on the expected future prices and locations if the consumer keeps on searching.

The equilibrium concept used is perfect Bayesian equilibrium: Platforms choose their fees a_{SE} and a_P optimally (except in section 4.3). Given a vector of fees a, advertisers set their price and advertising policy so as to maximize their profit given the other firms' strategies, participation decision as well as the stopping rule used by consumers. A consumer will use the search engine if and only if $v \geq v^*$. v^* and the stopping rule $R^*(.)$ are themselves best-responses to firms' strategies. I will focus on symmetric equilibria in pure strategies $(a^*, R^*(.), v^*, p^*, D^*)$. To highlight the fact that $R^*(.)$ depends on the expectation about future prices and locations, I will use the notation $R^*(p, \sigma^*)$, where $\sigma^* = (p^*, D^*)$.

Consumers have passive beliefs in the following sense: if a firm deviates from the equilibrium strategy (p^*, D^*), and this deviation is observed by a consumer, this consumer does not update his beliefs regarding other firms' strategy.

[7]Consumers are assumed to single home.

3 Analysis of the subgame $\Gamma(a)$

In this section I focus on the interactions on the search engine. The equilibrium on the other platform is derived in Wolinsky (1983) and Bakos (1997).

3.1 Consumer search

In equilibrium, when a consumer of type (v, ω) clicks on a link, the expected utility he gets from this click if he buys is

$$\int_{\omega-D^*}^{\omega+D^*} \frac{u(v, d(\omega, \theta), p^*)}{2D^*} d\theta = \int_0^{D^*} \frac{u(v, x, p^*)}{D^*} dx$$

Consumers regard each click as a random draw of a location θ from a uniform distribution, whose support is $[\omega - D^*; \omega + D^*]$. Indeed a firm located at a distance greater than D^* from ω would not appear on the results' page in equilibrium (the consumer would not be targeted). Suppose for now that all firms set the equilibrium price p^*. Then, after the first visit, the only way a consumer can improve his utility is by finding a closer firm. For $R^* \equiv R(p^*, \sigma^*)$ to be a reservation distance it must be such that a consumer is indifferent between continuing to search and buying the product:

$$\int_0^{R^*} \frac{u(v, x, p^*) - u(v, R^*, p^*)}{D^*} dx = s \tag{2}$$

The left-hand side of this equality is the expected improvement if a consumer decides to keep on searching after being offered a product at a price p^* and at a distance R^*. This expected improvement equals the search cost, so that the consumer is indifferent between buying or searching again. By totally differentiating (2), one gets

$$\frac{dR^*}{ds} = -\frac{D^*}{R^* \frac{\partial u(v, R^*, p^*)}{\partial d}} > 0, \quad \frac{dR^*}{dD^*} = -\frac{s^*}{R^* \frac{\partial u(v, R^*, p^*)}{\partial d}} > 0 \tag{3}$$

R^* is an increasing function of the equilibrium reach of advertising D^*: if consumers expect firms to try to reach a wide audience (by targeting many keywords), they adjust their stopping rule by being less demanding, because the expected improvement after a given offer is lower than with more precise targeting. R^* is also an increasing function of search costs: consumers are less demanding if it costs more to continue searching. Note also that in equilibrium, R^* does not depend on the equilibrium price p^*, because in equilibrium the expected price improvement due to an extra sample is always zero with quasi-linear utility functions.

Notice that when u takes the form (1), one can see that the optimal reservation distance does not depend on v, a feature that will be convenient in the derivation of the equilibrium.

Now, when a consumer samples a firm which has set an out-of-equilibrium price $p \neq p^*$, his belief about other firms' strategy and position does not change, and therefore his optimal stopping rule $R(p, \sigma^*)$ is such that accepting a price p at a distance $R(p, \sigma^*)$ gives the same utility as accepting a price p^* at a distance R^*, i.e $v - \phi(R(p, \sigma^*)) - p = v - \phi(R^*) - p^*$. Thus we have the following proposition.

Proposition 1 *Given other firms' expected strategy $\sigma^* = (p^*, D^*)$, a consumer accepts to buy a good at price p if and only if the selling firm is located at a distance less than $R(p, \sigma^*)$, with $R(p, \sigma^*)$ such that*

$$v - \phi(R(p, \sigma^*)) - p = v - \phi(R^*) - p^*$$

where R^* is given by (2).

Moreover, by the implicit function theorem, R is continuously differentiable and

$$\frac{dR(p, \sigma^*)}{dp} = -\frac{dR(p, \sigma^*)}{dp^*} = -\frac{1}{\phi'(R(p, \sigma^*))} < 0$$

3.2 Equilibrium

Suppose that firm θ sets a price p. Since it only has to pay for consumers who actually visit its link, firm θ's optimal targeting strategy is to appear to every consumer ω such that the expected profit made by θ through a sale to ω conditionally on ω clicking on θ's link is positive, i.e

$$p.Pr(\omega \text{ buys } \theta\text{'s product} | \omega \text{ clicks on } \theta\text{'s link}) - a_{SE} \geq 0 \tag{4}$$

where a_{SE} is the per-click fee paid to the search engine.

The next lemmas will enable us to derive the only symmetric equilibrium, by providing some necessary conditions. Let $E \equiv (p^*, D^*, R^*(.), v^*)$ be an equilibrium of the subgame $\Gamma(a)$.

For $v \geq v^*$, let $\delta(v, p^*) \equiv sup\{d \in [0, 1/2] \quad s.t. \quad u(v, d, p^*) \geq 0\}$. $\delta(v, p^*)$ is the largest distance d such that a consumer would buy at price p^* and at distance d if there was no other firm available.

Lemma 1 *In equilibrium, for every $v \geq v^*$, $\delta(v, p^*) \geq R^*(p^*, \sigma^*)$.*

Proof: Suppose that there is a consumer of type (v, ω), with $v \geq v^*$ such that $\delta(v, p^*) < R^*(p^*, \sigma^*)$. Let a firm be located in θ_1, with $\theta_1 \in (\omega + \delta(v, p^*), \omega + R^*(p^*, \sigma^*))$. Suppose that the consumer faces firm θ_1. Because $d(\omega, \theta_1) > \delta(v, p^*)$, the consumer would rather leave the market than buy from θ_1. But since $d(\omega, \theta_1) < R^*(p^*, \sigma^*)$, the consumer strictly prefers buying than visiting a new firm. This implies that the expected net value of a draw is negative for consumer (v, ω), which contradicts the fact that $v \geq v^*$, since v^* is such that the expected value of a draw is just zero. $\square$

Lemma 2 *Any symmetric profile of strategy $\sigma^* = (p^*, D^*)$ such that $D^* \neq R^*(p^*, \sigma^*)$ cannot be an equilibrium.*

Proof: This proof is in two stages: (1) if firms set $D^* < R(p^*, \sigma^*)$, then a firm can profitably deviate by targeting more consumers (2) if $D^* > R(p^*, \sigma^*)$, there is always at least one firm which can profitably deviate and lower its targeting distance.

1. Suppose that all firms have a targeting distance D^* smaller than $R^*(p^*, \sigma^*)$. Take a consumer ω and a firm θ such that $D^* < d(\theta, \omega) < R^*(p^*, \sigma^*)$. If θ were to deviate and choose to appear to consumer ω, then it would sell the good with probability equal to $P[v \geq p^* + \phi(d(\theta, \omega))|v \geq v^*]$ if ω clicked on its link. Now, from lemma 1, and since $d(\omega, \theta) < R^*(p^*, \sigma^*)$, we know that $P[v \geq p^* + \phi(d(\theta, \omega))|v \geq v^*] = P[\delta(v, p^*) \geq d(\theta, \omega)|v \geq v^*] = 1$. Thus it would be a profitable deviation.

2. Now suppose that all firms set $D^* > R^*(p^*, \sigma^*)$. Take a consumer ω, and denote $\overline{\theta}$ the firm which is located at a distance D^* from him. Since $d(\overline{\theta}, \omega) > R^*(p^*, \sigma^*)$, the probability that ω buys from $\overline{\theta}$ is zero. By reducing its reach, firm $\overline{\theta}$ can increase its profit. $\square$

Corollary 1 *If an equilibrium exists, it must be the case that consumers do not search more than once.*

This property of the equilibrium is at odds with what actually happens when people use search engines, but it should not been taken literally. Indeed, it is the result of two assumptions that make the model tractable: (i) that all consumers have the same search cost s, and (ii) that it is possible to target consumers perfectly. This property underlines an important insight, namely that targeting through keywords reduces the amount of search costs incurred in equilibrium.[8]

The next step in order to derive a symmetric equilibrium of the game is to study the best response of a firm when other firms play a symmetric strategy $\sigma^* = (p^*, D^*)$ with $D^* = R(p^*, \sigma^*)$.

Lemma 3 *Let θ be the location of a given firm on the circle. If:*

- *all the other firms play the strategy $\sigma^* = (p^*, D^*)$ where $D^* = R(p^*, \sigma^*)$, and*
- *consumers expect all firms to play $\sigma^* = (p^*, D^*)$ and thus play $R(p, \sigma^*)$,*

then, whatever price p that firm θ decides to set , the optimal advertising strategy is to set $D(p) = R^*(p, \sigma^*)$, i.e. a targeting distance equal to the reservation distance of consumers who face an "out of equilibrium" price.

The proof is very similar to the previous lemma's one, and is omitted.

Lemma 3 states that if a firm wants to deviate from a situation where all firms set a targeting distance equal to the "equilibrium" reservation distance, the deviation implies to set a scope of relevance equal to the "out of equilibrium" reservation distance. Thus, the deviation does not change the number of clicks per consumer, since they find it optimal to buy from the first firm they visit.

Now we can state an existence theorem and provide sufficient conditions for uniqueness.

Notice first that there always exists a "trivial"' equilibrium, in which firms set $D^* = 0$ and $p^* = \overline{v}$, and in which consumers do not search at all. I shall assume that when there is another equilibrium in which trade takes place, agents coordinate on the latter.

Two additional assumptions ensure existence (Assumption 1) and uniqueness (Assumption 2) of an equilibrium.

Assumption 1 *For any p, $R(p, p, 1/2) < 1/2$.*

[8] See section 4.2.

Under Assumption 1, if firms do not target specific keywords (i.e they target the whole circle), some consumers search more than once before buying. In particular, this assumption requires search costs not to be too large.

Assumption 2 *For all $d \in [0, 1/2]$, $\phi'(d) + d\phi''(d) \geq 0$.*

Assumption 2 does not rule out risk-loving behavior of consumers with respect to the quality of the match ($\phi'' < 0$), but restricts the extent of risk-loving. For instance, if $\phi(d) = -e^{-\alpha d}$, assumption 2 implies $\alpha \in (0, 2]$

Proposition 2 *Under Assumption 1, there exists a non trivial equilibrium of the game. If Assumptions 2 holds, this non-trivial equilibrium is unique.*

The complete proof is provided in the appendix.

Example, part I : In order to illustrate the previous analysis, it may be useful to examine an analytically simple example. Suppose that consumers' utility $u(v, d, p)$ is given by $u(v, d, p) = v - td^b - p$. As usual in models of spatial differentiation, t measures the intensity of consumer's preferences with respect to the characteristics of the products. The parameter b allows to represent different patterns of preferences. With $b > 1$, a consumer's utility decreases slowly when the distance d between ω and θ is small, and faster for higher values of d. On the other hand, when $b < 1$, starting from $d = 0$, a small increase of the distance leads to a big drop in utility. b also relates to consumers' attitude towards risk. Indeed, the relative risk aversion index with respect to the quality of the match is

$$I_u^R(d) \equiv d\frac{\frac{\partial^2 u(v,d,p)}{\partial d^2}}{\frac{\partial u(v,d,p)}{\partial d}} = b - 1$$

Therefore, the consumer is risk-averse for $b > 1$, risk-neutral for $b = 1$, risk-lover for $b < 1$.

In this case, equation (2) writes $R^* = \left(\frac{(b+1)sD^*}{bt}\right)^{\frac{1}{b+1}}$. Assumption 1 is thus satisfied whenever $s \leq \left(\frac{b}{(b+1)2^b}\right) t$, while assumption 2 holds irrespective of the parameters' values.

Using Proposition 1, one gets $R(p, \sigma^*)^b = R^{*b} + \frac{p^* - p}{t}$. By Lemma 2, in equilibrium $D^* = R^*$, and so $R^* = D^* = \left(\frac{(b+1)s}{bt}\right)^{\frac{1}{b}}$.

By Lemma 3, a firm's profit, if it sets a price p, is

$$\pi(p, \sigma^*, a_{SE}) = (p - a_{SE}) \left(\frac{(b+1)s}{bt} + \frac{p^* - p}{t} \right)^{\frac{1}{b}}$$

In the unique non-trivial symmetric equilibrium, we have

$$p^*(a_{SE}) = a_{SE} + (b+1)s$$

We see that in this example, the level of advertising (D^*) is an increasing function of the search costs. If consumers are more willing to accept lesser matches, firms will tend to target a broader set of keywords. The equilibrium price is also an increasing function of the search costs, as usual in search models.

The equilibrium price is also an increasing function of the risk-aversion parameter b. As consumers become more risk-averse, they are less willing to reject an offer and search again. Firms exploit this in equilibrium by raising their price.

4 Platform pricing

4.1 Monopolistic search engine

In this subsection, I assume that using the search engine is the only way to find a product for consumers. Let $v^*(a_{SE})$ be the lowest value of v such that a consumer is willing to use the search engine. $v^*(a_{SE})$ is such that $v^*(a_{SE}) - E[\phi(d)|d \leq R^*] - s - p^*(a_{SE}) = 0$. Since in equilibrium every consumer with $v \geq v^*(a_{SE})$ clicks only once, the search engine's profit is

$$\Pi^{SE}(a_{SE}) = a_{SE}\left(1 - F(v^*(a_{SE}))\right)$$

The optimal per-click fee a^*_{SE} is thus given by the formula

$$a^*_{SE} = \frac{1 - F(v^*(a_{SE}))}{v^{*\prime}(a^*_{SE})f(v^*(a_{SE}))} = \frac{1 - F(v^*(a_{SE}))}{p^{*\prime}(a^*_{SE})f(v^*(a^*_{SE}))}$$

This formula is reminiscent of the Lerner formula for monopoly pricing. The difference here is that a marginal increase of a_{SE} affects consumer participation only through the effect on the equilibrium price (the pass-through rate $p^{*'}(a_{SE})$).

4.2 Competition by another platform

Instead of assuming that consumers have to use the search engine in order to find a product, suppose now that there is another platform that allows consumers to search for products. That platform does not allow firms to target specific keywords, and so consumers draw randomly from the whole pool of firms.

Consumers do not have an intrinsic preference towards one platform or the other, and so they simply use the one that gives them the highest expected utility.[9]

In order to keep things as simple as possible, I focus on the uniform-linear case, in which v is uniformly distributed on $[0;\overline{v}]$ and $u(v, d, p) = v - td - p$.

The following proposition describes the equilibrium on the other platform.

Proposition 3 *(Wolinsky (1983), Bakos (1997)). In the uniform-linear case, the equilibrium price is* $p_P^* = \sqrt{st}$ *on the platform without targeting. Consumers' reservation distance is* $R_P^* = \sqrt{\frac{s}{t}}$*. The expected number of searches for a consumer is* $\frac{1}{2}/\sqrt{\frac{s}{t}}$*. The expected utility of a consumer of type* (v, ω) *is therefore*

$$EU_P(v, \omega) = v - 2\sqrt{st} \tag{5}$$

From the analysis in subsection 3.2, the situation on the search engine is as follows. The price is $p_{SE}^* = a_{SE} + 2s$, the reservation distance (which equals the targeting distance) is $R_{SE}^* = D_{SE}^* = \frac{2s}{t}$ ($R_{SE}^* \leq 1/2$ implies that $4s \leq t$). The expected utility of a consumer is therefore

$$EU_{SE}(v, \omega) = v - 4s - a_{SE} \tag{6}$$

By comparison of (5) and (6), one sees that two effects are at play here:[10]

- An *efficiency effect*: Using the search engine reduces the inefficiencies due to search costs and mismatch costs: $4s \leq 2\sqrt{st}$ for $t \geq 4s$, which increases consumers' expected utility.

[9]Assuming, in line with Armstrong (2006), that platforms are horizontally differentiated would lead to an equilibrium in which both platforms receive traffic, though the main insight would not be altered.

[10]These two effects do not depend on the fact that $u(v, d, p)$ is linear in d.

- An *internalization effect*: On the search engine, the targeting technology allows firms to target only the consumers that would buy conditional on learning the offer. The consequence is that a_{SE} plays the role of a marginal cost, whereas a_P is a fixed cost. a_{SE} is passed through consumers, whereas a_P is not.

We can use the reduced form expressions (5) and (6) to characterize the equilibrium fees chosen by both platforms.

Proposition 4 *In the uniform-linear case, if* $\overline{v} \geq 4(\sqrt{st} - s)$ *the optimal fee for the search engine is* $a^*_{SE} = 2\sqrt{st} - 4s > 0$. *In this case all consumers are indifferent between the two platforms. Consumers such that* $v \geq 2\sqrt{st}$ *use the search engine, while the others do not search at all.*[11]

If $\overline{v} < 4(\sqrt{st} - s)$, *the optimal fee for the search engine is* $a^*_{SE} = \frac{\overline{v}-4s}{2}$. *All consumers strictly prefer the search engine to the other platform. Consumers such that* $v \geq \frac{\overline{v}}{2} + 2s$ *use the search engine, while the others do not search at all.*

Proof: The search engine's profit is $\pi^{SE}(a) = a(1 - F(4s + a))$ as long as $4s + a \leq 2\sqrt{st}$. If $\pi'^{SE}(a) > 0$ for $a = 2\sqrt{st} - 4s$, then the search engine is constrained by competition, and its optimal price is such that consumers are indifferent between the two platforms. This occurs when $\overline{v} \geq 4(\sqrt{st} - s)$. Otherwise, the search engine is not constrained by competition, and charges the price that it would charge if it were a monopoly. □

Compared to a platform without targeted advertising, a search engine creates value by reducing both search costs and mismatch costs. It can then capture a fraction of this value, and attract all the active consumers. In this set-up, one clearly sees that the introduction of the targeting technology is welfare improving.

Given Proposition 4, one can look at the effect of a variation in the level of search costs and mismatch costs on the equilibrium profit of the search engine.

Proposition 5 *In the linear-uniform case, the search engine's profit is an increasing function of the mismatch cost* t, *and it is an inverted U-shaped function of the search cost* s.

Proof: To be written...

[11] The tiebreaking rule seems natural, because the search engine could always reduce its fee by an arbitrarily small amount.

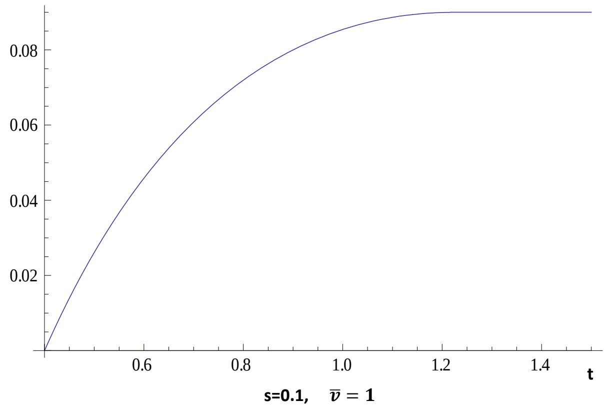

Figure 1: Search engine's profit as a function of t.

For low values of t, the search engine is constrained by the presence of the other platform. But as t increases, that platform becomes less attractive and the search engine can raise its price while keeping the consumers. For high values of t, the other platform is not competitive, and the search engine acts like a monopoly. Since, in the linear-uniform case, the expected utility of a consumer who uses the search engine does not depend on t, the search engine's profit does not vary with t (see figure 1).

Regarding s, there are two opposite effects. On the one hand, a rise in s benefits the search engine because it becomes relatively more attractive with respect to the other platform (since consumers search less on the search engine). This effect plays for low values of s (when the search engine actually competes with the other platform). When s is larger, the search engine is a monopoly on the market, and therefore a rise in s leads to less consumer participation (see figure 2).

4.3 Pricing through an auction

In the previous two subsections, I have assumed that the search engine is able to set whatever fee a_{SE} it wants. Although convenient, this assumption is at odds with what one observes in practice, where slots are allocated through an auction. Explicitly incorporating an auction stage in this game is very challenging, and here I present a reduced-form approach that captures the essential features of an auction and allows a meaningful discussion. As in the previous subsection, I assume that $u(v, d, p) = v - td - p$, and that v is uniformly distributed over

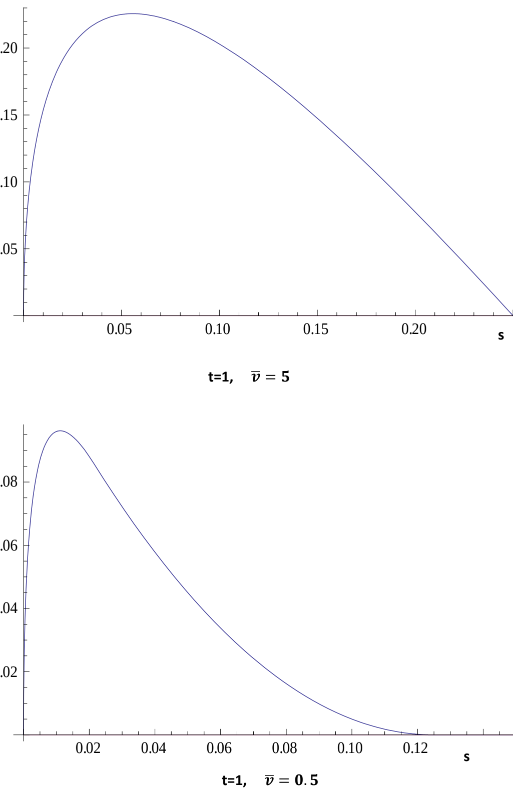

Figure 2: Search engine's profit as a function of s.

$[0;\overline{v}]$. In this case, the price as a function of a_{SE} is $p^* = 2s + a_{SE}$, the equilibrium targeting distance is $D^* = R^* = \frac{2s}{t}$, and consumers search only once, so that a consumer's expected utility, conditional on participating, is $v - 4s - a_{SE}$. I assume that the search engine is in a monopolistic situation.

Suppose that firms incur a fixed cost $C < 2s$ to be present on the search engine. Such a cost may entail monitoring tasks, frequent revisions of bids, and so on. Since there is a continuum of symmetric firms, the equilibrium advertising fee will be such that firms are left with no rent.

The zero-profit condition writes

$$(1 - F(4s + a_{SE}))[(p(a_{SE})) - a_{SE}] - C = 0 \tag{7}$$

which gives

$$a^*_{SE} = \overline{v}(1 - \frac{C}{2s}) - 4s$$

One sees that there are two channels through which the search cost s affects the equilibrium per-click fee. The first one is an increase in firms' mark-up, which pushes a^*_{SE} up. This is reflected in the term $-\frac{C}{2s}$. The other effect is a decrease in the number of consumers, and its consequence is a decrease in the per-click fee.

Therefore, a consumer who participates has an expected utility equal to

$$EU|_{v \geq v^*} = v + \overline{v}(\frac{c}{2s} - 1)$$

This expected utility is decreasing in the search costs, which is rather intuitive. A more interesting result is the following:

Proposition 6 *In the linear-uniform case, consumers' expected utility is increasing in C.*

The intuition is that as C increases, the auction results in a lower advertising fee, which benefits consumers.

Regarding the effect of search costs on the search engine's profit, we have the following proposition:

Proposition 7 *In the linear-uniform case, the search engine's profit is an inverted U-shaped function of the search cost s.*

Proof: The search engine's profit is

$$\pi^{SE} = \left(\overline{v} - 4s - \frac{C}{2s}\right) \frac{C}{2s\overline{v}}$$

The derivative of π^{SE} with respect to s is $\frac{C(C-s\overline{v})}{2s^3\overline{v}}$, which is positive for $s < C/\overline{v}$.□

The result in Proposition 7 is the same as in Proposition 5, but the explanation is different. Indeed, here, for low values of s, an increase in s does not improve the position of the search engine with respect to another platform (there is no other platform). The profit is increasing in s because firms are willing to bid more in order to access consumers, because they know that they will be able to charge a higher price. The negative correlation between s and π^{SE} for higher values of s has the same root as in Proposition 5, namely a lower participation.

Although the above analysis sheds some light on some effects of targeting through keywords, it does not allow to say much about the role of the search engine in terms of information transmission. In the following section I study a variant of the model in which the search engine reveals the information strategically.

5 Optimal matching mechanism

The assumption that the search engine does not behave strategically with respect to information revelation leaves aside interesting theoretical as well as practical issues. There is evidence that search engines pay a lot of attention to the way advertisements are displayed. The ranking of advertisements through a "quality score" illustrates this concern, as well as the use of a "broad match" technology aimed at matching consumers to firms when the keywords do not correspond exactly but are "close"' enough. Basically, with broad match, the search engine will display an advertisement even if the keyword has not been selected by the firm, provided it is regarded as relevant by the search engine. For instance, suppose that a firm selects only one keyword, namely "web hosting". If a consumer enters the keyword "web hosting company" or "webhost", then the firm's advertisement will appear on the consumer's screen. Google argues that one of the benefits brought by such a practice is that it saves time for firms: they no longer have to spend time and resources finding exactly what are the right keywords to use. The search engine will do that for them, using the available information on past queries and results in order to find relevant keywords.

Such practices may be regarded as an attempt to choose the accuracy of the matching system. For instance, putting large weights on the most relevant websites to a query improves the quality of the matching process, whereas applying a very loose "broad match" policy introduces some additional noise. Another example is the display of maps, indicating the physical location of firms. In March 2010, Google also launched an experiment consisting in displaying hotels' location and price in the sponsored results. In this section I will argue that a profit maximizing search engine has an incentive not to let firms target consumers as they wish. By introducing the appropriate level of noise in the process, the search engine may alleviate price competition between firms, thereby extracting more profit from them.

A convenient way to model the situation is to assume that the search engine is able to choose an accuracy level D. The following lemma

Lemma 4 *If the search engine has the possibility to choose the accuracy of the matching, in equilibrium it can extract firms' profit entirely.*

Proof: Let $q_i(p_i, p^*, D)$ be the quantity sold by firm i if it sets a price p_i, if other firms set a price p^*, and if the search engine chooses a level of accuracy D. Then firm i's profit is $p_i q_i(p_i, p^*, D) - aD$. In equilibrium, by setting $a = p^* q(p^*, p^*, D)/D$, the search engine extracts all the profit. $\square$

As is the case when firms cannot target specific keywords, their pricing strategy is independent of their advertising expenses, and so the search engine captures the whole profit.

It is now straightforward to see that the search engine will choose D so as to maximize firms' gross profit

The following proposition gives the optimal matching accuracy for the search engine. Recall that D^* is the equilibrium distance in the game in which firms choose their targeting strategy.

Proposition 8 *The optimal matching accuracy, from the search engine point of view, is $D^{SE} \geq D^*$. That is, the search engine will not improve the quality of the matching with respect to the "laissez-faire" situation.*

Before looking at the proof, let us discuss the intuition. If the search engine decides to improve the quality of the matching, that is, to set a lower value of D, a hold-up problem (the Diamond paradox) emerges. In this situation, firms set a price at least as high as the lowest value of v among participating consumers. Therefore these consumers, who also have to pay search costs, do not participate, a contradiction.

If the search engine sets a higher value of D, there are two competing effects. On the one hand, competition between firms is less intense, which leads to a higher price and hence a higher per-consumer profit. On the other hand, because consumers face higher prices, search and mismatch costs, the number of participating consumers decreases. If this drop in consumers participation is not too steep at $D = D^*$, then the search engine will optimally decide to lower the accuracy of the matching process.

6 Two-sided targeting

In the benchmark model, consumers are assumed to behave truthfully, in the sense that they enter the keyword that corresponds to their ideal brand. Although this behavior is an equilibrium strategy, one might be interested in richer patterns of communication. In this section I study a variant of the model in which communication is costly for consumers. More specifically, a consumer located in ω will enter a set of keywords that corresponds to an interval $\mathcal{K}_\omega = [\omega - L_\omega; \omega + L_\omega]$. The main assumption that I make here is that it is more costly for a consumer to be very accurate regarding what he is looking for than to be vague: for a given L_ω, the consumer incurs a cost $c(L_\omega)$, with $c' < 0$.

The matching technology is the following: a firm θ which targets a set $\mathcal{K}_\theta$ will belong to the set of possible matches for a consumer located in ω with a set of admissible keywords $\mathcal{K}_\omega$ if and only if $\mathcal{K}_\theta \cap \mathcal{K}_\omega \neq \emptyset$ or, equivalently , $D_\theta + L_\omega \geq d(\theta, \omega)$. (**mettre un dessin**)

This specification introduces a new trade-off for the consumer: choosing a small value of L_ω restricts the pool of potential offers, at a cost. Interestingly, the benefit of lowering L_ω will depend on firms' advertising strategies. Indeed, suppose that firms decide to target the whole circle. Then it is useless to be accurate when one enters one's query. On the other hand, if firms only target the keywords that match exactly their product, the marginal gain of lowering L_ω is much higher.

In order to obtain analytical expressions, I focus on the case in which $u(v, d, p) = v - td^b - p$.

In equilibrium, if firms choose D^* and p^*, and if consumers choose L^*, the optimal reservation distance of a consumer is such that

$$\int_0^{R^*} \frac{u(v,x,p^*) - u(v,R^*,p^*)}{D^* + L^*} dx = s$$

Using the functional form above, one gets

$$R^* = \left(\frac{(b+1)s(D^* + L^*)}{bt} \right)^{\frac{1}{b+1}} \tag{8}$$

Applying the same reasoning as in Lemma 2, we must have $R^* = D^* + L^*$ in equilibrium. Combined with (8), this leads to

$$R^* = \left(\frac{(b+1)s}{bt} \right)^{\frac{1}{b}} \tag{9}$$

This expression is the same as in the benchmark case, and so the equilibrium price is unchanged, at

$$p^*(a_{SE}) = a_{SE} + (1+b)s \tag{10}$$

Now let us look at possible deviations by a consumer. If a consumer decides to be less accurate in his query ($L > L^*$), his reservation distance R is given by

$$R = \left(\frac{(b+1)s}{bt}(D^* + L) \right)^{\frac{1}{1+b}} > \left(\frac{(b+1)s}{bt}(D^* + L^*) \right)^{\frac{1}{1+b}} = R^* \tag{11}$$

We also have

$$\frac{dR}{dL}|_{R=R^*, L=L^*} = \frac{1}{1+b} \in (0;1) \tag{12}$$

Equation (12) implies that if a consumer reduces the accuracy of his query, he becomes less demanding (because he expects that future draws will be of lower quality), but his reservation distance does not increase as fast as his accuracy decreases. Since in equilibrium $D^* + L^* = R^*$, we have, for $L > L^*$, $D^* + L > R$: by becoming less accurate, the consumer is now also in a position to refuse some of the offers that he receives (those at a distance between R and $D^* + L$). The expected number of clicks is now $(D^* + L)/R$, and the expected distance from the firm he eventually buys from is $\int_0^R \frac{x^b}{R} dx = \frac{R^b}{b+1}$.

Using expressions (9) and (11), the expected utility of a consumer who chooses $L > L^*$ is

$$E[U(L)|L > L^*] = v - \frac{t}{b+1}\left(\frac{(b+1)s}{bt}(D^*+L)\right)^{\frac{b}{1+b}} - s\frac{D^*+L}{\left(\frac{(b+1)s}{bt}(D^*+L)\right)^{\frac{1}{1+b}}} - p^*(a_{SE}) - c(L) \tag{13}$$

On the other hand, if the consumer decides to be more accurate than L^* ($L < L^*$), equation (12) tells us that he becomes more demanding, but the increase in his accuracy is larger than the decrease in his reservation distance. Consequently, the consumer will still search only once. The expected distance between the consumer and the firm he will eventually buys from is now $\int_0^{D^*+L} \frac{x^b}{D^*+L}dx = \frac{(D^*+L)^b}{b+1}$. The consumer's expected utility is then

$$E[U(L)|L < L^*] = v - \frac{t}{b+1}(D^*+L)^b - s - p^*(a_{SE}) - c(L) \tag{14}$$

For L^* to be an equilibrium, one must have

$$\frac{\partial E[U(L)|L > L^*]}{\partial L}|_{L=L^*} \leq 0, \quad \frac{\partial E[U(L)|L < L^*]}{\partial L}|_{L=L^*} \geq 0$$

where the first derivative is the right derivative at $L = L^*$ and the second one is the left derivative at $L = L^*$.

These two inequalities lead to an equality, namely

$$-c'(L^*) = s^{\frac{b-1}{b}}\left(\frac{b+1}{b}\right)^{-\frac{1}{b}} t^{\frac{1}{b}} \tag{15}$$

If c is such that the local maximum is also a global maximum, then the equilibrium accuracy is given by (15).

Because c is a decreasing convex function, one has the following comparative statics results:

Proposition 9 *The accuracy of consumers' queries increases as the mismatch cost t increases: $\partial L^*/\partial t > 0$.*

If consumers are risk-averse ($b > 1$), their queries are more accurate for higher search costs s ($\partial L^*/\partial s < 0$), but the reverse is true if they are risk-lover.

7 Conclusion

Search engines allow intent-related targeted advertising, and this paper illustrates the potential efficiency gains generated from firms' ability to target consumers: targeting leads to better matches on average, and to smaller expenses in search costs.

A profit maximizing search engine wants to soften price competition between firms in order to extract their profit. In some cases this implies maximizing the value of trade, because firms are able to capture a large part of consumers' surplus. In other instances, maximizing the price implies degrading the quality of the matching process in order to improve firms' bargaining power (through a worsening of consumers' outside option).

References

Anderson, S. P., and R. Renault (1999): "Pricing, Product Diversity, and Search Costs: A Bertrand-Chamberlin-Diamond Model," *RAND Journal of Economics*, 30(4), 719–735.

——— (2006): "Advertising Content," *American Economic Review*, 96(1), 93–113.

Armstrong, M. (2006): "Competition in Two-Sided Markets," *RAND Journal of Economics*, 37(3), 668–691.

Armstrong, M., J. Vickers, and J. Zhou (2009): "Prominence and consumer search," *RAND Journal of Economics*, 40(2), 209–233.

Athey, S., and G. Ellison (2007): "Position Auctions with Consumer Search," Working paper.

Bakos, Y. (1997): "Reducing Buyer Search Costs: Implications for Electronic Marketplaces," *Management Science*, 43(12), 1676–1692.

Batelle, J. (2005): *The Search: How Google and Its Rivals Rewrote the Rules of Business and Transformed Our Culture*. Nicholas Brealey Publishing.

Baye, M. R., and J. Morgan (2001): "Information Gatekeepers on the Internet and the Competitiveness of Homogeneous Product Markets," *American Economic Review*, 91(3), 454–474.

BERGEMANN, D., AND A. BONATTI (2010): "Targeting in Advertising Markets: Implications for Offline vs. Online Media," Working paper.

CAILLAUD, B., AND B. JULLIEN (2003): "Chicken & Egg: Competition among Intermediation Service Providers," *RAND Journal of Economics*, 34(2), 309–28.

DIAMOND, P. A. (1971): "A model of price adjustment," *Journal of Economic Theory*, 3(2), 156–168.

EDELMAN, B., M. OSTROVSKY, AND M. SCHWARZ (2007): "Internet Advertising and the Generalized Second-Price Auction: Selling Billions of Dollars Worth of Keywords," *American Economic Review*, 97(1), 242–259.

ESTEBAN, L., A. GIL, AND J. M. HERNANDEZ (2001): "Informative Advertising and Optimal Targeting in a Monopoly," *Journal of Industrial Economics*, 49(2), 161–80.

EVANS, D. S. (2008): "The Economics of the Online Advertising Industry," *Review of Network Economics*, 7(3), 359–391.

GOMES, R. (2011): "Optimal Auction Design in Two-Sided Markets," Discussion paper.

GROSSMAN, G. M., AND C. SHAPIRO (1984): "Informative Advertising with Differentiated Products," *Review of Economic Studies*, 51(1), 63–81.

HAGIU, A., AND B. JULLIEN (2010): "Why do intermediaries divert search?," Idei working papers, Institut d'Économie Industrielle (IDEI), Toulouse.

IYER, G., D. SOBERMAN, AND J. M. VILLAS-BOAS (2005): "The Targeting of Advertising," *Marketing Science*, 24(3), 461–476.

JOHNSON, J. (2008): "Targeted Advertising and Advertising Avoidance," Working paper.

KOHN, M. G., AND S. SHAVELL (1974): "The Theory of Search," *Journal of Economic Theory*, 9(2), 93–123.

ROBERT, J., AND D. O. STAHL (1993): "Informative Price Advertising in a Sequential Search Model," *Econometrica*, 61(3), 657–86.

Rochet, J.-C., and J. Tirole (2006): "Two-Sided Markets: A Progress Report," *RAND Journal of Economics*, 37(3), 645–667.

Roy, S. (2000): "Strategic segmentation of a market," *International Journal of Industrial Organization*, 18(8), 1279–1290.

Salop, S. (1979): "Monopolistic competition with outside goods," *Bell Journal of Economics*, 10(1), 141–156.

Stahl, D. O. (1989): "Oligopolistic pricing with sequential consumer search," *American Economic Review*, 79(4), 700–712.

Van Zandt, T. (2004): "Information Overload in a Network of Targeted Communication," *RAND Journal of Economics*, 35(3), 542–560.

Varian, H. R. (1980): "A Model of Sales," *American Economic Review*, 70(4), 651–59.

——— (2007): "Position auctions," *International Journal of Industrial Organization*, 25(6), 1163–1178.

Vives, X. (2001): *Oligopoly Pricing: Old Ideas and New Tools*, vol. 1 of *MIT Press Books*. The MIT Press.

White, A. (2008): "Search Engines: Left Side Quality versus Right Side Profits," Discussion paper.

Wolinsky, A. (1983): "Retail Trade Concentration Due to Consumers' Imperfect Information," *Bell Journal of Economics*, 14(1), 275–282.

——— (1984): "Product Differentiation with Imperfect Information," *Review of Economic Studies*, 51(1), 53–61.

A Proofs

A.1 Proof of Proposition 2

The equilibrium is obtained through the following steps:

1. Existence and uniqueness of an equilibrium targeting distance $D^* > 0$.

   This part follows from two lemmas, which are of particular importance and will be used in subsequent proofs.

   Lemma 5 *For every D, p and $p^{'}$, we have $R(p,p,D) = R(p^{'}, p^{'}, D)$.*

   Proof: From (2), $R(p,p,D)$ is given by

$$\int_0^{R(p,p,D)} \frac{\phi(R(p,p,D)) - \phi(x)}{D} dx = s$$

   We see that it does not depend on p.□

   Lemma 6 *Under assumption 1, and for any price p, the function $r : D \mapsto R(p,p,D)$ has two fixed points: 0 and $D^* \in (0; 1/2)$.*

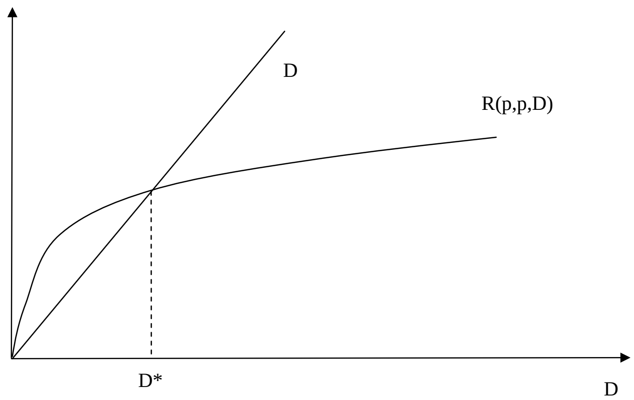

Figure 3: D versus $R(D)$

   Proof: From (2), we see that $r(D)$ is defined by

$$\int_0^{r(D)} \frac{\phi(r(D)) - \phi(x)}{D} dx = s$$

   Using the implicit functions theorem on the open interval $(0; 1/2)$, we get $r'(D) = \frac{s}{r(D)\phi'(r(D))}$. As D goes to zero, $r'(D)$ tends to $+\infty$, because $\lim_{D\to 0} r(D) = 0$ and $\phi'(.)$ is bounded and positive.[12] Moreover, $r(1/2) \leq 1/2$ (by assumption 1), and therefore there must be a $D^* \in (0; 1/2)$ such that $D^* = r(D^*)$. Such a D^* is unique if $r(.)$ is concave. Differentiating $r(D)$ a second time , one gets

$$r''(D) = -sr'(D)[\phi'(r(D)) + r(D)\phi''(r(D))][r(D)\phi'(r(D))]^{-2} \tag{16}$$

By assumption 2, the second term in brackets is positive, and therefore $r(.)$ is concave. In that case, one can see that $r(D)$ is above D when $D < D^*$, and below D otherwise. □

2. Existence and uniqueness of an equilibrium price strategy.

   A firm's profit equals $(p - a_{SE})R(p, p^*, D^*)$ if other firms play (p^*, D^*).

   First let's show that the profit is strictly quasi-concave in the firm's price if (C2) holds. A sufficient condition for that is that $1/R(p, \sigma^*)$ is convex in p (see Vives (2001) p.149). For notational convenience let us drop the arguments in $R(p, \sigma^*)$. From Proposition 1 and the implicit functions theorem, one gets $\frac{\partial R}{\partial p} = -\frac{1}{\phi'(R)}$. Straightforward computations show that $1/R(p, \sigma^*)$ is convex in p if and only if $2\phi'(R) \geq -R\phi''(R)$, which is the case if assumption 2 holds.

   Now that we know that the profit is strictly quasi-concave, and thus that the best response is a function, the following contraction argument ensures uniqueness of a symmetric equilibrium:

   Let $\pi(p, p^*) \equiv (p - a_{SE})R(p, p^*, D^*)$. Since we are looking for symmetric equilibria only, uniqueness is ensured if the best response mapping is a contraction for every firm.

   Using the fact that $\frac{\partial R}{\partial p}(p, p^*, D^*) = -\frac{\partial R}{\partial p}(p, p^*, D^*)$, straightforward computations show that

$$\frac{\partial^2 \pi}{\partial p^2} + \frac{\partial^2 \pi}{\partial p \partial p^*} = \frac{\partial R}{\partial p} < 0$$

   which is a sufficient condition for the best response mapping to be a contraction (see Vives (2001), p.47). There is thus a unique symmetric equilibrium. □

A.2 Proof of Proposition 8

Suppose that a consumer is of type (v, ω), and that firm θ sets a price p_θ while other firms play p^*. Three conditions must be satisfied for trade to occur between the consumer and the firm:

$$d(\theta, \omega) \leq D \qquad \text{(SED)}$$

$$v - \phi(d(\theta, \omega)) - p_\theta \geq 0 \qquad \text{(IR)}$$

$$d(\theta, \omega) \leq R(p_\theta, p^*, D) \qquad \text{(NS)}$$

[12] When $u(v, d, p) = v - td^b - p$ and $b < 1$, the assumption that ϕ' is bounded on $[0, 1]$ does not hold. Still, in that case, $r'(D) = D^{-\frac{b^2}{b+1}} \frac{s}{tb} \left(\frac{(b+1)s}{tb}\right)^{-\frac{b^2}{b+1}}$, and tends to $+\infty$ when D goes to 0.

Condition SED (for *search engine's D*) states that for a trade to happen, it must be the case that the firm is included in the pool of potential matches. Condition IR (*individual rationality*) ensures that buying the good provides a non-negative utility to the consumer. Finally, under condition NS (for *no-search*), the consumer prefers to buy than to continue searching.

Let v^* be the smallest value of v such that a consumer is willing to participate, given D. Let $\overline{x}(v,p,p^*,D)$ be the largest distance such that a consumer of type v buys at price p if other firms play p^*. $\overline{x}$ is the largest distance satisfying (SED), (IR) and (NS). Therefore $\overline{x}(v,p,p^*,D) = \min\{D, \phi^{-1}(v-p), R(p,p^*,D)\}$.

Firm θ's gross profit is then

$$\pi_\theta(p,p^*) = Dp\int_{v^*}^{\overline{v}}\int_0^{\overline{x}(v,p,p^*,D)}\frac{1}{D}f(v)dv = p\int_{v^*}^{\overline{v}}\overline{x}(v,p,p^*,D)f(v)dv \tag{17}$$

The next lemma simplifies the problem, by showing that $\overline{x}(v,p,p^*,D)$ cannot be equal to $\phi^{-1}(v-p)$ (unless it is also equal to D or $R(p,p^*,D)$).

Lemma 7 *For all $v \geq v^*$, if there exists $\overline{d} \leq D$ such that $v - \phi(\overline{d}) - p = 0$, then $\overline{d} \geq R(p,p^*,D)$.*

Proof: Suppose that $\overline{d} < R(p,p^*,D)$. Let $Z^*(v)$ be the expected value of a click (net of search costs) in equilibrium for a consumer of type v. Then

$$\overline{d} < R(p,p^*,D) \Longleftrightarrow Z^*(v) < v - \phi(\overline{d}) - p$$

Indeed, $\overline{d} < R(p,p^*,D)$ means that the consumer strictly prefers to buy than to search again, i.e the expected value of a click is smaller than the utility he gets if he buys the product immediately.

Now, we have $v - \phi(\overline{d}) - p = 0$, which implies that $Z^*(v) < 0$. But this contradicts the fact that $v \geq v^*$, because v^* is such that $Z^*(v^*) = 0$ and Z^* is increasing in v. □

Therefore, (17) rewrites

$$\pi_\theta(p,p^*) = p\int_{v^*}^{\overline{v}}\min\left(D, R(p,p^*,D)\right)f(v)dv = p\min\left(D, R(p,p^*,D)\right)[1-F(v^*)] \tag{18}$$

Let D^* be the fixed point of the function $D \mapsto R(p,p,D)$. D^* is the equilibrium level of advertising from section 3, and does not depend on p.

Lemma 8 *If the search engine chooses $D < D^*$, in any symmetric equilibrium, consumers do not participate.*

Proof: Suppose that $D < D^*$. Then, for every $\tilde{p}$, $R(\tilde{p},\tilde{p},D) > D$. (see Lemma 6) Therefore, at any symmetric strategy profile p, demand is inelastic around p. Each firm has an incentive to raise the price by ϵ, since such a deviation is not enough to trigger an additional search by consumers. □

If $D > D^*$, then $\min\left(D, R(p,p^*,D)\right) = R(p^*,p^*,D)$. Therefore the equilibrium price p^* must be such that

$$p^* \in argmax_p pR(p,p^*,D)[1-F(v^*)]$$

Since v^* depends on D, a firm's profit is

$$\pi^*_\theta(D) = p^*(D)R(p^*(D), p^*(D), D)[1 - F(v^*(D))]$$

By the envelope theorem,

$$\frac{\partial \pi^*_\theta(D)}{\partial D} = p^*(D)\frac{\partial R(p^*, p^*, D)}{\partial D}[1 - F(v^*(D))] - v^{*'}(D)f(v^*(D))p^*(D)R(p^*(D), p^*(D), D) \tag{19}$$

The first term is positive, and it corresponds to the fact that raising D enables firms to make a higher per-consumer profit. The second term takes into account the change in consumers' participation. We know that as D increases, both search costs and mismatch costs increase. The next lemma gives a sufficient condition for the equilibrium price to be increasing in D, in which case $v^{*'}(D) < 0$.

Lemma 9 *When $D > D^*$, if ϕ is convex, then the equilibrium price is an increasing function of D.*

Proof: The first order condition which determines the optimal price is

$$R(p(D), p(D), D) + p(D)\frac{\partial R}{\partial p}(p(D), p(D), D) = 0 \tag{20}$$

Given that $\frac{\partial R}{\partial p} = -\frac{\partial R}{\partial p(D)}$, totally differentiating (20) gives

$$\frac{dp(D)}{dD} = -\frac{\frac{\partial R}{\partial D}\left(1 + p(D)\phi''(R)(\phi'(R))^{-2}\right)}{\frac{\partial R}{\partial p}} \tag{21}$$

This last expression is non negative since $\frac{\partial R}{\partial D} > 0$ and $\frac{\partial R}{\partial p} < 0$.