Game of Platforms:
Strategic Expansion into Rival (Online) Territory*

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Abstract

Online platforms, such as Google, Facebook, or Amazon, are constantly expanding their activities, while increasing the overlap in their service offering. Motivated by this, we study an expansion game between two online platforms, offering different services to users for free, while selling user clicks to advertisers. Platforms decide whether or not to expand by adding the service already offered by the rival. Expansion decisions affect the partition of users in the market, which, in turn, affects platform prices and profits. We demonstrate that, in equilibrium, platforms may choose not to expand, even though expansion is costless. Such strategic "no expansion" decisions are due to quantity and price effects of changes in the user partition, and specifically changes in the degree of user multihoming, brought on by expansion.

Keywords: Media economics, entry, online platforms, two-sided markets.

JEL classification: D43, L10, L41

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1. Introduction

Google Plus, Google's social networking service, was introduced in 2011, in the wake of Buzz, Google's previous and quite unsuccessful attempt to expand into social networking. With 25 million users in one month, industry analysts initially dubbed Google Plus the "Facebook Killer", expecting Google Plus to be the next Facebook or Twitter. This narrative was quick to change. Most Google Plus users were not active, and in 2012 analysts and bloggers largely referred to Google Plus as a "Ghost Town", compared to its very active and lively counterpart, Facebook. Today, Google Plus is defined as a "social layer" on top of Google. Its active users are predominantly from the tech community, and use it mainly for aggregating, sharing and discussing news items related to their common interests. Still, Google Plus is far behind Facebook, with 28 million unique monthly visitors spending around 7 minutes on average on site, compared to Facebook's 142 million uniques, averaging almost 7 hours spent on the social network.[1] In the online world, where traffic and time spent equal money, Google Plus is hardly a success story.[2]

Google Plus is just one example of expansion by an online platform into a territory already occupied by a rival platform. Google, Facebook, Amazon, and Apple - the "big four" of the tech industry - are constantly expanding their activities, while increasing their overlap. Google and Facebook now compete both in social networking, where Facebook was the incumbent, and - since the recent introduction of Facebook Graph Search - also in search, Google's stronghold. Apple and Amazon compete in selling digital media and devices. More overlap between these platforms is found in cloud services, operating systems, smartphones, e-commerce, and the list goes on. With new services and products added each month, the overlap in these giants' activities will continue to increase, as they strive to provide a one-stop shop for their users.[3]

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[1] These figures from Nielsen are for desktop visitors in the US, in March 2013 (Wasserman 2013).

[2] Tassy 2011, Evans 2013, and Warren 2013 discuss Google Plus's evolution and current state.

[3] Also see The Economist, December 1st 2012.

And to what end? A major driver of these platforms' expansion is cultivating exclusive and intimate relationships with users that translate into large advertising revenues[4][5] (e.g., through improved ad targeting; see Kang 2012). Indeed, both Facebook and Google's revenue comes predominantly from advertising, and in 2013 Amazon launched its ad exchange, which was estimated to generate $835 million in 2014, by allowing retargeting of shoppers after they leave Amazon.com (Edwards 2012, Griffith 2012, and Taube 2013).

Motivated by this, we study optimal expansion strategies of online ad-financed platforms, thus adding to the growing literature on various strategic behaviors in platform markets.[6] Recent work has examined platform strategies such as openness and developer property rights (Parker and Van Alstyne 2008, Eisenmann et al. 2008, Boudreau 2010), compatibility with rival platforms (Casadesus-Masanell and Ruiz-Aliseda 2009), the choice of exclusive contracts versus multihoming (Hagiu and Lee 2011), tying (Choi 2010, Amelio and Jullien 2012), exclusion of some user types (Hagiu 2011), and platform investment in proprietary content (Hagiu and Spulber 2013), to name but a few.[7] In this work, we focus on expansion into services already offered by rival platforms, such that expansion may affect the overlap in platforms' user bases, or the *degree of multihoming*.

We model a market with two platforms, through which advertisers may reach potential buyers, also referred to as platform users. The platforms provide free services to users, generating

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4 Evans 2009 surveys the evolution of online advertising methods, provides some industry numbers, and discusses privacy concerns.

5 Google's ad revenue is now larger than that of the entire US print media, according to Edwards 2013.

6 The literature on multi-sided platform markets has largely focused on platforms' pricing strategies, under varying assumptions regarding market characteristics (see, for example, Caillaud and Jullien 2001 and 2003, Roche and Tirole 2002, 2003, and 2006, Parker and Van Alstyne 2005, Armstrong 2006, Hagiu 2006 and 2009b, Jullien 2008, Weyl 2010, Cabral 2011, Halaburda and Yehezkel 2013a and 2013b). More recently, the platforms literature has been evolving to consider strategies other than pricing.

7 Also see Eisenmann et al. 2006, Boudreau and Hagiu 2008, and Hagiu 2009a, who analyze case studies in their discussions of different strategies in platform markets.

revenues by selling user clicks to advertisers.[8] At the outset, each platform offers one service type, and buyers' optimally choose which platform(s) to use, inducing the initial *buyer partition*.

Platforms engage in an expansion game, where each platform strategically chooses whether or not to costlessly expand by adding the type of service already offered by its rival. The game proceeds in two stages. In the first stage, platforms make their expansion decisions, where these affect buyers' platform choice, determining the final buyer partition. In the second stage, platforms set prices per user click charged to advertisers, given the buyer partition. Advertisers then observe platform prices and the buyer partition, and choose where to place their ads, which, in turn, determines platforms' expected profits.

The expansion game is solved by backward induction. For each pair of expansion decisions, the buyer partition is derived, and platform pricing and profits follow. Platforms' optimal expansion decisions are then determined. To allow for a tractable analysis of this two-stage game, we simplify by assuming that the only network effect is a positive effect exerted by buyers on advertisers, whereas no network effect is exerted by advertisers on the buyer population, i.e., platform users neither suffer nor benefit from ads displayed on the platforms.

Endogeneity of the buyer partition is an important feature in our setting. The specifics of this endogeneity follow from the buyer model, which defines buyers' platform preferences and optimal choice behavior, given platforms' expansion decisions. The buyer model is maintained as a separate unit of analysis within our general framework, such that our setting is modular and can accommodate different buyer models seamlessly. The paper thus studies the effects of endogeneity of the buyer partition on optimal platform expansion decisions for a general buyer partition, considering different forms of endogeneity. Following the general analysis, a buyer model is proposed, analyzed and used to provide examples for the cases considered in the general framework. In the proposed model, endogeneity of the user partition results from an endogenous level of compatibility between the platforms; where both increases and decreases in compatibility following expansion are considered.

[8] The advertisers' side of the market fully subsidizes the buyers' side. This price structure is commonly assumed in the media platforms literature (e.g. Anderson and Coate 2005, Anderson et al. 2012, Reisinger 2012, Ambrus et al. 2013).

The basic motivation for expansion in our setting is improved ad targeting, modelled as an increase in users' ad-click probability, resulting from ad exposure on an additional service. When the buyer partition is exogenous, expansion increases click probability, thereby increasing the expected quantity of clicks sold to advertisers, leading to higher expected profits. However, when the buyer partition is endogenous, this benefit of expansion must be weighed against the effects of the change in the user partition.

We distinguish between two effects associated with the endogeneity of the partition of buyers – a quantity and a price effect. The former is a direct effect resulting from the change in user partition that may either increase or decrease the total number of potential buyers reached through the platform[9] following expansion, thereby affecting expected profits. The latter is an indirect effect, as equilibrium prices per click decrease in the degree of multihoming. This is because prices are optimally set such that advertisers place ads on both platforms, but do not pay double for reaching multihomers twice.

Considering the above effects of expansion we show that expansion is a dominant strategy whenever it decreases multihoming, while no-expansion may be optimal when user multihoming increases with expansion, i.e. when the price effect is negative. This allows us to characterize expansion equilibria for different types of endogeneity of the user partition. Specifically, when multihoming monotonically decreases with expansion the only equilibrium is symmetric expansion, but otherwise different types of equilibrium may arise depending on the magnitudes of the effects discussed. Notably, asymmetric expansion may be equilibrium even for symmetric platforms, and symmetric no-expansion may also be optimal, when the benefit of improved ad targeting for exclusive subscribers is small.

The following section 2 reviews the related literature. The general model is then set up in section 3, and analyzed in section 4. Section 5 proposes a buyer model, which is incorporated into the general framework in section 6, and used to demonstrate the general results. Concluding remarks and a discussion of managerial implications are offered in section 7.

[9] Note that both exclusive and multihoming users are reached, yet the two user types differ in their ad click probability.

2. Related Literature

Studying advertising-financed platforms, we relate to the literature on media platforms. This literature has examined equilibrium ad prices, levels of advertising, content differentiation, and platform entry (e.g., Anderson and Coate 2005, Crampes et al. 2009, Anderson et al. 2011 and 2012, Reisinger 2012, and Ambrus et al. 2013). Compared to these papers, we simplify by assuming that advertisers are homogeneous, and that the only network effect is exerted by potential consumers on advertisers, so as to allow for tractability in solving the platform expansion game.

While simplifying, the main features of our model are consistent with previous work in the media platforms literature. Advertisers in our model may multihome, as is commonly assumed in the literature (an exception is Reisinger 2012). We further allow for user multihoming, as in Anderson et al. 2011 and 2012, Ambrus et al. 2013, and Athey et al. 2013.

User multihoming creates some redundancy for multihoming advertisers in our model, and thus gives rise to pricing which follows the "principle of incremental pricing" defined in Anderson et al. 2011. Namely, prices are determined according to the incremental benefit of placing ads on an additional platform, thereby internalizing the redundancy for multihoming advertisers. A similar notion of redundancy in advertising arises in Athey et al. 2013 as a result of consumer switching.[10]

Note that in our model there is only partial redundancy from reaching multihoming users twice, as users' click-probability increases with ad exposure on an additional service. As a result, the degree of multihoming exerts a negative effect on prices. This is in contrast to Anderson et al. 2011, where the redundancy is full and the degree of multihoming does not directly affect ad pricing. On the other hand, and quite naturally, user exclusivity has a positive effect on prices in both papers.[11]

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[10] When tracking technologies are imperfect.

[11] In Athey et al. 2013 prices may either increase or decrease with consumer switching, as a result of the specific modelling of the switching process.

Within the media platforms literature, several papers have focused on platforms' endogenous differentiation (e.g., Gabszewicz et al. 2002, Dukes and Gal-Or 2003, Behringer and Filistrucchi 2009). Platforms in our model choose whether or not to expand by adding a second service already offered by their rival - our setting endogenizes both the number of services offered and their overlap. We therefore analyze a different notion of endogenous differentiation, characteristic of online platforms' expansion behavior.

Closely related, Eisenmann et al. 2011 study platform "envelopment", defined as "entry by one platform provider into another's market by bundling its own platform's functionality with that of the target's". Envelopment is thus essentially the same as the expansion behavior we study, yet the specific research questions and approach in the two papers are quite different. Namely, Eisenmann et al. build a typology of envelopment attacks based on the level of complementarity between the attacker and target platforms, deriving conditions for success of envelopment attacks. We, on the other hand, model a strategic expansion game between two platforms, so there is no "attacker" and "target" - both platforms may or may not expand, and conditions for different types of expansion equilibria are derived. Furthermore, while Eisenmann et al. take a very broad perspective, we focus on advertising-financed online platforms, which allows for a tractable model.

3. The Model

We model a market with two platforms, offering online services to *buyers* or *platform users,* in order to attract *advertisers* who would like users to click on their ads. Platforms are ad-financed, and advertising revenues are collected on a per click basis.

The focus of the model is platforms' strategic expansion behavior when the buyer partition is endogenous, and thus changes with expansion decisions. We analyze an entry or *expansion* game between the two platforms, where each platform may or may not expand by adding the service initially offered by the rival platform. Following expansion decisions, platforms set prices per user click charged to advertisers. Advertisers observe the partition of buyers resulting from platforms' expansion decisions, as well as platform prices, and choose their advertising strategy - placing ads on both platforms, on one of the platforms, or not advertising at all.

The model is constructed in a modular fashion. In this and the following section we consider an endogenous buyer partition with a general structure and characteristics, abstracting from the underlying buyer model which determines how expansion affects the partition of buyers. A possible buyer model which yields an endogenous user partition is presented in section 5.

3.1 Platforms - basic assumptions and notation

There are two platforms in the market and let $i \in \{1,2\}$ denote the platform index. At the outset, each platform provides one service, different from the one provided by its rival. Platform services are provided to users for free. A strategy for platform i is a couple (e_i, p_i), where $e_i \in \{E, \bar{E}\}$ represents the platform's expansion decision – either "expansion" denoted E or "no-expansion" denoted $\bar{E}$, and $p_i \in [0, \infty)$ is the price per user click charged to advertisers on the platform. Let (e_1, e_2) denote a pair of expansion decisions. We assume that platforms expand only by adding the service already offered by their rival.

Expansion is costless in the model. In reality, investment costs related with expansion are clearly positive and affect expansion decisions. We abstract away from such costs to focus on the effects of user partition endogeneity on platform expansion, identifying circumstances under which platforms will not expand, even when expansion is costless.

Platform profits are derived from user clicks sold to advertisers. We thus turn to introduce assumptions regarding buyers and advertisers in the market.

3.2 Buyers

There is a unit mass of buyers in the market. Buyers' choose which platform(s) they subscribe to, and this yields the *buyer partition* - a partition of the buyer population into three groups: exclusive users of platform 1 and 2, and multihomers, who subscribe to both platforms, using each platform's core service. This partition is an equilibrium of the buyer model, regarded in this and the following sections as a "black box" model, and presented in section 5.

The buyer partition is a function of platforms' expansion decisions (e_1, e_2), denoted $B^{e_1 e_2} \equiv \{b_1^{e_1 e_2}, b_2^{e_1 e_2}, b_{12}^{e_1 e_2}\}$, where $b_i^{e_1 e_2}$ is the group of platform i's exclusive subscribers and its mass, and $b_{12}^{e_1e_2}$ is the group of multihomers and its mass, for (e_1, e_2). We assume that the market is covered for any (e_1, e_2), i.e., $b_1^{e_1e_2} + b_2^{e_1e_2} + b_{12}^{e_1e_2} = 1$.[12]

We will consider both the *initial* and *final buyer partition*, as buyers choose platforms twice. Their initial choice is made before platforms decide on expansion, and without anticipating possible expansion. This results in the initial buyer partition - $B^{\bar{E}\bar{E}}$. Following expansion decisions, buyers may switch away from their initial subscription choices, which results in the final buyer partition - $B^{e_1e_2}$, where $e_1, e_2 \in \{\bar{E}, E\}$.

To illustrate the effect of platform expansion on the partition of buyers, we turn our attention to figure 1. The figure depicts the case of increased multihoming following platform expansion decisions. This is represented by the dashed ellipse, which shows the growth of the group of multihomers, at the expense of the groups of exclusive users of each platform. New services added by the platforms (when $e_i = E$) are enjoyed by their remaining exclusive users.

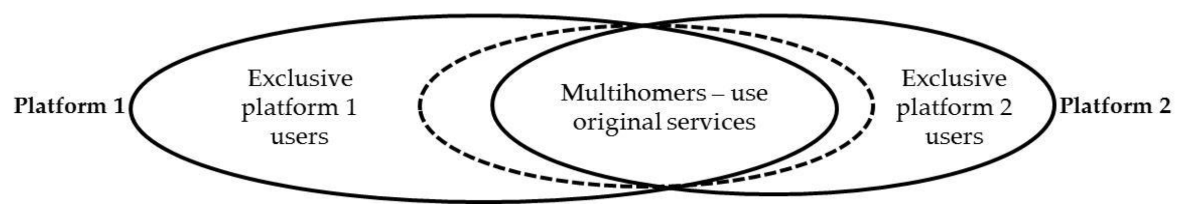

Figure 1: Endogenous buyer partition - the case of increased multihoming following expansion.

Buyers' ad click probability. The expected number of clicks generated by each group of buyers depends on its mass and click-through rate (henceforth, CTR). CTR is the probability that a user clicks on an ad, where ad exposure occurs via platform services. Let $\rho \in (0,1)$ denote users' ad click probability, or CTR, for ad exposure on one service. We assume that clicks are independent across services, such that a user exposed to an ad on two services will click exactly once with probability $2\rho(1-\rho)$, twice with probability ρ^2, and will not click at all with probability $(1-\rho)^2$. This represents the basic motivation for platform expansion – improving users'

[12] This assumption will allow us to gain traction in the analysis, with very little assumptions on the characteristics of the endogenous buyer partition. However, the assumption is not necessary for obtaining our main results.

overall click probability by increasing the number of contact points with the platform's user base.

3.3 Advertisers

There is a unit mass of homogeneous advertisers in the market. Advertisers' strategy is a choice of platform or platforms on which to place ads, denoted $\alpha \in A \equiv \{\{1\},\{2\},\{1,2\},\emptyset\}$.

The expected benefit of $\boldsymbol{\alpha}$. Advertisers benefit from *unique* user clicks on their ads, i.e., second clicks by users are considered redundant. This is represented by a constant value for a user's first ad click, normalized to 1, whereas the value of a user's second click is zero. We introduce the notation $\tilde{\rho} \equiv 2\rho - \rho^2$, which is the probability of *at least* one user click, when reaching the same user on two services. $\tilde{\rho}$ is thus the relevant click probability when considering the benefit of ad exposure on two services.

The expected benefit of α is the product of the relevant click probability and the mass of users reached, where platform i provides access to both its exclusive subscribers and multihoming users, who subscribe to one service from each platform. For $\alpha = \{i\}$, the expected benefit is simply $\rho_i b_i^{e_1 e_2} + \rho b_{12}^{e_1 e_2}$, where $\rho_i \equiv \rho_i(e_i)$ represents the probability of a unique click and depends on the platform's expansion decision, such that $\rho_i(\bar{E}) = \rho$ and $\rho_i(E) = \tilde{\rho}$. For $\alpha = \{1,2\}$ the expected benefit is $\rho_1 b_1^{e_1 e_2} + \rho_2 b_2^{e_1 e_2} + \tilde{\rho} b_{12}^{e_1 e_2}$, as multihomers are now reached through two services (and only unique clicks matter).

The expected cost of $\boldsymbol{\alpha}$. The expected cost is the amount charged by the platforms in α, which is, for each $i \in \alpha$, the product of price per click and the expected number of *clicks provided*. Considering the expected number of clicks provided by i, we assume that the platform perfectly tracks its exclusive users, and thus charges advertisers only for unique clicks by these users, $\rho_i b_i^{e_1 e_2}$. On the other hand, multihomers subscribe to only one service by the platform, and are not tracked outside of the platform. Each platform thus provides $\rho b_{12}^{e_1 e_2}$ expected clicks by these users.[13] Summarizing, each platform $i \in \alpha$ charges its advertisers a sum of $p_i\left[\rho_i b_i^{e_1 e_2} + \rho b_{12}^{e_1 e_2}\right]$.

[13] Multihoming advertisers are thus required to pay for redundant clicks. An alternative modelling choice is to assume that platforms identify multihoming users, and therefore the expected cost of their clicks is computed

The expected value of choice α is defined as the difference between the expected benefit and cost of α, and denoted $V^{\alpha} \equiv V\left(\alpha|(e_i, p_i)_{i=1,2}, B^{e_1e_2}\right)$:

(1) $V^{\alpha} =$

$$\begin{cases} (1-p_i)\left[\rho_i b_i^{e_1e_2} + \rho b_{12}^{e_1e_2}\right] & for\ \alpha = \{i\} \\ \left[\rho_1 b_1^{e_1e_2} + \rho_2 b_2^{e_1e_2} + \tilde{\rho} b_{12}^{e_1e_2}\right] - p_1\left[\rho_1 b_1^{e_1e_2} + \rho b_{12}^{e_1e_2}\right] - p_2\left[\rho_2 b_2^{e_1e_2} + \rho b_{12}^{e_1e_2}\right] & for\ \alpha = \{1,2\} \\ 0 & for\ \alpha = \emptyset \end{cases}$$

The important feature of V^{α} is that advertiser multihoming, or $\alpha = \{1,2\}$, entails some redundancy, as multihoming users are reached twice, and advertisers pay both platforms for their clicks. Specifically, when $\alpha = \{1,2\}$, access to multihoming users provides an expected benefit of $\tilde{\rho} b_{12}^{e_1e_2}$, at an expected cost of $(p_1 + p_2)\rho b_{12}^{e_1e_2}$, where $\rho < \tilde{\rho} < 2\rho$. Therefore, $V^{12} < V^1 + V^2$.

3.4 Platforms - profits

Each pair of platform expansion decisions (e_1, e_2) defines an expansion subgame, and determines $B^{e_1e_2}$ in the subgame. Platform i's expected profit in an expansion subgame (e_1, e_2), for price p_i, given the rival's price p_j, are denoted $\pi_i^{e_1e_2}(p_i|\, p_j)$, and given by:

$$(2)\ \pi_i^{e_1e_2}(p_i|\, p_j) = \begin{cases} p_i\left[\rho_i b_i^{e_1e_2} + \rho b_{12}^{e_1e_2}\right] & i \in \alpha \\ 0 & i \notin \alpha \end{cases}$$

Where the expected number of clicks, $\left[\rho_i b_i^{e_1e_2} + \rho b_{12}^{e_1e_2}\right]$, is comprised of $\rho_i b_i^{e_1e_2}$ clicks by exclusive subscribers, and $\rho b_{12}^{e_1e_2}$ clicks by multihomers, who use only the platform's core service, regardless of e_i.

3.5 Timeline

The timeline of the model is as follows:

1. Platforms make expansion decisions e_1 and e_2.

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according to a click probability of $0.5\tilde{\rho}$. Our main results continue to hold under this alternative assumption, as $V^{12} < V^1 + V^2$ continues to hold.

2. The final buyer partition $B^{e_1 e_2}$ is determined ($B^{e_1 e_2}$ is the equilibrium of the buyer model).
3. Platforms set prices per user click p_1 and p_2.
4. Advertisers choose the platform(s) on which they place their ads, α, and this determines platform profits.

3.6 Market equilibrium

We define market equilibrium for the simultaneous move expansion game, which represents an environment where platforms' development efforts are kept secret until new services are launched.

Definition 1: Market equilibrium is a couple $\langle \alpha^*, \ (e_i^*, p_i^*)_{i=1,2} \rangle$, such that:

1. Advertisers' platform choice is optimal, given platforms' expansion and pricing decisions, and the resulting $B^{e_1 e_2}$: $\alpha^* = argmax_{\alpha \in A} \{ V(\alpha | (e_i, p_i)_{i=1,2}, B^{e_1 e_2}) \}$.
2. Platform pricing is Nash equilibrium, given their expansion decisions, $B^{e_1 e_2}$, and α^*: $p_i^* = argmax \pi_i^{e_1 e_2} (p_i | p_j^*)$. Subgame equilibrium profits: $\Pi_i(e_1, e_2) \equiv \pi_i^{e_1 e_2} (p_i^* | p_j^*)$.
3. Platforms' expansion decisions are Nash equilibrium in the expansion game: $e_i^* = argmax \Pi_i (e_i, e_j^*)$.

The sequential move version of the platform expansion game is also considered, as it represents a market where platforms' development efforts are known. The sequential version is further used as a means of equilibrium selection, whenever multiple equilibria arise in the simultaneous move game.

The definition of market equilibrium for the sequential game will be similar, differing only in the equilibrium concept for optimal expansion decisions (item 3), which will be SGPE.

4. Analysis

We begin by deriving platform pricing and profits in a given expansion subgame, then proceed to derive optimal expansion rules, and characterize expansion equilibria.

4.1 Pricing Equilibrium

We show that equilibrium prices and profits are constrained by the degree of user multihoming in the final partition. The intuition for this result is as follows. In equilibrium, platforms set prices that induce advertiser multihoming (i.e., $\alpha^* = \{1,2\}$),[14] and internalize the resulting redundancy. Namely, multihoming advertisers suffer partial redundancy from reaching multihoming users through both platforms (recall that $V^{12} < V^1 + V^2$). Platforms internalize this redundancy by setting equilibrium prices according to the incremental benefit of advertising on an additional platform.[15]

Equilibrium prices thus decrease in the mass of multihomers, $b_{12}^{e_1 e_2}$, and increase in the mass of exclusive users, $b_i^{e_1 e_2}$. In other words, market power in the model stems from the degree of exclusivity, and decreases in the degree of multihoming.

The following proposition 1 provides a characterization of the pricing equilibrium, and the resulting platform profits, in a given subgame. (Throughout the proof, the superscript $e_1 e_2$ is omitted.)

Proposition 1: Given (e_1, e_2) and the resulting $B^{e_1 e_2}$, platform i sets its price at p_i^*, where:

$$(3)\ p_i^* = 1 - \frac{\rho^2 b_{12}^{e_1 e_2}}{\rho_i b_i^{e_1 e_2} + \rho b_{12}^{e_1 e_2}}$$

And its profits are given by:

$$(4)\ \pi_i^{e_1 e_2} = \rho_i b_i^{e_1 e_2} + \rho(1-\rho) b_{12}^{e_1 e_2}$$

Equilibrium prices decrease in the degree of multihoming, $b_{12}^{e_1 e_2}$, and increase in the degree of exclusivity, $b_i^{e_1 e_2}$.

Proof: Given (e_1, e_2) and B, advertisers place ads on both platforms whenever $V^{12} \geq V^1, V^2, 0$, and choose a single platform i whenever $V^i > V^{12}, V^j, 0$.[16] Solving $V^{12} \geq V^j$ we find that $\alpha^* = \{1,2\}$ whenever $p_i \leq p_i^*$. Furthermore, note that $V^i \geq V^j$ if and only if $p_i \leq \tilde{p}_i(p_j)$, where

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[14] Otherwise, there will be one platform with no advertisers and zero profits that will deviate to a lower price.

[15] This is in the spirit of the "principle of incremental pricing" (defined in Anderson et al. 2011).

[16] WLOG we assume that indifference between $\alpha = \{1,2\}$ and $\alpha = \{i\}$ is resolved in favor of $\alpha = \{1,2\}$.

$\tilde{p}_i(p_j) \equiv \frac{[\rho_i b_i - \rho_j b_j] + p_j[\rho_j b_j + \rho b_{12}]}{\rho_i b_i + \rho b_{12}}$. It is easily verified that $p_i^* = \tilde{p}_i(p_j^*)$, thus $V^1 = V^2 = V^{12} = \rho^2 b_{12}$ for $p_i = p_i^*$, $i = 1,2$.

We show that pricing at p_i^* is profit maximizing. First note that $i \in \alpha^*$ for all $p_i \leq p_i^*$, and the profit maximizing price in this region is clearly $p_i = p_i^*$. We now consider $p_i > p_i^*$. Pricing at $p_i > 1$ leads to $i \notin \alpha^*$ and zero profits, and is not profit maximizing. Therefore assume $p_i \in (p_i^*, 1)$: if $p_j \in (p_j^*, 1)$ then only one platform is chosen by advertisers - assume that i is chosen, i.e. $V^i \geq V^j$. This implies zero profits for j, and a profitable deviation to p_j^*. Alternatively, if $p_j \in (p_j^*, 1)$ and $p_j = p_j^*$ then $V^j > V^i$, thus $i \notin \alpha^*$ and i has a profitable deviation to $p_i = p_i^*$. We have thus shown that for any price $p_i \neq p_i^*$ there exists a profitable deviation to $p_i = p_i^*$. Nash equilibrium prices in a given subgame are thus $p_i = p_i^*$ for $i = 1,2$. Substituting for p_i^* in (2) yields the expression in (4)(4) for platforms' profits in subgame (e_1, e_2).

To see that p_i^* decreases in b_{12} and increases in b_i we examine the following first order derivatives:

$$(5)\ \frac{\partial p_i^*}{\partial b_{12}} = -\frac{\rho^2 \rho_i b_i}{(\rho_i b_i + \rho b_{12})^2} < 0$$

$$(6)\ \frac{\partial p_i^*}{\partial b_i} = \frac{\rho^2 \rho_i b_{12}}{(\rho_i b_i + \rho b_{12})^2} > 0$$

■

This interim result will play an important role in the analysis of platform expansion decisions.

4.2 Characterizing Expansion Equilibria

The next step of our analysis is the derivation of an optimal expansion rule based on the expected profits in each subgame, as given by expression (4). But first, we add notation for expansion effects on the buyer partition that will be useful down the road.

We introduce the notation $\Delta b_k^{e_i}|_{e_j} \equiv b_k^{Ee_j} - b_k^{\bar{E}e_j}$ for $k \in \{i, j, 12\}$, which represents the change in the mass of group k, resulting from i's expansion, given $e_j \in \{\bar{E}, E\}$. Platforms are assumed to exert symmetric effects when they expand, such that $\Delta b_k^{e_i}|_{\bar{E}} = \Delta b_k^{e_j}|_{\bar{E}}$ and $\Delta b_k^{e_i}|_E = \Delta b_k^{e_j}|_E$.

We henceforth focus on symmetric platforms, i.e. $b_1^{e_1e_2} = b_2^{e_1e_2}$ for $e_1 = e_2$, and symmetric expansion effects. Clearly, this may not be the case in most real world situations. But rather than restrict our analysis, this will demonstrate that platform asymmetry is not required to obtain non-trivial equilibrium outcomes (including equilibria with asymmetric expansion).

We proceed to derive an optimal expansion rule for platform i, given e_j. Solving ${\pi_i}^{Ee_j} \geq {\pi_i}^{\bar{E}e_j}$ (using (4)) yields the following condition:

$$(7)\ e_i|_{e_j} = E \iff \rho(1-\rho)b_i^{Ee_j} + \rho\Delta b_i^{e_i}|_{e_j} + \rho(1-\rho)\Delta b_{12}^{e_i}|_{e_j} \geq 0$$

The above condition represents the three effects of expansion: the CTR effect, and the quantity and price effects associated with changes in the user partition. Platform expansion decisions are thus based on weighing these effects against each other. Further discussion and intuition on these effects is hereby provided.

The CTR effect. The *CTR effect* refers to the increase in exclusive users' click probability, brought on by expansion. Specifically, expansion increases these users' click probability from ρ to $2\rho - \rho^2$ - an increase of $\rho(1-\rho)$ in click probability for the mass of $b_i^{Ee_j}$ exclusive users. The CTR effect is thus represented by the first term in (7), and is a positive effect, driving towards platform expansion. It immediately follows that when the user partition is exogenous, i.e. the partition does not change as a result of platform expansion, platforms will always expand and the equilibrium is (E, E). Lastly note that the CTR effect is non-monotonic in ρ, increasing in magnitude as ρ increases for $\rho < 0.5$, then decreasing in ρ for $\rho > 0.5$.

The quantity and price effects. The *quantity effect* is the *direct* effect of platform expansion - the change in user partition, which is endogenous in our setting. As expansion changes the mass of exclusive and multihoming users, it changes the expected number of clicks sold by the platform. The *price effect* of expansion is the *indirect* effect of the change in user partition, as changes in multihoming and exclusivity affect prices (see proposition 1). The quantity and price effects are jointly represented in the second and third term in (7).

It is important to note that changes in the mass of multihomers exert opposing quantity and price effects. Namely, an increase (decrease) in multihoming entails a positive (negative) quantity effect as the platform provides access to more (less) multihomers. At the same time, increased (decreased) multihoming leads to lower (higher) prices, implying a negative (positive) price effect.

Furthermore, our covered market assumption implies that changes in one group's mass will always be accompanied by corresponding changes to one or both of the other two user groups. Therefore the overall quantity and price effects of expansion will take into account changes in both exclusivity and multihoming.

We can now characterize platform expansion equilibria in the market. Our analysis will employ the following condition (8), obtained by arranging the inequality in (7) and substituting $b_i^{Ee_j} = b_i^{\bar{E}e_j} + \Delta b_i^{e_i}|_{e_j}$:

$$(8)\ e_i|_{e_j} = E \iff \frac{2-\rho}{1-\rho}\,\Delta b_i^{e_i}|_{e_j} + b_i^{\bar{E}e_j} + \Delta b_{12}^{e_i}|_{e_j} \geq 0$$

There are two main cases to consider, depending on the effect of a single platform's expansion on the level of multihoming in the market, these are: (1) $\Delta b_{12}^{e_i}|_{e_j} \leq 0$ and (2) $\Delta b_{12}^{e_i}|_{e_j} > 0$. Such increases or decreases in multihoming may be the result of endogenous compatibility costs incurred by multihoming users, as these are likely to be affected by platform expansion. The buyer model presented in section 5 includes endogenous compatibility costs as the driver of endogeneity of the user partition.

The analysis hereby proceeds considering expansion effects in general, allowing application of the general model to any framework of user preferences and choice. To gain traction in the analysis of the general model we consider expansion as providing an *advantage* over a non-expanded rival, at least weakly. This advantage implies that an expanded platform has a (weakly) larger mass of exclusive users than its rival, when the rival platform has not expanded. This is a resulting feature of the buyer model of section 5, taken here as an assumption. The previously mentioned assumptions - covered market and symmetry - will also play an important role in the upcoming analysis of optimal expansion strategies in our general setting.[17]

[17] These assumptions allow for the analysis of expansion strategies without specifying the buyer model. Given a specific buyer model these assumption are no longer needed, as the structure of endogeneity of $B^{e_1 e_2}$ follows from the model, and optimal expansion strategies are derived using condition (8).

The following Lemmas 1 and 2 consider optimal expansion strategies separately for the cases of decreased and increased multihoming following expansion, respectively. Lemma 1 states that whenever multihoming decreases with expansion, expansion is a dominant strategy. Intuitively, this result follows from the advantage associated with expansion, which implies that the decrease in multihoming is accompanied by an increase in exclusivity for the expanding platform (when the market is covered). In this case, the price effect is positive and large enough, such that the price effect and the always-positive CTR effect will always lead to expansion, even when the total quantity effect is negative (i.e., when the decrease in multihoming is larger than the increase in exclusivity).

Lemma 1: Expansion is a dominant strategy whenever it decreases multihoming, i.e., $\Delta b_{12}^{e_i}|_{e_j} \leq 0$ implies $e_i|_{e_j} \equiv E$.

Proof: $\Delta b_{12}^{e_i}|_{e_j} \leq 0$ and the covered market assumption imply that $\Delta b_i^{e_i}|_{e_j} + \Delta b_j^{e_i}|_{e_j} \geq 0$. Since an expanded platform has an advantage (at least weakly) over a non-expanded rival, $\Delta b_i^{e_i}|_{e_j} \geq \Delta b_j^{e_i}|_{e_j}$ and therefore it must be that $\Delta b_i^{e_i}|_{e_j} \geq 0$, whereas both $\Delta b_j^{e_i}|_{e_j} \geq 0$ and $\Delta b_j^{e_i}|_{e_j} < 0$ are possible. For $\Delta b_j^{e_i}|_{e_j} < 0$, a covered market implies $\Delta b_i^{e_i}|_{e_j} = \left|\Delta b_{12}^{e_i}|_{e_j}\right| + \left|\Delta b_j^{e_i}|_{e_j}\right|$, therefore $\Delta b_i^{e_i}|_{e_j} + \Delta b_{12}^{e_i}|_{e_j} \geq 0$ such that the inequality in (8) always holds. For $\Delta b_j^{e_i}|_{e_j} \geq 0$, a covered market implies $\Delta b_i^{e_i}|_{e_j} + \Delta b_j^{e_i}|_{e_j} = \left|\Delta b_{12}^{e_i}|_{e_j}\right|$. We substitute this into the LHS of the inequality in (8):

$$LHS = \frac{2-\rho}{1-\rho} \Delta b_i^{e_i}|_{e_j} + b_i^{\bar{E}e_j} - \left(\Delta b_i^{e_i}|_{e_j} + \Delta b_j^{e_i}|_{e_j}\right) = \frac{1}{1-\rho} \Delta b_i^{e_i}|_{e_j} + b_i^{\bar{E}e_j} - \Delta b_j^{e_i}|_{e_j} \geq 0.$$

The inequality always holds since $\Delta b_i^{e_i}|_{e_j} \geq \Delta b_j^{e_i}|_{e_j}$. We have thus shown that $e_i|_{e_j} \equiv E$ whenever $\Delta b_{12}^{e_i}|_{e_j} \leq 0$. ■

We turn to consider the case of increased multihoming following expansion. When both multihoming and exclusivity increase, the overall price effect may be either negative or positive, but the overall quantity effect is positive and large. The positive quantity and CTR effects dominate and platforms will optimally expand. On the other hand, when exclusivity decreases, the overall price effect is negative. The overall quantity effect is positive but small, as the decrease in exclusivity is smaller than the increase in multihoming, due to the expansion advantage. Hence, the expansion strategy will depend on the relative magnitudes of these and the CTR effect. Namely, when the CTR effect is relatively large (ρ is small), the quantity effect is large and the price effect small (when the decrease in exclusivity is small), then expansion is optimal, and otherwise equilibrium no-expansion may arise. This is summarized in Lemma 2.

Lemma 2: When i's expansion increases multihoming, i.e., $\Delta b_{12}^{e_i}|_{e_j} > 0$ -

1. If i's exclusivity increases then expansion is a dominant strategy: $\Delta b_i^{e_i}|_{e_j} \geq 0$ implies $e_i|_{e_j} \equiv E$.
2. If i's exclusivity decreases then expansion is optimal when the decrease in i's exclusivity is relatively small compared to j's, and the CTR is relatively small. Formally, for $\Delta b_{12}^{e_i}|_{e_j} > 0$ and $\Delta b_i^{e_i}|_{e_j} < 0$: $e_i|_{e_j} = E$ if and only if $\left|\Delta b_i^{e_i}|_{e_j}\right| \leq (1-\rho)\left[b_i^{\bar{E}e_j} - \Delta b_j^{e_i}|_{e_j}\right]$.

Proof: $\Delta b_{12}^{e_i}|_{e_j} > 0$ and the covered market assumption imply $\Delta b_i^{e_i}|_{e_j} + \Delta b_j^{e_i}|_{e_j} < 0$. Since an expanded platform enjoys a weak advantage over a non-expanded rival, we have $\Delta b_i^{e_i}|_{e_j} \geq \Delta b_j^{e_i}|_{e_j}$. It follows that $\Delta b_j^{e_i}|_{e_j} < 0$, whereas both $\Delta b_i^{e_i}|_{e_j} < 0$ and $\Delta b_i^{e_i}|_{e_j} \geq 0$ are possible. If $\Delta b_i^{e_i}|_{e_j} \geq 0$ then the inequality in (8) trivially holds.

Otherwise, if $\Delta b_i^{e_i}|_{e_j} < 0$, we substitute $\left|\Delta b_i^{e_i}|_{e_j}\right| + \left|\Delta b_j^{e_i}|_{e_j}\right| = \Delta b_{12}^{e_i}|_{e_j}$ into the LHS of the inequality in (8): $LHS = \frac{2-\rho}{1-\rho}\,\Delta b_i^{e_i}|_{e_j} + b_i^{\bar{E}e_j} - \Delta b_i^{e_i}|_{e_j} - \Delta b_j^{e_i}|_{e_j} = \frac{1}{1-\rho}\,\Delta b_i^{e_i}|_{e_j} + b_i^{\bar{E}e_j} - \Delta b_j^{e_i}|_{e_j}$, and therefore -

$$(9)\ e_i|_{e_j} = E \Leftrightarrow \left|\Delta b_i^{e_i}|_{e_j}\right| \leq (1-\rho)\left[b_i^{\bar{E}e_j} - \Delta b_j^{e_i}|_{e_j}\right]$$

■

Proposition 2 employs Lemmas 1 and 2 to provide a full characterization of expansion equilibria, when the buyer partition is endogenous, considering different forms of endogeneity. Expansion is said to *monotonically increase* multihoming when $b_{12}^{EE} > b_{12}^{E\bar{E}} > b_{12}^{\bar{E}\bar{E}}$ and to *monotonically decrease* multihoming when $b_{12}^{EE} < b_{12}^{E\bar{E}} < b_{12}^{\bar{E}\bar{E}}$ (recall that $b_{12}^{E\bar{E}} = b_{12}^{\bar{E}E}$).

Proposition 2: Expansion equilibria depend on the endogeneity of the buyer partition:

1. When expansion monotonically decreases multihoming the equilibrium is (E, E).
2. When expansion monotonically increases multihoming possible equilibria are: (E, E), $(\bar{E}, \bar{E})$, both $(\bar{E}, E)$ and $(E, \bar{E})$, or both (E, E) and $(\bar{E}, \bar{E})$.
3. When $b_{12}^{E\bar{E}} > b_{12}^{\bar{E}\bar{E}}$ and $b_{12}^{EE} < b_{12}^{E\bar{E}}$ the equilibrium is either (E, E) or both (E, E) and $(\bar{E}, \bar{E})$.
4. When $b_{12}^{E\bar{E}} < b_{12}^{\bar{E}\bar{E}}$ and $b_{12}^{EE} > b_{12}^{E\bar{E}}$ the equilibrium is either (E, E) or both $(\bar{E}, E)$ and $(E, \bar{E})$.

Proof: Follows immediately from Lemmas 1 and 2.■

We have thus shown that platforms may optimally choose not to expand, when expansion increases multihoming. This is because increased multihoming decreases market power, and increases price competition in our model. Interestingly, asymmetric expansion equilibria may arise even for symmetric platforms.

Whenever asymmetric equilibria arise in the simultaneous expansion game, the first mover will expand in the sequential game. This is because, in an asymmetric equilibrium, the expanded platform has a (weak) advantage and thus higher profits. When both (E, E) and $(\bar{E}, \bar{E})$ are equilibria in the simultaneous game, the SGPE of the sequential game is derived by comparing π_i^{EE} and $\pi_i^{\bar{E}\bar{E}}$ and will thus depend on the relative magnitudes of the effects of expansion.

Existence of the different equilibria mentioned in proposition 2 is shown in section 6, as the buyer model is analyzed for different cases of endogenous compatibility costs (incurred by multihomers), and the expansion equilibrium is derived for several numerical examples.

We conclude by considering the effects of changes in parameters, when expansion increases multihoming and decreases exclusivity. In this case, the expansion strategy depends on the relative magnitudes of expansion effects, and therefore parameter changes may result in a shift from equilibrium expansion to no-expansion. This is stated in the following corollary, which follows directly from Lemma 2.

Corollary 1: When expansion increases multihoming and decreases exclusivity for the expanded platform, the following changes in parameters may result in a switch from optimal expansion to no-expansion:

- Increases in the CTR, ρ.
- Increases in the expansion effect on own mass, $\left|\Delta b_i^{e_i}|_{e_j}\right|$.
- Decreases in the expansion effect on the rival's mass, $\left|\Delta b_j^{e_i}|_{e_j}\right|$.
- Decreases in the pre-expansion mass $b_i^{\bar{E}e_j}$.

As expected, increases in CTR lower the magnitude of the CTR effect when $\rho > 0.5$, and may therefore result in a switch to no-expansion. Such a switch may also result from larger decreases in own mass following expansion, as they decrease the magnitude of the positive quantity effect, and further strengthen the already-negative price effect. Furthermore, when the rival suffers a smaller decrease following expansion, then the expansion advantage is weakened, and more exclusive users are lost by the expanding platform, for a given increase in multihoming. Therefore a smaller user loss for the rival may result in a switch to no-expansion. Lastly, a smaller mass prior to expansion increases the relative magnitude of the effects associated with loss of exclusive users, such that decreases in the pre-expansion mass may lead to optimal no-expansion.

5. The buyer model

In this section we propose a buyer preference and choice model that yields the property of endogenous partition, central in our setting. This section thus provides a possible foundation for $B^{e_1e_2}$, and a test case for our general model.

We consider users characterized by preferences for platform service quality, who incur a compatibility cost when multihoming, and a switching cost when switching away from their initial platform choice following expansion decisions. The compatibility cost varies across the user population, and its distribution may change with platform expansion. The details of the buyer model are hereby provided.

Platform $i \in \{1,2\}$ offers service(s) of total quality q_i. At the outset each platform offers one service of quality q, such that $q_i = q$, and expansion implies adding a second service of quality $\Delta q \in (0, q)$. Therefore, $q_i = q$ for $e_i = \bar{E}$, and $q_i = q + \Delta q$ for $e_i = E$. Platforms' initial or *core* services are thus of higher quality than newly added services. Multihoming users subscribe to platforms' core services, thus enjoying an aggregate quality of $2q$.

Multihomers incur heterogeneous and endogenously determined compatibility costs, such that the compatibility cost for user $b \in B$ is $c_b^{e_1 e_2} \sim F^{e_1 e_2}\left[c_L^{e_1 e_2}, c_H^{e_1 e_2}\right]$, where $F^{e_1 e_2}$ is the PDF of compatibility costs. Note that both $F^{e_1 e_2}$ and its domain may change with platform expansion. This represents possible changes in platform compatibility resulting from the addition of the rival's core service, where compatibility may either increase or decrease with expansion, and the change in compatibility may vary across users.

Utility for user $b \in B$ is written as:

$$(10) \qquad \begin{aligned} u_i^b &= q_i \\ u_{12}^b &= 2q - c_b^{e_1 e_2} \end{aligned}$$

Where u_i^b is b's utility from subscription to a single platform i and u_{12}^b is b's utility from multihoming.

Users do not anticipate platform expansion, and therefore choose platforms twice. They make an initial subscription choice before expansion decisions are made, and may then switch to another choice following platform expansion. Switching to a new choice is costly, such that the utility for a switching user $b \in B$ from a new choice $i_{new} \in \{1,2,12\}$ is $u_{i_{new}}^b - s$, where $u_{i_{new}}^b$ is defined in (10) and $s \geq 0$ is the switching cost incurred.

To summarize, a user's initial platform choice is made given his individual realization of $c_b^{\bar{E}\bar{E}}$, and his final platform choice (following expansion decisions) is made given his realization of $c_b^{e_1 e_2}$ and the switching cost s.

Users' platform choices for each pair (e_1, e_2) are made so as to maximize utility, as defined above. We make two tie-breaking assumptions. First, we assume that indifference between the two platforms is resolved by a fair coin flip, i.e. users indifferent between platform 1 and 2 will choose each platform with probability 0.5. This represents an idiosyncratic platform preference, when platforms offer exactly the same quality level. Second, we assume that indifference between multihoming and choice of a single platform is resolved in favor of multihoming.

We proceed to define buyer equilibrium.

Definition 2: *Buyer equilibrium* is a choice of platform(s) for each buyer $b \in B$ given (e_1, e_2), such that each buyer's choice is utility maximizing, given $c_b^{e_1 e_2}$ and s.

A partition $B^{e_1 e_2}$ thus constitutes buyer equilibrium given (e_1, e_2). Endogeneity of $B^{e_1 e_2}$ is the result of the endogeneity of users' compatibility costs, which, in turn, represents endogenous inter-platform compatibility levels.

6. Applying the general framework: endogenous compatibility costs

In this section we use the proposed buyer model to provide a test case with numerical examples and further intuition for the general model. We therefore solve for buyer equilibrium before and after platform expansion, under a certain specification for $F^{e_1 e_2}$, chosen for its simplicity and realism.

For the following analysis we assume that $F^{\bar{E}\bar{E}}$ is a uniform distribution over $[0,1]$, and that it shifts either left or right with expansion decisions. Specifically, we consider users who draw a compatibility cost once at the outset from $F^{\bar{E}\bar{E}}$, and may then experience an increase or decrease in this cost, which depends on the pair of expansion decisions. All users experience the same increase or decrease in compatibility cost, such that each user maintains the same relative cost compared to his peers. Our symmetry assumption implies that $F^{E\bar{E}} = F^{\bar{E}E}$. For ease of notation let $n \in \{1,2\}$ denote the number of expanded platforms, and let $F^n \sim U[\Delta c^n, 1 + \Delta c^n]$ be the distribution of compatibility costs when n platforms expand, where Δc^n is either a positive or negative constant.

Next, solving for $B^{\bar{E}\bar{E}}$, group masses in the initial user partition are $b_1^{\bar{E}\bar{E}} = b_2^{\bar{E}\bar{E}} = \frac{1-q}{2}$ and $b_{12}^{\bar{E}\bar{E}} = q$.

When a single platform expands (WLOG, let $e_1 = E, e_2 = \bar{E}$) $c_b^{E\bar{E}} \sim U[\Delta c^1, 1 + \Delta c^1]$. If $\Delta c^1 > 0$ and s is not too high, initial multihomers with relatively high compatibility costs will switch to the expanded platform. On the other hand, if $\Delta c^1 < 0$ and s is not too high, initial subscribers of both platforms with a relatively low compatibility cost will switch to multihoming. For both $\Delta c^1 > 0$ and $\Delta c^1 < 0$, users from the non-expanded platform 2 with a high compatibility cost will switch to the expanded platform whenever $s \le \Delta q$, and will otherwise remain captive users of platform 2. Solving for $B^{E\bar{E}}$, we focus on the following cases, for $|\Delta c^1| > \Delta q$:[18]

(a) For $\Delta c^1 > 0$:

- For $s \le \Delta q$, users with $c_b^{E\bar{E}} \in (q - \Delta q + s, q + \Delta c^1]$ will switch from multihoming to the expanded platform 1, and all users of platform 2 will switch to 1.
- For $s \in (\Delta q, \Delta q + \Delta c^1]$, users with $c_b^{E\bar{E}} \in (q - \Delta q + s, q + \Delta c^1]$ will switch from multihoming to the expanded platform 1, and all users of platform 2 remain captive.

(b) For $\Delta c^1 < 0$, $s \le \min\{\Delta q, |\Delta c^1| - \Delta q\}$: initial platform 1 subscribers with $c_b^{E\bar{E}} \in (q - |\Delta c^1|, q - \Delta q - s]$ and initial platform 2 subscribers with $c_b^{E\bar{E}} \in (q - |\Delta c^1|, q - \Delta q]$ will switch to multihoming; the remaining platform 2 users will switch to platform 1.

When both platforms expand $c_b^{EE} \sim U[\Delta c^2, 1 + \Delta c^2]$, and there is symmetric switching away from multihoming when $\Delta c^2 > 0$ and towards multihoming when $\Delta c^2 < 0$. Solving for B^{EE}, we focus on the case of $s \le \min\{\Delta q, |\Delta c^2| - \Delta q\}$, for $|\Delta c^2| > \Delta q$:

(a) For $\Delta c^2 > 0$: users with $c_b^{EE} \in (q - \Delta q + s, q + \Delta c^2]$ will switch from multihoming to the expanded platforms, dividing equally between the two.

(b) For $\Delta c^2 < 0$: initial subscribers of platforms 1 and 2 for whom $c_b^{EE} \in (q - |\Delta c^2|, q - \Delta q - s]$ will switch from the expanded platforms to multihoming.

The resulting $B^{E\bar{E}}$ and B^{EE} are summarized in the following table 1 (for $|\Delta c^n| > \Delta q$ and $s \le \min\{\Delta q, |\Delta c^n| - \Delta q\}$, unless otherwise stated):

	$B^{E\bar{E}}$ $(n = 1)$	B^{EE} $(n = 2)$
(a) $\Delta c^n > 0$	$b_1^{E\bar{E}} = 1 - q + \Delta q + \Delta c^1 - s$ $b_2^{E\bar{E}} = 0$	$b_1^{EE} = b_2^{EE} = \frac{1-q+\Delta q+\Delta c^2-s}{2}$ $b_{12}^{EE} = q - \Delta q - \Delta c^2 + s$

[18] We do not conduct a full analysis of the buyer model for all possible parameter values, as our goal is to provide examples for the cases discussed in proposition 2.

	$b_{12}^{E\bar{E}} = q - \Delta q - \Delta c^1 + s$ For $s \in (\Delta q, \Delta q + \Delta c^1]$: $b_1^{E\bar{E}} = \frac{1-q}{2} + \Delta q + \Delta c^1 - s$ $b_2^{E\bar{E}} = \frac{1-q}{2}$ $b_{12}^{E\bar{E}} = q - \Delta q - \Delta c^1 + s$	
(b) $\Delta c^n < 0$	$b_1^{E\bar{E}} = 1 - q + \Delta q - \lvert\Delta c^1\rvert + 0.5s$ $b_2^{E\bar{E}} = 0$ $b_{12}^{E\bar{E}} = q - \Delta q + \lvert\Delta c^1\rvert - 0.5s$	$b_1^{EE} = b_2^{EE} = \frac{1-q+\Delta q-\lvert\Delta c^2\rvert+s}{2}$ $b_{12}^{EE} = q - \Delta q + \lvert\Delta c^2\rvert - s$

Table 1: Buyer partitions $B^{E\bar{E}}$ and B^{EE} for $|\Delta c^n| > \Delta q$ and $s \leq \min\{\Delta q, |\Delta c^n| - \Delta q\}$ (unless otherwise stated).

We now provide examples for the equilibrium outcomes discussed in proposition 2. The following subsections 6.1-6.4 correspond to cases 1-4 of proposition 2. For cases 1-4 we use $s \leq \min\{\Delta q, |\Delta c^n| - \Delta q\}$ and for case 4 we additionally consider $s \in (\Delta q, \Delta q + \Delta c^1]$.

6.1 Expansion monotonically decreases multihoming

Our buyer model has monotonically decreasing multihoming for $\Delta c^2 > \Delta c^1 > 0$. Using $B^{E\bar{E}}, B^{EE}$ from the above table in condition (8), we find that the inequality always holds for $e_j \in \{\bar{E}, E\}$ and therefore expansion is a dominant strategy. This also follows from Lemma 1. The equilibrium is therefore (E, E).

6.2 Expansion monotonically increases multihoming

The case of monotonically increasing multihoming corresponds to $\Delta c^1, \Delta c^2 < 0$ and $|\Delta c^2| - |\Delta c^1| > 0.5s$ in the buyer model. We use the above $B^{E\bar{E}}, B^{EE}$, and condition (8) to find:

- $e_i|_{\bar{E}} = E$ whenever $\rho \leq \rho_{\bar{E}}$, where $\rho_{\bar{E}} = \frac{3}{2} + \frac{\Delta q - |\Delta c^1| + \frac{1}{2}s}{1-q}$.
- $e_i|_E = E$ whenever $\rho \leq \rho_E$, where $\rho_E = \frac{2(1-q+\Delta q-|\Delta c^1|)+s}{1-q+\Delta q+|\Delta c^2|-2|\Delta c^1|}$.

Note that the thresholds $\rho_{\bar{E}}, \rho_E$ may be smaller than 1. Specifically, $\rho_{\bar{E}} < 1$ for $|\Delta c^1| > \frac{1-q+2\Delta q+s}{2}$, and $\rho_E < 1$ for $|\Delta c^2| > 1 - q + \Delta q + s$. We therefore show existence of the different equilibrium types mentioned in proposition 4, using a numerical example.

Numerical example: For parameter values $|\Delta c^1| = 0.2, |\Delta c^2| = 0.25, \Delta q = 0.1$ and $s = 0$, the relative size of the expansion thresholds $\rho_{\bar{E}}$ and ρ_E depends on q: (a) For $q = 0.89$: $\rho_{\bar{E}} = 0.59$ and $\rho_E = 0.33$, such that $\rho_E < \rho_{\bar{E}}$ and the equilibrium is (E, E) for $\rho \le \rho_E$, both $(E, \bar{E})$ and $(\bar{E}, E)$ for $\rho \in (\rho_E, \rho_{\bar{E}}]$, and $(\bar{E}, \bar{E})$ for $\rho > \rho_{\bar{E}}$ (see panel (a) of figure 2); (b) For $q = 0.83$: $\rho_{\bar{E}} = 0.91$ and $\rho_E > 1$, such that $\rho_{\bar{E}} < \rho_E$ and the equilibrium is (E, E) for $\rho \le \rho_{\bar{E}}$, and both $(\bar{E}, \bar{E})$ and (E, E) for $\rho \in (\rho_{\bar{E}}, 1]$ (see panel (b) of figure 2).

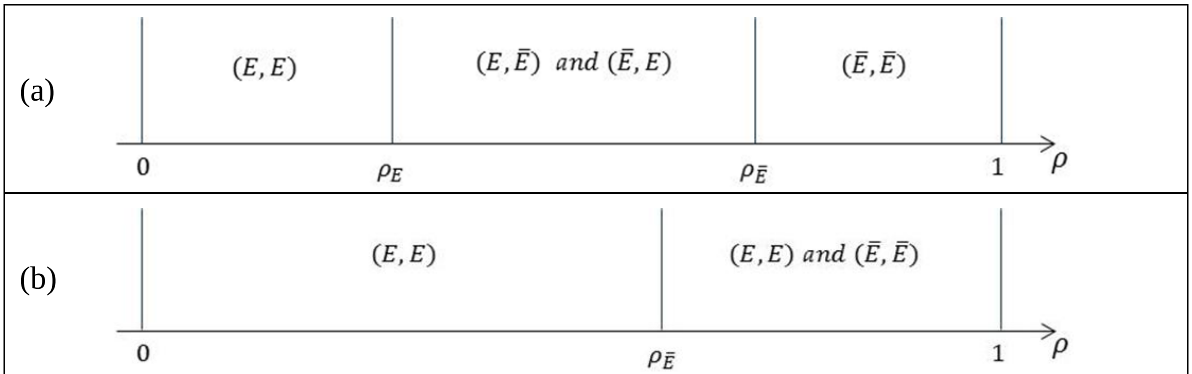

Figure 2. Equilibrium characterization for $|\Delta c^1| = 0.2, |\Delta c^2| = 0.25, \Delta q = 0.1, s = 0$: (a) $q = 0.89$ and (b) $q = 0.83$.

6.3 Expansion first increases and then decreases multihoming

This case corresponds to either $\Delta c^1, \Delta c^2 < 0$ and $|\Delta c^2| - |\Delta c^1| < 0.5s$, or to $\Delta c^1 < 0$ and $\Delta c^2 > 0$ in the buyer model. To show existence of the types of equilibrium discussed in proposition 2, we focus on the case of $\Delta c^1 < 0$ and $\Delta c^2 > 0$. Using $B^{E\bar{E}}, B^{EE}$, and condition (8) we find:

- $e_i|_{\bar{E}} = E$ whenever $\rho \le \rho_{\bar{E}}$, where $\rho_{\bar{E}}$ is defined in 6.2 above.
- $e_i|_E \equiv E$, as the inequality in (8) always holds for $e_j = E$ (also follows from Lemma 1).

Recall that $\rho_{\bar{E}} < 1$ for $|\Delta c^1| > \frac{1-q+2\Delta q+s}{2}$. Once again, a numerical example is provided to show existence of the equilibria discussed in proposition 2.

Numerical example: For parameter values $|\Delta c^1| = 0.35, \Delta c^2 = 0.2, \Delta q = 0.1,\ \ s = 0,$ and $q = 0.8$, the expansion threshold is $\rho_{\bar{E}} = 0.25$, such that the equilibrium is (E, E) for $\rho \leq \rho_{\bar{E}}$, and both $(\bar{E}, \bar{E})$ and (E, E) for $\rho \in (\rho_{\bar{E}}, 1]$ (the illustration is similar to the one in panel (b) of figure 2).

6.4 Expansion first decreases and then increases multihoming

This case corresponds to either $\Delta c^1, \Delta c^2 > 0$ and $\Delta c^1 > \Delta c^2$, or to $\Delta c^1 > 0$ and $\Delta c^2 < 0$ in the buyer model. To show existence of the relevant equilibrium types discussed in proposition 2, we focus on the case of $\Delta c^1 > 0$ and $\Delta c^2 < 0$, with $\Delta q < s < \Delta q + \Delta c^1$. Using $B^{E\bar{E}}, B^{EE}$, and condition (8) we find:

- $e_i|_{\bar{E}} \equiv E$, as the inequality in (8) always holds for $e_j = \bar{E}$ (also follows from Lemma 1).
- $e_i|_E = E$ whenever $\rho \leq \rho_E$, where $\rho_E = \frac{1-q+2\Delta q+2\Delta c^1-2s}{1-q+\Delta q+|\Delta c^2|+2\Delta c^1-3s}$.

Note that $\rho_E < 1$, since we assumed $s < |\Delta c^2| - \Delta q$. The equilibrium is therefore (E, E) for $\rho \leq \rho_E$, and both $(E, \bar{E})$ and $(\bar{E}, E)$ are equilibrium for $\rho \in (\rho_E, 1]$ (see figure 3).

Numerical example: For parameter values $\Delta c^1 = 0.1, |\Delta c^2| = 0.3, \Delta q = 0.1,\ \ s = 0.15,$ and $q = 0.1$, we have $\rho_E \approx 0.95$, and ρ_E decreases in q.

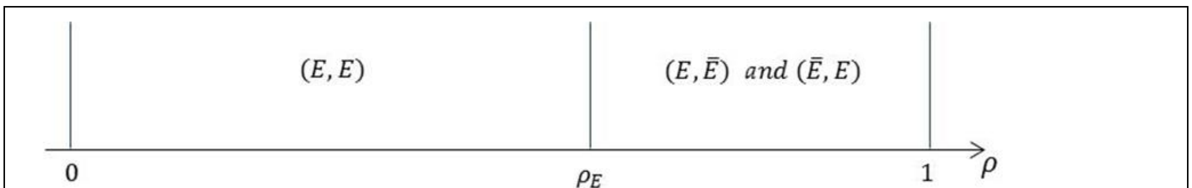

Figure 3. Equilibrium characterization for $\Delta c^1 = 0.1, |\Delta c^2| = 0.3, \Delta q = 0.1, s = 0.15,$ $q = 0.1$.

7. Discussion and managerial implications

We have presented a game theoretic framework for analysis of online platforms' expansion decisions, when the buyer partition is endogenous and thus responds to expansion. The analysis demonstrates that expansion may not be optimal when it increases the degree of user multihoming, as increased multihoming lowers platforms' market power.

When will expansion increase user multihoming? Different mechanisms or buyer models may be used to consider possible expansion effects on buyers' platform choice and multihoming behavior. The proposed buyer model suggests changes in inter-platform compatibility levels as the underlying cause for endogeneity of the buyer partition. Other mechanisms may consider possible effects of changes in buyers' choice sets, or expansion effects on users' perceptions of platform identity and service differentiation. The general framework may be further adapted to consider expansion into services that are substitutes or complements to the rival's service offering, as these are also expected to affect the partition of users and their multihoming behavior.

Our general framework is thus amenable to different market settings, and may be applied by managers even without fully specifying a buyer model, given forecasts on the partition of users following expansion. Whenever multihoming may increase – caution is advised, as expansion may decrease, rather than increase, profits.

Returning to the example of Google Plus - Does the model imply that Google should not have expanded into social networking? The short answer is "no". While not succeeding in stealing away (most) Facebook users, Google Plus did provide the platform with valuable social network data used to increase ad targetability. It seems that, for Google, the benefit of improved targeting was much larger than any effects of changes in the user partition.

This may not be the case for many platforms currently expanding into content streaming services. It is plausible that expansion into a video streaming service results in an "increased appetite" for streamed content, thereby increasing users' multihoming with competitors. For example, Amazon users who begin using Amazon Instant Video may increase their use of Netflix, Hulu, and other competitors. Expansion into content streaming should therefore take into account such strategic effects.

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