A More General Theory of Commodity Bundling*

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July 2012

Abstract

This paper discusses the incentive to bundle when consumer valuations are non-additive and/or when products are supplied by separate sellers. Whether integrated or separate, firms have an incentive to introduce a bundle discount when demand for the bundle is elastic relative to demand for stand-alone products. A simple formula is derived for when an integrated firm supplying substitute products wishes to offer a discount. When separate sellers coordinate on a bundle discount, they can use the discount to relax competition, which can harm welfare.

Keywords: price discrimination, bundling, discrete choice, oligopoly, common agency.

JEL codes: D11, D43, D82, D86, L13, L41.

1 Introduction

Bundling—the practice whereby consumers are offered a discount if they buy several distinct products—is widely used by firms, and is the focus of a rich economic literature. However, most of the existing literature discusses the phenomenon under relatively restrictive assumptions, namely a consumer's valuation for a bundle of several products is the sum of her valuations for consuming the items in isolation, and bundle discounts are only offered for products sold by the same firm. The two assumptions are related, in that when valuations are additive it is less likely that a firm would wish to reduce its price to a customer who also buys a product from another seller. This paper analyzes the incentive to engage in bundling when these assumptions are relaxed.

There are very many situations in which it is useful to model products as substitutes or complements. For instance, when visiting a city a tourist may gain some extra utility from visiting art gallery A if she has already visited art gallery B, but the incremental utility is likely to be smaller than if she were only to visit A. Joint purchase discounts (or premia) on products offered by separate sellers are rarer, though some examples include:

— A tourist may be able to buy a "city pass", so that she can visit all participating tourist attractions at a discount on the sum of individual entry fees. These could be organized either as a joint venture by the attractions themselves, or implemented by an intermediary which assembles its own bundles.

— Inter-firm bundling is prevalent in markets for transport services, as is the case with alliances between airlines or when neighboring ski-lifts offer a combined ticket.

— Products supplied by separately-owned firms are often marketed together with discounts for joint purchase. Thus, supermarkets and gasoline stations sometimes cooperate to offer a discount when both services are consumed, as do airlines and car rental firms.

— An independent academic journal might choose one subscription price when it is sold on a stand-alone basis to a library, and a lower price when it is sold as part of a "collection" alongside other journals.

— Pharmaceuticals are sometimes used as part of a "cocktail" with one or more drugs supplied by other firms. Drug companies can set different prices depending on whether the drug is used on a stand-alone basis or in a cocktail.

— Marketing data may reveal useful information about a potential customer's purchase history which can be used to determine a firm's price to the customer. For instance, information that the customer has chosen to buy firm 1's product may induce firm 2 to discount its own price.

— At a wholesale level, a manufacturer may offer a retailer a discount if the retailer does not stock a rival manufacturer's product. (Such contracts are sometimes termed "loyalty contracts".) This is a situation with a bundle premium instead of a discount.

The plan of the paper is as follows. In section 2, I present a general framework for consumer demand for two products which allows for non-additive valuations and the use of bundle discounts. Section 3 covers the case where an integrated firm supplies both products. I revisit the approach to bundling presented by John Long [10], which is used as a major ingredient for the analysis in section 3. Long's result is that the firm has an incentive to bundle when demand for the bundle is more elastic than demand for stand-alone products. Applying Long's general formula to the case of substitute products yields a simply formula governing when the firm wishes to introduce a bundle. Relative to the

situation with additive preferences, the integrated firm typically has a greater incentive to offer a bundle discount when products are substitutable. Because the purchase of one product can decrease a consumer's incremental utility from a second, the firm has a direct incentive to reduce the price for a second item, in addition to the rent-extraction motive for bundling familiar from the existing literature.

In section 4 I turn to the situation where products are supplied by separate sellers. With additive preferences, a firm has a unilateral incentive to offer a bundle discount when product valuations are negatively correlated. When there is full market coverage, so all consumers buy something, a firm has an incentive to offer a joint-purchase discount in a wide range of circumstances. When products are substitutes, whether a firm has a unilateral incentive to introduce a discount depends on the way that preferences are modelled. When there is a constant disutility of joint consumption, separate sellers typically wish to offer a joint-purchase discount: the fact that a customer has purchased the rival product implies that her incremental valuation for the firm's own item has fallen, and this usually implies that the firm would like to reduce its price to this customer. Alternatively, if a proportion of buyers only want a single item (for instance, a tourist in a city might only have time to visit a single museum) while other consumers have additive preferences, a seller would like, if feasible, to charge a premium when a customer also buys the rival product.

Finally, section 5 investigates coordinated discounts offered by separate sellers, which is currently the relevant case for several of the industries mentioned above. Specifically, I suppose that firms first agree on a bundle discount which they fund jointly, and subsequently choose prices without coordination. When valuations are additive, such a scheme will usually raise each firm's profit, and, at least in the example considered, its operation will also boost total welfare. However, when sellers offer substitute products, the negotiated discount overturns the innate substitutability of products, inducing firms to raise prices. The resulting "tariff-mediated" product complementarity can induce collusion which harms consumers and overall welfare.

This paper is not the first to investigate these issues. The incentive for an integrated seller to offer a discount for the purchase of multiple items is discussed by [1,10,12], among many others. [10,12] showed it is optimal to introduce a bundle discount whenever the distribution of valuations is statistically independent and valuations are additive, so that a degree of joint pricing is optimal even with entirely unrelated products. Except for

Long [10], these papers assume that valuations are additive.[1] While most analyses of bundling follow a diagrammatic exposition concentrating on the details of joint distribution of individual valuations, Long's analysis uses standard aggregate demand functions based on linear prices to determine when the firm wishes to offer a bundle discount. As well as arguably being more transparent, the beauty of Long's approach is that it applies equally to situations with non-additive preferences. Long's method is a precursor to the "demand profile" approach to nonlinear pricing emphasised in Wilson [17]. While Wilson shows how aggregate demand functions based on linear prices can often be used to derive optimal nonlinear tariffs when heterogeneity is captured by a scalar variable, he notes [17, section 4.3] that in richer models (such as the current bundling context) the use of aggregate demand functions cannot accurately generate optimal nonlinear tariffs. However, as shown by Long, these demand functions nevertheless determine the *incentive* to offer a bundle discount.

[9,14] study the incentive for a single-product monopolist to offer a discount if its customers also purchase a competitively-supplied product. Schmalensee supposes that two items are for sale to a population of consumers, and item 1 is available at marginal cost due to competitive pressure while item 2 is supplied by a monopolist. Valuations are additive, but are not independent in the statistical sense. If there is negative correlation in the values for the two items, the fact that a consumer buys item 1 is "bad news" for the monopolist, who then has an incentive to set a lower price to its customers who also buy item 1. Lewbel performs a similar exercise but allows the two items to be partial substitutes. In this case, the fact that a consumer buys item 1 is also bad news for the monopolist, and gives an incentive to offer a discount for joint consumption.

Calzolari and Denicolo [4] propose a model where consumers buy two products and each product is supplied by a single firm. Each firm potentially offers a nonlinear tariff which depends on a buyer's consumption of its own product and her consumption of the other firm's product. They find that the use of these tariffs can harm consumers compared to the situation in which firms base their tariff only on their own supply. Their model differs in two ways from the one presented in section 4 of this paper. First, in their model consumers have elastic demands, rather than unit demands, for the two products. Thus, they consider general nonlinear tariffs, while a firm in my model merely chooses a pair of

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[1] [16] analyzes an integrated firm's incentive to engage in bundling when products are either complements or substitutes. The analysis is carried out using a specific uniform example, and a consumer's valuation for the bundle is some constant proportion (greater or less than one, depending on whether complements or substitutes are present) of the sum of her stand-alone valuations. The focus of the analysis in [16] is on whether pure bundling is superior to linear pricing.

prices. Second, in my model consumers differ in a non-scalar way, and a consumer might like product 1 but not product 2, and can vary in the degree of substitutability between products. In [4], consumers differ by only a scalar parameter, and so all consumers view the two products when consumed alone as perfect substitutes.

Lucarelli *et al.* [11] discuss the case of pharmaceutical cocktails. Although the focus of their analysis is on situations in which firms set the same price for a drug, regardless of whether it is used in isolation or as part of a cocktail, they also consider situations where firms can set two different prices for the two kinds of uses. They document how a firm selling treatments for HIV/AIDS set different prices for similar chemicals depending on whether the drug was part of a cocktail or not. They estimate a demand system for colorectal cancer drugs, where there are at least 12 major drug treatments, 6 of which were cocktails combining drugs from different firms. Although in this particular market firms do not price drugs differently depending on whether the drug is used in a cocktail, they estimate the impact when one firm does engage in this form of price discrimination. They find that a firm will typically (but not always) reduce the price for stand-alone use and raise the price for bundled use.

Finally, negotiated bundling arrangements between separate firms are analyzed by Gans and King [7], who investigate a model with two kinds of products (gasoline and food, say), where each product is supplied by two differentiated firms. When all four products are supplied by separate firms which set their prices independently, there is no interaction between the two kinds of product. However, two firms (one offering each of the two kinds of product) can enter into an alliance and agree to offer consumers a discount if they buy both products from the alliance. (In their model, the joint pricing mechanism is similar to that used in section 5 below: firms decide on their bundle discount, which they agree to fund equally, and then set prices non-cooperatively.) Gans and King observe that when a bundle discount is offered for joint purchase of otherwise independent products, those products are converted into complements. In their model, in which consumer tastes are uniformly distributed, a pair of firms does have an incentive to enter into such an alliance, but when both pairs do this their equilibrium profits are unchanged from the situation when all four firms set independent prices, although welfare and consumer surplus fall.[2]

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[2][3] modifies this model so that rival suppliers of the same products are vertically rather than horizontally differentiated. They find that when two pairs of firms form an alliance all prices rise relative to the situation when all four products are marketed independently. This result resembles the analysis in section 5 below, where an agreed bundle discount can induce a form of collusion.

2 A Framework for Consumer Demand

Consider a market with two products, labeled 1 and 2, where a consumer buys either zero or one unit of each product (and maybe one unit of each). A consumer is willing to pay v_i for product $i = 1, 2$ on its own, and to pay v_b for the bundle of both products. (A consumer obtains payoff zero if she consumes neither product.) Thus a consumer's preferences are described by the vector (v_1, v_2, v_b), which varies across the population of consumers according to some known distribution.[3] A consumer's valuations are additive if $v_b = v_1 + v_2$, while she views the two products as partial substitutes when $v_b \leq v_1 + v_2$ and as partial complements if $v_b \geq v_1 + v_2$. Whenever there is free disposal, so that a consumer can discard an item without cost, we require that $v_b \geq \max\{v_1, v_2\}$ for all consumers.

Only deterministic selling procedures are considered in this paper.[4] Consumers face three prices: p_1 is the price for consuming product 1 on its own, p_2 is the price for product 2 on its own, and $p_1 + p_2 - \delta$ is the price for consuming the bundle of both products. Thus, δ is the discount for buying both products, which is zero if there is linear pricing or negative if consumers are charged a premium for joint consumption. A consumer chooses the option from the four discrete choices which leaves her with the highest surplus, so she will buy both items whenever $v_b - (p_1 + p_2 - \delta) \geq \max\{v_1 - p_1, v_2 - p_2, 0\}$, she will buy product $i = 1, 2$ on its own whenever $v_i - p_i \geq \max\{v_b - (p_1 + p_2 - \delta), v_j - p_j, 0\}$, and otherwise she buys nothing.

As functions of the three tariff parameters (p_1, p_2, δ), denote by Q_i the proportion of potential consumers who buy product $i = 1, 2$ (either on its own or as part of the bundle), and denote by Q_b the proportion who buy both products. It will also be useful to discuss demand when no discount is offered, so let $q_i(p_1, p_2) \equiv Q_i(p_1, p_2, 0)$ and $q_b(p_1, p_2) \equiv Q_b(p_1, p_2, 0)$ be the corresponding demand functions when $\delta = 0$. We will see that a firm's incentive to introduce a bundle discount is determined entirely by the properties of the "no-discount" demands. This is important insofar as these demand functions based on linear prices are easier to estimate from market data than the more hypothetical demands

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[3] In the analysis which follows, we assume that the stand-alone valuations (v_1, v_2) have a continuous marginal density with support on a compact rectangle in $\mathbb{R}_+^2$. Given (v_1, v_2), the distribution of v_b could be deterministic (as in Example 1 below) or discrete (as in Example 2). All we need to assume about the distribution of (v_1, v_2, v_b) is that it is sufficiently well behaved that the demand functions shortly defined are differentiable.

[4] Unlike the single-product case, when a monopolist sells two or more products it can often increase its profits if it is able to use stochastic schemes (e.g., where for a specified price the consumer obtains either product 1 or product 2 but she is not sure which one). See Pavlov [13] for a recent contribution to this topic, which studies cases with extreme substitutes (all consumers buy a single item) and with additive preferences. Pavlov finds in examples that the benefits of using stochastic schemes can be substantial with substitute products but much smaller with additive valuations.

Q_i and Q_b.[5]

Several properties of these demand functions follow automatically from the discrete choice nature of the consumer's problem, and are not contingent on whether the products are substitutes or complements. To illustrate, note that demand for a product is an increasing function of the bundle discount and a decreasing function of its price, i.e.,

$$Q_i(p_1, p_2, \delta) \text{ increases with } \delta \text{ and decreases with } p_i \text{ for } i = 1, 2 . \tag{1}$$

To see this, observe that a consumer buys product 1, say, if and only if

$$\max\{v_b - (p_1 + p_2 - \delta), v_1 - p_1\} - \max\{v_2 - p_2, 0\} \geq 0 . \tag{2}$$

(The first term on the left-hand side is the consumer's maximum surplus if she buys product 1—either in the bundle or on its own—while the second term is her maximum surplus if she does not buy the product.) Clearly, the set of such consumers is increasing (in the set-theoretic sense) in δ and decreasing in p_1. In the case of separate supply, analyzed in section 4, this implies that when a firm unilaterally introduces a bundle discount, its rival's profits will rise.

We also necessarily have Slutsky symmetry of cross-price effects.[6] Aggregate consumer surplus with tariff parameters (p_1, p_2, δ) is

$$V(p_1, p_2, \delta) \equiv \mathbb{E}[\max\{v_1 - p_1, v_2 - p_2, v_b - (p_1 + p_2 - \delta), 0\}] ,$$

where the expectation is taken with respect to the distribution of (v_1, v_2, v_b). Using an envelope argument, this surplus differentiates to give the demand functions:

$$\frac{\partial V}{\partial p_1} \equiv -Q_1 \; ; \; \frac{\partial V}{\partial p_2} \equiv -Q_2 \; ; \; \frac{\partial V}{\partial \delta} \equiv Q_b .$$

The symmetry of second derivatives of V implies that $\partial Q_i / \partial \delta \equiv -\partial Q_b / \partial p_i$ for $i = 1, 2$. Setting $\delta = 0$ implies that the impact of a small bundle discount on the total demand for a product is equal to the impact of a corresponding price cut on bundle demand, i.e.,

$$\left. \frac{\partial Q_i}{\partial \delta} \right|_{\delta=0} = -\frac{\partial q_b}{\partial p_i} . \tag{3}$$

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[5] The model of consumer preferences presented here is related to the small empirical literature which estimates discrete consumer choice when multiple goods are chosen simultaneously. For instance, see Gentzkow [8] who estimates the degree of complementarity between print and online newspapers. In his illustrative model in section 1.A, he supposes that the value of the bundle is the sum of the values of the two individual products plus a constant term (which could be positive or negative), which is similar to Example 1 discussed later in this paper.

[6] Any income effects exhibited by consumers are built in to the valuations (v_1, v_2, v_b). Indeed, income effects might be one reason why valuations are sub-additive, at least for large items: if a consumer purchases product 1, the reduction in income might reduce her willingness to pay for the second item.

This identity plays a key role when we analyze the profitability of introducing a discount.

Other demand properties depend on the substitutability or complementarity of products. To understand this, it is useful to describe the various "margins" between the four buying options. Specifically, if linear prices are used (i.e., $\delta = 0$) then when the price for product i is increased consumers might change their demand in four ways:

(i) instead of buying the bundle, a consumer buys only the other product j;

(ii) instead of buying product i on its own, a consumer buys nothing;

(iii) instead of buying product i on its own, a consumer buys product j on its own, or

(iv) instead of buying the bundle, a consumer buys nothing.

In the knife-edge case where valuations are additive only margins (i) and (ii) exist. More generally, if products are substitutes then margin (iv) does not exist. With substitutes, any consumer who chooses to buy the bundle at linear prices (p_1, p_2) would be happy to buy either product on its own if the bundle were not available. To see this, observe that a consumer with preferences (v_1, v_2, v_b) who buys the bundle at prices (p_1, p_2) satisfies $v_1 + v_2 - p_1 - p_2 \geq v_b - p_1 - p_2 \geq v_i - p_i$, where the first inequality follows from substitutability and the second is due to the superiority of the bundle to consuming product i on its own. Thus, $\min\{v_1 - p_1, v_2 - p_2\} \geq 0$. This implies that any consumer who optimally buys the bundle would instead buy a single item (if they change at all) rather than exit altogether when faced with a small price rise. A related observation is that with substitutes the set of consumers who buy *something* with linear prices (p_1, p_2) consists of those consumers with preferences satisfying $\max\{v_1 - p_1, v_2 - p_2\} \geq 0$. (Clearly, if $v_i \geq p_i$ then the consumer will buy something, since product i on its own yields positive surplus. Those consumers who buy the bundle lie inside this set since they satisfy $\min\{v_1 - p_1, v_2 - p_2\} \geq 0$.) In particular, the fraction of participating consumers depends only on the (marginal) distribution of the stand-alone valuations (v_1, v_2).

On the other hand, if products are complements then margin (iii) does not exist. With complements, any consumer who chooses to buy product i on its own at linear prices (p_1, p_2) has $v_j \leq p_j$. To see this, note that if a consumer with preferences (v_1, v_2, v_b) buys product i on its own at prices (p_1, p_2), then $v_i - p_i \geq v_b - p_1 - p_2 \geq v_1 + v_2 - p_1 - p_2$, where the first inequality follows from the superiority of consuming product i on its own to the bundle and the second is due to complementarity. We deduce that $v_j \leq p_j$, so that no consumer who optimally buys product i on its own would instead buy the other product on its own when faced with a small rise in p_i.[7]

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[7] This discussion is illustrated in a particular example in Figures 2A and 2B below.

As one would expect, the sign of cross-price effects depends on product substitutability or complementarity. A type-(v_1, v_2, v_b) consumer buys product 1 at linear prices (p_1, p_2) if and only if (2) holds. However, with substitutes (respectively, complements), the left-hand side in (2) is weakly increasing (respectively, decreasing) in p_2 for all (v_1, v_2, v_b), and hence the set of consumers who buy product 1 expands (respectively, contracts) with p_2. To see this, for example for the case of substitutes, note that the only way in which the left-hand side of (2) could strictly decrease with p_2 is if $v_b - p_1 - p_2 > v_1 - p_1$ and $v_2 - p_2 < 0$. However, since $v_b \leq v_1 + v_2$ this pair of inequalities are contradictory. To summarize:

Claim 1 *Suppose that $v_b \leq v_1 + v_2$ (respectively, $v_b \geq v_1 + v_2$) for all consumers. Then when linear prices are used, q_i weakly increases (respectively, decreases) with the other product's price p_j.*

Importantly, when products are substitutes and a bundle discount is offered, margin (iii) can be present and cross-price effects can be reversed. That is to say, a bundle discount can convert products which intrinsically are substitutes into complements. This observation will be a recurring theme in the following analysis.

3 Integrated Supply

3.1 Long's analysis revisited

In this section I recapitulate Long's analysis in [10], as the integrated-firm analysis throughout section 3 rests on this. Suppose that the market structure is such that an integrated monopolist supplies both products. Here, and in section 4 with separate supply, suppose that the constant marginal cost of supplying product i is equal to c_i. The firm's profit with bundling tariff (p_1, p_2, δ) is

$$\pi = (p_1 - c_1)Q_1 + (p_2 - c_2)Q_2 - \delta Q_b . \tag{4}$$

Consider the incentive to offer a bundle discount. Starting from any pair of linear prices (p_1, p_2), by differentiating (4) we see that the impact on profit of introducing a small discount $\delta > 0$ is

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\begin{aligned}
\left. \frac{\partial \pi}{\partial \delta} \right|_{\delta=0} &= \left\{ (p_1 - c_1) \frac{\partial}{\partial \delta}(Q_1 + Q_b) + (p_2 - c_2) \frac{\partial}{\partial \delta}(Q_2 + Q_b) - Q_b \right\} \Big|_{\delta=0} \\
&= -(p_1 - c_1) \frac{\partial q_b}{\partial p_1} - (p_2 - c_2) \frac{\partial q_b}{\partial p_2} - q_b ,
\end{aligned}
\tag{5}
$$

where the second equality follows from expression (3). Observe that this incentive depends only on the "no discount" demand function q_b.

Although Long also considers the asymmetric case, his analysis is greatly simplified when products are symmetric, and for the remainder of section 3 assume that $c_1 = c_2 = c$ and the same density of consumers have taste vector (v_1, v_2, v_b) as have the permuted taste vector (v_2, v_1, v_b). Since the environment is symmetric, for convenience we consider only tariffs which are symmetric in the two products. If the firm offers price p for either product and no bundle discount, write $x_s(p)$ and $x_b(p)$ respectively for the proportion of consumers who buy any single item and who buy the bundle. (Thus, $x_s(p) \equiv q_1(p, p) + q_2(p, p) - q_b(p, p)$ and $x_b(p) \equiv q_b(p, p)$.) From expression (5), a small discount is profitable with stand-alone price p in this symmetric setting if and only if

$$(p - c)x'_b(p) + x_b(p) < 0 \ . \tag{6}$$

Consider whether this inequality is satisfied at the most profitable linear price, p^*. Since p^* maximizes $(p - c)(x_s(p) + 2x_b(p))$, the first-order condition for p^* is

$$x_s(p^*) + 2x_b(p^*) + (p^* - c)(x'_s(p^*) + 2x'_b(p^*)) = 0 \ .$$

Taking this together with expression (6), we see that it is profitable to introduce a bundle discount if there is demand from both sources (i.e., $x_s(p^*)$, $x_b(p^*) > 0$) and bundle demand x_b is more elastic than single-item demand x_s at the most profitable linear price p^*. This discussion is summarized in this result:[8]

Proposition 1 *Suppose an integrated monopolist supplies two symmetric products and that at the most profitable linear price p^* there is positive single-item and bundle demand. Then the firm has an incentive to introduce a discount for buying the bundle whenever the demand for a single item is less elastic than the demand for the bundle, so that*

$$\frac{x_b(p)}{x_s(p)} \ \text{strictly decreases with } p \text{ at } p = p^* \ . \tag{7}$$

If expression (7) is reversed, then the firm would like if feasible to charge a premium for buying both items.

Condition (7) is intuitive: if the firm initially charges the same price for buying a single item as for buying a second item, and if demand for the latter is more elastic than demand

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[8] Long stated the result in the alternative, but equivalent, form whereby bundling was profitable if the ratio of total demand $x_s + 2x_b$ to the number of customers $x_s + x_b$ decreased when the price p increased. When products are not symmetric, Long shows using a similar analysis that the firm has an incentive to introduce a discount for buying the bundle whenever single-item demand is less elastic than bundle demand, in the sense that $q_b/(q_1 + q_2)$ strictly decreases with an equi-proportional amplification of price-cost mark-ups.

for the former, then the firm would like to reduce its price for buying a second item (and to increase its price for the first item).

Consider the knife-edge case where a consumer's valuation for the bundle is the sum of her stand-alone valuations, i.e., $v_b \equiv v_1 + v_2$. With additive valuations, if the firm offers the linear price p for buying either item the consumer's decision is simple: she should buy product i whenever $v_i \geq p$. Define

$$\Psi(p) \equiv \Pr\{v_2 \geq p \mid v_1 \geq p\} , \tag{8}$$

so that

$$\Psi(p) = \frac{x_b(p)}{x_b(p) + \frac{1}{2}x_s(p)}$$

and

$$\frac{x_b(p)}{x_s(p)} = \frac{1}{2} \cdot \frac{\Psi(p)}{1 - \Psi(p)} .$$

Proposition 1 implies, therefore, that the firm has an incentive to introduce a bundle discount if

$$\Psi(p) \text{ strictly decreases with } p . \tag{9}$$

Condition (9) holds, roughly speaking, if v_1 and v_2 are not "too" positively correlated.[9] In particular, a degree of bundling is profitable even if valuations are additive and statistically independent. As we explore in the next section, the more fundamental condition (7) is also useful for situations outside this additive case.

3.2 Bundling with substitute products

Using Long's condition (7), in this section I analyze in more detail the firm's incentive to bundle when preferences are sub-additive, and I assume throughout this section that $v_b \leq v_1 + v_2$ for all consumers. One advantage of assuming symmetry in the two products is that what is in general the three-dimensional nature of preferences reduces to two dimensions, since only the highest stand-alone valuation matters out of (v_1, v_2). With this in mind, given preferences (v_1, v_2, v_b), define

$$V_1 \equiv \max\{v_1, v_2\} ; \ V_2 \equiv v_b - V_1 , \tag{10}$$

so that V_1 is a consumer's maximum utility if she buys only one item and V_2 is her incremental utility from the second item. Note that $v_b = V_1 + V_2$, so that valuations are additive after this change of variables. With substitutes we have $V_2 \leq \min\{v_1, v_2\} \leq V_1$, and the

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[9]A sufficient condition for (9) to hold is that $\Pr\{v_1 \leq p \mid v_2 = p\}$ strictly increases with p.

support of (V_1, V_2) lies under the 45^0 line as shown on Figure 1. Faced with a linear price p for each item, the type-(V_1, V_2) consumer will buy a single item if $V_1 \geq p$ and $V_2 < p$, and she will buy both items if $V_2 \geq p$, as depicted on the figure.
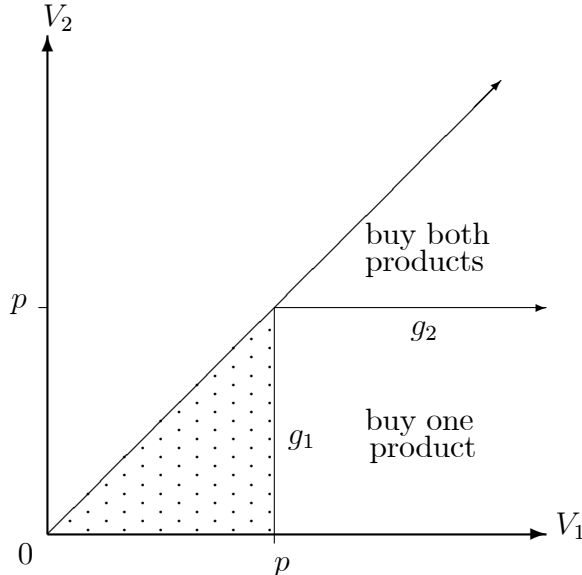

Figure 1: Pattern of demand with substitute products and linear price p

Similarly to (8), define

$$\Phi(p) \equiv \Pr\{V_2 \geq p \mid V_1 \geq p\} = \frac{\Pr\{V_2 \geq p\}}{\Pr\{V_1 \geq p\}} \ . \tag{11}$$

By examining Figure 1 we see that $x_b = (x_b + x_s)\Phi$, or

$$\frac{x_b(p)}{x_s(p)} = \frac{\Phi(p)}{1 - \Phi(p)} \ .$$

Therefore, when Φ is strictly decreasing Proposition 1 implies that the monopolist has an incentive to introduce at least a small bundle discount. In fact, we can obtain the following non-local result, which is our main result for integrated supply:

Proposition 2 *Suppose an integrated monopolist supplies two symmetric products which are substitutes (i.e., $v_b \leq x_1 + v_2$ for all consumers), and that at the most profitable linear price there is positive single-item and bundle demand. Then the most profitable bundling tariff for a monopolist involves a positive bundle discount if Φ in (11) is strictly decreasing.*

(All omitted proofs are found in the appendix.) The fundamental condition which makes bundling profitable for an integrated seller is (7), and this condition applies regardless of whether products are substitutes or not. However, the more transparent condition that Φ in (11) be decreasing only applies when products are substitutes.

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For $i = 1, 2$, write $G_i(p) = \Pr\{V_i \le p\}$ for the marginal c.d.f. for valuation V_i and $g_i(p) = G_i'(p)$ for the corresponding marginal density. (The densities g_1 and g_2 are the "measures" of the lines marked on Figure 1.) The condition that Φ is decreasing is equivalent to the respective hazard rates satisfying $g_1/(1 - G_1) < g_2/(1 - G_2)$.[10] Whenever this is the case, the firm has an incentive to introduce a bundle discount.

Proposition 2 applies equally to an alternative framework where a monopolist supplies a *single* product, and consumers consider buying one or two units of this product. Here, the parameter V_1 represents a consumer's value for one unit and V_2 is her incremental value for the second. Thus when all consumers have diminishing marginal utility (i.e., $V_2 \le V_1$) and Φ in (11) is decreasing, the single-product firm will offer a nonlinear tariff which involves a quantity discount. (However, this alternative interpretation of the model is not natural in the separate sellers context of sections 4 and 5, since we would have to assume that for some reason a supplier could sell only a single unit of the product to a consumer.)

A natural question is whether products being substitutes makes it more likely that the integrated firm wishes to introduce a bundle discount, relative to the same market but with additive valuations. Consider a market where the stand-alone valuations, v_1 and v_2, have a given (symmetric) distribution. We know from Proposition 1 that the firm has an incentive to offer a bundle discount whenever x_b/x_s is decreasing in the linear price p, which is equivalent to the condition that x_b/n decreases with p, where $n \equiv x_s + x_b$ is the fraction of consumers who buy something from the firm. Consider two scenarios: in scenario (a), each consumer's valuation for the bundle is additive, so that $v_b \equiv v_1 + v_2$, while in scenario (b) we have $v_b \le v_1 + v_2$. Write the fraction of consumers who buy both items at linear price p in scenario (a) as $x_b(p)$ and the corresponding fraction in scenario (b) as $\hat{x}_b(p)$. As discussed in section 2, n is exactly the same function in the two scenarios. Thus, if $\hat{x}_b/x_b$ (weakly) decreases with price, then whenever bundling is profitable under scenario (a) it is sure to be profitable under scenario (b) as well. It is plausible, though not inevitable, that bundle demand $\hat{x}_b$ is more elastic than demand x_b. Since $V_2 \le \min\{v_1, v_2\}$, it follows that $\hat{x}_b \le x_b$. Thus, for $\hat{x}_b$ to be more elastic we require that the slope $-\hat{x}_b'$ not be "too much" smaller than $-x_b'$.[11]

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[10] A sufficient condition for this to hold is that the likelihood ratio $g_1(p)/g_2(p)$ increases with p; that is to say, the "likelihood ratio" ordering of random variables is stronger then the "hazard rate" ordering.

[11] An example where the substitutability of products makes the firm *less* likely to engage in bundling is as follows. Suppose that $v_b = v_1 + v_2$ if $\min\{v_1, v_2\} \ge k$ and $v_b = \max\{v_1, v_2\}$ otherwise, where k is a positive constant. Thus, preferences are additive when both stand-alone valuations are high, while if one valuation does not meet the threshold k the incremental value for the second item is zero. With these preferences, whenever the linear price satisfies $p < k$ those consumers with $\min\{v_1, v_2\} \ge k$ will buy both items, and this set does not depend on p. Therefore, bundle demand $\hat{x}_b$ is completely inelastic for

Intuitively, when products are substitutes there is an extra motive to offer a bundle discount, relative to the additive case, which is to try to serve customers with a second item even though the incremental utility of the second item is lowered by the purchase of the first item. Once a customer has purchased one item, this is bad news for her willingness-to-pay for the other item, and this often gives the firm a motive to reduce price for the second item. With additive preferences, the only motive in this model to use a bundle discount is to extract information rents from consumers, and this motive vanishes if the firm knows consumer preferences. With sub-additive preferences, the firm may wish to offer a bundling tariff even when it knows the customer's tastes.

While with integrated supply sub-additive preferences merely give one additional reason to bundle, with separate sellers such preferences will often be the *sole* reason to offer a bundle discount, as discussed in section 4.

3.3 Special cases

In this section I describe two special cases to illustrate this analysis of bundling incentives, as well as some equilibrium bundling tariffs.

Example 1: Constant disutility of joint consumption.

Consider the situation in which for all consumers

$$v_b \equiv v_1 + v_2 - z \tag{12}$$

for a constant z. Here, to ensure free disposal we assume that the minimum possible realization of v_i is greater than z. With a linear price p_i for buying product i, the pattern of demand is as shown on Figure 2A for substitutes and on Figure 2B for complements.[12] The next result provides a sufficient condition for bundling to be profitable when products are substitutes (i.e., $z \geq 0$).

Claim 2 *Suppose that bundle valuations are given by (12) with $z \geq 0$. Suppose that each valuation v_i has marginal c.d.f. F and marginal density f, and the hazard rate $f(\cdot)/(1 - F(\cdot))$ is increasing. Then a monopolist has an incentive to offer a bundle discount when condition (9) holds.*

$p < k$, while in the corresponding example without substitution (i.e., setting $k = 0$), bundle demand is elastic. Whenever k is large enough that the equilibrium linear price is below k, the firm strictly lowers its profits if it introduces a bundle discount: it reduces its revenue from those who buy the bundle without any compensating boost to demand.

[12]These two figures illustrate the discussion in section 2 about how margin (iii) is absent with substitutes and margin (iv) is absent with complements.
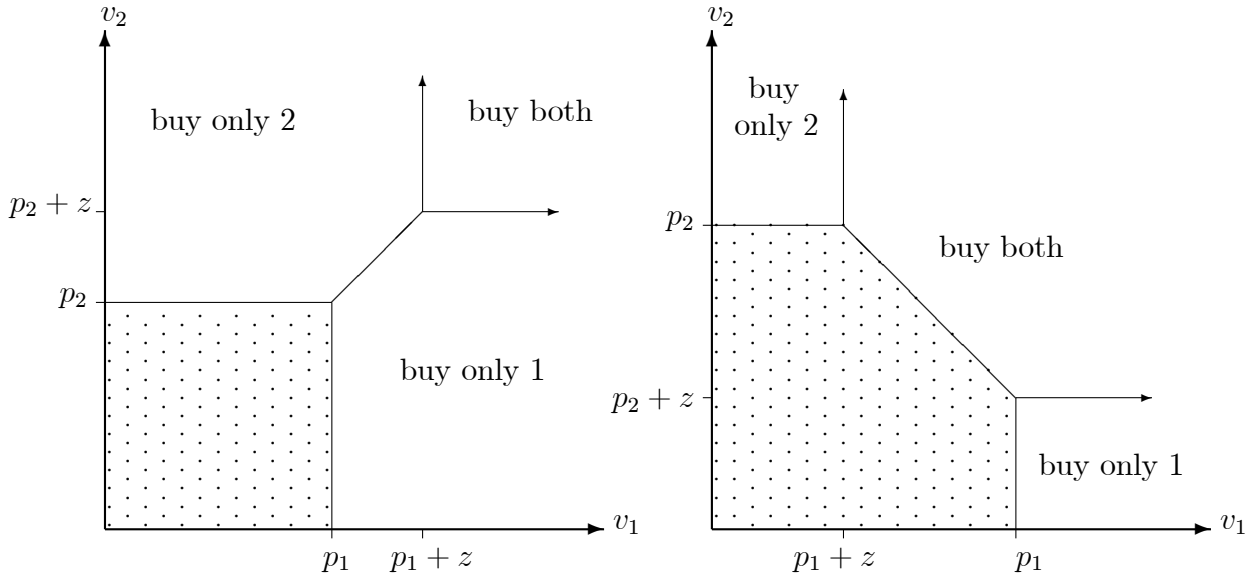
Figure 2A: Pattern of demand $(z > 0)$ Figure 2B: Pattern of demand $(z < 0)$

To illustrate, suppose that (v_1, v_2) is uniformly distributed on the unit square $[1, 2]^2$, and that $z = \frac{1}{4}$ and $c = 1$.[13] Then an integrated monopolist which uses linear prices will choose $p = \frac{73}{48} \approx 1.521$, generating profit of around 0.407. Total welfare, measured as profit plus consumer surplus, here is about 0.604. At this price, around 73% of potential consumers buy something, although only 5% buy both products. The most profitable bundling tariff can be calculated to be

$$p \approx 1.594 \; ; \; \delta \approx 0.380 \; , \tag{13}$$

which generates profit of about 0.449, and about 66% of potential consumers buy something but now 28% buy both items. Total welfare here is approximately 0.643, which is a substantial increase over the linear pricing regime, although consumers in aggregate are slightly worse off. This bundle discount is large enough to outweigh the innate substitutability of the products (i.e., $\delta > z$), and faced with this bundling tariff consumers now view the two products as complements rather than substitutes. (The resulting pattern of demand looks similar to that shown on Figure 2B.)

Although a useful condition has not been found to ensure when a discount is profitable in the case of complements $(z < 0)$, in examples this is often the case. For instance, consider the same example as just presented, except that $z = -\frac{1}{4}$ instead of $z = \frac{1}{4}$. Here, the most profitable linear price is $p = \frac{121}{80} \approx 1.521$, which generates profit of about 0.657 and welfare 0.987. At this price about 77% of consumers buy something, and about 51% buy the bundle. One can check that x_b/x_s is strictly decreasing at this most profitable

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[13]The detailed calculations involved in this example can be found in the accompanying online appendix.

linear price, and Proposition 1 implies that the firm has an incentive to introduce a bundle discount. In fact, in this example the most profitable bundling tariff involves *pure* bundling, and no consumers buy just a single item. The most profitable bundle price is $P \approx 2.987$, which generates profit of 0.719 and higher welfare of 1.048. At this tariff about 73% of consumers choose to buy something (in which case they buy the bundle).

Example 2: Time-constrained consumers.

A natural reason why products might be substitutes is that some buyers are only able to consume a restricted set of products, perhaps due to time constraints.[14] To that end, suppose that the stand-alone valuations (v_1, v_2) are distributed in the population in some manner, and given (v_1, v_2) a consumer's bundle valuation is

$$
v_b = \begin{cases} v_1 + v_1 & \text{with probability } \lambda \\ \max\{v_1, v_2\} & \text{with probability } 1 - \lambda \ . \end{cases} \tag{14}
$$

The interpretation is that with probability $1 - \lambda$ a consumer is time-constrained and can only buy one item, while otherwise they are unconstrained. (See Figure 3 for an illustration.)
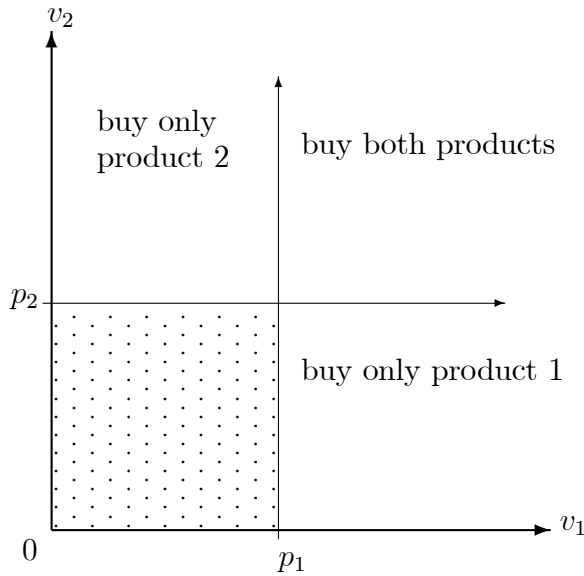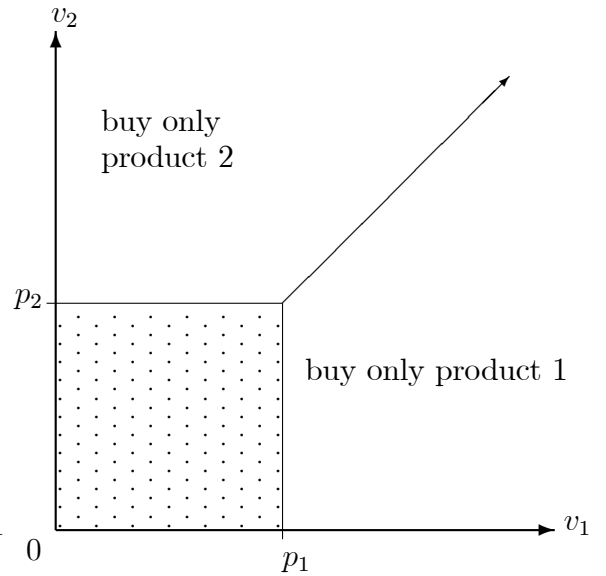

Figure 3A: Unconstrained consumers   Figure 3B: Time-constrained consumers

Given the distribution for (v_1, v_2), let $\Psi(\cdot)$ be as defined in (8). It is straightforward to show $\Phi(p) = \lambda\Psi(p)/(2 - \Psi(p))$, so that Φ is decreasing if and only if Ψ is. Proposition 2 therefore implies that when some (but not all) consumers are time-constrained, an integrated firm has an incentive to offer a bundle discount if and only if (9) holds, i.e., under

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[14][15] also analyzes the situation where an exogenous fraction of consumers wish to buy a single product.

the same condition as when consumers have additive preferences. The reason is that when the firm offers a bundle discount this only affects the λ unconstrained consumers, and the sign of the impact on profit is just as if all consumers had additive preferences.[15]

4 Separate Sellers

4.1 General analysis

I turn now to the situation where the two products are supplied by separate sellers, where for $i = 1, 2$ firm i sells product i. In contrast to the integrated seller case, here there is no expositional advantage in assuming that products are symmetric, and we no longer make that assumption. Suppose that the sellers set their tariffs simultaneously and non-cooperatively. (The next section discusses a setting in which firms coordinate on the bundle discount.)

When firms offer linear prices—i.e., prices which are not contingent on whether the consumer also purchases the other product—firm i chooses its price p_i^* given its rival's price to maximize $(p_i - c_i)q_i$, so that

$$(p_i^* - c_i)\frac{\partial q_i}{\partial p_i} + q_i = 0 \ . \tag{15}$$

In some circumstances, a firm can condition its price on whether a consumer also buys the other firm's product. For instance, a museum could ask a visitor to show her entry ticket to the other museum to claim a discount. Suppose now that firm i offers a discount $\delta > 0$ from its price p_i^* to those consumers who purchase product j as well. (Those consumers who only buy product i continue to pay p_i^*.) Then firm i's profit is

$$\pi_i = (p_i^* - c_i)Q_i - \delta Q_b \ , \tag{16}$$

and from (3) the impact on profit of a small discount is equal to

$$\left.\frac{d\pi_i}{d\delta}\right|_{\delta=0} = -(p_i^* - c_i)\frac{\partial q_b}{\partial p_i} - q_b \ . \tag{17}$$

(Again, this incentive depends on properties of the "no discount" demand function q_b.) When a firm has positive single-item and bundle demand, so that $0 < q_b < q_i$, then if firm's bundle demand is more elastic than its total demand, so that $\frac{-1}{q_i}\frac{\partial q_i}{\partial p_i} < \frac{-1}{q_b}\frac{\partial q_b}{\partial p_i}$, then (15) implies that (17) is strictly positive. In this case, offering a discount for joint purchase will increase the firm's profit.

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[15]Although whether the firm has an incentive to offer a bundle discount does not depend on λ, the most profitable prices will do so.

Thus, discounts for joint purchase can arise even when products are supplied by separate firms and when a firm chooses and funds the discount unilaterally. The reason is straightforward: since the own-price elasticity of bundle demand is higher than that of total demand, a firm wants to offer a lower price to those consumers who also buy the other product. As expression (1) shows, the introduction of a discount will also benefit the rival firm.

We summarise this discussion as:

Proposition 3 *Suppose firm i has positive single-item and bundle demand at the equilibrium linear prices. Then firm i has an incentive to offer a discount to those consumers who buy product j whenever demand for the bundle is more elastic than total demand for firm i's product, in the sense that*

$$\frac{q_b(p_1, p_2)}{q_i(p_1, p_2)} \text{ strictly decreases with } p_i . \tag{18}$$

If expression (18) is reversed, so that q_b/q_i increases with p_i, then firm i would like if feasible to charge its customers a premium if they buy product j.

The crucial difference between condition (18) and the corresponding condition (7) with integrated supply is that with a single seller both prices are increased, whereas with separate sellers only one price rises. As discussed in section 2, with substitute products and linear pricing, a firm competes on the three margins (i)–(iii). Broadly speaking, condition (18) requires that margins (ii) and (iii) together are less significant, relative to the size of associated demand, than margin (i).

It is possible that one firm has an incentive to offer a discount when a customer also buys the other firm's product, but the other firm does not.[16] However, it may well be that both firms choose to offer such a discount. If firm $i = 1, 2$ offers the price p_i when a consumer only buys its product and the price $p_i - \delta_i$ when she also buys the other product, a consumer who buys the bundle pays the price $p_1 + p_2 - \delta_1 - \delta_2$. The issue then arises as to how the combined discount $\delta = \delta_1 + \delta_2$ is implemented. For instance, a consumer might have to buy the two items sequentially, and firms cannot simultaneously require proof of purchase from the other seller when they offer their discount. However, there are at least two natural ways to implement this inter-firm bundling scheme. First, the bundle discount could be implemented via an electronic sales platform which allows consumers

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[16] One simple way this can happen is when one firm sells to *all* consumers at the equilibrium linear prices, while the other does not. Clearly, the latter firm has nothing to gain from making its price contingent on whether its customers buy the other product, while the former may have such an incentive.

to buy products from several sellers simultaneously. Sellers choose their prices contingent on which other products (if any) a consumers buys, a website displays the total prices for the various combinations, and firms receive their stipulated revenue from the chosen combination. With such a mechanism there is no need for firms to coordinate their tariffs. Second, there may be "product aggregators" present in the market who assemble their own bundles from products sourced from separate firms and retail these bundles to final consumers. In the two-product case discussed in this paper, aggregators bundle the two products together and each firm chooses a wholesale price for its product contingent on being part of the bundle. If the aggregator market is competitive, the price of the bundle will simply be the sum of the two wholesale prices. Again, there is no need for firms to coordinate their prices.

A major difference between this inter-firm bundling discount and the discount offered by an integrated supplier is that with separate sellers the discount is chosen non-cooperatively. A bundle is, by definition, made up of two "complementary" components, namely, firm 1's product and firm 2's product, and the total price for the bundle is the sum of each firm's component price $p_i - \delta_i$. When a firm considers the size of its own discount δ_i, it ignores the benefit this discount confers on its rival. Thus, as usual with separate supply of complementary components, double marginalization will result and the overall discount $\delta = \delta_1 + \delta_2$ will be too small from the perspective of industry profit.

If it is somehow feasible for firms to charge a premium when its customer buy the other firm's product, then another rather trivial equilibrium always exists: each firm stipulates that a consumer can only buy its product *or* the other product, i.e., firms set $\delta = -\infty$. The resulting pattern of demand is as shown on Figure 3B above, and firms choose their stand-alone prices accordingly. This is an equilibrium because if one firm requires is customers to buy its product alone or not at all, there is nothing the other firm can do to unravel this exclusivity requirement. It is often implausible that firms would wish to engage in this form of artificially intense competition, even if it were feasible. Nevertheless, one situation in which this strategy might be attractive for a firm is if it wishes to deter entry or induce exit of a relatively weak rival. Of course, in many situations, a consumer can hide her purchase from a rival firm, in which case a firm cannot feasibly levy a premium when a customer buys another supplier's product.

4.2 Special cases

In this section, I analyze in more depth various special cases where separate sellers have an incentive to introduce a joint-purchase discount. Consider first the situation where

consumer valuations are additive, so that firms do not compete with each other when linear prices are used:

Proposition 4 *Suppose that valuations are additive, i.e., $v_b = v_1 + v_2$. Starting from the situation where firms set equilibrium linear prices (p_1^*, p_2^*), firm i has an incentive to offer a discount to its customers who buy the other product whenever $\Pr\{v_j \leq p_j^* \mid v_i\}$ strictly increases with v_i.*

Whenever the valuations are negatively correlated in the strong sense that $\Pr\{v_j \leq p \mid v_i\}$ increases with v_i, then, a firm has an incentive to offer a discount for joint purchase. Somewhat counter-intuitively, those firms which offer products which appeal to very different kinds of consumer (boxing and ballet, say) may wish to offer discounts to consumers who buy the other product.

In the oligopoly context, it is sometimes reasonable to consider situations with full coverage, so that all consumers buy something over the relevant range of linear prices.[17] (This is relevant when the minimum possible realizations of v_1 and v_2 are sufficiently high.) When the outside option of zero is not relevant for any consumer's choice, all that matters for demand is the distribution of *incremental* utilities, and given the triple (v_1, v_2, v_b) define new variables

$$\hat{v}_1 \equiv v_b - v_2 \;;\; \hat{v}_2 \equiv v_b - v_1 \tag{19}$$

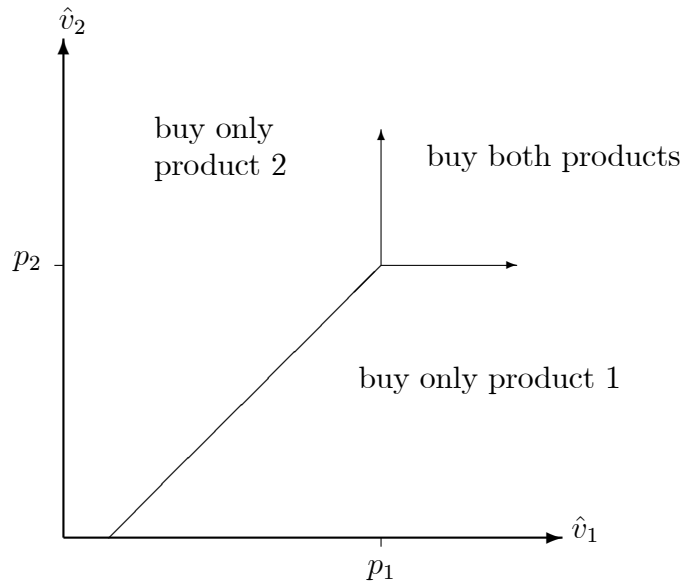for the incremental valuation for product i given the consumer already has product j.

Figure 4: Pattern of demand with full consumer coverage

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[17]This is not a useful special case to consider in the context of integrated supply. For instance, [2] shows how a monopolist will typically wish to exclude some consumers when consumers have multi-dimensional private information (as they do here).

As shown on Figure 4, a consumer will buy both items with linear prices (p_1, p_2) provided that $\hat{v}_1 \geq p_1$ and $\hat{v}_2 \geq p_2$, and otherwise she will buy product 1 instead of product 2 when $\hat{v}_1 - p_1 \geq \hat{v}_2 - p_2$. In particular, margins (ii) and (iv) discussed in section 2 are not present, and this may boost the incentive to offer a bundle discount. Write $G_i(\hat{v}_i \mid \hat{v}_j)$ for the distribution function for $\hat{v}_i$ conditional on $\hat{v}_j$, and write $g_i(\hat{v}_i \mid \hat{v}_j)$ for the associated conditional density function.

Proposition 5 *Suppose that at the equilibrium linear prices there is full consumer coverage, and firm i has positive single-item and bundle demand. If the incremental valuations (19) satisfy*

$$\frac{g_i(\hat{v}_i \mid \hat{v}_j)}{1 - G_i(\hat{v}_i \mid \hat{v}_j)} \text{ strictly increases with } \hat{v}_i \text{ and weakly increases with } \hat{v}_j , \qquad (20)$$

then firm i has an incentive to offer a discount to its customers who buy the other product.

It is somewhat reasonable to suppose that the hazard rate in (20) increases with $\hat{v}_i$. That the hazard rate weakly increases with $\hat{v}_j$ is perhaps less economically natural, but includes independence of incremental valuations as a particular case.

We next consider the impact of inter-firm bundling in the examples with non-additive valuations introduced in section 3.3.

Example 1. Here, the pattern of consumer demand was illustrated in Figures 2A and 2B. Write $H_i(v_i \mid v_j)$ for the c.d.f. for v_i conditional on v_j and $h_i(v_i \mid v_j)$ for the associated conditional density. The next result describes when a firm has a unilateral incentive to offer a bundle discount or premium. (The proof of the claim is similar to that for Proposition 5 and omitted.)

Claim 3 *Suppose that bundle valuations are given by (12) and the stand-alone valuations satisfy*

$$\frac{h_i(v_i \mid v_j)}{1 - H_i(v_i \mid v_j)} \text{ strictly increases with } v_i . \qquad (21)$$

(i) If $z > 0$ and $h_i/(1 - H_i)$ in (21) weakly increases with v_j, then seller i has an incentive to offer a discount to its customers who buy the rival's product;
(ii) If $z < 0$ and $h_i/(1 - H_i)$ in (21) weakly decreases with v_j, then seller i has an incentive to charge a premium, if feasible, to its customers who buy the rival's product.

It is economically intuitive that when products are substitutes $(z > 0)$, the firm has an incentive to offer a discount when its customers purchase the rival product. If the potential

customer purchases the other product, this is bad news for the firm as the customer's incremental value for its product has been shifted downwards by z, and this provides an incentive to offer a lower price. Likewise, when products are complements ($z < 0$) the firm will likely wish to offer a higher price when a customer has bought the other product.

Consider the same specific example as presented in section 3—that is, (v_1, v_2) uniform on $[1, 2]^2$, $z = \frac{1}{4}$ and $c = 1$—applied to the case with separate sellers. The equilibrium linear price is $p = \frac{81}{56} \approx 1.446$ and industry profit is about 0.399. Around 9% of consumers buy both items with this linear price, and 80% buy something. Total welfare with linear pricing is about 0.658. The equilibrium non-cooperative bundling tariff is

$$p_1 = p_2 \approx 1.476 \; ; \; \delta = \delta_1 + \delta_2 \approx 0.101 \; . \tag{22}$$

Here, the combined bundle discount, $\delta = \delta_1 + \delta_2$, is only about one quarter the size of the discount with integrated supply in (13), reflecting the discussion in section 4.1 that separate firms will non-cooperatively choose too small a discount. Now, around 14% of consumers buy both items, and industry profit rises to 0.421 while total welfare rises to 0.665. Relative to the outcome with linear pricing, here consumers in aggregate are harmed, but total welfare rises, when firms unilaterally offer a discount. Note that the equilibrium linear price lies between the two discriminatory prices when firms engage in this form of price discrimination.[18] Intuitively, when firms offer a bundle discount, this reduces the effective degree of substitution between products, which in turn relaxes competition between firms and induces them to raise price.

Example 2. Consider next the situation in which some consumers are time constrained. Then we can obtain the following result:

Claim 4 *Suppose that stand-alone valuations v_1 and v_2 are independently distributed, where v_i has distribution function $F_i(\cdot)$ and density $f_i(\cdot)$, and where for each i the hazard rate $f_i(\cdot)/(1 - F_i(\cdot))$ is strictly increasing. Suppose that bundle valuations are given by (14). When the two products are supplied by separate sellers, sellers have no incentive to offer a discount to those consumers who buy the rival product. They would, if feasible, like to charge their customers a higher price if they also buy the rival product.*

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[18]This is not surprising in the light of the analysis of Corts [5], who shows that when the two firms wish to set their lower price to the same group of customers (the "weak" market, which in this example is the set of consumers who buy both products), then the equilibrium non-discriminatory price lies between the two discriminatory prices. However, we cannot apply Corts' result directly, since his argument relies on there being no cross-price effects across the two consumer groups, which is not the case here.

Intuitively, because of competition with the rival firm, demand from the time-constrained group of consumers is more elastic than demand from the unconstrained group. Each firm holds a monopoly position over the unconstrained group, and would like to exploit this position over those consumers if feasible. Comparing Examples 1 and 2 shows that the precise manner in which products are substitutes is important for a firm's incentive to offer a bundling discount.

5   Coordinated Discounts

The analysis to this point has considered the two extreme cases where there is no tariff coordination between separate sellers (section 4), and where there is complete tariff coordination (section 3). The problem with complete coordination is that any competition between rivals is eliminated. As discussed in section 4, though, the problem with a policy which permits no coordination between sellers is that the resulting bundle discount may be inefficiently small or non-existent. It would be desirable to obtain the efficiency gains which often accrue to bundling without permitting the firms to collude over their regular prices.

One way this might be achieved is if firms first negotiate an inter-firm bundle discount, the funding of which they agree to share, and then compete by choosing their stand-alone prices non-cooperatively. Specifically, suppose the two firms are symmetric and consider the following joint pricing scheme: firms first coordinate on bundle discount δ, and if firm $i = 1, 2$ sets the stand-alone price p_i then the price for buying both products is $p_1 + p_2 - \delta$ and firm i receives revenue $p_i - \frac{1}{2}\delta$ when a bundle is sold.

Consider first the case where valuations are additive, so that competition concerns are absent. Firm i's profit under this scheme is

$$\pi_i = (p_i - c)Q_i - \tfrac{1}{2}\delta Q_b \ , \tag{23}$$

where each firm's price is a function of the agreed discount δ as determined by the second-stage non-cooperative choice of prices. The impact of introducing a small $\delta > 0$ on firm i's equilibrium profit is therefore equal to

$$\left.\frac{d\pi_i}{d\delta}\right|_{\delta=0} = -(p^* - c)\frac{\partial q_b}{\partial p_i} - \tfrac{1}{2}q_b \tag{24}$$

$$+ \left(\left.\frac{dp_i}{d\delta}\right|_{\delta=0}\right)\left(\left.\frac{\partial}{\partial p_i}[(p_i - c)q_i]\right|_{p_i=p_j=p^*}\right) + \left(\left.\frac{dp_j}{d\delta}\right|_{\delta=0}\right)\left(\left.\frac{\partial}{\partial p_j}[(p_i - c)q_i]\right|_{p_i=p_j=p^*}\right) \ . \tag{25}$$

(Recall that p^* is most profitable linear price when $\delta = 0$.) Here, the right-hand side of (24) represents the direct effect of introducing the discount, keeping stand-alone prices

unchanged, while the terms in (25) reflect the indirect effect of the discount on the firm's profit via its impact on the two prices, p_i and p_j. Note that both terms in (25) vanish: the first because p^* is the optimal price for firm i when firms choose linear prices (i.e., p^* maximizes $(p_i - c)q_i$), and the second because a change in the other firm's price has no impact on a firm's demand when there is no bundling discount and valuations are additive (i.e., q_i does not depend on p_j when valuations are additive). Following the discussion in section 3.1, in the symmetric additive case expression (24) is positive if and only if (9) holds.

To summarize:

Proposition 6 *Suppose that products are symmetric and valuations are additive. Consider the coordinated bundling scheme whereby firms choose prices non-cooperatively, and if firm $i = 1, 2$ sets the stand-alone price p_i then the price for buying the bundle is $p_1 + p_2 - \delta$ and firm i receives revenue $p_i - \frac{1}{2}\delta$ when a bundle is sold. If condition (9) holds, for small discount $\delta > 0$ this scheme increases each firm's profit relative to the situation where the products are sold independently ($\delta = 0$).*

This result suggests that a coordinated bundling scheme of this form could be profitable for many pairs of suppliers, even if they supply unrelated products. Proposition 6 could be seen as a "separate seller" analogue of the result for integrated monopoly derived by [10,12], who showed with additive preferences that when condition (9) was satisfied it was profitable for a monopolist to introduce a bundle discount.

To illustrate, consider the specific case of additive valuations where stand-alone valuations (v_1, v_2) are uniformly distributed on $[1, 2]^2$ and the marginal cost of each product is $c = 1$. This is a special case of Example 1 above, with z set equal to zero. Without coordination on the inter-firm discount (so $\delta = 0$) firms set price $p^* = \frac{3}{2}$. The resulting payoffs in the market are reported in the first row of Table 1. If firms first coordinate on δ and then choose price non-cooperatively, one can check that the most profitable discount is $\delta \approx 0.384$, which implements the higher price $p \approx 1.669$. The corresponding payoffs are reported in the second row of this table. In this example, then, allowing the firms to coordinate on an inter-firm discount boosts profit, harms consumers in aggregate, and (slightly) increases overall welfare. In this example welfare would be boosted with a modest coordinated discount, but firms "overshoot" and choose too deep a discount. (In this example, welfare would be maximized if firms agreed on the moderate discount $\delta \approx 0.198$.) Because valuations are additive and statistically independent, a firm would not wish unilaterally to introduce a bundle discount in this market.

	industry profit	consumer surplus	welfare
linear pricing ($\delta = 0$)	0.500	0.250	0.750
coordinated discount ($\delta = 0.384$)	0.544	0.209	0.753

Table 1: Market outcomes with and without coordination on discount ($z = 0$)

While the operation of the joint-pricing scheme appears relatively benign when values are additive, this can be reversed when firms offer substitutable products. Consumers benefit, and total welfare rises, when firms are forced to set low prices due to products being substitutes. However, a negotiated inter-firm discount can reduce the effective substitutability of products and relax competition between suppliers. To illustrate this effect, modify the preceding example so that $z = \frac{1}{4}$. The impact of partial coordination in this case is reported in Table 2. As derived in section 4.2, with linear pricing firms choose price $p \approx 1.446$ and the resulting payoffs are given in the first row. When firms coordinate on the bundle discount and then choose stand-alone prices non-cooperatively, their most profitable choice is $\delta \approx 0.390$, which implements price $p \approx 1.588$, and payoffs are given in the second row. In contrast to Table 1, now total welfare falls when firms coordinate on the discount, reflecting the high prices which are then induced. For comparison, the third row reports payoffs when firms choose their joint-purchase discount non-cooperatively, when the equilibrium tariff is (22) above. For firms and consumers, the resulting outcome is intermediate between the outcomes with linear pricing and with a coordinated discount; however it generates the highest welfare level of the three regimes. At least in this example, a modest bundle discount enhances welfare, but when firms coordinate on the discount, they choose too deep a discount from a welfare perspective.

	industry profit	consumer surplus	welfare
linear pricing ($\delta = 0$)	0.399	0.259	0.658
coordinated discount ($\delta = 0.390$)	0.449	0.202	0.651
non-cooperative discount ($\delta = 0.1$)	0.421	0.244	0.665

Table 2: Market outcomes with and without coordination on discount ($z = \frac{1}{4}$)

Thus, the apparently pro-consumer policy of coordinating to offer a discount for joint purchase may act as a device to sustain collusion. This suggests that negotiated inter-firm discounting schemes operated by firms supplying substitutable products should be viewed with some suspicion by antitrust authorities, although non-cooperative discounting as analyzed in section 4 may be welfare-enhancing.

6　Conclusions

This paper has extended the standard model of bundling to allow products to be partial substitutes and for products to be supplied by separate sellers. With monopoly supply, building on Long's approach in [10], we typically found that the firm has an incentive to offer a bundle discount in at least as many cases as with the traditional model with additive valuations. Sub-additive preferences give the firm an additional reason to offer a bundle discount, which is to better target a low price for a second item at those customers who are inclined (with linear prices) to buy a single item.

When products were supplied by separate firms, we found that a firm often has a unilateral incentive to offer a joint-purchase discount when their customers buy rival products. In such cases, inter-firm bundle discounts are effected without any need for coordination between suppliers. The two principal situations in which a firm might wish to do this are (i) when product valuations are negatively correlated in the population of consumers, and (ii) when products are substitutes in such a way that bundle demand is more elastic than total demand. While product substitutability makes bundle demand *smaller* than it would otherwise be, it need not make such demand more elastic. Plausible kinds of substitution lead firms to offer either a joint-purchase discount or (if feasible) a joint-purchase premium. In an example we saw that when firms price discriminate in this manner, relative to the uniform pricing regime, equilibrium profits are higher and welfare rises. One reason why profits rise is that when firms offer an inter-firm bundle discount, this mitigates the innate substitutability of their products and competition is relaxed.

Historically, this form of price discrimination was not often observed. In many cases, in order to condition price on a purchase from a rival supplier, a firm would need a "paper trail" such as a receipt from the rival. One problem with this system is that customers are then encouraged to visit the rival firm first, and because of transaction and travel costs, this might mean that fewer customers would actually come to the firm. A second problem is that it is hard for two firms to offer such discounts, since a customer might have to visit the firms sequentially. However, as discussed in section 4, these two related problems can nowadays often be overcome with modern ways of selling, and we may see greater use of this kind of contingent pricing in future.

A more traditional way to implement inter-firm bundling is for firms to coordinate aspects of their pricing strategy. In this paper I examined one particular kind of coordination, which is where firms agree on a joint purchase discount, and subsequently choose their prices non-cooperatively. Because a bundle discount mitigates the innate substitutability

of rival products, separate sellers can use this mechanism to lessen rivalry in the market. Thus, firms often have an incentive to explore joint pricing schemes of this form, and regulators have a corresponding incentive to be wary.

In future work it would be useful to extend the analysis in this paper in at least two directions. First, what happens if the products in question are intermediate products? It may be that the framework studied here could sometimes be extended to situations where rival manufacturers potentially supply products to a retailer, which then supplies one or both products to final consumers. If products are partial substitutes, might a manufacturer have an incentive to charge a lower price if the retailer also chooses to supply the rival product? This would then result in the opposite pricing pattern to the "loyalty pricing" schemes which worry antitrust authorities. Second, it would be interesting to explore whether a "large" firm has an incentive to exclude smaller firms from its internal bundling policies, with the aim of driving these rivals out of the market. In a famous antitrust case concerning ski-lifts in the Aspen resort, described in [6], one small ski-lift operator successfully sued a larger operator for not permitting it to participate in its multi-mountain ski-pass scheme.

APPENDIX

Proof of Proposition 2: We already know that choosing $\delta > 0$ is more profitable than choosing $\delta = 0$ when expression (7) holds, which in turn is true when (11) is strictly decreasing. Therefore, it remains to rule out the possibility that a tariff with a quantity *premium* is optimal. So suppose to the contrary that the firm makes greatest profit by charging P_1 for the first item and $P_2 > P_1$ for the second. By modifying Figure 1 to allow $P_2 > P_1$, one sees that the firm's profit takes the additively separable form

$$(1 - G(P_1))(P_1 - c) + (1 - G(P_2))\Phi(P_2)(P_2 - c)$$

where we write $G(p) \equiv \Pr\{V_1 \leq p\}$. This profit is therefore greater than when the firm offers either of the linear prices P_1 and P_2. That is to say

$$(1-G(P_1))(P_1-c)+(1-G(P_2))\Phi(P_2)(P_2-c) \geq (1-G(P_1))(P_1-c)+(1-G(P_1))\Phi(P_1)(P_1-c)$$

or

$$(1 - G(P_2))\Phi(P_2)(P_2 - c) \geq (1 - G(P_1))\Phi(P_1)(P_1 - c) \ , \tag{26}$$

and

$$(1-G(P_1))(P_1-c)+(1-G(P_2))\Phi(P_2)(P_2-c) \geq (1-G(P_2))(P_2-c)+(1-G(P_2))\Phi(P_2)(P_2-c)$$

27

or

$$(1 - G(P_1))(P_1 - c) \geq (1 - G(P_2))(P_2 - c) \ . \tag{27}$$

Since (11) is strictly decreasing, (26) implies that

$$(1 - G(P_2))\Phi(P_2)(P_2 - c) > (1 - G(P_1))\Phi(P_2)(P_1 - c) \ ,$$

which contradicts expression (27). Thus, the most profitable tariff involves $P_2 < P_1$. ∎

Proof of Claim 2: From Figure 2A we see that with linear price p for each product we have

$$x_b(p) = (1 - F(p + z))\Psi(p + z) \ ; \ x_s(p) = (1 - F(p))(2 - \Psi(p)) - x_b(p) \ ,$$

and so (11) is given by

$$\Phi(p) = \frac{x_b(p)}{x_s(p) + x_b(p)} = \frac{(1 - F(p + z))\Psi(p + z)}{(1 - F(p))(2 - \Psi(p))} \ .$$

Differentiating shows that Φ is strictly decreasing with p if and only if

$$\frac{\Psi'(p)}{2 - \Psi(p)} + \frac{\Psi'(p + z)}{\Psi(p + z)} < \frac{f(p + z)}{1 - F(p + z)} - \frac{f(p)}{1 - F(p)} \ .$$

Since F is assumed to have an increasing hazard rate, the right-hand side of the above is non-negative, while if condition (9) holds then the left-hand side is strictly negative. Therefore, Φ is strictly decreasing and Proposition 2 implies the result. ∎

Proof of Proposition 4: Let $F_i(v_i)$ and $f_i(v_i)$ be respectively the marginal c.d.f. and the marginal density for v_i, and let $H(v_i) \equiv \Pr\{v_j \leq p_j^* \mid v_i\}$, where p_j^* is firm j's equilibrium linear price. Then

$$q_i(p_i, p_j^*) = 1 - F_i(p_i) \ ; \ q_b(p_i, p_j^*) = \int_{p_i}^{\infty} (1 - H(v_i))f_i(v_i)dv_i \tag{28}$$

and

$$-\frac{\partial q_i}{\partial p_i} = f_i(p_i) \ ; \ -\frac{\partial q_b}{\partial p_i} = (1 - H(p_i))f_i(p_i) \ .$$

Since H is assumed to be strictly increasing in v_i, it follows from (28) that

$$q_b(p_i, p_j^*) < (1 - H(p_i))(1 - F_i(p_i))$$

and so

$$-\frac{1}{q_i}\frac{\partial q_i}{\partial p_i} = \frac{f_i(p_i)}{1 - F_i(p_i)} < -\frac{1}{q_b}\frac{\partial q_b}{\partial p_i}$$

and Proposition 3 implies the result. ∎

Proof of Proposition 5: Let $h_j(\hat{v}_j)$ denote the marginal density for $\hat{v}_j$. From Figure 4 we see that

$$q_b = \int_{p_j}^{\infty} (1 - G_i(p_i \mid \hat{v}_j))h_j(\hat{v}_j)d\hat{v}_j \ ; \ -\frac{\partial q_b}{\partial p_i} = \int_{p_j}^{\infty} g_i(p_i \mid \hat{v}_j)h_j(\hat{v}_j)d\hat{v}_j \ .$$

From assumption (20) we obtain

$$-\frac{\partial q_b}{\partial p_i} = \int_{p_j}^{\infty} \frac{g_i(p_i \mid \hat{v}_j)}{1 - G_i(p_i \mid \hat{v}_j)}(1 - G_i(p_i \mid \hat{v}_j))h_j(\hat{v}_j)d\hat{v}_j \geq \frac{g_i(p_i \mid p_j)}{1 - G_i(p_i \mid p_j)}q_b \ .$$

Similarly, the demand for product i on its own satisfies

$$q_i - q_b = \int_0^{p_j} (1 - G_i(\hat{v}_j + p_i - p_j \mid \hat{v}_j))h_j(\hat{v}_j)d\hat{v}_j \ ; \ -\frac{\partial [q_i - q_b]}{\partial p_i} = \int_0^{p_j} g_i(\hat{v}_j + p_i - p_j \mid \hat{v}_j)h_j(\hat{v}_j)d\hat{v}_j \ .$$

From assumption (20) we obtain

$$\begin{aligned} -\frac{\partial [q_i - q_b]}{\partial p_i} &= \int_0^{p_j} \frac{g_i(\hat{v}_j + p_i - p_j \mid \hat{v}_j)}{1 - G_i(\hat{v}_j + p_i - p_j \mid \hat{v}_j)}(1 - G_i(\hat{v}_j + p_i - p_j \mid \hat{v}_j))h_j(\hat{v}_j)d\hat{v}_j \\ &< \int_0^{p_j} \frac{g_i(p_i \mid \hat{v}_j)}{1 - G_i(p_i \mid \hat{v}_j)}(1 - G_i(\hat{v}_j + p_i - p_j \mid \hat{v}_j))h_j(\hat{v}_j)d\hat{v}_j \\ &\leq \frac{g_i(p_i \mid p_j)}{1 - G_i(p_i \mid p_j)}[q_i - q_b] \ . \end{aligned}$$

It follows that

$$-\frac{1}{[q_i - q_b]}\frac{\partial [q_i - q_b]}{\partial p_i} < \frac{g_i(p_i \mid p_j)}{1 - G_i(p_i \mid p_j)} \leq -\frac{1}{q_b}\frac{\partial q_b}{\partial p_i} \ .$$

Since demand $q_i - q_b$ is less elastic than q_b, it follows that the elasticity of q_i, which is an average of the elasticities of $q_i - q_b$ and q_b, is also smaller than q_b, and so Proposition 3 implies the result. ∎

Proof of Claim 4: Consider firm 1's incentive to offer a discount. From Figure 3A, given independence of v_1 and v_2, the elasticity of total demand from the λ unconstrained consumers is equal to elasticity of bundle demand from these consumers, and equal to $f_1(p_1)/(1 - F_1(p_1))$.

From Figure 3B, the demand from the time-constrained consumers, say y, is

$$y = F_2(p_2)(1 - F_1(p_1)) + \int_{p_2}^{\infty} f_2(v_2)(1 - F_1(v_2 + p_1 - p_2))dv_2$$

and so

$$
\begin{aligned}
-\frac{\partial y}{\partial p_1} &= F_2(p_2)f_1(p_1) + \int_{p_2}^{\infty} f_2(v_2)f_1(v_2 + p_1 - p_2)dv_2 \\
&= F_2(p_2)f_1(p_1) + \int_{p_2}^{\infty} f_2(v_2)(1 - F_1(v_2 + p_1 - p_2))\frac{f_1(v_2 + p_1 - p_2)}{1 - F_1(v_2 + p_1 - p_2)}dv_2 \\
&> F_2(p_2)f_1(p_1) + \frac{f_1(p_1)}{1 - F_1(p_1)} \int_{p_2}^{\infty} f_2(v_2)(1 - F_1(v_2 + p_1 - p_2))dv_2 \\
&= F_2(p_2)f_1(p_1) + \frac{f_1(p_1)}{1 - F_1(p_1)}(y - F_2(p_2)(1 - F_1(p_1))) \\
&= \frac{f_1(p_1)}{1 - F_1(p_1)}y .
\end{aligned}
$$

Here, the inequality follows from the assumption that the hazard rate is strictly increasing. It follows that

$$
-\frac{1}{y}\frac{\partial y}{\partial p_1} > \frac{f_1(p_1)}{1 - F_1(p_1)} ,
$$

and so (total) demand from the time-constrained consumers is more elastic than total demand from the unconstrained consumers. Putting the two groups of consumers together, we see that total demand is therefore more elastic than bundle demand. Invoking Proposition 3 completes the proof. ∎

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