INVESTMENT INCENTIVES IN TWO-SIDED PLATFORMS*

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Abstract. We study investment incentives in proprietary and open platforms. We find that for a given a level of platform adoption, investment incentives are stronger in proprietary platforms. However, open platforms may receive larger investment because they benefit from user innovation and may lead to wider adoption, which raises the return to investment. Application prices do not affect equilibrium investment and adoption in proprietary platforms. The same is not true for open platforms. In particular, we derive conditions under which an increase in application prices leads to an increase in both, user and developer adoption in open platforms. We also study a mixed duopoly model and examine how the price structure and investment incentives of the proprietary platform are affected by investment in the open platform.

Keywords: Two-Sided Markets, Platform Investment, Network Effects, Open Source, User Innovation, Complementarity (JEL O31, L17, D43).

1. Introduction

While proprietary and open source software have coexisted since the early days of the computing industry, competition between these two modes of development has intensified dramatically following the surge of the Internet in the mid-1990s. Prominent examples include Windows vs. Linux, MS Office vs. Open Office, Safari vs. Firefox, MS Internet Server vs. Apache, and more recently, Apple's iOS vs. Google's Android. The coexistence of these two diametrically opposed modes of platform governance has sparked a thriving literature on open source examining why individuals and profit-maximizing firms might choose to contribute to open source development (see Lerner and Tirole, 2005; von Krogh and von Hippel, 2006; Fershtman and Gandal, 2011, for recent surveys).

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While insightful and enlightening, theoretical developments on open source have fallen short of fully embracing the modeling breakthroughs offered by the literature on two-sided platforms of the past decade (e.g., Caillaud and Jullien, 2003; Rochet and Tirole, 2003, 2006; Armstrong, 2006; Hagiu, 2006a; Spulber, 2006; Weyl, 2010). Likewise, while the literature on two-sided platforms has studied some aspects of open platforms, the most distinctive feature of open source (namely, user and developer innovation) has not been considered.[1]

In this paper, we bring together work on these two streams of work to address the following questions: how are the incentives to invest in platform quality affected by the degree of platform openness? which of these two modes of governance leads to investments closer to the social optimum? and how are incentives to invest in platform quality moderated by competition between proprietary and open two-sided platforms?

We set up a model of a platform that brings together users (buyers) and developers (sellers) of applications. Users are heterogeneous in their willingness to pay for access to the platform. Developers are also heterogeneous in that they bear different costs of developing applications. A proprietary platform chooses how much to invest in platform quality and sets access prices for each side of the market. An open platform may be accessed for free and users and developers may invest in improving its quality. Regardless of whether the platform is proprietary or open, after users and developers have accessed the platform, developers compete to sell applications to users. As in Dixit and Stiglitz (1977), we assume that users prefer product variety but consider applications as interchangeable.

Our model has three distinctive features. First, there is endogenous investment in platform quality, which determines the value of the platform for consumers. User investment in open platforms is endogenous, in contrast with previous works on user innovation, which assume it is exogenously given. Second, along with the case of *substitute* applications whose value decreases with the number of applications available, we study the mirror case of *complement* applications. Finally, we do not assume that users are price takers. Rather, users may have bargaining power to negotiate application prices. User bargaining power may emanate from structural features or from price sensitiveness due to the presence of substitute product categories (such as pirated versions of the software).

[1]To this literature, an open platform is one that is free to access, and a proprietary platform is one that sets access prices, possibly different than zero.

We divide the analysis into two parts. We first examine models of proprietary and open monopoly platforms; that is, we consider incentives to invest by proprietary and open platforms in isolation from each other and compare equilibrium outcomes. In the second part of the paper, we analyze a duopoly model with direct competition between both types of platforms.

In the case of a monopoly proprietary platform, we find that the structure of access prices depends on the nature of cross-payments between users and developers, which in turn is affected by the technical characteristics of applications (whether they are substitutes or complements), and by the bargaining power of users and developers. The platform may choose to subsidize one side of the market or no side at all. Users (developers) are more likely to be subsidized in equilibrium when cross-payments are high (low).

In contrast with access prices, user and developer entry and platform quality do not depend on the cross-payments between users and developers. The platform provider internalizes the effect of cross-payments through access prices, which leads to the same levels of entry and investment, regardless of the price of applications.

We also find that there is insufficient entry compared to the first best and that the size of the inefficiency grows with the rate at which users' and developers' entry costs increase with entry. Finally, proprietary platforms would choose to invest in quality at the socially optimal level if entry was efficient. However, because entry is inefficient, investment in quality winds up being inefficient also.

In the case of a monopoly open platform, we find that entry is inefficient. An open platform provides the right incentives for user entry when the price of applications is zero, but in this case, it provides insufficient incentives to developers. As a consequence, user and developer entry are suboptimal due to the existence of indirect network effects. On the developer side, the platform provides the right incentives for developer entry when developers have full bargaining power and applications are *substitutes*, but in this case it provides insufficient incentives to users. When applications are *complements*, incentives for developer entry are insufficient even if developers have full bargaining power, because in this case, a developer's entry decision depends on its average contribution to user utility instead of its marginal contribution.

We also find that users and developers invest less in platform quality than what would be socially optimal. Users do not take into account the effect of their quality investments in other users' utility. Moreover, they recognize that the more they invest, the more expensive applications become. Developer investment is inefficient because developers do not appropriate the full increase in platform quality.

An increase in developer bargaining power (i.e. application prices) may lead to an increase in both user and developer adoption. In particular, an increase in the bargaining power of developers will lead to higher user adoption (and consumer surplus), if developers would be subsidized if the platform was proprietary. Likewise, a decrease in developer bargaining power will lead to higher developer adoption (and developer profits), if users would be subsidized if the platform was proprietary.

Our comparison of proprietary and open platforms shows that, holding all else constant, open platforms always provide stronger incentives for entry to at least one side of the market. However, a proprietary platform may provide stronger incentives for one side if it chooses to subsidize it. On the other hand, taking user and developer access as given, proprietary platforms provide better incentives for investment. Nonetheless, open platforms may end up with higher investment and quality than proprietary platforms because (i) open platforms allow users to innovate, which is impossible in proprietary platforms, and (ii) they may lead to more entry by users and/or developers, which would raise the returns to investment and could offset the investment incentives disadvantage.

From a welfare point of view, proprietary and open platforms are both inefficient, but the reasons for inefficiency differ. In general terms, proprietary platforms are welfare superior to open platforms if investments in platform quality are more important than the effect of product variety on user utility, and vice versa.

Finally, we study competition between a proprietary and an open platform. We find equilibrium prices depend on the equilibrium relation between entry and investments. In particular, the effect of investment incentives on prices depends on whether increases in user and developer investment in the open platform lead to higher or lower profits for the proprietary platform, and on whether increases in user and developer entry lead to increases or decreases in user and developer investment. For example, suppose that (i) an increase in the number of users in the proprietary platform lowers investment incentives in the open platform; (ii) developers multi-home, so that an increase in the number of developers in the proprietary platform increases investment incentives in the open platform; and

(iii) an increase in the investment in the open platform leads to lower profits for the proprietary platform. Then, the proprietary platform provider will lower user access price and will raise developer access price, compared to a situation in which there is no investment.

In contrast to the case of a monopolist proprietary platform, the bargaining power of users matters when determining equilibrium entry, even if there are no investment incentives. This is because, when developers multi-home, entry depends partially on the revenues they obtain from selling applications in the open platform, which depend on their bargaining power.

Finally, we find that equilibrium investment in the proprietary platform increases relative to the investment in the open platform as the equilibrium market share of the proprietary platform increases, and as the bargaining power of developers in the application market decreases.

1.1. **Related literature.** Our paper contributes to the literature on multisided markets and the economics of open source. Seminal work by Caillaud and Jullien (2003) studies competition between match-making intermediaries with indirect network effects in a model with agents on both sides choosing platform membership simultaneously. Multiple equilibria exist because membership on either side depends on the expected number of members on the other side. The efficient industry structure may be a monopoly or a duopoly and an efficient equilibrium always exists, but inefficient equilibria exist too. While our platforms also exhibit indirect network effects and both sides choose simultaneously whether to adopt, and while we find that multiple equilibria exist, our model best captures the economics of hardware-software platforms where developers sell applications to users, rather than matchmaking.

A large share of the extant literature on two-sided platforms studies pricing in the presence of network effects (e.g., Spulber, 1996; Rochet and Tirole, 2003, 2006; Armstrong, 2006; Hagiu, 2006a; Casadesus-Masanell and Ruiz-Aliseda, 2008; Weyl, 2010). In general terms, the structure of equilibrium prices depends on the relative size of demand elasticities and cross-group externalities, the costs of serving each side of the market, the market structure, and whether end-users single-home or multi-home. Although we focus on the incentives to invest in platform quality, we also derive the access prices charged by proprietary platforms in equilibrium and obtain results congruous with the existing literature. Closer to our setting, Hagiu (2006b) and Economides and Katsamakas (2006b) compare proprietary and open platforms. These papers model open platforms as free-access platforms. While we also assume zero access prices to open platforms, we allow for user and developer innovation to improve platform quality.

Incentives to invest in quality by proprietary and open platforms have not been analyzed in detail before. Hagiu (2007), Belleflamme and Peitz (2010) and Lin, Li, and Whinston (2011) study sellers' investment incentives. Our work is closer to Economides and Katsamakas (2006a) who examine incentives to invest in a one-sided platform with one application developer. These authors compare proprietary and open source operating systems (OSs). In a proprietary OS, quality-enhancing investments are made by the platform owner; in an open OS, investments are made by the application developer and advanced users.[2] They find that the incentives to invest in the application are generally larger when the platform is open, and that investment in the open source OS is larger if there are strong reputation effects from participation in open source development, and/or a significant part of the open source users are developers.

Rather than one-sided operating systems, we consider two-sided platforms. In our setting, the proprietary platform chooses access prices for two sides and may subsidize one to better exploit indirect network effects. Moreover, we allow for endogenous platform adoption by users and developers and, contrary to Economides and Katsamakas (2006a), in our model there is always a large number of users and developers. We do not consider the role of reputation from participation in open source development on developers' and users' incentives to invest. Our analysis thus shows that such reputational concerns are not necessary for an open platform to obtain higher investment than a proprietary one.

The early literature on open source was concerned with explaining why individual developers contributed to open source projects allegedly for free (Lerner and Tirole, 2005; von Krogh and von Hippel, 2006; Fershtman and Gandal, 2011, present excellent surveys). The most common explanations were: altruism, personal gratification, peer recognition, and career concerns. We do not consider social preferences or career concerns. Rather, we focus on self-interested agents and examine the value of investments in the platform to the very users and developers who make those investments.

[2]Advanced users are users who invest in platform quality to maximize reputation.

While the contributions of individual users have played a crucial role for the success of many open source projects, the same is true of contributions by developers and commercial firms. Shah (2006) investigates the effects of sponsorship of open source projects by commercial firms and finds that voluntary developers tend to contribute less, have different motivations for contributing, and take on fewer code maintenance tasks than in the absence of such sponsorship. To the best of our knowledge, ours is the first theoretical paper on open platforms that, in addition to endogenous user innovation, explicitly considers the incentives by developers to contribute to improve the quality of the platform.

Papers examining competition between open source and proprietary software have considered duopoly models of a profit-maximizing, proprietary firm and a community of not-for-profit/non-strategic open source user-developers selling at zero price (Mustonen, 2003; Bitzer, 2004; Gaudeul, 2005; Casadesus-Masanell and Ghemawat, 2006; Economides and Katsamakas, 2006b; Lee and Mendelson, 2008; Casadesus-Masanell and Llanes, 2011). These papers, however, assume that investment incentives are exogenously given (generally, investment in open source is a function of the number of users). An exception is Llanes and de Elejalde (2009), who assume investment is performed by sellers of complementary goods. In addition, for the most part, the literature on mixed duopoly presents models of one-sided firms. We contribute work in this area by endogenizing user and developer investment incentives and by considering interactions between two-sided platforms.

The rest of the paper is organized as follows. In Section 2 we present the model. Section 3 characterizes the socially optimal allocation. In Sections 4 and 5 we study the cases of proprietary and open platforms, and in Section 6 we compare outcomes across the two governance modes. Section 7 studies equilibrium investment in a mixed duopoly where a proprietary and an open platform compete for users. Section 8 concludes.

2. The model

We model a two-sided monopoly platform that brings together application developers and users.[3] There is a continuum of potential users and developers. Users

[3]More generally, our model applies to any technology platform allowing the interaction between sellers and buyers.

demand applications and run them on the platform. The platform may be software (e.g., an operating system), hardware (e.g., a DVD player), or a combination of the two (e.g., a video game console). Users have quasilinear utility functions. The indirect utility of user i is

$$u(i) = v(n, x, z) - \int_0^n \rho(j)\, dj - h(i) - p^u - \sigma\, z(i), \tag{1}$$

where n is the measure of available applications, x is the investment in platform quality by the platform provider or by developers, $z(i)$ is user i's investment in platform quality (user innovation), σ is the marginal cost borne by users when investing in platform quality, $h(i)$ is a user-specific adoption cost, p^u is the platform access fee for users, and $\rho(j)$ is the price of application j.[4] As we discuss below, user investments may be positive only when the platform is open.

Function $v(n, x, z)$ is the gross utility of consuming n applications when the platform has received quality investments x and z. We follow the usual convention of representing derivatives through subscripts (e.g., $v_{nx} = \frac{\partial^2 v(n,x,z)}{\partial n\, \partial x}$). Users prefer higher quality platforms and application variety ($v_x > 0$, $v_z > 0$, and $v_n > 0$). Investments in platform quality and the measure of applications are complements (v_{nx}, $v_{nz} \geq 0$). When $v_{nn} = 0$ applications are independent in that consuming more of any one application does not affect the marginal utility of consuming any other application. The cases $v_{nn} < 0$ and $v_{nn} > 0$ correspond to applications being substitutes and complements. When $v_{nn} < 0$, we have $v(n_1, x, z) + v(n_2, x, z) > v(n_1 + n_2, x, z)$ and applications detract from each other. The reverse is true for complements.

Without loss of generality, let $h(0) = 0$. Consumers are ordered according to cost so that $h_i > 0$. Let m be the measure of users entering the market.

Each developer may produce one application. Developer j's profits are

$$\pi(j) = \rho(j)\, m - c(j) - p^d - \kappa\, x(j), \tag{2}$$

where m is the measure of users, $c(j)$ is a developer-specific development cost, κ is the marginal cost of investing in platform quality, $x(j)$ is developer j's investment in platform quality, and p^d is the platform access fee for developers. Developer investments $x(j)$ may be positive only when the platform is open. Developers are ordered according to cost so that $c_j > 0$. Assume $0 \leq c(0) \leq v_n(0, x, z)$, which

[4]Function h may also be interpreted as a taste differentiation parameter or transportation cost.

means that having a positive number of applications is always desirable from a social point of view.

Prices are determined through Nash bargaining. In particular, suppose the surplus from buying an application (i.e. the surplus generated by an interaction between a user and a developer) is θ. Then, the equilibrium price will be $\alpha\,\theta$, where $\alpha \in [0,1]$ represents the bargaining power of developers. If $\alpha = 1$, developers are price setters. If $\alpha = 0$, users have all the bargaining power and there can be no cross-payments between both sides. In a market in which software piracy is pervasive, for example, α will be close to zero, and prices will tend to be low.

The platform may be proprietary or open source (hereinafter referred to as "open"). When the platform is proprietary, investment is performed by the platform provider and access prices may be different from zero (positive or negative). When the platform is open, investment is performed by developers and users, and access prices are zero. As noted, the existing literature on open platforms in multisided markets has only considered the zero-price dimension of open source, and has not studied the implications of open source on the incentives for innovation. We include this important aspect of open platforms to our model and analysis.

3. Social planner

We begin by solving for the first-best allocation. The social planner chooses m, n, x, and z to maximize the sum of indirect utility and profits:

$$\begin{aligned} W &= \int_0^m u(i)\,di + \int_0^n \pi(j)\,dj, \\ &= m\,v(n,x,z) - \int_0^m h(i)\,di - \int_0^n c(j)\,dj - \sigma\,z - \kappa\,x. \end{aligned}$$

The equations characterizing the first best (obtained straightforwardly differentiating W with respect to m, n, x, and z) are:

$$v = h, \qquad m\,v_n = c, \qquad m\,v_x = \kappa, \qquad m\,v_z = \sigma.$$

4. Proprietary platform

The timing of the game is the following: (i) the platform provider chooses x, p^u, and p^d, (ii) users and developers decide whether to join the platform, and (iii) developers and users bargain over $\rho(j)$, and users choose how many applications to buy. The equilibrium concept is subgame perfect equilibrium, and we solve the model recursively.

Since users and developers cannot invest in platform quality, equations (1) and (2) become

$$\begin{aligned} u(i) &= v(n,x,0) - \int_0^n \rho(j)\,dj - h(i) - p^u, \text{ and} \\ \pi(j) &= \rho(j)\,m - c(j) - p^d. \end{aligned}$$

In the third stage, developers and users bargain over the price of applications, $\rho(j)$. The price is determined differently when applications are substitutes and complements. When applications are substitutes, the maximum price an application developer may charge is v_n (if the price of any application was greater than the marginal value of the last application, users would be better off not consuming that application). The actual price paid by users is determined through bargaining and is equal to $\alpha\, v_n$.

When applications are complements, the maximum price is no longer v_n. To see this, note that if price was v_n, the total cost of a bundle of n applications would be larger than its gross utility to users ($n\, v_n > v(n,x,0) - v(0,x,0)$), and thus users would be better off not buying any application. Therefore, in equilibrium we must have $\int_0^n \rho(j)dj \leq v(n,x,0) - v(x,0,0)$. In a symmetric equilibrium, the maximum application price is $(v(n,x,0) - v(0,x,0))/n$. Let $w(n,x,z) = (v(n,x,z) - v(0,x,z))/n$, which is increasing in n when applications are complements ($w_n = (v_n - w)/n > 0$). The actual price paid by users is determined through bargaining and equal to $\alpha\, w$.

In the second stage, users and developers choose whether to access the platform. The marginal entrants, m and n, satisfy $v(n,x,0) - n\,\rho^* = h(m) + p^u$, and $m\,\rho^* = c(n) + p^d$, where ρ^* represents the third-stage equilibrium application price. From here, we obtain the inverse demand functions:

$$p^u = v(n,x,0) - n\,\rho^* - h(m), \tag{3}$$

$$p^d = m\,\rho^* - c(n). \tag{4}$$

Since ρ^* does not depend on m, $\partial\, p^u/\partial\, m = -h_m < 0$ with substitutes and complements. With substitutes, $\partial\, p^d/\partial\, n = \alpha\, m\, v_{nn} - c_n$, which is always negative. With complements, $\partial\, p^d/\partial\, n = \alpha\, m\, w_n - c_n$, which is negative if and only if $n\, c_n > \alpha\, m\,(v_n - w)$. We will assume that this holds whenever n solves (4), which in turn means there is only one (m,n) pair verifying (3) and (4) for a given set of prices p^u and p^d.

The following lemma shows that application prices do not affect the equilibrium allocation m, n, x arising from platform choices. Nonetheless, application prices do affect the level of access fees.

Lemma 1 (Neutrality of cross-payments). *In a proprietary platform, the equilibrium allocation m, n, and x is the same, regardless of the size of cross-payments between users and developers (application prices).*

Proof. The platform chooses m, n, and x to maximize total profits, $m\,p^u + n\,p^d - \kappa\,x$. There is a one-to-one correspondence between m, n, and p^u, p^d. Replacing prices by inverse demand functions in the profit function we obtain $m\,(v - n\,\rho^* - h(m)) + n\,(m\,\rho^* - c(n)) - \kappa\,x$. Rearranging terms, profits can be rewritten as $m\,v - m\,h(m) - n\,c(n) - \kappa\,x$, which clearly does not depend on ρ^*. ■

The platform provider internalizes the effect of cross-payments on m, n, and x, and chooses p^u and p^d to neutralize their effect. For Lemma 1 to hold, two conditions must be satisfied. First, the platform provider must be able to price both sides of the market. If the platform provider could not price one side of the market (for example, if platform access cannot be verified for one side), then it would not be able to transfer utility from one side to the other. Second, the market must exhibit pure membership externalities.[5] As we show in Section 5, Lemma 1 does not hold for open platforms. Thus, while the nature of cross-payments does not matter for proprietary platforms, it does play a role for open platforms.

In the first stage, the platform provider chooses x, p^u, and p^d to maximize profits $m\,p^u + n\,p^d - \kappa\,x$. The following proposition characterizes the equilibrium.

Proposition 1 (Proprietary platform). *An equilibrium exists and is unique. The measure of users and developers (m, n), and the investment in platform quality (x), satisfy $v = h + m\,h_m$, $m\,v_n = c + n\,c_n$, and $m\,v_x = \kappa$. When applications are substitutes, $\rho^* = v_n$, $p^u = m\,h_m - \alpha\,n\,v_n$, and $p^d = n\,c_n - (1-\alpha)\,m\,v_n$. When applications are complements, $\rho^* = w$, $p^u = m\,h_m - \alpha\,n\,w$, and $p^d = n\,c_n - m\,(v_n - \alpha\,w)$.*

Proof. According to Lemma 1, the platform provider solves the following problem:

$$\max_{m,n,x} \quad m\,v(n,x,0) - m\,h(m) - n\,c(n) - \kappa\,x.$$

[5] See Armstrong (2006) and Rochet and Tirole (2006) for further details.

The first order conditions are $v = h + m\,h_m$, $m\,v_n = c + n\,c_n$ and $m\,v_x = \kappa$. Substituting the first two expressions in the inverse demand functions, we obtain the optimal access prices stated in the proposition. ■

Free entry implies that the marginal user and developer obtain zero utility and profit in equilibrium. Therefore, the net utility of user $i < m$ in equilibrium is $u(i) = h(m) - h(i)$, and the profit of developer $j < n$ is $\pi(j) = c(n) - c(j)$.

The condition determining x in the proprietary platform is the same as that of the first best. Therefore, if m, n, and z were set at their socially optimal levels, investment would be optimal. A proprietary platform sets access prices in order to capture the full increase in user surplus due to an increase in x, and thus has the correct incentives to invest in product quality.

However the conditions determining m and n are different from those of the first best, which means that x will be set at an inefficient level. Efficiency requires that the value of the platform is equal to the entry cost of the marginal user ($v = h$), and that the marginal benefit of the marginal application is equal to the entry cost of the marginal developer, ($m\,v_n = c$). The platform provider does not fully internalize the marginal benefits of increases in m and n, and thus sets prices that lead to insufficient entry.

Turning to the analysis of prices, the platform adjusts p^u and p^d so that users obtain gross utility v and developers obtain a net revenue of $m\,v_n$. Developers may be subsidized in equilibrium only when α is low, and thus, the prices they obtain from applications are also low. When $\alpha = 1$ developers are not subsidized. Likewise, users may be subsidized when $\alpha > 0$ but are not subsidized when $\alpha = 0$. When $\alpha = 0$, gross utility is already equal to v and no further adjustments through p^u are needed.

Equilibrium access prices can be written using formulas for price elasticity of demand. In particular, $\varepsilon^m_{p^u} = \frac{1}{m\,h_m}\,p^u$ is the price elasticity of user demand and $\varepsilon^n_{p^d} = \frac{1}{n\,c_n - m\,n\,\rho^*_n}\,p^d$ is the price elasticity of developer demand. As shown by the previous literature (Armstrong, 2006; Rochet and Tirole, 2003, 2006), equilibrium prices can be expressed as a combination of elasticities and indirect network effects:

$$\frac{p^u + n\,\rho^*}{p^u} = -\frac{1}{\varepsilon^m_{p^u}}, \quad \text{and} \quad \frac{p^d + m\,(v_n - \rho^* - n\,\rho^*_n)}{p^d} = -\frac{1}{\varepsilon^n_{p^d}}.$$

Here, we show that the technical characteristics of applications (whether applications are complements or substitutes) and the nature of payments between users and developers (application prices) affect indirect network effects, i.e. the structure of access prices depends on the ability of developers to charge users for the use of their applications.

5. Open platform

We now turn to the case of an open platform. In open platforms, investment in quality is decentralized: each user and developer chooses independently how much to invest in platform quality. By their very nature, open platforms have unstructured entry and investment. Therefore, m, n, $z(i)$, and $x(j)$ are determined simultaneously in the first stage. Application prices ($\rho(j)$) are set in a second stage.

Since access to the platform is free, equations (1) and (2) become:

$$\begin{aligned} u(i) &= v(n,x,z) - \int_0^n \rho(j)\,dj - h(i) - \sigma\,z(i), \text{ and} \\ \pi(j) &= \rho(j)\,m - c(j) - \kappa\,x(j), \end{aligned}$$

where $x = \int_0^n x(j)\,dj$ and $z = \int_0^m z(i)\,di$.

In open platforms, cross-payments between users and developers matter because they affect their incentives to join the platform and invest in platform quality. Because application prices are determined differently when they are substitutes and complements, we study both cases separately. Proposition 2 summarizes the equilibrium choices of users and developers when applications are substitutes.

Proposition 2 (Open platform with substitute applications)**.** *An equilibrium exists, but there may be multiple equilibria. In any equilibrium, quality investments by users and developers (z, x), satisfy* $v_z - \alpha\,n\,v_{nz} = \sigma$ *and* $\alpha\,m\,v_{nx} = \kappa$*. The measure of users and developers (m, n) solves* $h = v - \alpha\,n\,v_n$ *and* $c = \alpha\,m\,v_n$*. Application prices are* $\rho = \alpha\,v_n$*.*

Proof. By the arguments brought forward in Section 4, application price is $\alpha\,v_n$. In the first stage, users and developers choose whether to enter and how much to invest in improving the quality of the platform. In choosing how much to invest, users solve

$$\max_{z(i)} v(n,x,z) - \alpha\,n\,v_n(n,x,z) - h(i) - \sigma\,z(i),$$

and developers solve

$$\max_{x(j)} \alpha\,m\,v_n(n,x,z) - c(j) - \kappa\,x(j).$$

The first order conditions yield $v_z - \alpha\, n\, v_{nz} = \sigma$ and $\alpha\, m\, v_{nx} = \kappa$. Finally, free entry implies that the marginal user and developer obtain zero utility and profit. The marginal agents do not invest in platform innovation. Therefore, in equilibrium we must have $v - \alpha\, n\, v_n - h(m) = 0$, and $\alpha\, m\, v_n - c(n) = 0$. ■

In equilibrium, users obtain $u(i) = h(m) - h(i) - \sigma\, z(i)$, and developers earn $\pi(j) = c(n) - c(j) - \kappa\, x(j)$. Because $h_i > 0$ and $c_j > 0$, larger equilibrium entry by users and/or developers implies more user utility and developer profit.

Open source does not force agents to invest in platform quality. If $v - \alpha\, n\, v_n > h(j)$, user j finds it optimal to enter and there will be entry until $v - \alpha\, n\, v_n = h$. Likewise, developers will enter until $\alpha\, m\, v_n = c$. Marginal entrants do not invest in platform quality.

There is an efficiency trade-off between the conditions determining m and n. As α increases, the condition determining n gets closer to the socially optimal one, but the condition determining m moves away from the socially optimal one. The condition determining m is the same as that of the first best only when $\alpha = 0$. Note however, that even in this case, m will be suboptimal because n determined inefficiently. Likewise, the condition determining n is the same as that of the first best only when $\alpha = 1$, but n will be suboptimal because m is determined inefficiently.

As we show in Lemma 2, an increase in α may lead to an increase (or decrease) in both m and n, because of the existence of indirect network effects. For example, the positive effect of the increase in n on user utility may more than offset the negative effect of the increase in application prices, thereby leading to a higher m.

Lemma 2 (Barganing power and entry in open platforms)**.** *Suppose x and z are fixed. Then,*

$$\frac{dm}{d\alpha} = -\frac{n\, c_n - (1-\alpha)\, m\, v_n}{D} \qquad \text{and} \qquad \frac{dn}{d\alpha} = \frac{m\, h_m - \alpha\, n\, v_n}{D},$$

where

$$D = \frac{c_n\, h_m}{v_n} - \alpha \left(\frac{m\, h_m\, v_{nn}}{v_n} + (1-\alpha)\, v_n - \alpha\, n\, v_{nn} \right).$$

Proof. The equilibrium conditions are $v - \alpha\, n\, v_n - h(m) = 0$ and $\alpha\, m\, v_n - c(n) = 0$. Holding x and z constant, the total differential with respect to α is

$$\begin{aligned} h_m \frac{dm}{d\alpha} + (v_n - \alpha\, v_n + \alpha\, n\, v_{nn}) \frac{dn}{d\alpha} + n\, v_n &= 0, \\ \alpha\, v_n \frac{dm}{d\alpha} + (\alpha\, m\, v_{nn} - c_{nn}) \frac{dn}{d\alpha} + m\, v_n &= 0, \end{aligned}$$

which yields a system of two equations and two unknowns. Solving this system, we obtain $dm/d\alpha$ and $dn/d\alpha$. ■

A sufficient condition for $D > 0$ is $v_{nnn} < 0$. To see this, note that the denominator can be rewritten as

$$D = \frac{(1-\alpha)\, c_n\, h_m}{v_n} + \alpha \left(\frac{h_m\,(c_n - m\, v_{nn}) - v_n^2}{v_n} + \alpha\,(v_n + n\, v_{nn}) \right).$$

The first term is always positive. The second-order condition of the social planner problem implies that $h_m\,(c_n - m\, v_{nn}) - v_n^2 > 0$. We are left with $v_n + n\, v_{nn}$, which is positive when $v_{nnn} < 0$. Note that the denominator may be positive even if $v_{nnn} > 0$, as long as v_{nnn} is not too large.

If the denominator is positive, $dm/d\alpha > 0$ if and only if $n\, c_n - (1-\alpha)\, m\, v_n < 0$, and $dn/d\alpha > 0$ if and only if $m\, h_m - \alpha\, n\, v_n > 0$. Therefore, if a *proprietary* platform would choose to subsidize *developers*, the number of *users* would increase with α in an *open* platform. Otherwise, $dm/d\alpha < 0$. Likewise, if a proprietary platform would choose to subsidize *users*, the number of *developers* would decrease with α in an open platform. Otherwise, $dn/d\alpha > 0$. In addition, note that the effects depend on the level of α. Other things equal, $dm/d\alpha > 0$ is more likely for low α and $dn/d\alpha < 0$ is more likely for high α.

Intuitively, when α is very low, developers' revenue from the sale of applications is too low and there is too little entry in the developer side, which hurts consumer utility and leads to low entry in the user side. An increase in α leads to more developer entry, which in turn benefits users and improves user entry. This is exactly the same case in which a proprietary platform would choose to subsidize developers. A similar intuition applies to $dn/d\alpha < 0$ when $p^u < 0$.

Equilibrium investments do not depend on the exact distribution of investments among agents. As long as $v_z - \alpha\, n\, v_{nx} > \sigma$, any user will find it optimal to increase its investment in platform quality. In equilibrium, $v_z - \alpha\, n\, v_{nx} = \sigma$, regardless of who is investing. Likewise, in equilibrium $\alpha\, m\, v_{nx} = \kappa$.

Incentives for user investment are insufficient for two reasons. First, when deciding how much to invest, users do not take into account the effect that that investment has on the utility of other users. Second, users take into account that application prices v_n increase with z, which dampens their incentives to invest. Likewise, the incentives for developer investments are insufficient because developers do not fully internalize the effect of an increase in x in user utility.

We turn now to the case of complement applications. Proposition 3 summarizes the equilibrium choices of users and developers in this case.

Proposition 3 (Open platform with complement applications)**.** *An equilibrium exists, but there may be multiple equilibria. In any equilibrium, quality investments by users and developers (z, x), satisfy* $v_z - \alpha\, n\, w_z = \sigma$ *and* $\alpha\, m\, w_x = \kappa$*. The measure of users and developers (m, n) solves* $h = v - \alpha\, n\, w$ *and* $c = \alpha\, m\, w$*. Application prices are* $\rho = \alpha\, w$*.*

Proof. The proof follows similar steps than the proof of Proposition 2, taking into account that the price of applications is now equal to $\alpha\, w$. ■

Most of the intuitions developed above for substitutes also apply to complements. An important difference is that in the case of complements, the condition for n is inefficient *even if* $\alpha = 1$. The reason is that when applications are complements, revenues of developers depend on their applications' *average* contribution to consumer gross utility instead of their marginal contribution. Therefore, in the case of complements, even if m, x, and z were set at their optimal levels, developer entry would be inefficient.

6. Comparison

In this section, we compare the equilibrium conditions determining m, n, x, and z for proprietary and open platforms. Table 1 presents a summary of results. We will compare entry and investment incentives analyzing one condition at a time, holding everything else constant. Obviously, all variables are jointly determined, thus there are interactions that we are not considering in a one-on-one comparison.

The first difference between proprietary and open platforms is that α does not play a role in the former, but matters for the latter. As noted, a proprietary platform provider internalizes the effect of α on user and developer entry, and chooses access prices so that the desired level of entry occurs regardless of the distribution of bargaining power between both sides of the market.

	Substitutes	Complements
Welfare optimum	$v = h$ $m\,v_n = c$ $m\,v_z = \sigma$ $m\,v_x = \kappa$	
Proprietary platform	$v = h + m\,h_m$ $m\,v_n = c + n\,c_n$ $z = 0$ $m\,v_x = \kappa$	
Open platform	$v - \alpha\,n\,v_n = h$ $\alpha\,m\,v_n = c$ $v_z - \alpha\,n\,v_{nz} = \sigma$ $\alpha\,m\,v_{nx} = \kappa$	$v - \alpha\,n\,w = h$ $\alpha\,m\,w = c$ $v_z - \alpha\,n\,w_z = \sigma$ $\alpha\,m\,w_x = \kappa$

Table 1. Comparison of results

Turning to entry, we see that whether proprietary platforms provide higher or lower incentives than open platforms for user entry depends on the comparison between $m\,h_m$ and $\alpha\,n\,v_n$ in the substitutes case, and between $m\,h_m$ and $\alpha\,n\,w$ in the complements case. If α is sufficiently close to zero, open platforms provide stronger incentives for user entry. However, even when $\alpha = 1$, we cannot guarantee that proprietary platforms provide stronger incentives for user entry. Recall that the equilibrium access price for users is $p^u = m\,h_m - \alpha\,n\,v_n$ (substitutes) and $p^u = m\,h_m - \alpha\,n\,w$ (complements). Thus, incentives for user entry are stronger in proprietary platforms compared to open platforms when users are subsidized in equilibrium ($p^u < 0$).

A similar comparison can be made for the relative strength of incentives for developer entry. Proprietary platforms provide stronger incentives when $n\,c_n$ is less than $(1-\alpha)\,m\,v_n$ (substitutes) or $(1-\alpha)\,m\,w$ (complements), which is the same condition determining whether developers are subsidized by the proprietary platform provider.

Finally, we compare equilibrium investment in open vs. proprietary platforms. Incentives for z are nonexistent in the case of proprietary platforms for the simple reason that users cannot contribute quality improvements. Comparing x's we see that the condition that determines the equilibrium quality investment for an open platform is $\alpha\, m\, v_{nx} = \kappa$ (substitutes) or $\alpha\, m\, w_x = \kappa$ (complements) and for the proprietary platform the condition is $m\, v_x = \kappa$. Holding everything else constant (i.e., given m, n, and assuming $z = 0$), equilibrium investment in an open platform is lower than in a proprietary platform, even if $\alpha = 1$. In the case of complements, this follows from $w_x < v_x$ which, as we have argued above, always holds. In the case of substitutes, even though v_{nx} could be larger than v_x from a mathematical point of view, it is only reasonable to assume that $v_{nx} < v_x$. To understand why, note that if the model had a discrete number of developers, then v_{nx} would be defined as $v_x(n, x, z) - v_x(n-1, x, z)$, which is always smaller than $v_x(n, x, z)$.

In any case, investment may be larger in an open platform compared to a proprietary one. The reason is that m, n, and x are determined jointly, and open platforms also have user innovation. For instance, n may be larger when the platform is open, which could lead to stronger incentives to invest in the open platform, as long as v_{nnx} is positive and sufficiently large. Similar arguments can be made for m and z. Indeed, in Section 6.1 we provide an example where a platform that is open has larger investment in quality than if the platform was proprietary.

6.1. **Example.** The following example illustrates that investment in platform quality may be larger when the platform is open. The example also demonstrates that a proprietary platform may wind up encouraging more user and developer entry than an open platform.

To have fair a comparison between investments by a proprietary platform provider and developers in an open platform, we assume that there is no user innovation. Let $v(x, n) = x^a\, n^b$, where $0 < a < 1$ and $0 < b < 1$. It is easy that the assumption $b < 1$ implies that applications are substitutes. We assume that $2\,a + b < 1$. This guarantees that the second order conditions for profit maximization are satisfied. Note that investment in platform quality and the measure of applications, $v_{nx} > 0$. We let $h(i) = i$, $c(j) = j$, and $\kappa = 1$.

Using the equations in Table 1, we derive equilibrium adoption and investment. The social planner's solution is:

$$m^s = \left(a^a\, b^{-\frac{b}{2}}\right)^{\frac{1}{1-2\,a-b}}, \qquad n^s = \left(a^a\, b^{\frac{2\,a-1}{2}}\right)^{\frac{1}{1-2\,a-b}}, \qquad x^s = \left(a^{1-b}\, b^b\right)^{\frac{1}{1-2\,a-b}}.$$

For the proprietary platform, the equations are:

$$m^p = \frac{m^s}{2^{\frac{1-a}{1-2\,a-b}}}, \qquad n^p = \frac{n^s}{2^{\frac{1-a}{1-2\,a-b}}}, \qquad x^p = \frac{x^s}{2^{\frac{1-a}{1-2\,a-b}}}.$$

For the open platform, we have:

$$m^o = \left(a^a\,(\alpha\,b)^{\frac{a+b}{2}}\,(1-\alpha\,b)^{\frac{a+b-2}{2}}\right)^{\frac{1}{1-a-b}}, \quad n^o = \left(a^a\,(\alpha\,b\,(1-\alpha\,b))^{\frac{1}{2}}\right)^{\frac{1}{1-a-b}},$$

$$x^o = \left(a^{1-b}\,(\alpha\,b\,(1-\alpha\,b))^{\frac{1}{2}}\right)^{\frac{1}{1-a-b}}.$$

It is easy to find parameter values for which $x^p < x^o$, $m^p > m^o$, or $n^p > n^o$.

7. Duopoly

In this section we extend the model to analyze competition between a proprietary and an open platform. For concreteness, we will focus on the case of substitute applications, but similar results hold for the case of complements.

There is one unit mass of single-homing users, indexed by $i \in [0,1]$. User i's utility of consuming n applications in a proprietary and in an open platform is

$$\begin{aligned} u^p(i) &= v(n^p, x^p, 0) - \int_0^{n^p} \rho^p(j)\,dj - p^u - h^p(i), \text{ and} \\ u^o(i) &= v(n^o, x^o, z) - \int_0^{n^o} \rho^o(j)\,dj - \sigma z(i) - h^o(i), \end{aligned}$$

where superscripts p and o indicate whether the variable or function refers to the proprietary or the open platform.

As in previous sections, the proprietary platform has no user innovation and access to the open platform is free. To guarantee that the market is covered, we assume that $\min_i h^p(i)$ and $\min_i h^o(i)$ are sufficiently small. The optimal choice of platform by users depends (partly) on $h(i) = h^p(i) - h^o(i)$, which measures the difference in the cost of learning how to use the proprietary vs. the open platform. Assume $h_i > 0$, with $\lim_{i\to 0} h(i) = -\infty$ and $\lim_{i\to 1} h(i) = \infty$. Let m indicate the indifferent user. Then, m is the measure of users choosing the proprietary platform, and $1-m$ is the measure choosing the open platform.

Developers multi-home. Thus we assume that it is inexpensive to adapt applications to run on the two platforms. Even though the measure of applications is the same for both platforms, equilibrium application prices may differ across platforms because these depend on platform quality investments, and on user and developer innovation.

The timing is as follows: (i) the proprietary platform provider chooses p^u, p^d, and x^p, (ii) users and developers choose which platform to join, and users and developers joining an open platform choose $x^o(j)$ and $z(i)$, and (iii) users and developers bargain over application prices $\rho^p(j)$ and $\rho^o(j)$. The timing reflects the fact that proprietary platforms are developed before they become accessible to users and developers, but that adoption and development are contemporaneous in open platforms. The equilibrium concept is subgame perfect equilibrium, and we solve the model recursively.

7.1. **Equilibrium entry by users and developers.** In the third stage, users and developers bargain over application prices. The price of applications running on the proprietary platform is $\alpha\, v_n^p$ and that of those running on the open platform is $\alpha\, v_n^o$.

In the second stage, the marginal user and developer satisfy $h(m) = v^p - v^o - n\,\alpha\,(v_n^p - v_n^o) - p^u$ and $c(n) = \alpha\,(m\,v_n^p + (1-m)\,v_n^o) - p^d$, from which we can obtain the indirect demand functions:

$$p^u = v^p - v^o - n\,\alpha\,(v_n^p - v_n^o) - h, \tag{5}$$

$$p^d = \alpha\,(m\,v_n^p + (1-m)\,v_n^o) - c. \tag{6}$$

The optimal investments by users and developers are

$$v_z^o - \alpha\,n\,v_{nz}^o = \sigma, \tag{7}$$

$$\alpha\,(1-m)\,v_{nx}^o = \kappa. \tag{8}$$

In the first stage, the platform provider chooses p^u, p^d, and x^p to maximize profits, taking into account that the second-stage equilibrium levels of m, n, x_o, and z are functions of p^u, p^d, and x^p. We now turn to examining these choices.

7.2. **Pricing and investment.** To better understand the equilibrium choices in the first stage, it is helpful to study a generalization of the model which we later specialize to the functional forms in equations (5) to (8). In particular, let

$$\begin{aligned} m &= M\,(p^u, n, x^o, x^p, z), \\ n &= N\,(p^d, m, x^o, x^p, z), \\ x^o &= X\,(m, n, z), \quad \text{and} \\ z &= Z\,(n, x^o). \end{aligned}$$

These equations show that changes in prices and investment have direct and indirect effects. For example, a change in p^u affects m directly, but it also affects n and x^o indirectly through m, which in turn affect m, n, x^o and z, and so on. To calculate the total effect of changes in p^u, we calculate the total differential of M, N, X, and Z with respect to p^u, and solve for dm/dp^u, dn/dp^u, dx^o/dp^u, and dz/dp^u.[6]

In the first stage, the proprietary platform provider solves

$$\max_{p^u,p^d,x^p} p^u\, m(p^u,p^d,x^p) + p^d\, n(p^u,p^d,x^p) - \kappa\, x^p,$$

where $m(p^u,p^d,x^p)$ and $n(p^u,p^d,x^p)$ are the measure of users and developers arising from the equilibrium of the second-stage. Proposition 4 characterizes the equilibrium p^u, p^d, and x^p.

Proposition 4 (Duopoly pricing and investment)**.** *Equilibrium prices are*

$$\begin{aligned} p^u &= -\left(1-\Delta^m_{x^o}\, X_m\right)\frac{m}{M_{p^u}} + \left(N_m + \Delta^n_{x^o}\, X_m\right)\frac{n}{N_{p^d}}, \\ p^d &= \left(M_n + \Delta^m_{x^o}\, X_n + \Delta^m_z\, Z_n\right)\frac{m}{M_{p^u}} - \left(1-\Delta^n_{x^o}\, X_n - \Delta^n_z\, Z_n\right)\frac{n}{N_{p^d}}, \end{aligned}$$

and investment solves

$$-M_{x^p}\frac{m}{M_{p^u}} - N_{x^p}\frac{n}{N_{p^d}} = \kappa,$$

where $\Delta^m_{x^o} = \frac{M_{x^o}+M_z Z_{x^o}}{1-X_z Z_{x^o}}$, $\Delta^n_{x^o} = \frac{N_{x^o}+N_z Z_{x^o}}{1-X_z Z_{x^o}}$, $\Delta^m_z = \frac{M_z+M_{x^o} X_z}{1-X_z Z_{x^o}}$, *and* $\Delta^n_z = \frac{N_z+N_{x^o} X_z}{1-X_z Z_{x^o}}$.

Proof. The first order conditions are $m+p^u\,\frac{dm}{dp^u}+p^d\,\frac{dn}{dp^u} = 0$, $p^u\,\frac{dm}{dp^d}+n+p^d\,\frac{dn}{dp^d} = 0$ and $p^u\,\frac{dm}{dx^p} + p^d\,\frac{dn}{dx^p} - \kappa = 0$. The optimal choices depend on the derivatives of $m(p^u,p^d,x^p)$ and $n(p^u,p^d,x^p)$ with respect to p^u, p^d, and x^p. We will show how to obtain dm/dp^u (the other derivatives are obtained similarly). The total differential of equations M, N, X, and Z with respect to p^u are

$$\begin{aligned} \frac{dm}{dp^u} &= M_{p^u} + \frac{dn}{dp^u}\, M_n + \frac{dx^o}{dp^u}\, M_{x^o} + \frac{dz}{dp^u}\, M_z, \\ \frac{dn}{dp^u} &= \frac{dm}{dp^u}\, N_m + \frac{dx^o}{dp^u}\, N_{x^o} + \frac{dz}{dp^u}\, N_z, \\ \frac{dx^o}{dp^u} &= \frac{dm}{dp^u}\, X_m + \frac{dn}{dp^u}\, X_n + \frac{dz}{dp^u}\, X_z, \quad \text{and} \\ \frac{dz}{dp^u} &= \frac{dn}{dp^u}\, Z_n + \frac{dx^o}{dp^u}\, Z_{x^o}, \end{aligned}$$

[6] See the proof of Proposition 4 for more details.

which constitutes a system of four equations with four unknowns. Solving for dm/dp^u, we obtain

$$\frac{dm}{dp^u} = \frac{1 - N_{x^o}(X_n + X_z Z_n) - X_z Z_{x^o} - N_z(Z_n + X_n Z_{x^o})}{D} M_{p^u},$$

where

$$\begin{aligned} D &= 1 - (M_z N_m + N_z) Z_n - (X_n + (M_z X_m + X_z) Z_n) N_{x^o} \\ &\quad + (X_m(-1 + N_z Z_n) - N_m(X_n + X_z Z_n)) M_{x^o} \\ &\quad - (N_z X_n + M_z(X_m + N_m X_n) + X_z) Z_{x^o} \\ &\quad - (N_m - N_m X_z Z_{x^o} + (N_{x^o} + N_z Z_{x^o}) X_m) M_n. \end{aligned}$$

Introducing the derivatives in the first order conditions for the proprietary platform and solving for p^u, p^d, and x^p, we obtain the result stated in the proposition. ■

The proposition says that the equilibrium prices are affected by the indirect effects arising from the relationship between m, n, x^o, and z. To illustrate this, consider p^u. If investments x^o and z were fixed (so that $X_m = 0, X_n = 0, Z_m = 0$, and $Z_n = 0$), we would have

$$p^u = -\frac{m}{M_{p^u}} + N_m \frac{n}{N_{p^d}}.$$

Suppose now that only z is fixed ($Z_m = 0$ and $Z_n = 0$). Then, we would have

$$p^u = -\frac{m}{M_{p^u}} + N_m \frac{n}{N_{p^d}} - \left(-\frac{M_{x^o}}{M_{p^u}} m - \frac{N_{x^o}}{N_{p^d}} n\right) X_m. \tag{9}$$

The expression inside the parenthesis measures the change in revenues on the user and developer side caused by a change in x^o. X_m measures the change in x^o caused by a change in m.[7] Therefore, the new term measures the indirect effect of a change in m on profits as it operates through x^o.

Last, consider what happens when we introduce user innovation. Price becomes

$$p^u = -\frac{m}{M_{p^u}} + N_m \frac{n}{N_{p^d}} - \left(-\frac{\Delta^m_{x^o}}{M_{p^u}} m - \frac{\Delta^n_{x^o}}{N_{p^d}} n\right) X_m. \tag{10}$$

[7]For instance, $-\frac{M_{x^o}}{M_{p^u}}$ measures the change in p^u caused by a change in x^o, holding m and n constant. Multiplying this ratio by m, we obtain the change in revenues from the user side caused by a change in x^o.

The difference between (9) and (10) is that M_{x^o} and N_{x^o} are replaced by $\Delta^m_{x^o}$ and $\Delta^n_{x^o}$, which capture the indirect effects on prices caused by the interaction between x^o and z. This interaction augments (dampens) the effects of x^o on prices when x^o and z are complements (substitutes). Finally, p^d has additional terms related to the effects of a change in n on z. These terms do not appear in the formula for p^u because m does not enter directly into Z.

In summary, the effect of investments in the open platform on p^u and p^d depends on whether increases in x^o and z lead to higher or lower profits on the user and developer sides, and on whether increases in m and n lead to increases or decreases in x^o and z.

The equations in Proposition 4 can be written as modified Lerner equations:

$$\frac{p^u + a_{mn}}{p^u} = -(1-b_m)\,\varepsilon^m_{p_u} \qquad \text{and} \qquad \frac{p^d + a_{nm}}{p^d} = -(1-b_n)\,\varepsilon^n_{p_d},$$

where a_{mn} is the externality that users impose on developers, a_{nm} is the externality that developers impose on users, b_m is the indirect effect of changes in m operating through x^o, and b_n is the indirect effect of changes in n operating through x^o and z.

Note finally that the simple expression determining x^p coincides with the formula that we would have if there was no interaction with the open platforms.

We now specialize Proposition 4 to the functional forms introduced above. To simplify the exposition, we assume that there is no user innovation.[8] The pricing formulas are:

$$\begin{aligned} p^u &= m\,h_m - \alpha\,n\,(v^p_n - v^o_n) + (m\,v^o_x - \alpha\,n\;v^o_{nx})\,\frac{v^o_{nx}}{(1-m)\,v^o_{nxx}} \\ p^d &= n\,(c_n - \alpha\,v^o_{nn}) - (1-\alpha)\,m\,(v^p_n - v^o_n) - (m\,v^o_x - \alpha\,n\;v^o_{nx})\,\frac{v^o_{nnx}}{v^o_{nxx}}. \end{aligned}$$

Substituting in (5) and (6), we obtain equations determining the optimal measure of users and developers:

$$\begin{aligned} h + m\,h_m &= (v^p - v^o) - (m\,v^o_x - \alpha\,n\;v^o_{nx})\,\frac{v^o_{nx}}{(1-m)\,v^o_{nxx}}, \\ c + n\,c_n &= m\,(v^p_n - v^o_n) + \alpha\,(v^o_n + n\,v^o_{nn}) + (m\,v^o_x - \alpha\,n\;v^o_{nx})\,\frac{v^o_{nnx}}{v^o_{nxx}}. \end{aligned}$$

[8]Introducing user innovation leads to complex formulas with little gain in insight over the formulas that we have presented in Proposition 4. The analysis can be obtained from the authors upon request.

To understand these equations, suppose first that investment in the open platform was fixed ($X_m = 0, X_n = 0$). Then, we would have:

$$\begin{aligned} h + m\, h_m &= (v^p - v^o) \\ c + n\, c_n &= m\,(v^p_n - v^o_n) + \alpha\,(v^o_n + n\, v^o_{nn}). \end{aligned}$$

In contrast to the case of a monopolist proprietary platform, α now matters in determining the equilibrium n (even in the absence of investment in the open platform). Entry on the developer side depends partly on the revenues that developers obtain on the open platform, which depend on α.

Next, we interpret the terms related to investment in the open platform (x^o). In short, these capture the effects on profits due to the interactions between m and x^o and between n and x^o. A little algebra yields:

$$\begin{aligned} X_m &= \frac{v^o_{nx}}{(1-m)\, v^o_{nxx}}, \\ X_n &= -\frac{v^o_{nnx}}{v^o_{nxx}}, \quad \text{and} \\ \frac{M_{x^o}}{M_{p^u}}\, m + \frac{N_{x^o}}{N_{p^d}}\, n &= m\, v^o_x - \alpha\, n\, v^o_{nx}. \end{aligned}$$

X_m is always negative: an increase in m decreases the market share of the open platform, and therefore lowers the incentives to invest in it. X_n may be positive or negative depending on the sign of v^o_{nnx} which is positive (negative) when n and x act as complements (substitutes) on application price $\rho^o = v^o_n$. If $v^o_{nnx} > 0$, then X_n is positive: an increase in n leads to a higher v^o_{nx}, and therefore, more incentives to invest. The opposite is true when $v^o_{nnx} < 0$.

Finally, as we discussed above, $-\frac{M_{x^o}}{M_{p^u}}\, m - \frac{N_{x^o}}{N_{p^d}}\, n$ measures the change in profits due to a change in x^o. The sign of this expression is ambiguous. To see this, consider $\frac{M_{x^o}}{M_{p^u}} = v^o_x - \alpha\, n\, v^o_{nx}$ which may be positive or negative: on the one hand, an increase in x^o raises the quality of the open platform which lowers user demand for the proprietary platform, but on the other, it leads to higher application prices for the open platform, which has the opposite effect. If $\frac{M_{x^o}}{M_{p^u}} > 0$, an increase in the quality of the open platform leads to a higher demand for the proprietary platform, i.e. to higher profits.

Consider now $\frac{N_{x^o}}{N_{p^d}} = -\alpha\,(1-m)\, v^o_{nx}$. This expression is always negative: an increase in the quality of the open platform leads to higher entry by developers, and therefore to higher profits for the proprietary platform. This seemingly counter-intuitive result is due to multi-homing. The proprietary platform provider gains more on the developer side when there is more entry, and entry is partly determined by developers' revenue on the open platform. If developers single-homed, the sign of this expression would likely be negative.

We conclude that $-\frac{M_{x^o}}{M_{p^u}}\, m - \frac{N_{x^o}}{N_{p^d}}\, n$ may be positive or negative, i.e. taking into account investment incentives in the open platform may lead to higher or lower prices for the proprietary platform. For example, suppose that $X_n > 0$, i.e. an increase in m leads to an increase in x^o. Further suppose that an increase in x^o lowers the proprietary platform's revenue on the user side ($-\frac{M_{x^o}}{M_{p^u}}\, m < 0$) and increases its revenue on the developer side ($-\frac{N_{x^o}}{N_{p^d}}\, n > 0$). We know that $X_m < 0$, i.e. an increase in m leads to a decrease in x^o, holding everything else constant. Then, if the effect on the user side is larger (smaller) than the effect on the developer side, p^u will increase (decrease) relative to the case without investment, and p^d will decrease (increase) with respect to the case without investment. Therefore, the total effect of investment incentives on prices may be asymmetric.

Finally, the equation determining x^p is $m\, v_x^p = \kappa$, from which we obtain the following ratio in equilibrium:

$$\alpha\, \frac{1-m}{m} = \frac{v_n^p}{v_{nx}^o}.$$

Therefore, equilibrium investment in the proprietary platform increases relative to the investment in the open platform as the equilibrium market share of the proprietary platform increases, and as the bargaining power of developers decreases.

8. Conclusion

We have examined a model of a proprietary and an open source, two-sided platform to study equilibrium investment in quality. The analysis has provided answers to three important questions that had not been tackled before: (i) how are the incentives to invest in platform quality affected by the degree of platform openness? (ii) which of these two modes of governance leads to investment closer to the social optimum? and (iii) how are incentives to invest in platform quality moderated by competition between proprietary and open two-sided platforms?

To the first question, our answer is twofold. First, while the nature of cross payments between users and developers plays a role in determining user and developer investments in open platforms (in the sense that the larger the bargaining power of agents in one side of the platform, the more that side is willing to invest), these play no role in monopoly proprietary platforms. Through access prices, a proprietary platform ensures that a particular level of investment takes place, regardless of how much users pay for applications. Second and relatedly, the effect of investments in quality on application prices plays an important role in determining the equilibrium efforts to innovate in open platforms. The stronger the effect, the less users (and the more developers) will want to invest.

We also present simple rules to determine the effects of a change in the bargaining power of developers *vis-a-vis* users on the number of users and developers in open platforms. In particular, an increase in the bargaining power of developers will lead to higher user adoption (and higher consumer surplus) in an open platform if developers would be subsidized if the platform was proprietary. Likewise, a decrease in the bargaining power of developers will lead to higher developer adoption (and higher developer profits) if users would be subsidized if the platform was proprietary.

To the question of social optimality, we find that a proprietary platform would invest efficiently if adoption by users and developers was efficient. Lower than efficient entry, however, implies that investment is always lower than what a social planner would choose. Free riding implies that investment is always socially suboptimal in open platforms. Nonetheless, investment may be larger than in the case of proprietary platforms due to larger entry. Therefore, open platforms may lead to investments in platform quality closer to social efficiency.

Finally, to the question on incentives to invest and competition, we find that a proprietary platform cannot fully internalize the effects of cross-payments on entry and investment when in competition for users against an open platform. As a consequence, the extent to which users have bargaining power affects equilibrium entry. This is because when developers multi-home, entry depends partially on the revenues they obtain from selling applications in the open platform, which depend on their bargaining power. Likewise, equilibrium investment in the proprietary platform increases relative to investment in the open platform as developers' bargaining power in the application market decreases.

We hope to have provided a solid first step to better understand incentives to invest in proprietary and open platforms. An obvious next step is to extend the model to study mixed modes of governance (e.g., a platform open to one side only or an open platform with coordinated investment by developers). Having presented a thorough analysis of two extreme modes of governance, we leave these extensions for future work.

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