Timing of discovery and the division of profit with complementary innovations

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Abstract

This paper studies the optimal division of profit with complementary innovations. We identify circumstances in which the research should be conducted sequentially, targeting one innovation after another in a prespecified order, even if each innovation can be achieved independently of the others. We then consider the implementation of this sequential pattern of investment in a market equilibrium with specialized research firms. Firms are involved in a war of attrition, where each has an incentive to wait for the others to achieve their innovations before sinking its own R&D investment. We show that the optimal policy must reward early innovators more generously than late ones to speed up the innovative process.

Keywords: patents; complementary innovations; war of attrition; standard setting organization

JEL classification number: O3

1 Introduction

In many high-tech industries, such as telecommunications, biotechnology, and software, innovative technological knowledge is typically modular in nature. Various complementary innovations must be assembled together to operate a new technology. For example, software programs often comprise a number of functions, each of which may be protected by separate patents or copyrights.

When innovations are complementary, patent policy must determine not only the total size of the reward, but also its division among the firms that contribute to the discovery. In this paper, we consider the timing of innovations as a possible determinant of the division of profit. We argue that, all else equal, early innovators (i.e. those who discover the first components of a complex technology) should obtain more than late ones.

One may wonder whether it is at all possible to condition the innovators' reward on the timing of their discovery. For example, Gilbert and Katz (2011) argue that, in the current patent system, the adjudication of disputes depends only on what patents are in force at any given point in time. Based on this observation, they take it as an axiom that the division of profit should not depend on the order in which innovations are achieved.

However, it can be argued that the current patent system already rewards early and late innovators differently. When innovations are strictly complementary, early patents are worthless until late innovators also succeed. However, they expire earlier, since the patent lifetime is finite and the same for all patents. This means that late innovators are effectively more strongly protected than early ones, an outcome that is not necessarily socially desirable.

More to the point, the division of profit is not always determined mechanically. Often, there is scope for policy. Consider, for instance, the case that a product is found infringing many patents, and damages must be awarded. In this case, the courts can consider the timing of innovations as a possible determinant of the division of the total damages among different patent holders. Similarly, the timing of innovations can be taken into account by antitrust authorities and the courts when deciding the proper meaning of the obligation to license patents on FRAND terms (i.e., fair, reasonable and non discriminatory terms).[1] These decisions are important not only in themselves, but also because they indirectly affect the terms of licensing contracts.

The development of the CDMA technology for mobile phones provides an excellent illustration of some of these issues. The CDMA technology is a new, more efficient way of permitting a number of users to share a common channel to transmit voice and data. When Qualcomm started to develop the CDMA tecnology, the project was deemed extremely risky. According to a physics professor at Berkeley, this technology was "against the laws of physics". No wonder, initially other firms were reluctant to invest in the complementary components that are needed to manufacture the final product. But Qualcomm's eventual success spurred complementary innovation.[2] Qualcomm itself developed a com-

[1]This obligation is typically imposed by standards setting organizations on firms that want to have their proprietary technology included in a technical standard. However, the true meaning of this obligation is so vague that often litigation occurs. For a discussion, see for instance Baumol and Swanson (2005) and Schmalensee (2009).

[2]It seems reasonable to conjecture that the development of CDMA might had been faster if the complementary technology had become available more quickly.

plete 3G technical standard, CDMA2000, in which it owns more than 90% of essential patents. Europe adopted a different standard, W-CDMA, in which Qualcomm's share of essential patents is much lower.[3] Today several CDMA-based 3G standards compete for adoption in which the core technology developed by Qualcomm is complemented by different sets of alternative components. Qualcomm must now license its essential patents on FRAND terms, but the exact interpretation of this obligation is controversial and has spurred extensive litigation.

In this paper, we make two related claims. First, we argue that, even if all innovations could be achieved simultaneously, it may be better to conduct the research sequentially, targeting one innovation after another in a pre-specified order. The intuitive explanation is as follows. When complementarity is strict and innovation is uncertain, there is a risk that even if a component is invented, R&D investment may prove completely wasteful because other complementary innovations are not achieved. Conducting the research sequentially minimizes this risk, which instead would compound if all innovative components were targeted simultaneously.

Now, if only one firm could achieve all the innovations, it would have any incentive to organize the research efficiently, with no need of policy intervention. However, when different firms must be involved (for instance, because each has a comparative advantage in the search for a different component), there arises a coordination problem: Who invests first? Our second claim is that

[3] CDMA2000 did not ensure backward compatibility with the 2G European standard (GSM). In addition, European manufacturers were concerned about Qualcomm's almost exclusive control of CDMA2000: see Cabral and Salant (2007).

the optimal resolution to this problem requires early innovators to be rewarded more generously than late ones – provided that all innovative components are needed to operate the new technology and all are equally difficult to invent. The intuition here is that early innovators bear more risk than late ones, as the latter can wait until other complementary components are achieved before sinking their own R&D investments. As a result, if all innovators were rewarded equally, firms would engage in a game of waiting (a "war of attrition"), and innovation would be delayed.

It is important to stress that this conclusion does not rest on innovation being inherently sequential – an assumption that is not made here. Innovation is sequential when the search for follow-on innovations can not start until after a basic innovation has been achieved. The literature has shown that in this case there is a positive dynamic externality, so basic innovators should obtain extra protection as a reward for opening the way to the follow-ons.[4] However, here we focus on the case that all innovations can be achieved simultaneously and independently from each other. In this framework, each innovator exerts a positive externality on the others. All innovators are symmetric under this respect, so none deserves special protection on this ground.

While several papers have studied patent policy with complementary innovations, they have focused on different issues. Shapiro (2001) discusses the pricing externalities and the hold-up problem possibly arising with complementary innovations. Ménière (2008) and Denicolò and Halmenschlager (2009) analyze the

[4] See Scotchmer (2004) for an excellent discussion of the intertemporal externality arising with sequential innovation.

problem of whether all or only some of the innovative components should be granted patent protection. Clark and Konrad (2008) analyze firms' incentive to accumulate portfolios of complementary patents, and Gilbert and Katz (2011) study how the division of profit should depend on the strength of such portfolios. Choi and Gerlach (2011) focus on the case in which some components are easier to achieve than others. They argue that too many firms have an incentive to target the easiest innovations first, creating a duplication-of-effort effect, whereas the optimal policy requires different firms to specialize in different innovations. None of these contributions consider the timing of innovations as a possible determinant the optimal division of profit from complementary innovations.

The rest of the paper is organized as follows. The next section presents the model. Section 3 focuses on two special cases, which help isolate the main effects at work in the model. Section 4 finds the first best social optimum, showing that a sequential pattern of research is preferable to simultaneous investment. Section 5 characterizes the market equilibrium. Section 6 analyzes the second best optimal policy and proves the main result of the paper. Section 7 offers some concluding remarks.

2 Model assumptions

There are two strictly complementary innovations in prospect, denoted 1 and 2. Each innovation has zero stand-alone value, and both must be obtained in order to develop a new product. The time horizon is infinite: once invented, the new product can be commercialized forever.

The product market. For simplicity, the product market is assumed to be stationary. We assume that the demand for the new product is rectangular, as in Green and Scotchmer (1995). This allows me to abstract from static monopoly deadweight losses and to focus on the specific issues created by the complementarity between the two innovations. The flow revenue from the innovation, net of production costs, is denoted by v. This is also the hypothetical flow profit that would accrue to fully protected patent holders.

All firms discount future profits at the constant discount rate $r > 0$, which is also the social discount rate. Thus, the discounted social value of the new technology is v/r.

Innovative activity. If both innovations could be achieved by a single firm, the division of profit would be irrelevant. Therefore, we assume that only one firm (labelled firm 1) can invest in innovation 1 and a different firm (firm 2) can uniquely invest in innovation 2. This assumption captures the notion that firms have a comparative advantage in the search for different innovative components.

Innovation is uncertain. Firms choose both the timing and the level of their R&D investments. Each R&D project involves a fixed cost F, and a variable cost that determines the probability of success. For simplicity, we assume that the variable cost is quadratic: to innovate with probability x_i, firm i must invest αx_i^2. The parameter α captures the difficulty of achieving the innovation. The parameters α and F are the same for both firms, which therefore are *ex ante* symmetric.

Research projects are uncorrelated, so the probability that both innovations are achieved is $x_1 x_2$. To guarantee that all the optimization problems considered below entail interior solutions, we assume that $\alpha > \frac{1}{2}\frac{v}{r}$. Moreover, we assume that $2F < \frac{v}{r}$ to avoid trivial cases in which there is no investment in R&D.

Timing. For ease of exposition, it is convenient to take time as discrete; when necessary, however, we shall let the length of time periods go to zero, working in continuous time.

For simplicity, we assume that each firm can invest only in one period. Once a firm has invested, uncertainty over the success of the R&D project is resolved instantaneously.[5] This implies that if a firm invests and fails, the other firm will have no incentive to invest in subsequent periods.

Within each period t, the timing of moves is as follows. Provided it has not invested so far, firm i first decides whether to sink the fixed research cost F or not. After sinking F and observing whether the other firm has in its turn sunk F or not, firm i then decides its variable R&D investment αx_i^2, and innovates with probability x_i. The outcome of a firm's innovative activity is observable by both firms; thus, when a firm chooses its variable R&D investment, it knows whether it is investing earlier, later, or simultaneously to the other firm.

Patent policy. Firms get no payoff unless both innovations are achieved. If and when they are, the new product can be marketed and firms obtain a positive reward. In the absence of patent protection, however, the new product could be

[5] This assumption simplifices the analysis, but is restrictive. It would be interesting to extend the analysis to the case that firms can keep investing until they succeed, as in the model of patent races of Lee and Wilde (1980).

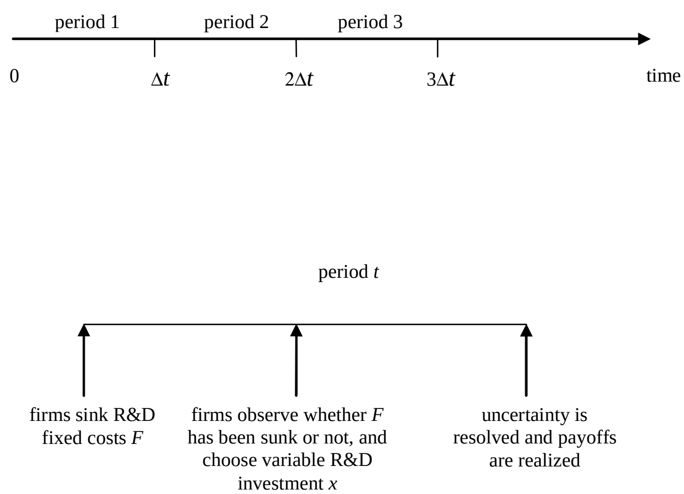

Figure 1: Time and timing of moves within periods

perfectly imitated and any profit would be competed away. Thus, the innovators' reward is determined by patent policy.

To fix ideas, we assume that patent life is infinite[6] and that patent policy determines the breadth of protection. Following Gilbert and Shapiro (1990), we interpret patent breadth simply as the flow of profit each innovator can obtain. We assume that the policy-maker specifies a flow profit s, the same for both firms, if the two innovations are achieved simultaneously. If instead innovations are achieved sequentially, the reward for the first innovative component (i.e., the early innovator) is s_E, and that for the second (i.e. the late innovator) is s_L.[7] The variables s, s_E, and s_L may be interpreted as the royalty rates patent-

[6] This assumption is analytically convenient, since it ensures that the game played by the two firms is stationary. However, it is not crucial for the paper's results.

[7] I assume that s, s_E, and s_L are time invariant. This rules out the possibility that the policymaker conditions the rewards on the period in which the innovations are achieved. A possibe justification of this assumption is that the timing of the start of the patent races is not verifiable. I rule out also the possibility that s_E and s_L may depend on the time lag between

holders are allowed to charge. This formulation is flexible enough to allow the policy-maker to condition the reward on the timing of innovations, as discussed in the introduction. However, the policy-maker may well choose to set $s_E = s_L$, or even $s = s_E = s_L$. In other words, whether and how the reward should be conditioned on the timing of discovery is determined endogenously. Clearly, the policy-maker cannot distribute more profit than the innovation produces, so $2s \leq v$ and $s_E + s_L \leq v$.

Social welfare. Having abstracted from monopoly deadweight losses, we assume that the policy-maker chooses patent policy so as to maximize expected consumer surplus. This assumption is a convenient shortcut to capture the social costs of patent protection, and ensures that full patent protection, i.e. $s = \frac{1}{2}v$ or $s_E + s_L = v$, cannot possibly be optimal.[8]

3 Two simplified variants of the model

To isolate different economic effects that are at work in the model, in this section we consider two simplified variants. In the first, the probability of success of each R&D project is taken to be constant. In the second, the two firms must invest in a pre-specified order.

the two innovations.

[8] Had I assumed that the policymaker maximizes the sum of consumer surplus and firms' profits, the optimal policy would always involve maximum patent protection, i.e. $2s = v$ and $s_E + s_L = v$. However, the paper's main conclusion, namely, that $s_E > s_L$ at the optimum, would continue to hold.

3.1 Indivisible research projects

With indivisible research projects, each firm can develop its innovation with a fixed probability $\bar{x}$ by investing F. Thus, each firm effectively chooses only the timing of its R&D investment.

When a firm invests, the optimal continuation strategy for the other is obvious: not to invest at all if the first firm failed, or if $\bar{x}\frac{s_L}{r} < F$; invest immediately if the first firm has succeeded and $\bar{x}\frac{s_L}{r} \geq F$. Therefore, a necessary condition for the existence of an equilibrium where firms do invest in R&D is

$$\frac{s_L}{r} \geq \frac{F}{\bar{x}}. \tag{1}$$

Because the optimal continuation strategy is trivial, the game effectively becomes a pure war of attrition in which the firm that invests first "concedes." we begin from the case in which firms play non degenerate mixed strategies. To deal with that case, it is convenient to treat time as a continuous variable, so that the probability that firms invest simultaneously is negligible. After developing the analysis of this case, we show that a policy that induces firms to invest simultaneously cannot possibly be optimal in this variant of the model.

In the war of attrition, a strategy for firm i is a probability distribution $G_i(s)$ that gives the probability that firm i concedes (i.e. invests) at time $t \leq s$, provided the other firm has not invested yet. A pure strategy, where firm i invests with probability one at some $t = \hat{t}_i$, corresponds to a degenerate distribution where $G_i(s) = 0$ for $s < \hat{t}_i$ and $G_i(s) = 1$ for $s \geq \hat{t}_i$. Since firms are symmetric and the game is stationary, we focus on stationary symmetric equilibria, as in the original paper by Maynard Smith (1974).[9] Thus, strategies are exponential distributions $G(s) = 1 - e^{-zs}$, where z is the hazard rate. Finding a mixed strategies equilibrium then boils down to finding the value of z.

We denote by E the firm that invests first and by L the follower. Assuming that condition (1) holds, and given the optimal continuation strategy for the other firm, firm E's expected profit, as of the time of its R&D investment, is $\pi_E = \left(\bar{x}^2 \frac{s_E}{r} - F\right)$. The follower's profit, by contrast, is $\pi_L = \left(\bar{x} \frac{s_L}{r} - F\right)$, since the follower invests only if the leader has succeeded. Clearly, no firm will ever want to invest first unless $\pi_E \geq 0$, or

$$\frac{s_E}{r} \geq \frac{F}{\bar{x}^2}. \tag{2}$$

The calculation of the equilibrium is now entirely standard. For firms to be willing to randomize, the expected payoff from immediate investment, π_E, must equal that obtained by waiting until the other firm invests. In equilibrium, this is $\int_0^\infty [1 - G(t)]\, z\pi_L e^{-rt} dt = \frac{z}{z+r}\pi_L$. Equating these payoffs one obtains:

$$z^* = \frac{r\left(\bar{x}^2 s_E - rF\right)}{\bar{x}\left(s_L - \bar{x}s_E\right)}. \tag{3}$$

The numerator is positive if (2) holds. Therefore, a necessary condition for z^* to be positive is

$$s_L \geq \bar{x}s_E. \tag{4}$$

If this condition did not hold, each firm would want to invest first, and the

[9] For an analysis of asymmetric equilibria in war of attrition with private information, see, for instance, Nalebuff and Riley (1984).

only symmetric equilibrium would involve both firms simultaneously investing at $t = 0$. We shall discuss this possible outcome later.

Now consider the problem of the policy-maker, who maximizes expected consumer surplus:

$$\begin{aligned} W &= \int_0^\infty [1 - G(t)]^2 \, 2z \frac{\bar{x}^2(v - s_E - s_L)}{r} e^{-rt} dt \\ &= \frac{2z}{2z + r} \frac{\bar{x}^2(v - s_E - s_L)}{r}, \end{aligned} \tag{5}$$

where z is given by (3), under constraints (2) and (4) (which taken together imply (1)). The solution can be easily characterized by noting that z should be maximized, and the total reward $s_E + s_L$ minimized, under the constraints (2) and (4). It follows that the optimal policy is

$$\begin{aligned} s_E^+ &= \frac{rF}{\bar{x}^2} \\ s_L^+ &= \frac{rF}{\bar{x}}. \end{aligned} \tag{6}$$

Intuitively, the expected rewards just cover the fixed R&D costs, so firms get zero net profits. Firms invest immediately, so the expected delay of innovation vanishes. For future reference, notice that under the optimal policy (6), expected social welfare is:

$$W^+ = \bar{x}^2 \frac{v}{r} - (1 + \bar{x})F. \tag{7}$$

Equations (6) demonstrate that the first innovator must be compensated more than the second one. The intuitive reason is that the firm that invests first bears more risk, as in addition to the intrinsic uncertainty of its own R&D project, it faces the risk that the project of the other firm may also fail (in which case its own innovative component would be worthless). The late innovator, by contrast, can wait until the first component has been invented before sinking its R&D investment. Thus, he faces only the uncertainty associated with his own R&D project. As a result, if both innovators obtained the same reward, each firm would prefer to be second and both would have an incentive to delay their R&D investment. The optimal policy rewards the first innovator more than the second one, eliminating any socially wasteful delay.

We conclude this subsection by verifying that social welfare cannot be increased by inducing both firms to invest simultaneously. The reason is that if firms invest simultaneously, their profit would be $\pi = \left(\bar{x}^2 \frac{s}{r} - F\right)$, for both would face double uncertainty. To ensure that $\pi \geq 0$ (otherwise, firms would not invest) the policy-maker must set $s \geq \frac{rF}{\bar{x}^2}$. This implies that social welfare then cannot exceed $\bar{x}^2 \frac{v}{r} - 2F$, which is lower than the value guaranteed by the optimal policy found above (eq. 7).

3.2 Pre-specified order of R&D investment

Now suppose firms can only invest in R&D in a pre-specified order. We denote again by E the firm that invests first and by L the firm that invests second. Firm E invests at time $t = 0$ and firm L follows suit at $t = 1$. Crucially, L can observe whether E succeeded or not before choosing its R&D investment. Like in the general model, R&D investment comprises a fixed and a variable component, with the probability of success depending on the latter.

To find the equilibrium of the game we proceed backwards. Clearly, firm L does not invest if E has not succeeded. If E's innovation has been achieved, firm L's expected profit is:

$$\pi_L = x_L \frac{s_L}{r} - \alpha x_L^2 - F. \tag{8}$$

The optimal level of R&D investment then is:

$$x_L^* = \frac{s_L}{2\alpha r}. \tag{9}$$

Let us now move back to the first stage of the game. Anticipating L's R&D effort, E's expected profit is:

$$\pi_E = x_E x_L^* \frac{s_E}{r} - \alpha x_E^2 - F. \tag{10}$$

E's optimal choice is:

$$x_E^* = \frac{s_E s_L}{4\alpha^2 r^2}. \tag{11}$$

This completes the determination of the subgame perfect equilibrium of the game. In equilibrium, firms' profit are:

$$\pi_L^* = \frac{s_L^2}{4\alpha r^2} - F, \tag{12}$$

and:

$$\pi_E^* = \frac{s_E^2 s_L^2}{16\alpha^3 r^4} - F. \tag{13}$$

Using (9) and (11), expected consumer surplus is:

$$W = \frac{s_E s_L^2}{8\alpha^3 r^4}(v - s_E - s_L). \tag{14}$$

The optimal policy can then be easily calculated as:[10]

$$s_E^* = \frac{v}{4}$$
$$s_L^* = \frac{v}{2}. \quad (15)$$

Unlike the optimal solution found in the previous variant of the model, now the second innovator obtains more than the first one. The intuition here is as follows. Because the second innovator invests only if the first one has succeeded, his R&D investment does not depend on the profit accruing to the first innovator. In other words, an increase in s_E increases only x_E^*, not x_L^*. An increase in s_L, by contrast, has a positive effect on the R&D investments of both firms. This means that an increase in the reward to the late innovator has a greater incentivizing effect than an increase in the reward to the first one.

4 The complete model: First best optimum

The previous section has highlighted two countervailing effects. On the one hand, when firms choose the timing of their R&D investment, it is desirable to reward the first innovator more than the second one in order compensate the former for the extra risk he takes. On the other hand, rewarding the second innovator produces more powerful incentives. We now turn to the complete model, where both effects are at work, to determine which one prevails and under what circumstances.

[10] In order for firms' profit to be non-negative, the following conditions must hold:

$$v > 4r\sqrt{\alpha F},$$
$$v > 4\sqrt{2}r\sqrt[4]{\alpha^3 F}.$$

We begin, in this section, by characterizing the first best solution that would be chosen by a policy-maker with full control of the R&D decisions of both firms. Initially, we take time to be discrete, but then we move to the case of continuous time by taking the limit as the length of the period Δt goes to zero.

First of all, we show that even if each innovation can be achieved independently of the other, a benevolent social planner chooses a sequential pattern of R&D, targeting one innovation after another in a pre-specified sequence. The proof is very simple. Since delaying R&D investment is socially costly, a benevolent social planner who invests in both innovations simultaneously would invest immediately (i.e., at $t = 0$), obtaining an expected net benefit of:

$$x_1 x_2 \frac{v}{r} - \alpha x_1^2 - \alpha x_2^2 - 2F. \tag{16}$$

Alternatively, the social planner can conduct the research sequentially: in this case he invests in innovation 1, say, at $t = 0$, and in innovation 2 at $t = 1$ only if innovation 1 has been achieved. (Clearly, the order in which innovations are targeted is a matter of indifference.) The expected net social benefit from this policy is:

$$x_1 x_2 \frac{\frac{v}{r}}{(1 + r\Delta t)} - \alpha x_1^2 - F - x_1 \frac{\alpha x_2^2 - F}{(1 + r\Delta t)}. \tag{17}$$

since now the gross benefit $\frac{v}{r}$ is achieved one period later, but the R&D costs for the second innovation are borne only if the first one has already been achieved, i.e., with probability x_1.

Inspection of (16) and (17) reveals that if Δt is small enough, a sequential pattern of research strictly dominates the simultaneous investment solution as long as $x_1 < 1$. The intuitive reason is that R&D investment targeted at innovation i is risky for two reasons: because it may fail, and because even if it is successful, it may prove completely useless as the complementary innovation j is not achieved. The former risk is inevitable, but the latter compounds if both innovations are targeted simultaneously. Targeting one innovation after another in a pre-specified sequence minimizes this latter risk.

Having established that a sequential pattern of R&D investment is optimal, the socially optimal levels of R&D investment are obtained by straightforward maximization of (17). Letting Δt go to zero, one gets:

$$\hat{x}_1 = \frac{v^2}{4\alpha r^2} \tag{18}$$

and

$$\hat{x}_2 = \frac{v}{2\alpha r}. \tag{19}$$

5 The complete model: Equilibrium

Now we proceed to characterize the sub-game perfect Markov equilibrium of the multi-stage game of complete information played by the two firms. In this section we take s, s_E and s_L as exogenous; in the next section we shall analyze the optimal choice of these policy variables.

Since firms are symmetric and the game is stationary, we focus on stationary symmetric sub-game perfect Markov equilibria. (The interested reader is referred to Appendix A for formal definitions of strategies, payoff functions, and equilibria.) A few properties of the equilibrium follow immediately from our assumptions. First, it is clear that no firm will ever invest if the other has already invested and failed. Because the game is stationary, this implies that in equilibrium x_i can take on only three possible values: x_S, x_E and x_L, which denote the variable R&D investment when a firm invests simultaneously to, earlier, or later than the other, respectively. By symmetry, they are the same for both firms; in addition, they are time independent since the game is stationary and we look for Markov equilibria.

Lemma 1. *In equilibrium, if a firm invests in R&D in period t and succeeds, then the other firm follows suit, investing in period $t+1$.*

Proof. From the stationarity of the game and the assumption that the discount rate is positive, $r > 0$, it follows immediately that either firm j invests in period $t+1$, or it does not invest at all. But if firm j's best strategy is never to invest after firm i succeeds, firm i will in turn never invest. ■

The next lemma shows that there is no equilibrium in which innovations are achieved simultaneously.

Lemma 2. *In equilibrium, firms never succeed in the same period.*

Proof. To prove the lemma, consider the subgame that starts when both firms have sunk the fixed R&D cost in the same period t. In this subgame, there is no equilibrium where firms make positive variable R&D investment. That is, $x_S = 0$, implying that firms can never innovate simultaneously.

To show this, suppose that both firms have sunk their fixed R&D costs in period t. Firm i's payoff, as of period t, is (letting Δt go to zero and omitting time indices)

$$x_i x_j \frac{s}{r} - \alpha x_i^2,$$

so its best response function is

$$x_i = \min \left[x_j \frac{s}{2\alpha r}, 1 \right].$$

Since $s \leq \frac{v}{2}$, from the assumption $\alpha > \frac{1}{2}\frac{v}{r}$ it follows $\alpha > rs$. This implies that the best response function of firm i is

$$x_i = x_j \frac{s}{2\alpha r}.$$

This is a straight line with a slope lower than one. Since the other firm is symmetric, this immediately implies that the unique equilibrium is $x_i = x_j = 0$.

■

Lemma 2 means that a sequential pattern of R&D investment is not only socially desirable; it is, indeed, the only possible equilibrium outcome with positive R&D investment. The intuition is that with complementary innovations a firm investing to obtain innovation i exerts a positive externality on the firm that targets innovation j, but this positive externality is not internalized. When firms sink their fixed R&D cost simultaneously, they get trapped in a zero-investment equilibrium where firm i anticipates that innovation j will not be achieved, making innovation i worthless, and vice versa. The assumption that $\alpha \geq \frac{v}{2r}$ implies that there are not enough resources to support a more optimistic equilibrium.[11]

[11] When this inequality is reversed, in addition to the zero-investment equilibrium there is

In view of the above results, we can focus on equilibria in which a firm invests *early* and the other invests in the next period, i.e., *late*. If both succeed, the former will obtain s_E and the latter s_L. The equilibrium pattern of R&D investments depend on s_E and s_L as follows. Let

$$G(s_L) = \frac{1}{2}s_L^2 + \frac{1}{2}\sqrt{s_L^4 - 8Fr\alpha s_L^2 + 64Fr^4\alpha^3 + 16F^2r^2\alpha^2} - 2Fr\alpha.$$

Proposition 1. (i) *If* $\pi_L^* < 0$, $\pi_E^* < 0$, *or both (region A in Figure 2), firms never invest in R&D .*

(ii) *If* $\pi_L^*, \pi_E^* \geq 0$ *and* $s_E s_L \geq G(s_L)$ *(region B in Figure 2), there are two asymmetric equilibria, with firm 1 (resp., firm 2) investing in period 1 and firm 2 (resp., firm 1) investing in period 2. In the limit as* Δt *goes to zero, firms invest immediately and their R&D investments are* x_E^* and x_L^*.

(iii) *If* $\pi_L^*, \pi_E^* \geq 0$ and *and* $s_E s_L < G(s_L)$ *(region C in Figure 2), there is a unique symmetric mixed-strategy equilibrium. Conditional on no investment to date, firm i invests in period t with a constant probability* $z_i\Delta t$. *If it succeeds, firm j follows suit. In the limit as* Δt *goes to zero, the instantaneous probability of investment is*

$$z = \frac{r\pi_E}{\pi_L - \pi_E} \geq 0, \tag{20}$$

where

$$\pi_E = \frac{s_E^2 s_L^2}{16r^4\alpha^3} - F \tag{21}$$

also an equilibrium in which both firms set $x = 1$. An equilibrium with positive, simultaneous R&D investment may also arise when the R&D cost functon is not quadratic: see Shapiro (2007) and Denicolò (2007).

and

$$\pi_L = \frac{s_E s_L (s_L^2 - 4\alpha r F)}{16 r^4 \alpha^3}. \tag{22}$$

The levels of R&D investment are still given by x_E^* and x_L^*.

Proof. See Appendix B. ■

Part (i) of Proposition 1 simply says that firms do not invest in R&D if the prospective reward is too low, i.e. such that $\pi_E < 0$, or $\pi_L < 0$, or both. Part (ii) characterizes the equilibrium arising in region B, where $\pi_E \geq \pi_L \geq 0$ and hence each firm wants to invest first (or is indifferent between being first or second). In this case a coordination problem arise: both firms want to invest in period 1, but if they both sink the fixed R&D cost in period 1, they end up being trapped in a no innovation equilibrium (Lemma 2), and hence cannot recoup the fixed R&D cost. The resolution to this coordination problem is an asymmetric equilibrium in which one firm invests in period 1 and the other in period 2. In the limit as Δt goes to zero, however, there is no delay in R&D activity and uncertainty is resolved immediately.

When instead $\pi_L > \pi_E \geq 0$, i.e., in region C, each firm wants the other to invest first. However, delaying investment is costly because future profits are discounted. Firms are therefore involved in a war of attrition, which has no symmetric equilibrium in pure strategies (Maynard Smith, 1974). In a symmetric mixed strategy equilibrium, in each period firms invest with instantanesous probability z if no one has invested earlier. Since the game is stationary, it is clear that the instantaneous probability of investment, conditional on no investment to date, is constant over time. It can be determined as in Section 3 above.

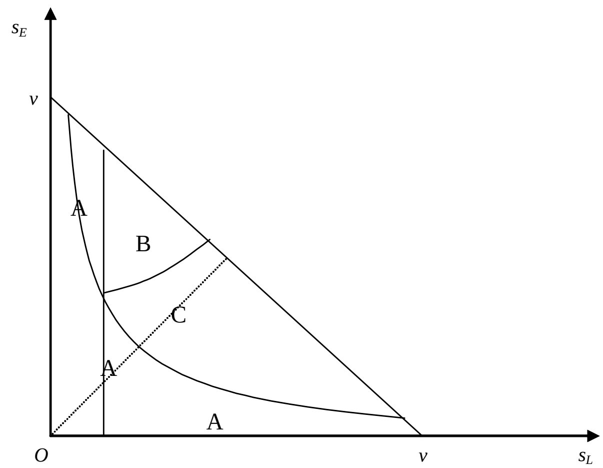

Figure 2: The three regions of Proposition 1

For firms to be willing to randomize, the expected payoff from immediate investment, π_E, must equal that obtained by waiting until the other firm invests. In equilibrium, this is $\int_0^\infty [1 - G(t)]\, z\pi_L e^{-rt} dt = \dfrac{z}{z+r}\pi_L$. Equating these payoffs and exploiting symmetry yields (20). Since $\pi_E < \pi_L$, investment in research is now delayed. To be precise, the expected waiting time to discovery is $\frac{1}{2z} > 0$. The hazard rate is $2z$ since investment by a firm (if successful) immediately triggers investment by the other.

6 Optimal patent policy

Having characterized the equilibrium for any possible value of s, s_E and s_L, now we turn to the optimal choice of these variables. In view of Lemma 2, the value of s is irrelevant and so we can focus exclusively on the optimal choice of

s_E and s_L.

The policy-maker maximizes expected consumer surplus. When $\pi_E \geq \pi_L$, firms invest in R&D immediately and so expected consumer surplus is

$$W = \frac{x_E x_L (v - s_E - s_L)}{r} \tag{23}$$

When $\pi_E < \pi_L$, however, the timing of R&D investment is stochastic and follows a Poisson process with hazard rate $2z$. Consequently, investment in research is delayed and the present value of expected consumer surplus is

$$\begin{aligned} W &= \frac{2z}{2z + r} \frac{x_E x_L (v - s_E - s_L)}{r} \\ &= \frac{2\pi_E}{\pi_E + \pi_L} \frac{x_E x_L (v - s_E - s_L)}{r} \end{aligned} \tag{24}$$

The factor $\frac{2\pi_E}{\pi_E + \pi_L}$ captures the social cost of delayed R&D investment. It is an index of the inefficient delay due to the war of attrition between the two firms. The lower is the ratio between the payoff obtained by investing early and late, the longer firms will delay their R&D investments on average. Patent policy affects both the expected waiting time to R&D investment and the probability that the innovation will be achieved when firms invest.

Summarizing, the policy-maker now chooses s_E and s_L to maximize (23) or (24), whichever applies, anticipating that the firms' R&D efforts are given by (9) and (11), and subject to the firms' participation constraints $\pi_E, \pi_L \geq 0$. Even though we have assumed a simple functional form for the R&D cost function, this problem does not have a manageable closed-form solution. However, we can prove the following:

Proposition 2. *At the optimum, early innovators obtain a greater reward* *than late innovators:* $s_E \geq s_L$*, where the inequality is strict if* $F > 0$*.*

Proof. See Appendix B. ■

The intuitive explanation is that the firm that invests early faces a risk that its R&D investment will prove wasteful because the other innovative component is not achieved eventually. By contrast, the firm that invests late can wait until the other innovative component is available before sinking its R&D investment. This means that if the policy-maker wants to guarantee to early innovators the same expected reward as to late innovators, it must set $s_E > s_L$. Of course, this is only an intuition, since the policy-maker is not constrained to set $\pi_E = \pi_L$. However, any deviation from such a policy is socially costly in that it entails delayed investments in R&D. To avoid excessive delay, the policy-maker must guarantee to early innovators an expected profit not too much lower than that obtained by late innovators. Proposition 2 shows that this, too, requires $s_E > s_L$, in order to compensate the early innovator at least partially for the extra risk he takes.

7 Concluding remarks

We have shown that early innovators should be rewarded more generously than late innovators, all else equal. But how can the optimal policy be implemented? Under the current patent system, if a firm operating the new technology infringes on both patents, both patent-holders have the power to block the marketing of the new product and so they would get the same per period profit. In principle the firm that innovates first might be granted longer patent protection, but in practice the patent life is fixed and finite. And since in the real world there is typically some delay between the first and the second innovation, early innovators will get less than late ones.

There is, however, one important instance in which patent policy can be fine tuned. Standard setting organizations often require firms holding intellectual property rights that are essential for a technical standard – typically patents – to license on fair, reasonable and non discriminatory (FRAND) terms. The exact meaning of the FRAND formula is controversial, however, so antitrust authorities and the courts are constantly called upon to resolve disputes between patent owners and their licensees. In adjudicating such disputes, antitrust authorities and the courts can exercise a lot of judicial discretion, and nothing prevents them from conditioning the royalty rate that patent-holders are allowed to charge on the timing of their innovations.

From this perspective, this paper may be viewed as a contribution to the heated debate on the proper interpretation of the FRAND obligation. Two main approaches have been proposed so far, which may be called *ex post* and *interim*. The *ex post* approach implictly or explicitly posits that all innovations have already been made and a standard has already been selected. From this viewpoint, all patents that are essential to a standard are indistinguishable and so should be treated equally. Therefore, the *ex post* approach implies a numeric proportionality rule to divide the aggregate royalty among patent-holders, whereby each patent-holder should get a share of aggregate royalties equal to its share of essential patents.[12]

The *interim* approach, by contrast, assumes that innovations have already been made but a technical standard has not been selected yet. It contends that the goal of the FRAND obligation is to prevent the exploitation of any market power firms may have acquired just because of the inclusion of their technology in a standard. Since certain patents may be essential for any conceivable standard, others may have had substitutes of lesser quality at the time the technical standard is adopted, and yet others may have had perfect substitutes, not all patents that are essential *ex post* should be treated equally. According to the *interim* approach, FRAND licensing terms should reflect the different degrees of market power different patent owners had before a particular standard was adopted.[13]

While the *interim* approach is conceptually better grounded than the *ex post* approach, neither is truly compelling. Patents are granted to reward innovative activity and the most appropriate criterion to determine the aggregate FRAND royalty rate and its division among various patent owners is the maximisation of expected social welfare as of before R&D investments are sunk. According to this *ex ante* approach, in determining whether or not a particular licensing strategy breaches a FRAND obligation, antitrust authorities and the courts

[12] It is claimed that a numeric proportionality rule is simple and minimizes transaction costs: see e.g. Farrell et al. (2007)

[13] Two possible ways of assessing this *ex ante*, legitimate market power have been proposed in the economics literature. One maintains that FRAND royalty rates should reproduce the outcome of a hypothetical auction run by the standard setting organization during the standard setting process, the winner of which would have become the standard. The second approach applies cooperative game theory solutions like the Shapley value. Both approaches imply that FRAND royalty rates should depend on the value of the patents and the substitutability or complementarity relationships between them: see Layne-Farrar, Padilla and Schmalensee (2007).

should take into account the effect that the rule they adopt would have on firms' R&D investment decisions.

This paper provides an illustration of the *ex ante* approach. In our setting, innovative components are symmetric in that each has zero stand-alone value and no substitutes, so both the *ex post* and the *interim* approach would entail a symmetric division of profit. But we have seen that this need not be the optimal way of stimulating R&D investment. When firms innovate sequentially, the optimal policy should allow the first innovator to charge a greater royalty rate than the second in order to prevent excessive delay in R&D investment. The intuitive reason is that early innovators bear more risk than late innovators, who wait until the other complementary components are available before sinking their R&D investments, and should get an extra reward for the extra risk they take on.The simple model analyzed in this paper can be extended in various directions. For example, a sequential pattern of R&D investment need not always be optimal. If the value of the new technology is very large as compared to R&D costs, it may be both desirable and profitable to invest simultaneously in all innovative components, especially when innovative activity takes time and innovations do not occur instantaneously. However, if the timing of innovations is uncertain and depends on R&D investment, it may still be desirable to reward early innovators more than late innovators if this stimulates firms' effort at the least social cost.

Another issue left for future research is the case where innovative components are heterogeneous. If a sequential pattern of R&D investment is indeed optimal, there arises the question of the most appropriate order in which the various innovative components should be targeted. Intuition suggests that those components that are most important and/or most difficult to achieve should be targeted first. This might provide an additional rationale for rewarding early innovators more generously.

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Appendix A

Formally, the game can be described as follows. Let $y_{i,t}$ be a binary variable that takes on a value of 1 if firm i sinks the fixed R&D cost in period t and 0 if it does not. Define $Y_{i,t} = \sum_{k=1}^{t} y_{i,k}$; this variable records whether firm i has already sunk F ($Y_{i,t} = 1$) or not ($Y_{i,t} = 0$). Similarly, let $h_{i,t}$ be a binary variable that takes on a value of 1 if firm i succeeds in period t and 0 if it does not, and let $H_{i,t} = \sum_{k=1}^{t} h_{i,k}$. Since we focus on subgame perfect Markov equilibria, we summarize the history of the game as of the beginning of period $t+1$ through four state variables, i.e., $\mathcal{H}_t = \{Y_{1,t}, Y_{2,t}, H_{1,t}, H_{2,t}\}$. It is also convenient to define $\mathcal{H}_{t+\frac{1}{2}} = \{\mathcal{H}_t, y_{1,t+1}, y_{2,t+1}\}$; $\mathcal{H}_{t+\frac{1}{2}}$ summarizes the history of the game after firms have decided whether or not to sink the fixed R&D cost in period $t+1$, but before the period $t+1$ variable R&D investments are made and uncertainty is resolved.

A pure Markov strategy for firm i specifies an action $y_{i,t+1} \in \{0,1\}$ as a function of $\mathcal{H}_t$ and an action $x_{i,t+1} \in [0,1]$ as a function of $\mathcal{H}_{t+\frac{1}{2}}$. Our assumptions restrict the set of feasible actions as follows. First, $Y_{i,t} = 1$ implies $y_{i,t+1} = 0$: each firm can invest in R&D only once. Second, $y_{i,t+1} = 0$ implies $x_{i,t+1} = 0$: a firm cannot make any variable R&D investment in periods where it has not sunk the fixed R&D cost.

Mixed Markov strategies are defined in the standard way. Note that since the R&D cost function is quadratic and firms are risk neutral, firms never randomize over the variable R&D investment x_i. However, firms can randomize between sinking the fixed R&D cost or not. We denote by $z_{i,t}\Delta t$ the probability that $y_{i,t}=1$.

Firms' per-period payoffs $\pi_{i,t}\Delta t$ are defined as follows:

$$\pi_{1,t}\Delta t = \begin{cases} 0 \text{ if } H_{1,t}H_{2,t}=0 \\ s\Delta t \text{ if } H_{1,t}H_{2,t}=1 \text{ and } \Psi_t=0 \\ s_E\Delta t \text{ if } H_{1,t}H_{2,t}=1 \text{ and } \Psi_t>0 \\ s_L\Delta t \text{ if } H_{1,t}H_{2,t}=1 \text{ and } \Psi_t<0, \end{cases} \tag{25}$$

where $\Psi_t=\sum_{k=1}^{t} H_{1,k}-H_{2,k}$ is positive if firm 1 has innovated before firm 2, negative if firm 1 has innovated after firm 2, and zero if firms have innovated simultaneously (or no firm has yet innovated by period t). $\pi_{2,t}\Delta t$ is defined similarly. Total discounted payoffs are

$$\Pi_i=\sum_{t=1}^{\infty}\frac{\pi_{i,t}\Delta t-\alpha x_{i,t}^2}{(1+r\Delta t)^t}. \tag{26}$$

Appendix B

Proof of Proposition 1. Let consider the subgame that starts when one firm has invested in R&D in period t, with the other following suit, i.e., in period $t+1$. Proceeding backward, we start from the firm that invests late. Since this firm invests only if the other innovative component has already been achieved, its objective function is (for simplicity we take the limit as Δt goes to zero) $x_L\frac{s_L}{r}-\alpha x_L^2-F$, and so it would choose

$$x_L=\frac{s_L}{2\alpha r}$$

provided that $s_L \geq 2r\sqrt{\alpha F}$ (if this inequality is reversed, firms do not invest in R&D and all payoffs vanish). Next consider the firm that invests early. Its expected profit is $x_E x_L \frac{s_E}{r} - \alpha x_E^2 - F$. Since it anticipates that x_L is given by the above expression if inequality $s_L \geq 2r\sqrt{\alpha F}$ holds, this firm will choose

$$x_E = \frac{s_E s_L}{4\alpha^2 r^2}$$

provided that $s_E s_L \geq 4r^2\alpha\sqrt{\alpha F}$ (again, firms do not invest in R&D if this inequality is reversed). Substituting back into the profit function we get (21) and (22).

The characterization of the mixed equilibrium of the war of attrition is derived in the text following Proposition 1. ■

Proof of Proposition 2. First of all we show that the optimal patent policy never entails $\pi_E > \pi_L$. The proof is by contradiction. When $\pi_E > \pi_L$, by Proposition 1 firms invest in R&D immediately and the policy-maker's objective function (i.e., expected consumers surplus) becomes

$$W = \frac{s_E s_L^2 (v - s_E - s_L)}{8r^4\alpha^3}$$

The solution to this problem is $s_E = \frac{v}{4}$and $s_L = \frac{v}{2}$. Substituting into (8) and (10) and then back into the profit functions gives $\pi_E = \frac{v^4}{1024\alpha^3 r^4} - F$ and $\pi_L = \frac{16\alpha r^2 v^2 (v^2 - 16\alpha r^2 F)}{512\alpha^3 r^4}$. Simple algebra shows that $\pi_E < \pi_L$when $\alpha > \frac{1}{4}\frac{v}{r}$. This means that the policy-maker will never want to choose a policy such that $\pi_E > \pi_L$.

Thus, at the optimum it must be $\pi_E \leq \pi_L$. Inspection of (24) reveals that when $\pi_E = 0 < \pi_L$, social welfare vanishes. It follows that the solution to the social problem entails either $\pi_E = \pi_L$(in which case firms invest immediately) or $0 < \pi_E < \pi_L$(in which case R&D investments are delayed).

Consider the case $\pi_E = \pi_L$ first. Using (8) and (10), condition $\pi_E = \pi_L$ becomes

$$\frac{s_E s_L^2 (s_E - s_L)}{16\alpha^3 r^4} = F(1 - x_E)$$

whence it immediately follows that $s_E \geq s_L$, with a strict inequality whenever $F > 0$.

Consider next the case $0 < \pi_E < \pi_L$. In general, the social problem is decomposable and can be divided into two stages: in the first stage, the policy-maker chooses the optimal aggregate reward $\hat{s} = s_E + s_L$; in the second stage, it chooses the optimal division of the profit between the early and the late innovator. Consider the second stage, where the optimal combination of s_E and s_L is sought, for any fixed given $\hat{s}$. Using $s_L = \hat{s} - s_E$, we can express social welfare W as a function of s_E only. Differentiating and evaluating the derivative at $s_E = \frac{\hat{s}}{2}$ we get

$$\frac{dW}{ds_E} \mid_{s_E = \frac{\hat{s}}{2}} = \frac{\hat{s}^2 (16\alpha^2 r^2 + \hat{s}^2)(v - \hat{s})(\hat{s}^4 - 256\alpha^3 r^4 F)}{4\alpha^2 r^2 (\hat{s}^4 - 128\alpha^3 r^4 F - 8\alpha r \hat{s}^2 F)^2} F$$

The condition $s_E s_L \geq 4r^2 \alpha \sqrt{\alpha F}$, which must hold for π_E to be positive, guarantees that $\hat{s}^4 > 256\alpha^3 r^4 F$. It follows that the derivative is non negative, and is strictly positive when $F > 0$. This implies that at the optimum $s_E \geq s_L$, with a strict inequality when $F > 0$. ■