Strategic Search Diversion and Intermediary Competition

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Abstract

We study search diversion by competing intermediaries connecting consumers with third-party stores (sellers). Search diversion is a strategic instrument that enables intermediaries to trade-off total consumer traffic for higher revenues per individual consumer visit. It is particularly useful in contexts in which the stores or products which are most sought-after by consumers are not the ones that yield the highest revenues for the intermediary that provides access to them. First, we show that intermediaries have stronger incentives to divert search when store entry is endogenous and intermediaries cannot perfectly price discriminate among stores. This is because intermediaries' incentives are driven by the marginal stores, which benefit the most from search diversion. Second, competition among intermediaries can lead to *more* search diversion relative to monopoly when consumers multihome and stores singlehome: in this case, intermediaries' incentives are driven by store preferences. Conversely, competition leads to less search diversion when consumers singlehome and stores multihome: in this case, intermediaries seek to maximize consumer surplus.

1 Introduction

Search diversion occurs when intermediaries giving consumers access to various products or third-party sellers (stores) deliberately introduce noise in the search process through which consumers find the products or stores they are most interested in. This practice is widespread among both offline and online intermediaries. Retailers often place the most sought-after items at the back or upper floors of their stores (e.g. bread and milk at supermarkets, iPods and iPhones at Apple Stores); shopping malls design their layout so as to maximize the distance travelled by visitors (cf. Elberse et al. (2007)). E-commerce sites and search engines (e.g. Amazon, eBay, Yahoo) design their websites in order to divert users' attention from the products they were initially looking for, towards discovering products they might be interested in - and eventually buy (cf. Petroski (2003), Shih et al. (2007)). And all advertising-supported media (from offline magazines to online portals) are purposefully designed so as to expose readers to advertisements, even though they are primarily interested in content.

While search diversion may lead to higher intermediary revenues per consumer "visit" (or "impression" - in the language of online ad-media), it reduces the overall attractiveness of the intermediary to consumers and therefore also leads to lower consumer traffic (i.e. total mumber of visits).

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All intermediaries listed above face this fundamental tradeoff. A 2007 analysis of online portals and social networks summarizes it as follows:

"If you optimize for page views, you're going to make decisions, both strategic and tactical, that increase page views. This results in things like split-stories on news sites, way too many screens on social networking sites, and perhaps worst of all, an entire approach focused on 'keeping people on the site.' The problem with this approach is that it quickly gets out of hand. Instead of providing a single value proposition, the site provides countless propositions, thus diluting the value of each one. In almost all cases of optimizing for page views, the experience of the user takes a back seat. You optimize for the quarterly numbers, not user satisfaction. If you optimize for the user experience, on the other hand, you have to take a longer approach. Sure, you might not get as much advertising revenue in the short term as the site with more page views, but you're adding more long-term value."[1]

The basic economic logic of search diversion was first analyzed by Hagiu and Jullien (2011), using a model with a monopoly intermediary that offers consumers access to two stores, whose affiliation is exogenously given. There remain however several important issues raised by the use of search diversion as a strategic design tool. How does an intermediary's incentive to divert search change when it must attract both consumers *and* stores (as opposed to just consumers)? What is the effect of competition between intermediaries on their optimal levels of search diversion? In particular, should increased competitive pressure - for consumers or for stores - lead to more or less search diversion? The purpose of this paper is to present a formal model in order to address these questions.

In our model each intermediary offers consumers access to two "stores", 1 and 2, which can be interpreted as either products owned by the intermediary or third-party sellers. First, consumers must decide which intermediary to visit, and stores must decide which intermediary to affiliate with (i.e. to make themselves available on). Second, consumers must search on an intermediary in order to find the affiliated stores. The intermediary is paid a fee r whenever a consumer visits an affiliated store. Each consumer is primarily interested in one of the two stores. Store 1 is more popular, i.e. preferred by more consumers. The key decision made by the intermediary is the quality of the search service, i.e. how easy it makes it for any given consumer to find her preferred store. Specifically, we identify the search quality with the probability q that a consumer finds her preferred store on the first try. The lower the search quality q, the more "search diversion" is induced by the intermediary, i.e. the higher the probability that consumers will first be diverted to their less preferred store.

In this context, endogenizing store entry has two effects. First, it creates an additional incentive to divert search relative to the basic trade-off between consumer traffic (total number of consumers who visit) and higher revenues per consumer visit studied in Hagiu and Jullien (2007). Specifically, when the intermediary cannot price-discriminate among stores, its profits are aligned with those of the marginal - i.e. less popular - store. Search diversion is then a strategic instrument that helps reduce the variance of profits among stores and thereby improve the intermediary's rent-extraction

[1] "Should Designers Optimize for Page Views or User Experience?" http://bokardo.com/archives/should-designers-optimize-for-page-viewsor-user-experience/

power. Second, endogenous store entry introduces genuine "two-sidedness" in the intermediaries' optimization problems. Two-sidedness plays a central role when we introduce competition among intermediaries. Our analysis reveals that the effect of competition on intermediaries' incentives to divert search critically depends on which of the two sides - stores or consumers - is more difficult to attract. When stores affiliate with both intermediaries (i.e. when stores multihome) but consumers affiliate with one intermediary only (i.e. consumers singlehome), search diversion is driven by consumer preferences, which implies that it is generally *lower* than what a monopoly intermediary would choose. Conversely, when consumers multihome but stores singlehome, intermediaries' choices are entirely driven by store preferences, which can lead to *more* search diversion relative to the monopoly case.

Our modelling set-up is best interpreted as a stylized representation of online intermediaries for commercial products, such as Amazon, Bing Shopping, iTunes, Netflix, etc. All of these intermediaries use recommender systems to provide consumers with a search service, by directing them to products or third-party online sites which are most likely to best suit consumers' (revealed) preferences. Preferences are inferred by the intermediaries based on users' profiles, past browsing and shopping history and comparison with users that have similar profiles. The intermediaries' revenues come from third-party online sites they generate leads for (Bing Shopping, Kaboodle.com, ThisNext.com) or directly through the margins they make when they sell their own products (Amazon, iTunes, Netflix). Users on the other hand can access and use the intermediaries' services for free. Most of these intermediaries blur the line in their recommendations between the sites or products which objectively correspond to users' preferences and those that generate the highest revenues for the intermediary (cf. Steel (2007)). The difference between the search quality q and 1 is meant to capture precisely this type of degradation of the search service provided to users.

Table 1 below summarizes several intermediation contexts to which our model applies.

Intermediary	**Store 1**	**Store 2**	**q**	**r**
Independent online recommenders (e.g. Bing Shopping, Kaboodle, ThisNext)	Most popular products or shopping sites	Less popular products or shopping sites	Probability that recommender shows a given consumer the product or site that bestsuits her preferences (as inferred by the intermediary)	Referral or advertising fee from 3rd-party sites to which the intermediary directs traffic
E-commerce sites with built-in recommenders (e.g. Amazon, iTunes, Netflix)	Most popular products or content	Other products or content	Same as above	Margin made on various products
Brick-and mortar retailers (e.g. Apple Store, Target, Wal-Mart)	Most popular products (e.g. iPods, bread, milk)	Less popular products	Ease and convenience of navigating the store (e.g. q is lower when the most popular products are at the back of the store)	Margin made on various products
Shopping malls	Anchor stores	Other stores	Ease and convenience of navigating the mall (e.g. q is lower when anchor stores are far from the main access points and from each other)	Rent plus percentage of revenues charged by mall developer to stores

1.1 Related literature

Our paper builds upon the model of search diversion introduced by Hagiu and Jullien (2011). We extend their analysis in two important directions: endogenizing store affiliation decisions and introducing competition among intermediaries (Hagiu and Jullien (2011) focus exclusively on a monopoly intermediary with exogenously given store participation).

We contribute to the strategy and economics literature on two-sided platforms by introducing a key design decision (search quality) that many platforms/intermediaries have to make, but has not been formally studied. Indeed, most of the existing work on two-sided platforms focuses on pricing strategies (Armstrong (2006), Economides and Katsamakas (2006), Eisenmann et al. (2006), Parker and Van Alstyne (2005), Rochet and Tirole (2006), Spulber (2006), Weyl (2010)) and market outcomes (Caillaud and Jullien (2003), Hossain et al. (2011), Zhu and Iansiti (2011)) in the presence of indirect network effects. Our paper is aligned with an emerging body of work aiming to expand the formal study of platforms to design decisions (e.g. Boudreau (2010), Parker and Van Alstyne (2008) who study openness choices).

At a broader level, several articles have pointed out that intermediaries have to make design compromises between the interests of their two sides (e.g. Kaplan and Sawhney (2000) in a survey of B2B business models; Evans and Schmalensee (2007) in an overview of markets featuring two-sided platforms). To the best of our knowledge however, this issue has not received formal modelling treatment.

The remainder of the paper is organized as follows. Section 3 lays out the modeling set-up and analyzes the monopoly intermediary case, endogenizing store affiliation decisions. Section 4 tackles the scenario with competing intermediaries and focuses on two polar market equilibria: one in which both stores multihome while consumers singlehome and one in which all consumers multihome while both stores affiliate exclusively with one intermediary. We conclude in section 5.

2 Monopoly intermediary with exogenous store affiliation

In this section we lay out the foundation for our analysis using a variant of the model in Hagiu and Jullien (2011). We build upon it in subsequent sections by adding novel elements: endogenous store entry and competition among intermediaries.

There is a monopoly intermediary which allows a unit mass of consumers to access two stores (or products), 1 and 2, which are *already* affiliated with the intermediary. This corresponds to settings in which there exist long-standing affiliation contracts between stores and the intermediary or in which the intermediary simply owns the stores. For a consumer to access a store, the consumer must first affiliate with (i.e. visit) the intermediary and then find the store through a search process which we describe below.

For the sake of concision, we work with two stores throughout the paper. It is straightforward (though more complicated) to extend our analysis to many stores: all of our main results would go through.

2.1 Consumers

Ex-ante, i.e. before affiliating with the intermediary, consumers only differ in their location x, uniformly distributed on a Hotelling segment $[0, 1]$. The intermediary is located at 0. When consumer x affiliates with the intermediary, she incurs transportation costs tx, where $t > 0$.

Ex-post, i.e. after deciding whether or not to affiliate with the intermediary, consumers differ along two dimensions: preferences for stores and search costs.

Along the first dimension, there are two types of consumers. Type 1 consumers make up a fraction α of the population and derive *net* utilities u^H from visiting store 1 and u^L from visiting store 2. Type 2 consumers make up the remaining fraction $(1 - \alpha)$ and derive net utilities u^H from visiting store 2 and u^L from visiting store 1, where $0 < u^L < u^H < 1$ are exogenously given.[2] Store

[2] u^i should be interpreted as encompassing the utility of just "looking around" the store plus the expected utility of actually buying something, *net* of the price paid. Hagiu and Jullien (2011) also treat the case with endogenous store prices. Here we assume for simplicity store prices are exogenously fixed and work directly with net utilities throughout. Also, there is no substitutability/complementarity between stores. The corresponding cases are discussed in Hagiu and Jullien (2011).

1 is more popular than store 2, i.e. $\alpha \geq 1/2$.

Along the second dimension, consumers are differentiated in their unitary search cost c, which they incur whenever they visit a store. When both stores are affiliated with the intermediary, consumers can only visit them *sequentially* and therefore perform at most two rounds of search. Search costs c are distributed on $[0,1]$ according to a twice continuously differentiable cumulative distribution function F. The distribution F is independent of the distribution of types $(\alpha, 1-\alpha)$. Also, from an *ex-ante* perspective, a consumer located at any position x perceives the same *ex-post* probability distributions of search costs $F(.)$ and store preferences $(\alpha, 1-\alpha)$.

In addition to search costs and store benefits, each consumer derives standalone net utility $u_0 > 0$ from visiting the intermediary. We assume $u_0 > t$, so that the intermediary always covers the entire consumer market. Our analysis is easily extended to the case with partial consumer market coverage as we briefly discuss at the end of this section.

2.2 The intermediary

The intermediary is assumed to derive exogenously fixed revenues $r > 0$ *for each consumer visit* to a store. The fee r is exogenously fixed, perhaps through some earlier bargaining game which is not modeled in this section (it will be in the next sections, where r is endogenized). [Note that r could correspond to a referral fee paid by the stores to the intermediary if they are independent, or to the margin made by the intermediary on the stores' products when they are owned by the intermediary.] In the online appendix we investigate the effects of allowing the intermediary to derive different fees from the two stores (r_1 and r_2). There are no fees charged by the intermediary to consumers.[3]

Once they have affiliated with the intermediary, consumers learn their type 1 or 2, i.e. their favorite store. Consumers cannot however identify and access a store without the intermediary's help. The intermediary observes each affiliated consumer's type (1 or 2) but not her search cost c. The intermediary can then choose to direct consumers to either one of the two stores.

The intermediary's design technology allows it to choose a probability $q \in [0,1]$ with which it directs *any* given consumer to her preferred store (store j for type $j \in \{1,2\}$) in the *first* round of search. We call this probability the *quality of the search service* provided by the intermediary and we say that the intermediary *diverts* a fraction $(1-q)$ of consumers, i.e. sends them to their less preferred store first. Once a consumer has visited and identified one store in the first round of search, she knows for sure the identity of the other store, although she needs to incur her search cost c again if she wants to visit it. The focus of our paper is on the intermediaries' choice of q. We assume that q can be costlessly set to any value between 0 and 1.

The timing of the game we consider in this section is as follows:

1. The intermediary announces q publicly and credibly

[3] This assumption fits all of our motivating examples. The effect of allowing the intermediary to charge fixed access fees to consumers is explored in Hagiu and Jullien (2011).

2. Consumers decide whether or not to visit (affiliate with) the intermediary

3. Affiliated consumers learn their type (1 or 2) and their search cost c and then engage in search for stores.

Two aspects of this set-up deserve mention. First, we have assumed that consumers have no choice but to follow the intermediary's direction in the first round of search. Alternatively, one could assume that consumers can choose between heeding the recommendation and searching independently, which would lead to the imposition of the additional constraint $q \geq 1/2$. Second, the intermediary chooses one q for all consumers, whereas Hagiu and Jullien (2011) allow for the possibility that the intermediary offers a different q_i for each consumer type $i \in \{1, 2\}$. Both of these assumptions are made for convenience and do not change our analysis in any meaningful way.

2.3 The consumer search process

In stage 3, consumers affiliated with the intermediary (all of them for $u_0 > t$) must decide whether to search or not. When the intermediary has chosen $q < 1$, consumers anticipate that they may be diverted. A consumer of either type with low search costs, i.e. $c \leq u^L$, always conducts two rounds of search no matter what store she is directed to in the first round. She derives net utility $u^H + u^L - 2c \geq 0$. A consumer with intermediate search costs - i.e. $u^L < c \leq u^H$ - continues to search only if she is diverted in the first round, since she then knows for sure that she will find her preferred store in the second round, when she will obtain utility $u^H - c \geq 0$. The consumer's net utility is then $u^H + (1-q)\, u^L - (2-q)\, c$. This is positive for $c \leq u(q)$, where:

$$u(q) \equiv \frac{u^H + (1-q)\, u^L}{2-q} \leq u^H$$

is increasing in q and represents the average utility per search of a consumer with search cost above u^L. Finally, consumers with search cost above $u(q)$ do not engage in the search process at all.

2.4 Optimal search diversion

A consumer with search cost below u^L always visits both stores. Consumers with search costs between u^L and $u(q)$ expect to visit $2-q$ stores on average. Given that all consumers affiliate, the intermediary's profits are $rX(q)$, where:

$$X(q) \equiv qF\left(u^L\right) + (2-q)\, F(u(q)) \tag{1}$$

is the expected number of visits per consumer. Since the revenue r per visit is exogenous, the intermediary's profits are maximal when the total number of consumer visits is maximal. Thus the intermediary's optimal choice of q in stage 1 is:

$$q^X \equiv \max_q X(q) \tag{2}$$

This optimization problem involves a straightforward trade-off between *total* consumer participation in the search process and the average number of visits to a store *per consumer*. Indeed, reducing q induces a consumer with search costs above u^L to visit more stores $(2-q)$, but it reduces the overall mass of consumers, $F(u(q))$, who engage in search. This tradeoff is analyzed at length in Hagiu and Jullien (2011).

Let us briefly discuss the case in which not all consumers affiliate with the intermediary, which occurs when $u_0 < t$. In this case, the number of affilated consumers given q is:

$$D(q) \equiv \min\left\{1, \frac{u_0 + \int_0^{u^L}\left(u^H + u^L - 2c\right) f(c)\,dc + \int_{u^L}^{u(q)}\left(u^H + (1-q)\,u^L - (2-q)\,c\right) f(c)\,dc}{t}\right\} \tag{3}$$

where the long term on the right is obtained by taking the expected value of consumer utility from the perspective of stage 2. It is easily seen that consumer traffic to the intermediary is increasing in the quality of the search service: $D'(q) \geq 0$.

The intermediary's profits are now $rX(q)\,D(q)$, so that the optimal level of search quality:

$$\widehat{q}^X \equiv \arg\max_q \left[X(q)\,D(q)\right]$$

is larger than $q^X = \arg\max_q [X(q)]$, the level chosen under full consumer market coverage. This simple insight is relevant for the analysis that follows. Since consumers unambiguously prefer *less* search diversion (i.e. higher q), the more elastic consumers' demand for the intermediary's services, the larger the pressure on the intermediary to reduce search diversion by increasing q. As we will see in the next section however, at least one of the stores may prefer *more* search diversion, which means that in general, the intermediary's choice of q has to solve a conflict of interest between its two sides.

In the online appendix we also discuss the case in which the intermediary receives different fees from the two stores, r_1 and r_2. The key (and highly intuitive) insight that emerges is that the optimal quality of search $q^X(r_1, r_2)$ is lower than the one with equal revenues r if and only if $r_1 < r_2$. In other words, the intermediary diverts search more when it derives higher revenues from the less popular store relative to the more popular store.

3 Endogenous store affiliation

We now introduce the first of two major novelties relative to Hagiu and Jullien (2011): we endogenize stores' decisions to affiliate with the intermediary. This implies stores are third-party entities not owned by the intermediary.

3.1 Timing and additional assumptions

Specifically, the timing of the game we consider in this section is as follows:

1. The intermediary publicly commits to q, which is observed by all players.

2. The intermediary sets the store fee r, which is observed by stores but not by consumers.

3. Stores and consumers simultaneously decide whether or not to affiliate with the intermediary.

4. Consumers observe store affiliation decisions, learn their type (1 or 2) and their search cost c and make search decisions. Store sales and intermediary revenues are realized.

Separating the choices of q and r has no effect on the solution of this game: we did it solely for the sake of consistency with the competing intermediaries case, where this assumption does make a difference as we discuss in section 4 below. Furthermore, we continue to assume $u_0 > t$, so that all consumers always affiliate with the intermediary at stage 3, which means that whether or not consumers observe the store fee r is irrelevant here. Once again however, this assumption matters in the case with competing intermediaries.

Importantly, the intermediary cannot price discriminate and therefore sets the same fee r for both stores. Nothing would change if instead we allowed the intermediary to engage in a more complicated bargaining game with stores. The only thing that matters is that the intermediary cannot extract the entire surplus from stores. Otherwise the analysis would be identical to the case with exogenous store affiliation (and possibly different revenues extracted by the intermediary from each store) discussed above.

The stage 4 consumer search process when both stores are affiliated with the intermediary is exactly the same as in the previous section. If only one store is affiliated, then every consumer is directed to that store right away with probability 1. In this case, a consumer with search cost c conducts at most one round of search. If her type matches the store's type then she searches if and only if $c \leq u^H$ and obtains net utility $u^H - c$. If her type does not match the store's type, she searches if and only if $c \leq u^L$ and obtains $u^L - c$.

We also need to specify stores' payoffs. For each store, affiliation with the intermediary requires a fixed investment $k > 0$ (writing a contract, designing products or a website to be compatible or available with the intermediary's service, etc.). Each store makes the same net profits π per consumer visit. In the online appendix, we also investigate the effect of allowing stores to make different net profits on different consumer types (i.e. $\pi^H > \pi^L$) and the intermediary to charge per sales fees as opposed to per consumer visit (or per click) fees.

Store i's profits when both stores affiliate with the intermediary are the sum of profits derived from visits by type i consumers (for whom store i is the favorite) and profits derived from visits by diverted type j consumers, net of fixed costs and intermediary fees. Using the fact that the consumer market is entirely covered, the expressions of stores' profits are:

$$\begin{aligned} \Pi_1^S &= (\pi - r)\left\{\alpha F(u(q)) + (1-\alpha)\left[qF\left(u^L\right) + (1-q)F(u(q))\right]\right\} - k \\ \Pi_2^S &= (\pi - r)\left\{(1-\alpha)F(u(q)) + \alpha\left[qF\left(u^L\right) + (1-q)F(u(q))\right]\right\} - k \end{aligned} \quad (4)$$

When store 1 alone affiliates, its payoffs are $(\pi - r)\left[\alpha F\left(u^H\right) + (1-\alpha)F\left(u^L\right)\right] - k$. And when store 2 alone affiliates with the intermediary, its payoffs are $(\pi - r)\left[(1-\alpha)F\left(u^H\right) + \alpha F\left(u^L\right)\right] - k$.

We need two additional assumptions for technical reasons.

Assumption 1 *k is not too large.*

The precise meaning of this assumption is detailed in the appendix, but the need for it is easily understood: it ensures that the monopoly intermediary always finds it profitable to induce both stores to affiliate with it. This is indeed the case we wish to focus on in order to determine the effect of endogenous store entry on the intermediary's incentives to divert search: search diversion is meaningless if only one store affiliates.

Furthermore, note that the model with endogenous store affiliation decisions encompasses two externalities: a two-sided *indirect* network effect between stores and consumers (participation on one side affects the value of affiliation on the other side), and a *direct* network effect between stores. The latter arises whenever there is some search diversion (i.e. whenever $q < 1$): if store j decides to affiliate, it will be sought by some consumers who would not search in its absence, and some of these new searchers are diverted to store $i \neq j$, their less preferred store. Thus, the presence of one store on the intermediary may benefit the other store. This externality between stores can potentially create multiple equilibria due to coordination failure in the presence of network effects. We therefore also assume:

Assumption 2 *In stage 3, stores always coordinate on a Pareto-optimal equilibrium from their joint perspective.*

This assumption ensures that whenever there are two alternative equilibria, one in which both stores affiliate with the intermediary and one in which none affiliates, the former prevails.

3.2 Optimal search diversion

In the appendix we show formally that, given assumptions 1 and 2, the intermediary necessarily induces both stores to affiliate in equilibrium. Here, we simply derive and provide the intuition for our results assuming that the intermediary finds it profitable to attract both stores.

Since store 1 is more popular than store 2 ($\alpha \geq 1/2$), we have $\Pi_1^S \geq \Pi_2^S$. Thus, given search quality q and participation by all consumers, the intermediary induces both stores to affiliate if and only if $\Pi_2^S \geq 0$. The intermediary's optimization problem is then to maximize revenues $rX(q)$ subject to this constraint. The constraint is binding, i.e. r is set so that:

$$\Pi_2^S = (\pi - r)\left[(1 - \alpha q) F(u(q)) + \alpha q F\left(u^L\right)\right] - k = 0$$

Given this fee, it is always an equilibrium for both stores to accept the offer and affiliate with the intermediary. Replacing the resulting r in the expression of intermediary profits, we obtain the intermediary's optimal choice of q:

$$q^M \equiv \arg\max_q \left\{\pi X(q) - K\left(\frac{F(u(q))}{X(q)}\right)\right\} \tag{5}$$

where $X(q)$ is defined in (1) above and:

$$K(x) \equiv \frac{k}{\alpha - (2\alpha - 1)x} \tag{6}$$

Note that $\pi X(q)$ is the joint revenue of the intermediary and the stores. Consequently, the term $K(F(u(q))/X(q))$ represents the net surplus that the intermediary must leave to stores in order to obtain affiliation by both. Given that $F(u(q))/X(q) > 1/2$, this surplus is larger than $2k$, the stores's affiliation costs.

The corresponding optimal fee r^M chosen by the intermediary is given by:

$$\left(\pi - r^M\right)\left[\left(1 - \alpha q^M\right) F\left(u\left(q^M\right)\right) + \alpha q^M F\left(u^L\right)\right] = k \tag{7}$$

Of course, the previous reasoning is heuristic and ignores the possibility that the intermediary might wish to induce only store 1 to affiliate. The following lemma (proven in the appendix) confirms that (q^M, r^M) do indeed characterize the optimal strategy for the intermediary.

Lemma 1 *Under assumptions 1 and 2, (q^M, r^M) defined by (5) and (7) are the optimal choices for a monopoly intermediary and they induce affiliation by both stores.*

The new feature that arises with endogenous store affiliation decisions is that search diversion affects not only the number of consumer visits, but also the surplus that has to be left to stores. This leads the intermediary to depart from the optimal choice q^X derived in the preceding section with exogenous store affiliation (expression 2). Since $K(.)$ is increasing, we obtain the following proposition:

Proposition 1 *A monopoly intermediary chooses a lower level of search quality when store affiliation is endogenous than when it is exogenously given, i.e. $q^M \leq q^X$.*

Proof. Note that:

$$\frac{X(q)}{F(u(q))} = q\left(\frac{F\left(u^L\right)}{F(u(q))} - 1\right) + 2$$

is decreasing in q because $F\left(u^L\right)/F\left(u\left(q\right)\right)-1<0$ and $F\left(u\left(q\right)\right)$ is increasing in q. Thus $K\left(F\left(u\left(q\right)\right)/X\left(q\right)\right)$ is increasing in q. This implies that any maximizer of $\pi X\left(q\right)-K\left(F\left(u\left(q\right)\right)/X\left(q\right)\right)$ is smaller than any maximizer of $\pi X\left(q\right)$. ∎

The result in Proposition 1 identifies a novel source of incentives for the intermediary to divert search, relative to the analysis in Hagiu and Jullien (2011). The intermediary wishes to reduce the profit differential between the two stores while maintaining the participation of the less profitable store 2. To see this, note that the sum of stores' *gross* profits (before paying the intermediary's fees) is $\pi X\left(q\right)-2k$, whereas the difference between these profits is $\pi\left(2\alpha-1\right)q\left[F\left(u\left(q\right)\right)-F\left(u^L\right)\right]$, which is strictly increasing in q for $\alpha>1/2$. If the intermediary could extract the entire surplus from stores (by price discriminating among stores or charging two-part tariffs with a fixed fee R and a variable fee r), it would maximize $\pi X\left(q\right)$ and therefore set $q=q^X$. But when it cannot price discriminate and is restricted to a unique variable fee r, the intermediary must leave $\pi\left(2\alpha-1\right)q\left[F\left(u\left(q\right)\right)-F\left(u^L\right)\right]$ on the table. In this context, reducing q below q^X allows it to enhance its ability to extract rents from stores. Fundamentally, this is because the intermediary's profit maximization places a higher weight on maximizing revenues coming from store 2, which benefits the most from search diversion.

It is straightforward to show that this result is unchanged if instead of charging variable fees r the intermediary could only charge fixed access fees R.[4] The only thing that matters is that the intermediary cannot extract the entire surplus from stores. For this reason, the incentive for search diversion identified here is reminiscent of the classic problem of quality choice by a monopoly firm (cf. Spence (1975)). Just like the monopolist's quality choice is driven by the *marginal* as opposed to the *average* consumer, so is our intermediary's choice of q driven by the marginal store, which prefers a lower q than the infra-marginal store.[5]

In the online appendix we show that allowing for partial coverage of the consumer market does not change these conclusions and has the same effect on search diversion as in the case with exogenous store affiliation.

4 Competition between intermediaries

We maintain the same structure of consumer preferences (with $u_0>t$) and the same two stores as in the previous section with endogenous store affiliation. The difference is that now there are two competing intermediaries, A and B, one at each end of the Hotelling $[0,1]$ segment. Each intermediary $i\in\{A,B\}$ chooses a level of search quality $q_i\in[0,1]$ and a per-click fee $r_i\geq 0$ that it charges to all affiliated stores. Consumers and stores can in principle affiliate with one or both intermediaries.

[4] The intermediary would then solve $\max_{R,q}2R$ subject to $\pi\left\{(1-\alpha)F\left(u\left(q\right)\right)+\alpha\left[qF\left(u^L\right)+(1-q)F\left(u\left(q\right)\right)\right]\right\}-R\geq k$. This is equivalent to $\max_q\left\{2\pi\left[\alpha X\left(q\right)-(2\alpha-1)F\left(u\left(q\right)\right)\right]\right\}$. Since $F\left(u\left(q\right)\right)$ is increasing in q, the solution to this optimization problem is also strictly lower than $q^X=\arg\max_q X\left(q\right)$.

[5] See Weyl (2010) for an analysis of Spencian distorsions due to pricing by two-sided platforms.

Timing

The timing is similar to the one in the previous section:

1. Intermediaries A and B simultaneously and credibly announce q_A and q_B. All players observe (q_A, q_B).

2. Intermediaries simultaneously and credibly announce their fees r_A and r_B. Only stores and intermediaries observe (r_A, r_B).

3. Stores and consumers simultaneously decide which intermediary(ies) to affiliate with. The resulting store affiliations are (S_A, S_B), while the resulting consumer affiliations are (D_A, D_B), where $S_i \in \{\emptyset, \{1\}, \{2\}, \{1,2\}\}$ and $D_i \in [0,1]$ for $i = A, B$.i[6]

4. Consumers observe store affiliations with intermediaries, learn their type (1 or 2) and their search cost c. Affiliated consumers make their search decisions. Store sales and intermediary revenues are realized.

There are two aspects of this timing structure which warrant discussion. First, the reason for separating the choices of q_i and r_i between the first two stages of the game is largely technical. Our characterization of competitive equilibria below would be the same if we worked with the entire space of simultaneous (q_i, r_i) deviations. The difference is that the set of equilibrium conditions would be significantly more complicated, which is why we opted for the simpler set-up. Second, the assumption that consumers observe (q_A, q_B) but not (r_A, r_B) has further simplifying virtues: it implies that consumer affiliation decisions at stage 3 only depend on (q_A, q_B). It is also quite reasonable: in most real-world settings, it is unlikely that consumers observe *and* understand the precise terms of the pricing relationships between intermediaries and affiliated stores.

Equilibrium

Given (q_A, q_B) chosen by intermediaries in stage 1, an equilibrium $\Omega(q_A, q_B)$ of the *subgame starting at stage 2* consists of fees $r_i^*(q_i, q_j)$ chosen by intermediaries at stage 2, consumer affiliation demands $D_i^*(q_i, q_j)$ and store affiliation decisions $S_i^*(r_i, q_i, q_j)$ realized at stage 3 for $i \neq j \in \{A, B\}$, such that:[7]

- at stage 2, $r_i^*(q_i, q_j)$ is intermediary i's optimal choice given that intermediary j chooses $r_j^*(q_j, q_i)$, consumer affiliations are equal to $D_A^*(q_A, q_B)$ and $D_B^*(q_B, q_A)$ and store affiliation decisions are given by $S_A^*(r_A, q_A, q_B)$ and $S_B^*(r_B, q_B, q_A)$

- at stage 3, $S_A^*(r_A, q_A, q_B)$ and $S_B^*(r_B, q_B, q_A)$ result from each store making optimal affiliation decisions, given the other store's decision, intermediary fees equal to r_A and r_B and consumer affiliations equal to $D_A^*(q_A, q_B)$ and $D_B^*(q_B, q_A)$

[6] For example, $S_i = \{1, 2\}$ means that both stores affiliate with intermediary i.

[7] Consumer demand for intermediary i, $D_i^*(q_i, q_j)$, does not depend on (r_i, r_j) because consumers do not observe (r_i, r_j). This also implies that stores' decisions whether or not to affiliate with intermediary i depend on q_i, q_j and r_i, but not on r_j, for $i \neq j \in \{A, B\}$.

- at stage 3, $D_A^*(q_A, q_B)$ and $D_B^*(q_B, q_A)$ result from consumers making optimal affiliation decisions, given store affiliations equal to $S_A^*(r_A^*(q_A, q_B), q_A, q_B)$ and $S_B^*(r_B^*(q_B, q_A), q_B, q_A)$.

An equilibrium for the full game starting at stage 1 is then a pair (q_A^*, q_B^*) such that there exists a stage 2 equilibrium $\Omega(q_A^*, q_B^*)$ in which intermediary A's profits are higher than in *any* equilibrium $\Omega(q_A, q_B^*)$ and intermediary B's profits are higher than in *any* equilibrium $\Omega(q_A^*, q_B)$, for all $(q_A, q_B) \in [0,1]^2$.

In what follows we analyze two different versions of this model, depending on whether or not consumers obtain the standalone utility u_0 from every intermediary they affiliate with. These two versions are designed to lead to two polar equilibrium outcomes: i) each consumer affiliates with one intermediary only (i.e. singlehomes), while both stores multihome; ii) all consumers multihome, while the two stores singlehome with the same intermediary. In both cases, the consumer search behavior at stage 4 is unchanged from the previous sections. The differences occur in stage 3 and earlier.

4.1 Stores multihome and consumers singlehome

In this subsection we assume that u_0 can be decomposed as $u_0 = v_0 - s$, where v_0 is the gross standalone utility obtained by a consumer from visiting an intermediary, while s is the corresponding cost. Crucially, standalone utilities v_0 are assumed to be perfect substitutes across the two intermediaries, i.e. if a consumer visits both intermediaries, her standalone utility is just v_0. This scenario occurs when the two intermediaries offer the same standalone service, e.g. news and weather reports for an e-commerce site. By contrast, the cost s has to be incurred for every intermediary a consumer affiliates with: this can be thought of as the fixed cost of registering and/or learning how to deal with a given intermediary. We make the following assumption:

Assumption 3 *v_0 and s are large enough so that every consumer affiliates with exactly one intermediary in stage 3.*

We also maintain assumptions 1 and 2 from the previous section. The only difference relative to the monopoly case is the exact upper bound on k needed for assumption 1 (the details are relegated to the appendix). Intuitively, here assumption 1 guarantees that stores' fixed affiliation costs are low enough so that they will multihome in any equilibrium.

At stage 3, consumers' affiliation decisions depend on the identity and the number of stores they *expect* to find on each intermediary. The expected utility for a consumer affiliating with intermediary $i \in \{A, B\}$ when the intermediary has set q_i and the consumer anticipates the set of stores affiliated with i to be S_i is $u_0 + V(q_i, S_i)$, where $V(q_i, S_i)$ is defined as follows:

$$\begin{aligned} V(q_i, \{1,2\}) &\equiv \int_0^{u^L} \left(u^H + u^L - 2c\right) f(c)\, dc + \int_{u^L}^{u(q_i)} \left(u^H + (1-q_i)\, u^L - (2-q_i)\, c\right) f(c)\, dc \\ V(q_i, \{1\}) &\equiv \alpha \int_0^{u^H} \left(u^H - c\right) f(c)\, dc + (1-\alpha) \int_0^{u^L} \left(u^L - c\right) f(c)\, dc \\ V(q_i, \{2\}) &\equiv (1-\alpha) \int_0^{u^H} \left(u^H - c\right) f(c)\, dc + \alpha \int_0^{u^L} \left(u^L - c\right) f(c)\, dc \\ V(q_i, \emptyset) &\equiv 0 \end{aligned}$$

Note that the consumer's expected utility when she expects both stores to affiliate, $V(q_i, \{1,2\})$, is strictly increasing in q_i. By contrast, $V(q_i, \{1\})$ and $V(q_i, \{2\})$ are independent of q_i. Furthermore, since $\alpha \geq 1/2$, we have $V(q_i, \{2\}) \leq V(q_i, \{1\}) < V(1, \{1,2\})$ for all $q_i \in [0,1]$.

At stage 3, when consumers expect the sets of stores affiliated with the intermediaries to be (S_i, S_j) and (q_i, q_j) were chosen in stage 1, consumer affiliation with intermediary $i \neq j \in \{A, B\}$ is:

$$d_i(q_i, q_j, S_i, S_j) = \frac{1}{2} + \frac{1}{2t}\left[V(q_i, S_i) - V(q_j, S_j)\right]$$

Suppose that at stage 2 both intermediaries find it profitable to set their fees such that both stores multihome in the ensuing affiliation equilibrium. We will rigorously show in the proof of Lemma 2 below that this is necessarily the case in any equilibrium for the entire game. At stage 3, total consumer affiliation with intermediary $i \in \{A, B\}$ must then be:

$$D_i^*(q_i, q_j) \equiv d_i(q_i, q_j, \{1,2\}, \{1,2\}) = \frac{1}{2} + \frac{1}{2t}\left[V(q_i, \{1,2\}) - V(q_j, \{1,2\})\right]$$

The profits Π_{li}^S obtained by store $l \in \{1,2\}$ from its affiliation with intermediary $i \in \{A, B\}$ are then:

$$\begin{aligned} \Pi_{1i}^S &= (\pi - r_i)\left[(1-(1-\alpha)\, q_i)\, F(u(q_i)) + (1-\alpha)\, q_i F\left(u^L\right)\right] D_i^*(q_i, q_j) - k \\ \Pi_{2i}^S &= (\pi - r_i)\left[(1-\alpha q_i)\, F(u(q_i)) + \alpha q_i F\left(u^L\right)\right] D_i^*(q_i, q_j) - k \end{aligned}$$

Note that the expressions of store profits are similar to (4) in section 3, except for the partial consumer market coverage term $D_i^*(q_i, q_j)$.

Given $D_i^*(q_i, q_j)$ and because stores can multihome, store l's decision to affiliate or not with intermediary i at stage 3 depends only on the expected profit Π_{li}^S. Store 1 is more popular than store 2, therefore $\Pi_{1i}^S \geq \Pi_{2i}^S$. Consequently, given $D_A^*(q_A, q_B)$ and $D_B^*(q_B, q_A)$, both stores affiliate with intermediary i in stage 3 if and only if store 2 is willing to affiliate with intermediary i, i.e. $\Pi_{2i}^S \geq 0$ for $i = A, B$.

Our assumptions on the the structure of the game imply that stage 3 consumer affiliation with intermediary $i \in \{A, B\}$ is not affected by the *actual* choice of fee r_i but only by its *expected* value in a stage 2 equilibrium given (q_A, q_B). If such an equilibrium involves both stores multihoming then in stage 2 intermediary i necessarily sets the fee r_i at the level that solves:

$$\max_{r_i} \{r_i X(q_i) D_i^*(q_i, q_j)\} \quad \text{subject to } \Pi_{2i}^S \geq 0$$

As in the previous section, the intermediary's revenues are the product of the per-click fee r_i, the total number of consumers who affiliate $D_i^*(q_i, q_j)$ and the expected number of store visits (clicks) per affiliated consumer $X(q_i)$. Store 2's participation constraint is binding and equivalent to:

$$(\pi - r_i)\left[(1 - \alpha q_i) F(u(q_i)) + \alpha q_i F\left(u^L\right)\right] D_i^*(q_i, q_j) = k \tag{8}$$

The analysis is therefore very similar to the one in the monopoly section with endogenous store affiliation. In stage 1 intermediary $i \in \{A, B\}$ solves:

$$\max_{q_i} \left\{\pi X(q_i) D_i^*(q_i, q_j) - K\left(\frac{F(u(q_i))}{X(q_i)}\right)\right\}$$

Recall that $K(.)$ was defined in (6) above and represents the net surplus that the intermediary must leave to stores when it cannot perfectly price discriminate.

Let then q_s^C solve:

$$q_s^C = \arg\max_{q_i} \left\{\pi X(q_i) D_i^*\left(q_i, q_s^C\right) - K\left(\frac{F(u(q_i))}{X(q_i)}\right)\right\} \tag{9}$$

In the appendix we prove the following lemma:

Lemma 2 *Under assumptions 1, 2 and 3, $\left(q_s^C, q_s^C\right)$ defined by (9) is the unique symmetric equilibrium of the game starting in stage 1.*

The proof of the lemma is conceptually straightforward. The only part which requires a somewhat lengthy discussion and is not contained in the analysis above is proving that *any* equilibrium involves both stores multihoming. This is guaranteed by Assumption 1.

Using (9) and the fact that in the symmetric equilibrium $D_i^*\left(q_s^C, q_s^C\right) = 1/2$, the first order condition determining q_s^C is equivalent to:

$$\pi X'(q) - 2\frac{d}{dq}\left[K\left(\frac{F(u(q))}{X(q)}\right)\right] + \frac{\pi X(q)}{t}\frac{\partial V(q, \{1, 2\})}{\partial q} = 0$$

which we can compare to the first-order condition determining q^M, the monopoly intermediary's choice when store entry is endogenous (cf. (5) above):

$$\pi X'(q) - \frac{d}{dq}\left[K\left(\frac{F(u(q))}{X(q)}\right)\right] = 0$$

There are two terms that drive a wedge between q_s^C and q^M. First, the positive term $\frac{\pi X(q)}{t}\frac{\partial V(q,\{1,2\})}{\partial q}$ represents the expected effect due to competition for consumers, which tends to lead to *less* search diversion (higher search quality) compared to a monopoly intermediary. Because this term is positive and decreasing in t, there is *less* search diversion in equilibrium when competition among intermediaries for consumers is *more* intense, i.e. when t is *smaller*.

Second, the negative term $-\frac{d}{dq}\left[K\left(\frac{F(u(q))}{X(q)}\right)\right]$ is due to the fact that the fixed cost for a store (k) must be recouped on a smaller demand with competing platforms (1/2 instead of 1), which tends to lead to more search diversion. Note that $\alpha = 1/2$ renders this term equal to 0, leading to the following proposition:

Proposition 2 *For α sufficiently close to $1/2$, competition between intermediaries with singlehoming consumers induces less search diversion than a monopoly intermediary, i.e. $q_s^C > q^M$.*

This proposition confirms the common intuition one might have: competition between intermediaries should lead to better search quality. Fixing the levels of consumer participation, each intermediary acts as a monopoly on the other side of the market (because stores multihome) and therefore extracts monopoly rents from stores' participation. But the value that can be captured from stores depends on the mass of affiliated consumers. Since consumers singlehome, intermediaries have to compete to attract consumers, which leads to less search diversion. This logic extends similar results obtained for equilibrium prices with Bertrand competition between competitive bottlenecks (Caillaud and Jullien (2003), Armstrong (2006), Crampes et al (2010)). The key difference is that here the instrument for competition is the level of search diversion instead of the price of the service.

As we will see however, this intuition no longer applies when the nature of two-sided competition between intermediaries is changed.

4.2 Stores singlehome and consumers multihome

We continue to decompose u_0 as $u_0 = v_0 - s$, but in this subsection we assume that consumers derive the standalone utility u_0 from *each* intermediary they affiliate with (unlike the previous subsection, in which u_0 could only be enjoyed once). This corresponds to situations in which intermediaries offer different standalone services (e.g. one e-commerce site could post sports results on its site, while the other shows movie reviews). Since $u_0 > t$, this implies:

Remark 1 *In any stage 2 equilibrium $\Omega(q_A, q_B)$ all consumers affiliate with both intermediaries:*

$$D_A^*(q_A, q_B) = D_B^*(q_B, q_A) = 1 \text{ for all } (q_A, q_B)$$

Note however that, although each consumer is affiliated with both intermediaries, she need not search on both of them in stage 4.

Throughout this sub-section, we make the following assumption (its precise meaning is detailed in the appendix):

Assumption 4 *k is not too small*

This assumption guarantees that stores do not want to join both intermediaries given that consumers multihome. Importantly, we show in the appendix that this assumption is compatible (i.e. can be satisfied at the same time) with assumption 1, so that all equilibria derived in the paper co-exist for a non-empty range of parameter values.

The profits derived by the two stores when they both *exclusively* affiliate with an intermediary which has chosen (q,r) in stages 1 and 2 are then:

$$\begin{aligned}\Pi_1^S(q,r) &\equiv (\pi - r)\left\{\alpha F(u(q)) + (1-\alpha)\left[qF\left(u^L\right) + (1-q)F(u(q))\right]\right\} - k,\\ \Pi_2^S(q,r) &\equiv (\pi - r)\left\{(1-\alpha)F(u(q)) + \alpha\left[qF\left(u^L\right) + (1-q)F(u(q))\right]\right\} - k\end{aligned}$$

Since all consumers multihome, the characterization of equilibria for the full game is easier than in the previous subsection. It is contained in the following lemma (proven in the appendix):

Lemma 3 *Under assumptions 2 and 4, there exists an equilibrium in which both intermediaries set $\left(q_m^C, r_m^C\right)$ and both stores affiliate exclusively with the same intermediary if and only if the following three conditions hold: a) $r_m^C = 0$; b) $\Pi_i^S\left(q_m^C, 0\right) \geq \Pi_i^S(1,0)$ for $i = 1,2$; c) there exists no $q \in [0,1)$ such that $\Pi_1^S(q,0) \geq \Pi_1^S\left(q_m^C, 0\right)$ and $\Pi_2^S(q,0) \geq \Pi_2^S\left(q_m^C, 0\right)$ with at least one strict inequality.*

Note that both intermediaries make 0 profits in all equilibria characterized by this lemma (condition a)). This is not surprising: since consumers multihome, the intermediaries find themselves competing a la Bertrand for store affiliation, which leads to $r_m^C = 0$. Condition b) ensures that no store wants to deviate by affiliating exclusively with the inactive intermediary, given the other store's choice. We show in the appendix that this condition also guarantees that no store wants to deviate by multihoming. Finally, condition c) ensures that neither intermediary can find a level of search quality which would then enable it to attract both stores exclusively at a positive fee.

To reach a more useful characterization of the possible equilibria, we further assume that when both stores affiliate exclusively with an intermediary, their profits are single-peaked in q, the level of search quality chosen by that intermediary. Denote by q_{m1}^C and q_{m2}^C the respective maximizers of the two stores' profit expressions:

$$\begin{aligned}q_{m1}^C &\equiv \arg\max_q \Pi_1^S(q,0) = \arg\max_q \left\{(1-\alpha)X(q) + (2\alpha - 1)F(u(q))\right\}\\ q_{m2}^C &\equiv \arg\max_q \Pi_2^S(q,0) = \arg\max_q \left\{\alpha X(q) - (2\alpha - 1)F(u(q))\right\}\end{aligned}$$

Since $F(u(q))$ is increasing in q, the expressions above imply that $q_{m2}^C < q^X < q_{m1}^C$ for all q when $\alpha > 1/2$. In other words, the less popular store has stronger preferences for search diversion (it prefers a lower quality of search q).

Let us also define:

$$q^L_{m1} \equiv \min\left\{q \geq 0, \Pi^S_1(q,0) \geq \Pi^S_1(1,0)\right\}$$

the minimum level of search quality that store 1 is willing to accept in order to stay exclusive with the same intermediary as store 2 (rather than going exclusive with the other intermediary and obtaining $\Pi^S_1(1,0)$). It is easily seen that q^L_{m1} is smaller than q^C_{m1} when the latter is smaller than 1.

With these additional notations, the following proposition follows directly from Lemma 3:

Proposition 3 *If $q^C_{m1} = 1$ then the unique equilibrium with stores singlehoming and consumers multihoming is $q^C_m = 1$. If $0 \leq q^C_{m2} < q^C_{m1} < 1$ then there exists a range of equilibria: $q^C_m \in \left[\max\left(q^C_{m2}, q^L_{m1}\right), q^C_{m1}\right]$.*

Proof. First, note that any equilibrium q cannot be lower than q^L_{m1} since that would violate condition b) in Lemma 3. This implies that when $q^C_{m1} = 1$ the only possible equilibrium is $q = 1$. Second, note that any equilibrium q must belong to the interval $\left[q^C_{m2}, q^C_{m1}\right]$. Indeed, any $q < q^C_{m2}$ is strictly Pareto-dominated from the two stores' perspective by q^C_{m2}, while any $q > q^C_{m1}$ is Pareto-dominated by q^C_{m1} (condition c) in Lemma 3). Finally, any $q \in \left[\max\left(q^C_{m2}, q^L_{m1}\right), q^C_{m1}\right]$ clearly satisfies conditions b) and c) in Lemma 3, so is an equilibrium. ■

The various cases described in Proposition 3 are illustrated in figures 1-3 below.

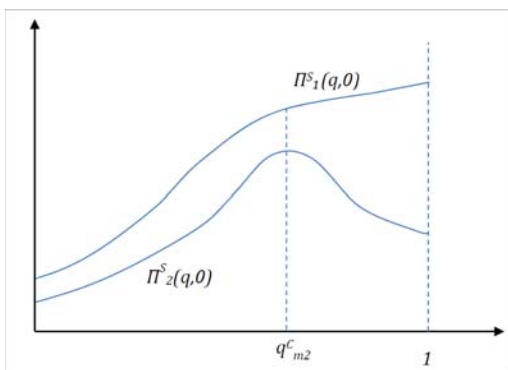

Figure 1

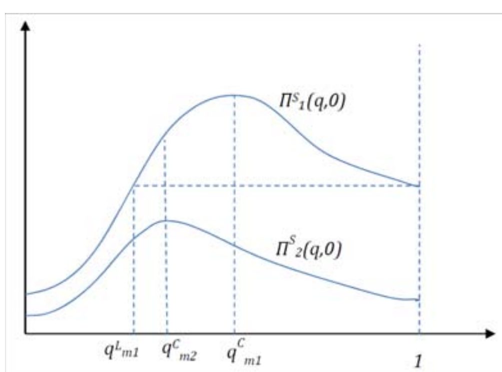

Figure 2

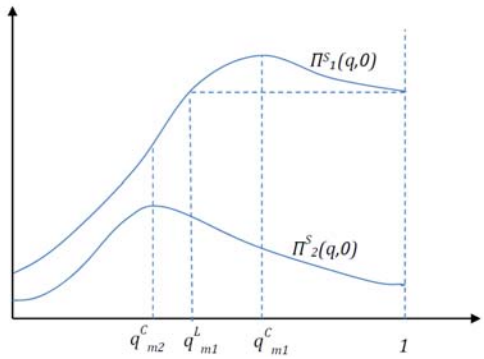

Figure 3

The key insight contained in Proposition 3 is that the equilibrium level of search diversion when consumers multihome and stores singlehome is entirely driven by store preferences. Note indeed that the range of possible equilibrium values for q_m^C is independent of the intensity of competition on the consumer side of the market (t). This is of course due to the fact that consumers multihome and therefore intermediaries must concentrate all of their competitive efforts on stores.

Recall that stores disagree on the optimal level of search quality: store 2 prefers a lower level (i.e. more diversion) than store 1. The equilibrium level of search quality can be anywhere in-between. Fundamentally, this is due to the positive externalities among stores' affiliation decisions, which lead them to prefer coordinating on the same intermediary when both intermediaries divert search. The chosen intermediary is then the product of an uncoordinated choice, which may favor either one of the two stores.

Comparing stores' preferred levels of search quality with the level chosen by a monopoly intermediary, we have:

$$q_{m2}^C < q^M < q_{m1}^C$$

for all $\alpha > 1/2$.[8] Combining these inequalities with Proposition 2 and recalling the definitions of q_{m1}^C, q_{m2}^C and q_s^C above, we have:

Corollary 1 *If $q^X < 1$ then for all α greater than but sufficiently close to $1/2$, $q_{m1}^L < q_{m2}^C < q^M < q_{m1}^C < q_s^C$.*

Proof. The only inequalities not already proven are $q_{m1}^L < q_{m2}^C$ and $q_{m1}^C < q_s^C$. They are obtained by noting that q_{m1}^C, q_{m2}^C and q^M all tend to q^X when $\alpha \to 1/2$, while q_s^C and q_{m1}^L are bounded away from q^X. ■

[8] The right inequality follows from $q_{m1}^C > q_X$. The left inequality ($q_{m2}^C < q^M$) is obtained by writing:

$$q^M = \arg\max_q \left\{ X(q) \left[\pi - \frac{k}{\alpha X(q) - (2\alpha - 1) F(u(q))} \right] \right\}$$

Corrolary 3 along with Propositions 2 and 3 make it clear that, depending on its nature, competition among intermediaries has opposite effects on the level of search diversion relative to a monopoly intermediary. When consumers multihome, competition between intermediaries leads to *more* search diversion relative to a monopolist with positive probability (i.e. there is a positive measure of competitive equilibria involving more search diversion). By contrast, when consumers singlehome, competition among intermediaries leads to less search diversion (q_s^C).

5 Conclusion and managerial implications

Our study of search diversion by intermediaries has yielded several important insights that should be of managerial interest. First and at the most basic level, search diversion is a strategic instrument that enables intermediaries to trade-off total user traffic for higher revenues per individual user visit. It is particularly useful when the most sought-after stores or products are not the ones that yield the highest revenues for the intermediary that provides access to them. This is why all advertising-supported platforms (e.g. magazines, online portals, search engines) do *not* provide users with the easiest and clearest possible access to the "objective" content they seek. Instead, they compromise the user search experience to at least some degree by ensuring that they stumble upon advertisements. Similarly, even recommender systems such as Netflix's, Amazon's or Apple's operate a compromise between recommending the products which best fit a consumer's preferences (conditional on the information available to the intermediary) and those that yield the highest margins for the intermediary.

Second, intermediaries' incentives to strategically divert user search are stronger when the affiliation of stores is endogenous and intermediaries cannot price discriminate among stores. In this case, intermediaries find it optimal to divert search beyond what the basic tradeoff described above indicates, in order to improve their rent-extraction power vis-a-vis stores. Indeed, search diversion reduces the variance of revenues among the various stores (it transfers revenues from infra-marginal stores to the marginal stores). As a result, intermediaries can extract larger fees from stores. This insight has a clear empirical implication which can be tested: intermediaries who control or own stores (products) should induce less search diversion relative to intermediaries who do not.

Third, the interdependence between affiliation decisions on the two sides of the market - consumers and stores - implies that the effect of competition between intermediaries on the equilibrium level of search diversion is determined by the nature of competition. When stores multihome and consumers singlehome, intermediaries compete mostly for consumers, which leads them to reduce search diversion relative to the monopoly case. Conversely, when consumers multihome and stores singlehome, intermediaries focus their competitive efforts on attracting stores, which may lead them to divert search even more than a monopolist would. Again, these results suggest both strategy recommendations and empirical tests: the response of intermediaries' search diversion incentives to competitive pressure should depend on which side of the market becomes more competitive.

A broader implication of our analysis is that design decisions by two-sided platforms and intermediaries (search effectiveness being but one specific example) are fundamentally driven by their quest to improve rent-extraction power and can favor one side or the other of the market, depending on the precise nature of competition. One should then not be surprised that intensified competition may lead to design decisions which go against consumers' preferences - but instead favor third-party sellers, advertisers, or whatever the relevant second side might be. A striking example was the privacy compromise made by Microsoft with the design of its Internet Explorer 8 web browser in favor of advertisers and against users' interests. The episode was reported by the Wall Street Journal: "As the leading maker of Web browsers, the gateway software to the Internet, Microsoft must balance conflicting interests: helping people surf the Web with its browser to keep their mouse clicks private, and helping advertisers who want to see those clicks. In the end, the product planners lost a key part of the debate. The winners: executives who argued that giving automatic privacy to consumers would make it tougher for Microsoft to profit from selling online ads. Microsoft built its browser so that users must deliberately turn on privacy settings every time they start up the software." (Wingfield (2010))

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6 Appendix

6.1 Assumptions

The precise assumptions required for all of our proofs are as follows:

- Assumption 1 for the monopoly section 3:

$$\frac{\left[(1-\alpha)F\left(u^H\right)+\alpha F\left(u^L\right)\right]^2}{\alpha F\left(u^H\right)+(1-\alpha)F\left(u^L\right)} > \frac{k}{\pi} \tag{10}$$

- Assumption 1 for the duopoly section 4.1:

$$\frac{1}{2}\left[1-\frac{1}{t}\left(\begin{array}{c}\alpha\int_0^{u^L}\left(u^L-c\right)f(c)\,dc\\ +(1-\alpha)\int_0^{u^H}\left(u^H-c\right)f(c)\,dc\end{array}\right)\right]\frac{\left[(1-\alpha)F\left(u^H\right)+\alpha F\left(u^L\right)\right]^2}{\alpha F\left(u^H\right)+(1-\alpha)F\left(u^L\right)} > \frac{k}{\pi} \tag{11}$$

- Assumption 4 for the duopoly section 4.2:

$$\alpha(1-\alpha)\left[F\left(u^H\right)-F\left(u^L\right)\right] < \frac{k}{\pi} \tag{12}$$

Note that (11) implies (10). There exists $\varepsilon > 0$ such that all of these 3 assumptions are verified for $\alpha \in [1/2, 1/2+\varepsilon]$ and t very large if the following condition holds:

$$\frac{F\left(u^H\right)+F\left(u^L\right)}{4} > \frac{k}{\pi} > \frac{F\left(u^H\right)-F\left(u^L\right)}{4}$$

6.2 Proof of Lemma 1

Recall that $u_0 > t$ implies that all consumers affiliate with the intermediary for any choice of q and r. We need to show two things:

i) at $\left(q^M, r^M\right)$ the intermediary sells to both stores

ii) the intermediary cannot profitably deviate from $\left(q^M, r^M\right)$.

Define $\hat{r}$ such that:

$$(\pi - \hat{r})\left[\alpha F\left(u^H\right) + (1-\alpha) F\left(u^L\right)\right] = k$$

$\hat{r}$ is the highest fee that store 1 is willing to pay when it anticipates to be the only store affiliated with the intermediary.

For point i), suppose the intermediary sets q^M and r^M (minus some small ε). If $r^M \leq \hat{r}$ then affiliation by both stores is the unique stage 3 equilibrium. Indeed, store 1 affiliates in any equilibrium implying that store 2 affiliates as well. If $r^M > \hat{r}$, there are two possible equilibria: either both stores affiliate or none affiliates. But our assumption that stores coordinate on a Pareto undominated equilibrium rules out the no affiliation equilibrium.

For point ii), note that by the definition of $\left(q^M, r^M\right)$ the intermediary cannot profitably deviate to any (q, r) such that both stores affilate. Let us consider then a deviation which results in affiliation of only one store. Clearly, the intermediary cannot do better in any such deviation than in the scenario in which it sets $r = \hat{r}$ and only store 1 affiliates. Thus the intermediary's deviation profit cannot be greater than:

$$\hat{r}\left[\alpha F\left(u^H\right) + (1-\alpha) F\left(u^L\right)\right] = \pi\left[\alpha F\left(u^H\right) + (1-\alpha) F\left(u^L\right)\right] - k$$

By definition of q^M, we have:

$$\pi X\left(q^M\right) - K\left(\frac{F\left(u\left(q^M\right)\right)}{X\left(q^M\right)}\right) \geq \pi X(1) - K\left(\frac{F(u(1))}{X(1)}\right)$$

But:

$$\pi X(1) - K\left(\frac{F(u(1))}{X(1)}\right) = \pi\left(F\left(u^H\right) + F\left(u^L\right)\right) - k\frac{F\left(u^H\right) + F\left(u^L\right)}{\alpha F\left(u^L\right) + (1-\alpha) F\left(u^H\right)}$$

Comparing this with the maximal profit obtained when only store 1 affiliates, we have:

$$\begin{aligned} & \pi X(1) - K\left(\frac{F(u(1))}{X(1)}\right) - \left\{\pi\left[\left(\alpha F\left(u^H\right) + (1-\alpha) F\left(u^L\right)\right)\right] - k\right\} \\ = \quad & \pi\left[(1-\alpha) F\left(u^H\right) + \alpha F\left(u^L\right)\right] - k\frac{\alpha F\left(u^H\right) + (1-\alpha) F\left(u^L\right)}{\alpha F\left(u^L\right) + (1-\alpha) F\left(u^H\right)} \end{aligned}$$

The last expression is positive by (10).

■

6.3 Proof of Lemma 2

The proof of the Lemma is in 4 steps.

Step 1 *Fix (q_i, q_j) chosen in stage 1. Then, in the game starting at stage 2 and for any S_j:*

- *If $D_i(q_i, q_j, \{1,2\}, S_j)$ is an equilibrium affiliation demand for intermediary i then:*

$$(1-\alpha q_i) F(u(q_i)) + \alpha q_i F(u^L) \geq \alpha F(u^H) + (1-\alpha) F(u^L)$$

or:

$$\pi \left[X(q_i) - \alpha F(u^H) - (1-\alpha) F(u^L)\right] D_i(q_i, q_j, \{1,2\}, S_j) \geq K\left(\frac{F(u(q_i))}{X(q_i)}\right) - k$$

- *If $D_i(q_i, q_j, \{1\}, S_j)$ is an equillibrium affiliation demand for intermediary i then:*

$$(1-\alpha q_i) F(u(q_i)) + \alpha q_i F(u^L) < \alpha F(u^H) + (1-\alpha) F(u^L)$$

and:

$$\pi \left[X(q_i) - \alpha F(u^H) - (1-\alpha) F(u^L)\right] D_i(q_i, q_j, \{1\}, S_j) \leq K\left(\frac{F(u(q_i))}{X(q_i)}\right) - k$$

Proof of Step 1

If the equilibrium of the game starting at stage 2 involves intermediary i obtaining affiliation demand $D_i(q_i, q_j, \{1,2\}, S_j)$ then in this equilibrium both stores affiliate with intermediary i in stage 3 and the fee r_i set by intermediary i in stage 2 must be such that store 2 is exactly indifferent between affiliating or not when it anticipates store 1 to affiliate and consumer affiliation to be $D_i(q_i, q_j, \{1,2\}, S_j)$:

$$(\pi - r_i)\left[(1-\alpha q_i) F(u(q_i)) + \alpha q_i F(u^L)\right] D_i(q_i, q_j, \{1,2\}, S_j) = k$$

Intermediary i's profits in this equilibrium are therefore equal to:

$$r_i X(q_i) D_i(q_i, q_j, \{1,2\}, S_j) = \pi X(q_i) D_i(q_i, q_j, \{1,2\}, S_j) - K\left(\frac{F(u(q_i))}{X(q_i)}\right)$$

Let r_i' denote the highest fee that store 1 is willing to pay when it anticipates to be the only store affiliated with intermediary i and consumer affiliation to be equal to $D_i(q_i, q_j, \{1,2\}, S_j)$:

$$(\pi - r_i')\left[\alpha F(u^H) + (1-\alpha) F(u^L)\right] D_i(q_i, q_j, \{1,2\}, S_j) = k$$

Since consumers do not observe the choice of r_i in stage 2, $D_i(q_i, q_j, \{1,2\}, S_j)$ is an equilibrium only if the intermediary cannot profitably deviate from r_i while consumer affiliation demand stays fixed at $D_i(q_i, q_j, \{1,2\}, S_j)$.

Suppose then that:

$$(1-\alpha q_i) F(u(q_i)) + \alpha q_i F\left(u^L\right) < \alpha F\left(u^H\right) + (1-\alpha) F\left(u^L\right)$$

and

$$\pi\left[X(q_i) - \alpha F\left(u^H\right) - (1-\alpha) F\left(u^L\right)\right] D_i(q_i, q_j, \{1,2\}, S_j) < K\left(\frac{F(u(q_i))}{X(q_i)}\right) - k$$

The first inequality implies $r_i < r_i'$. Thus, if intermediary i deviated to r_i', it would induce affiliation by store 1 only in stage 3, while consumer affiliation would remain unchanged. The intermediary's deviation profits would then be:

$$\pi\left[\alpha F\left(u^H\right) + (1-\alpha) F\left(u^L\right)\right] D_i(q_i, q_j, \{1,2\}, S_j) - k$$

and by the second inequality these profits are strictly higher than the intermediary's equilibrium profits $\pi X(q_i) D_i(q_i, q_j, \{1,2\}, S_j) - K\left(\frac{F(u(q_i))}{X(q_i)}\right)$. Thus, $D_i(q_i, q_j, \{1,2\}, S_j)$ could not be an equilibrium, which is a contradiction.

We have thus proven that $D_i(q_i, q_j, \{1,2\}, S_j)$ is an equilibrium only if one or both of the last two inequalities above does not hold.

The proof for $D_i(q_i, q_j, \{1\}, S_j)$ is very similar. If the equilibrium of the game starting at stage 2 involves intermediary i obtaining affiliation demand $D_i(q_i, q_j, \{1\}, S_j)$ then in this equilibrium only store 1 affiliates with intermediary i in stage 3 and the fee r_i set by intermediary i in stage 2 must be such that store 1 is exactly indifferent between affiliating or not when it anticipates to be the only affiliated store and consumer affiliation to be $D_i(q_i, q_j, \{1,2\}, S_j)$:

$$(\pi - r_i)\left[\alpha F\left(u^H\right) + (1-\alpha) F\left(u^L\right)\right] D_i(q_i, q_j, \{1\}, S_j) = k$$

Intermediary i's profits in this equilibrium are therefore equal to:

$$r_i\left[\alpha F\left(u^H\right) + (1-\alpha) F\left(u^L\right)\right] D_i(q_i, q_j, \{1\}, S_j) - k$$

Let r_i' be the highest fee that store 2 is willing to pay when it anticipates both stores to affiliate with intermediary i and consumer affiliation to be $D_i(q_i, q_j, \{1\}, S_j)$:

$$(\pi - r_i')\left[(1-\alpha q_i) F(u(q_i)) + \alpha q_i F\left(u^L\right)\right] D_i(q_i, q_j, \{1\}, S_j) = k$$

If $r_i' \geq r_i$ then affiliation by store 1 only with intermediary i would never be an equilibrium. We therefore must have $r_i' < r_i$, which is equivalent to:

$$(1-\alpha q_i) F(u(q_i)) + \alpha q_i F\left(u^L\right) < \alpha F\left(u^H\right) + (1-\alpha) F\left(u^L\right)$$

Furthermore, suppose that:

$$\pi\left[X(q_i)-\alpha F\left(u^H\right)-(1-\alpha)F\left(u^L\right)\right]D_i\left(q_i,q_j,\{1\},S_j\right)>K\left(\frac{F\left(u\left(q_i\right)\right)}{X\left(q_i\right)}\right)-k$$

If intermediary i deviated from r_i by setting r_i' (slightly below) then both stores affiliating with intermediary i would be the only equilibrium in stage 3 given that consumer affiliation stays unchanged at $D_i(q_i,q_j,\{1\},S_j)$. But then the last inequality implies that this deviation would be strictly profitable, which is a contradiction. Thus, we have proven that $D_i(q_i,q_j,\{1\},S_j)$ is an equilibrium only if:

$$(1-\alpha q_i)F\left(u\left(q_i\right)\right)+\alpha q_iF\left(u^L\right)<\alpha F\left(u^H\right)+(1-\alpha)F\left(u^L\right)$$

and:

$$\pi\left[X(q_i)-\alpha F\left(u^H\right)-(1-\alpha)F\left(u^L\right)\right]D_i\left(q_i,q_j,\{1\},S_j\right)\leq K\left(\frac{F\left(u\left(q_i\right)\right)}{X\left(q_i\right)}\right)-k$$

■

Step 2 *If intermediary i chooses $q_i=1$ in stage 1 then any equilibrium of the game starting in stage 2 necessarily involves both stores affiliating with intermediary i.*

Proof of Step 2

Suppose by contradiction that given $q_i=1$ and q_j chosen in stage 1, only store 1 affiliates with intermediary i in the ensuing equilibrium. Then the equilibrium consumer affiliation demand for intermediary i must be $D_i(1,q_j,\{1\},S_j)$ and the fee r_i must be such that:

$$(\pi-r_i)\left[\alpha F\left(u^H\right)+(1-\alpha)F\left(u^L\right)\right]D_i\left(1,q_j,\{1\},S_j\right)\geq k$$

implying that intermediary i's profits are smaller than:

$$\pi\left[\alpha F\left(u^H\right)+(1-\alpha)F\left(u^L\right)\right]D_i\left(1,q_j,\{1\},S_j\right)-k$$

Suppose now that intermediary i deviates to r_i' in stage 2, where r_i' is the solution to:

$$(\pi-r_i')\left[(1-\alpha)F\left(u^H\right)+\alpha F\left(u^L\right)\right]D_i\left(1,q_j,\{1\},S_j\right)=k$$

Since consumers do not observe the choice of r_i', their behavior in stage 3 is unchanged, so that, given r_i', both stores affiliate with intermediary i in the stage 3 equilibrium. The intermediary's deviation profits are then:

$$\pi\left[F\left(u^H\right)+F\left(u^L\right)\right]D_i\left(1,q_j,\{1\},S_j\right)-k\frac{F\left(u^H\right)+F\left(u^L\right)}{(1-\alpha)F\left(u^H\right)+\alpha F\left(u^L\right)}$$

The deviation is therefore profitable if and only if:

$$\pi\left[(1-\alpha)F\left(u^H\right)+\alpha F\left(u^L\right)\right]D_i\left(1,q_j,\{1\},S_j\right)\geq k\frac{\alpha F\left(u^H\right)+(1-\alpha)F\left(u^L\right)}{(1-\alpha)F\left(u^H\right)+\alpha F\left(u^L\right)}$$

which is equivalent to:

$$\pi D_i\left(1, q_j, \{1\}, S_j\right) \geq k \frac{\alpha F\left(u^H\right)+(1-\alpha) F\left(u^L\right)}{\left[(1-\alpha) F\left(u^H\right)+\alpha F\left(u^L\right)\right]^2} \tag{13}$$

But we have $D_i\left(1, q_j, \{1\}, S_j\right) \geq \frac{1}{2}+\frac{V_1-V_{12}(1)}{2t}$: the worst possible case for intermediary i is when intermediary j attracts both stores ($S_j=\{1,2\}$) and sets $q_j=1$. This yields

$$D_i\left(1, q_j, \{1\}, S_j\right) \geq \frac{1}{2t}\left[t-\alpha \int_0^{u^L}\left(u^L-c\right) f(c)\, dc-(1-\alpha) \int_0^{u^H}\left(u^H-c\right) f(c)\, dc\right]$$

Inequality (13) above is therefore verified if:

$$\frac{\pi}{2}\left[1-\frac{1}{t}\left(\alpha \int_0^{u^L}\left(u^L-c\right) f(c)\, dc+(1-\alpha) \int_0^{u^H}\left(u^H-c\right) f(c)\, dc\right)\right] \geq k \frac{\alpha F\left(u^H\right)+(1-\alpha) F\left(u^L\right)}{\left[(1-\alpha) F\left(u^H\right)+\alpha F\left(u^L\right)\right]^2} \blacksquare$$

which is ensured by (11).

Thus, given $(q_i=1, q_j)$, the equilibrium of the game starting at stage 2 necessarily involves both stores affiliating with intermediary i.■

Step 3 *If intermediary i chooses q_i in stage 1 and induces affiliation by one store only (store 1) in the game starting at stage 2*[9] *then it can profitably deviate to $q_i=1$.*

Proof of Step 3

Suppose that $q_i<1$ and that in the ensuing equilibrium only store 1 affiliates with intermediary i. Thus, affiliation demand for intermediary i in this equilibrium is $D_i\left(q_i, q_j, \{1\}, S_j\right)$, where $S_j \in \{\emptyset, \{1\}, \{1,2\}\}$. As in Step 2 above, intermediary i's profits are then necessarily smaller than:

$$\pi\left[\alpha F\left(u^H\right)+(1-\alpha) F\left(u^L\right)\right] D_i\left(q_i, q_j, \{1\}, S_j\right)-k$$

Suppose now that intermediary i deviates to $q_i=1$ in stage 1. From step 2, we know that any ensuing equilibrium necessarily involves both stores affiliating with intermediary i, so that affiliation demand for intermediary i in this deviation has the form $D_i\left(1, q_j, \{1,2\}, S_j^{\prime}\right)$, where the set $S_j^{\prime}$ of stores that affiliate with intermediary j may be different than S_j. Furthermore, intermediary i's stage 2 choice of $r_i^{\prime}$ in the deviation equilibrium must satisfy:

$$\left(\pi-r_i^{\prime}\right)\left[(1-\alpha) F\left(u^H\right)+\alpha F\left(u^L\right)\right] D_i\left(1, q_j, \{1,2\}, S_j^{\prime}\right)=k$$

Thus, the intermediary's deviation profits are:

$$\pi\left[F\left(u^H\right)+F\left(u^L\right)\right] D_i\left(1, q_j, \{1,2\}, S_j^{\prime}\right)-k \frac{F\left(u^H\right)+F\left(u^L\right)}{(1-\alpha) F\left(u^H\right)+\alpha F\left(u^L\right)}$$

[9] If an intermediary chooses to induce affiliation by one store only, it is clearly more profitable to have store 1 than store 2.

and the deviation is profitable if and only if:

$$\begin{aligned} & \pi \left[F\left(u^H\right)+F\left(u^L\right)\right] D_i\left(1, q_j,\{1,2\}, S_j^{\prime}\right) \\ & -\pi\left[\alpha F\left(u^H\right)+(1-\alpha) F\left(u^L\right)\right] D_i\left(q_i, q_j,\{1\}, S_j\right) \\ \geq\ & k \frac{\alpha F\left(u^H\right)+(1-\alpha) F\left(u^L\right)}{(1-\alpha) F\left(u^H\right)+\alpha F\left(u^L\right)}\end{aligned}$$

which is equivalent to:

$$\begin{aligned} & \pi\left[F\left(u^H\right)+F\left(u^L\right)\right]\left[D_i\left(1, q_j,\{1,2\}, S_j^{\prime}\right)-D_i\left(q_i, q_j,\{1\}, S_j\right)\right] \\ & +\pi\left[(1-\alpha) F\left(u^H\right)+\alpha F\left(u^L\right)\right] D_i\left(q_i, q_j,\{1\}, S_j\right) \\ \geq\ & k \frac{\alpha F\left(u^H\right)+(1-\alpha) F\left(u^L\right)}{(1-\alpha) F\left(u^H\right)+\alpha F\left(u^L\right)}\end{aligned}$$

Recall that:

$$D_i\left(1, q_j,\{1,2\}, S_j^{\prime}\right)=\frac{1}{2}+\frac{1}{2 t}\begin{cases}V_{12}(1)-V_{12}\left(q_j\right) \geq 0 & \text{if } S_j^{\prime}=\{1,2\} \\ V_{12}(1)-V_1>0 & \text{if } S_j^{\prime}=\{1\}\end{cases}$$

$$D_i\left(q_i, q_j,\{1\}, S_j\right)=\frac{1}{2}+\frac{1}{2 t}\begin{cases}V_1-V_{12}\left(q_j\right) & \text{if } S_j=\{1,2\} \\ 0 & \text{if } S_j=\{1\}\end{cases}$$

Suppose that $D_i\left(1, q_j,\{1,2\}, S_j^{\prime}\right)<D_i\left(q_i, q_j,\{1\}, S_j\right)$. From the two expressions above it is apparent that the only way this could happen is if $S_j=\{1,2\}$ and $S_j^{\prime}=\{1\}$. But $D_i\left(1, q_j,\{1,2\},\{1\}\right)<D_i\left(q_i, q_j,\{1\},\{1,2\}\right)$ implies $D_j\left(q_j, 1,\{1\},\{1,2\}\right)>D_j\left(q_j, q_i,\{1,2\},\{1\}\right)$.

On the other hand, since both $D_j\left(q_j, 1,\{1\},\{1,2\}\right)$ and $D_j\left(q_j, q_i,\{1,2\},\{1\}\right)$ are equilibria (following different interemediary choices in stage 1), Step 1 implies that:

$$\left(1-\alpha q_j\right) F\left(u\left(q_j\right)\right)+\alpha q_j F\left(u^L\right)<\alpha F\left(u^H\right)+(1-\alpha) F\left(u^L\right)$$

and:

$$\begin{aligned}\pi\left[\begin{array}{c}X\left(q_j\right)-\alpha F\left(u^H\right) \\ -(1-\alpha) F\left(u^L\right)\end{array}\right] D_j\left(q_j, q_i,\{1,2\},\{1\}\right) & \geq K\left(\frac{F\left(u\left(q_j\right)\right)}{X\left(q_j\right)}\right)-k \\ & \geq \pi\left[\begin{array}{c}X\left(q_j\right)-\alpha F\left(u^H\right) \\ -(1-\alpha) F\left(u^L\right)\end{array}\right] D_j\left(q_j, 1,\{1\},\{1,2\}\right)\end{aligned}$$

which implies $D_j\left(q_j, 1,\{1\},\{1,2\}\right) \leq D_j\left(q_j, q_i,\{1,2\},\{1\}\right)$, a contradiction.

Thus, we must have: $D_i\left(1, q_j,\{1,2\}, S_j^{\prime}\right) \geq D_i\left(q_i, q_j,\{1\}, S_j\right)$. Consequently, for intermediary i's deviation to be profitable it is sufficient that:

$$\pi\left[(1-\alpha) F\left(u^H\right)+\alpha F\left(u^L\right)\right] D_i\left(q_i, q_j,\{1\}, S_j\right) \geq k \frac{\alpha F\left(u^H\right)+(1-\alpha) F\left(u^L\right)}{(1-\alpha) F\left(u^H\right)+\alpha F\left(u^L\right)}$$

As in Step 2, we have:

$$D_i\left(q_i, q_j, \{1\}, S_j\right) \geq \frac{1}{2} + \frac{V_1 - V_{12}(1)}{2t}$$

The deviation is thus profitable if:

$$\frac{\pi}{2}\left[1 + \frac{V_1 - V_{12}(1)}{t}\right]\left[(1-\alpha)F\left(u^H\right) + \alpha F\left(u^L\right)\right] \geq k\frac{\alpha F\left(u^H\right) + (1-\alpha)F\left(u^L\right)}{(1-\alpha)F\left(u^H\right) + \alpha F\left(u^L\right)}$$

which is guaranteed by (11).■

Step 4

Steps 1 through 3 above imply that in any candidate equilibrium both stores multihome (i.e. affiliate with both intermediaries). Thus, in any equilibrium, both intermediaries necessarily choose $\left(q_s^C, r_s^C\right)$ defined by (9) and (8) with $q_i = q_s^C$. All we have left to verify is that intermediary i does indeed find it profitable to induce both stores to affiliate with it in the subgame starting in stage 2, after the intermediaries have chosen $q_A = q_B = q_s^C$ in stage 1 (instead of setting r_i such that only store 1 affiliates). This is true if:

$$\begin{aligned}
& \pi X\left(q_s^C\right) D_i\left(q_s^C, q_s^C, \{1,2\}, \{1,2\}\right) - K\left(\frac{F\left(u\left(q_s^C\right)\right)}{X\left(q_s^C\right)}\right) \\
\geq\ & \pi\left[\alpha F\left(u^H\right) + (1-\alpha)F\left(u^L\right)\right] D_i\left(q_s^C, q_s^C, \{1,2\}, \{1,2\}\right) - k
\end{aligned}$$

where $D_i\left(q_s^C, q_s^C, \{1,2\}, \{1,2\}\right) = \frac{1}{2}$.

By definition of q_s^C:

$$\begin{aligned}
& \pi X\left(q_s^C\right) D_i\left(q_s^C, q_s^C, \{1,2\}, \{1,2\}\right) - K\left(\frac{F\left(u\left(q_s^C\right)\right)}{X\left(q_s^C\right)}\right) \\
\geq\ & \pi X(1) D_i\left(1, q_s^C, \{1,2\}, \{1,2\}\right) - K\left(\frac{F\left(u^H\right)}{X(1)}\right)
\end{aligned}$$

We then have[10]:

$$\begin{aligned}
& \pi X(1) D_i\left(1, q_s^C, \{1,2\}, \{1,2\}\right) - K\left(\frac{F\left(u^H\right)}{X(1)}\right) \\
& - \left\{\pi\left[\alpha F\left(u^H\right) + (1-\alpha)F\left(u^L\right)\right] D_i\left(q_s^C, q_s^C, \{1,2\}, \{1,2\}\right) - k\right\} \\
\geq\ & \pi\left[X(1) - \alpha F\left(u^H\right) - (1-\alpha)F\left(u^L\right)\right] D_i\left(1, q_s^C, \{1,2\}, \{1,2\}\right) - \left[K\left(\frac{F\left(u^H\right)}{X(1)}\right) - k\right] \\
=\ & \pi\left[(1-\alpha)F\left(u^H\right) + \alpha F\left(u^L\right)\right]\frac{1}{2} - k\frac{\alpha F\left(u^H\right) + (1-\alpha)F\left(u^L\right)}{(1-\alpha)F\left(u^H\right) + \alpha F\left(u^L\right)}
\end{aligned}$$

[10] This is because:

$$D_i^*\left(1, q_s^C\right) = \frac{1}{2} + \frac{V_{12}(1) - V_{12}\left(q_s^C\right)}{2t} \geq \frac{1}{2} = D_i^*\left(q_s^C, q_s^C\right)$$

which is non-negative if and only if:

$$\frac{\pi}{2} \geq k \frac{\alpha F\left(u^H\right) + (1-\alpha) F\left(u^L\right)}{\left[(1-\alpha) F\left(u^H\right) + \alpha F\left(u^L\right)\right]^2} \tag{14}$$

which is always the case under (11).

This allows us to conclude that intermediary i will indeed find it profitable to induce both stores to participate in stage 2 when $\left(q_s^C, q_s^C\right)$ are chosen in stage 1.

■

6.4 Proof of Lemma 3

Suppose that there exists an equilibrium in which the two intermediaries choose $\left(q_m^C, r_m^C\right)$ in stages 1 and 2, inducing both stores to affiliate with intermediary A exclusively. Profits for the two stores are then:

$$\begin{aligned} \Pi_1^S\left(q_m^C, r_m^C\right) &= \left(\pi - r_m^C\right)\left\{\alpha F\left(u\left(q_m^C\right)\right) + (1-\alpha)\left[q_m^C F\left(u^L\right) + \left(1-q_m^C\right) F\left(u\left(q_m^C\right)\right)\right]\right\} - k \\ \Pi_2^S\left(q_m^C, r_m^C\right) &= \left(\pi - r_m^C\right)\left\{(1-\alpha) F\left(u\left(q_m^C\right)\right) + \alpha\left[q_m^C F\left(u^L\right) + \left(1-q_m^C\right) F\left(u\left(q_m^C\right)\right)\right]\right\} - k \end{aligned}$$

For this to be an equilibrium we must have:

- (i) $r_m^C = 0$ (otherwise B can profitably deviate to $\left(q_m^C, r_m^C - \varepsilon\right)$ with ε small)
- (ii) $\Pi_i^S\left(q_m^C, 0\right) \geq 0$ for $i = 1, 2$ (individual store rationality)
- (iii) $\Pi_i^S\left(q_m^C, 0\right) \geq \Pi_i^S(1, 0)$ for $i = 1, 2$ (neither store can profitably deviate by going exclusive with intermediary B[11])
- (iv) $\Pi_1^S\left(q_m^C, 0\right) \geq \pi\left\{\alpha F\left(u^H\right) + (1-\alpha)\left[q_m^C F\left(u^L\right) + \left(1-q_m^C\right) F\left(u\left(q_m^C\right)\right)\right]\right\} - 2k$ (store 1 cannot profitably deviate by affiliating to both intermediaries[12])
- (v) $\Pi_2^S\left(q_m^C, 0\right) \geq \pi\left\{(1-\alpha) F\left(u^H\right) + \alpha\left[q_m^C F\left(u^L\right) + \left(1-q_m^C\right) F\left(u\left(q_m^C\right)\right)\right]\right\} - 2k$ (store 2 cannot profitably deviate by affiliating to both intermediaries)

Note that (i) and (iii) correspond exactly to a) and b) in the text of the Lemma. We still have to prove that c) must also hold. Suppose that it does not, i.e. that there exists q such that $\Pi_1^S(q, 0) \geq \Pi_1^S\left(q_m^C, 0\right)$ and $\Pi_2^S(q, 0) \geq \Pi_2^S\left(q_m^C, 0\right)$ with at least one strict inequality. Because of iii), we must have $q < 1$. Then, since $\Pi_1^S(q, 0)$ and $\Pi_2^S(q, 0)$ are continuous and $f(.)$ is atomless, we can also find $\widehat{q} < 1$ sufficiently close

[11] Indeed, if store 1 for instance goes exclusive with intermediary B, then in stage 4 type-1 consumers with search cost $c \in \left(u^L, u^H\right]$ search on intermediary B only, while type-2 consumers with search cost $c \in \left(u^L, u^H\right]$ search on intermediary A only. Consumers with search cost $c \leq u^L$ of either type search on both intermediaries.

[12] If store 1 affiliates with both intermediaries while store 2 stays exclusive with A, then in stage 4 all type-1 consumers with $c \leq u^H$ search on intermediary B and therefore visit store 1. No type-2 consumer searches on intermediary B. Type-2 consumers with search cost $c \leq u^L$ visit store 1 for sure on intermediary A, while type-2 consumers with search cost $c \in \left(u^L, u\left(q_m^C\right)\right]$ visit store 1 on intermediary A with probability $\left(1 - q_m^C\right)$.

to q such that $\Pi_1^S(\widehat{q},0) > \Pi_1^S\left(q_m^C,0\right)$ and $\Pi_2^S(\widehat{q},0) > \Pi_2^S\left(q_m^C,0\right)$. If intermediary B deviates to $\widehat{q}$ in stage 1 (while A still sets q_m^C), then in the game starting at stage 2 intermediary B will always be able to attract both stores exclusively by setting $r > 0$ small enough and therefore obtaining positive profits. This deviation would therefore be strictly profitable for B.

We have thus proven that $\left(q_m^C, r_m^C\right)$ is an equilibrium only if a), b) and c) in the text of the Lemma hold.

Conversely, suppose that a), b) and c) hold, both intermediaries set $\left(q_m^C, r_m^C\right)$ in stages 1-2 and both stores affiliate exclusively with intermediary A in stage 3. We will show that this is an equilibrium, i.e. neither the stores nor the intermediaries can profitably deviate.

We first prove that (b) (which is the same as (iii) above) implies (iv) and (v) above. Indeed, suppose that (iv) does not hold while (iii) does. This means that for store 1 it is more profitable to affiliate with both intermediaries than either to stay exclusive with intermediary A or to exclusively affiliate with intermediary B. These last two conditions are equivalent to:

$$\alpha\pi\left[F\left(u^H\right) - F\left(u\left(q_m^C\right)\right)\right] > k$$

and:

$$(1-\alpha)\left(1-q_m^C\right)\pi\left[F\left(u\left(q_m^C\right)\right) - F\left(u^L\right)\right] > k$$

Multiplying the former inequality by $(1-\alpha)$, the latter by α and summing them, we obtain:

$$\alpha(1-\alpha)\pi\left\{F\left(u^H\right) - F\left(u^L\right) - q_m^C\left[F\left(u\left(q_m^C\right)\right) - F\left(u^L\right)\right]\right\} > k$$

which implies:

$$\alpha(1-\alpha)\pi\left[F\left(u^H\right) - F\left(u^L\right)\right] > k$$

This conflicts with assumption 4. The same contradiction can be obtained by assuming (v) does not hold while (iii) does. We have thus shown that (iii), (iv) and (v) hold, which means that given $\left(q_m^C, r_m^C = 0\right)$ it is an equilibrium for both stores to affiliate exclusively with intermediary A in stage 3.

Next, neither intermediary can profitably deviate from $r_m^C = 0$ in stage 2 given the stage 1 choices. Finally, condition c) ensures that neither intermediary can profitably deviate from q_m^C in stage 1.

■