Innovation Beyond Patents: Technological Complexity as a Protection against Imitation*

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Abstract

A large portion of innovators do not patent their inventions. This is a relative puzzle since innovators are often perceived to be at the mercy of imitators in the absence of legal protection. In practice, innovators however invest actively in making their products technologically hard to reverse-engineer. We consider the dynamics of imitation and investment in such protection technologies, both by the innovator and by imitators. We show that it can justify high level of profits beyond patents and can account for the differences across sectors in the propensity to patent. Surprisingly, in general, the protection technologies that yield the highest profits for the innovator are expensive and do not protect well. Our model also allows us to draw conclusions on the dynamics of mobility of researchers in innovative industries.

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1 Introduction

Contrary to popular belief, a large share of innovations is *not* protected by patents. Moser (2011) documents that most innovations presented at 19th Century World Fairs were not patented at all (e.g., 89 percent for the 1851 fair). More recently, influential surveys of managers such as those by Levin et al. (1987) and Cohen et al. (2000) remark that the legal protection conferred by patents is by far less preferred than other means of protection, such as secrecy or lead time. Cohen et al. (2000) actually show that this is true both for product and process innovations and that the trend seems to accelerate over time. Moreover, all these papers note that the propensity to patent varies a lot by sector.

It may appear somewhat puzzling that innovators do not rely more heavily on patents since they can be seen as helplessly at the mercy of rampant imitation in the absence of legal protection. Even if imitation is considered a time-consuming process, this lag between innovation and imitation is most often viewed as exogenously given, a characteristic of the product or of the sector for instance. In this paper, in line with the previously mentioned evidence, we argue that innovators in many industries are not at all as helpless as is commonly argued. We focus on the fact that they can partly control the speed and extent of imitation by making their technologies harder to reverse-engineer. The main point of our paper is to show that the *dynamics* of investment in such protective measures, by the innovator and by imitators, can explain both why firms can collect high profits without patenting and can also account for the differences across sectors in the propensity to patent.

Concrete examples abound of how firms can strategically invest to hamper imitation efforts. Ichijo (2010) illustrates this for some consumer electronics products: "Sharp has put tremendous efforts into making imitation of its LCD TV sets time consuming and difficult. Various initiatives at Kameyama are aimed at increasing complexities (...) in order to make imitation difficult". A similar behavior can also be observed in some high-tech manufacturing industries. The software industry is full of obfuscation strategies and tools designed to interfere with reading of the machine code or its decompilation. Not only software, but also hardware can be actively protected. For example, it is quite typical in the semiconductor industry to encase some of the important circuitry in epoxy blocks so that electronics are destroyed if someone tries to open them.[1] It is not unusual either to design the integrated circuits to have pieces that are seemingly unused but are required for the operation. Note that in the CIS survey, complexity of products is listed as a strategy alongside secrecy and lead time to collect profits from innovations, and it appears to be very popular (as emphasized for instance in Cassiman and Veugelers

[1] Another common way to reverse-engineer electronics and circuits is to use x-ray images and work out what components have been used. For this reason, firms try to hinder these imitation efforts by positioning parts in such a way that the x-ray recognition is hampered.

(2002)). Finally, paying high wages to reduce researcher mobility is also an important way to hamper imitation.

We argue in this paper that the common wisdom that free-riding by imitators is extremely harmful for an innovator misses two important aspects that our model with investments in protective measures does capture. First, free-riders find themselves in a similar situation to that of the innovator once they have imitated a protected innovation. Thus, the original innovator benefits from the incentive of imitators to keep imitation barriers high for those who have not yet imitated. Second, the innovator also benefits from the incentive that imitators have to free-ride on each other. If it is anticipated that the next imitator to enter will not actively pursue protective measures, all remaining imitators have incentives to delay their entry in the hope of benefiting from the imitation effort of the next one who happens to enter.

All these ideas are formalized in an infinite-horizon model in which the original innovator faces a potentially large pool of ex ante identical imitators who are initially inactive. At every period, imitators who have not yet reversed-engineered the innovator's technology decide whether to do so at some (possibly low) imitation cost c_i. If they do, they also decide whether or not to pay a given one-time protection cost c_p. If all previous entrants have paid c_p, the cost of reverse-engineering for the remaining imitators is c_i. If at least one of them did not, the innovation becomes freely available. Protection technologies are characterized by their cost, c_p, and the strength they confer, c_i.

We find in this context that the innovator can earn substantial rents, even in a very unfavorable environment, where for instance imitation is instantaneous. Such high (post-innovation) rents may also be well above those attained by imitators. Surprisingly, the protection technologies that tend to yield a high payoff for the innovator are expensive and do not protect very well (relatively high c_p and small c_i). The intuition behind this result is as follows. The fact that the protection technology is relatively expensive means that the innovator uses it but, upon entry, imitators do not. Thus, as soon as the first imitator enters, the knowledge necessary to reproduce the technology enters the public domain and all remaining imitators enter for free. This creates a strong incentive for imitators to try to free-ride on other imitators' reverse engineering efforts and thus delay entry, which leaves potentially very high profits to the innovator. Our theory can shed light on why innovation was observed to flourish in sectors where legal protection did not exist, such as in the software industry.

We focused our previous discussion on the most profitable protection technology, but we characterize in the paper the symmetric mixed-strategy equilibria for arbitrary technologies. This leads us to characterize a theoretically interesting pattern where typically a series of preemption games is followed with some probability by a waiting game. Imitators are involved in a series of preemption games taking place quasi-instantaneously at the outset of the game. All the imitators who happen to enter in this phase pay the protection cost, but fear mis-coordination and thus mix at each instant between waiting and imitating. They pursue protection in the hope of securing some rents, anticipating that the initial phase of massive entry will be followed with some probability by a waiting game played by the imitators left to enter. Such a game involving delayed imitation arises because once a sufficient number of imitators have entered, the protection cost is too large relative to the post-entry payoff, and hence the next imitator to enter does so without paying for protection. These imitators thus engage in a waiting game and delay entry in the hope that another imitator enters before them.

The survey evidence we mentioned earlier points to large variations across sectors of the propensity to patent. We can then use our previous results to characterize how profits vary as a function of the protection technology (c_p and c_i) and how they compare to profits under patents. Unfortunately, we do not currently have data on protection technologies, be it their cost or the extent of protection they confer, to check even informally the validity of our predictions. We nevertheless point out that, according to surveys of managers (Cohen et al. 2000), the sectors for which examples of protection technologies come most readily to mind, electronic components and semiconductors, are amongst the sectors where patents are judged to be least effective.[2]

We characterize how payoffs of the innovator vary with c_p for a given level of c_i. We find an interesting pattern characterized by discontinuities where profits decrease and then discontinuously jump upwards. A somewhat loose intuition for this pattern is that as c_p increases and crosses certain thresholds, the number of imitators who attempt to enter in the preemption phase decreases by one unit, to the benefit of the innovator. As a consequence, for a given value of c_i and a given level of profits from patents, the range of values of c_p such that innovators choose not to patent can take the form of a disconnected set of intervals, suggesting that empirical work needs to be done with particular care.

We also examine a different dimension of protection that appears essential in practice. Knowledge is often diffused through scientists'mobility, so investing in protection can be seen in this light as paying key researchers sufficiently high wages so as to prevent them from leaving the firm. We capture this idea in a stylized variant of our model where we add competition for researchers following imitation. We show that the results are very close to the results of our base model, except that the cost of protection becomes endogenous in this setting. We view this as providing a micro foundation for the protection cost c_p.

There is now a relatively large theoretical literature on how there can be innovation in the absence of patents (see e.g., Boldrin and Levine (2002, 2007, 2008)). What differentiates our paper is our focus on strategic protection measures such as technology complexity, an aspect that is entirely ignored by such a literature despite its empirical relevance. For instance, Maurer and Scotchmer (2002) as well as Henry and Ponce (2011)

[2] 21.3 percent for electronic components and 26.7 for semiconductors. Note that the software industry is not part of the survey.

emphasize trading of knowledge as a source of post-innovation rents rather than complexity. In another vein, Anton and Yao (1994, 2002) analyze how a financially-constrained innovator with an innovative idea can earn substantial post-innovation rents even if her idea can be expropriated when revealed to either of two firms capable of commercializing it. In turn, Baccara and Razin (2007) analyze the incentives to disclose ideas when there is possibly more than one innovator with the same idea and patenting is not feasible.

There is also a literature on the choice between patenting and secrecy, as pioneered by Gallini (1992). Note that we attempt to explain more broadly strategic choices outside patents, of which secrecy is just one example. Furthermore, the mechanisms and questions examined are significantly different from ours. Horstmann, MacDonald and Slivinski (1985) insist on the signaling dimension of patents in an environment where innovators have private information on the value of imitation for potential imitators, whereas Gallini (1992) is interested in analyzing the optimal trade-off between patent length and breadth. Kultti et al. (2007) deal with the comparison between patenting and secrecy in a setting with multiple independent discoveries and where the idea becomes public under secrecy with a certain exogenous probability. Anton and Yao (2004) is closer to our work in the sense that the innovator can take strategic actions to decrease competition even when she chooses secrecy. Indeed, the innovator can choose a level of disclosure: more disclosure signals a better innovation and thus makes the imitator less aggressive in the product market. In our paper, the strategy available to the innovator to deter imitation is of a different kind, and furthermore we insist on the importance of dynamics.

Our paper also contributes to the literature on entry games with an infinite horizon of play. Our game exhibits a theoretically interesting pattern of a series of preemption games followed by a waiting game. Our approach towards analyzing continuous-time preemption games builds upon Fudenberg and Tirole (1985), except that we have more than two players (possibly) mixing over more than two actions. The existence of equilibrium coordination failures directly relates our work to that of Dixit and Shapiro (1986), Cabral (1993, 2004), Vettas (2000) and Bertomeu (2009). Vettas (2000) is of particular relevance because he finds the remarkable result that the payoff expected by an incumbent first increases then decreases as more firms become active in the market. A similar nonmonotonicity result is derived in our setting, even though we allow flow profits to strictly decrease in the number of firms active in the market, unlike Vettas (2000). Our focus on continuous time allows us to dispense with his assumption, showing that his insights carry over to settings in which decisions can be made very often.

We conclude our discussion of the related literature by observing that it is not usual to find entry timing games that display both preemption and waiting motives as play unfolds. Important exceptions are Sahuguet (2006) and Park and Smith (2008). Our preemption-then-waiting result resembles that in Sahuguet's (2006) analysis of volunteering for heterogeneous tasks under incomplete information about others' preferences.

Besides our focus on complete information, we differ from his analysis in several other dimensions, especially in the questions analyzed (i.e., public good provision vs. innovation protection and imitation).

The remainder of the paper is organized as follows. In Section 2 we introduce the model. In Section 3 we solve for the equilibrium entry and protection decisions. In Section 4 we draw conclusions on the level of profits of the innovator in the absence of patents and discuss specific examples. In Section 5 we examines protection of knowledge through restraining worker mobility. All proofs are presented in the appendix.

2 Model

We analyze a discrete-time game that lasts infinitely many periods of length $\Delta > 0$ each. The time variable is denoted by $t = 0, \Delta, 2\Delta, ...$ All players have the same per-period discount factor δ^{Δ}. We will focus on the case in which Δ is positive but converges to zero, i.e., the continuous-time limit of the game.

The game involves one innovator and $n-1 \geq 2$ (ex ante identical) potential imitators. Prior to the start of the game, the innovator has discovered a new technology. The imitators can then decide in each period whether to imitate or stay out of the market an additional period. We consider the dynamics of imitation of this technology. The cost of imitation depends on the strategic choices made by the innovator and the imitators who previously entered. In any period t, we refer to the players who have already entered the market as "insiders", whereas we refer to the imitators who have not yet entered as the "outsiders".

The innovator at time $t = 0$ and the imitators upon entry need to decide whether to invest in protection. Protection technologies are characterized by two parameters c_i and c_p. We denote $c_p > 0$ for the one-time cost that needs to be incurred to achieve protection. In any period, if the innovator and all insiders incurred the protection cost $c_p > 0$, the outsiders who decide to enter need to incur imitation cost $c_i > 0$. This one-time cost c_i gives instantaneous access to the same technology. However, if one of the insiders did not pay c_p upon entry, then imitation becomes costless for all outsiders. We assume that the costs c_p and c_i remain fixed throughout the game, in particular they are independent of the number of firms active in the market.

In each period, an outsider can therefore choose among three actions:

- to imitate and pay the protection cost, an action denoted by p
- to imitate and not pay the protection cost, an action denoted by u
- not to imitate and wait another period, an action denoted by w

Per-period profits depend on the number of firms who have entered. We denote π_j for the per-period individual profit if $j \in \{1, ..., n\}$ firms (including the innovator) hold the technology.[3] Denoting the rate at which profits are discounted by firms by r, let $\Pi_j \equiv \pi_j / r$ represent the value of a perpetual stream of discounted profits collected by a firm when a total of $j \in \{1, ..., n\}$ firms hold the technology and no further entry takes place. We assume π_j and thus Π_j are decreasing.

We mostly focus, in particular in Section 3, on the case where $\Pi_n > c_i$. This corresponds to a situation where all firms will eventually enter the market: even if $n-1$ firms are already on the market and all the insiders and the innovator paid the protection cost c_p, imitation is still profitable. Note that this is a priori the worst-case scenario for innovation in the absence of legal protection since the protection technology does not offer much of a guarantee. At the end of the section, we consider the case $\Pi_n < c_i$ and show that the result are very similar, up to a notational change.

We allow for mixed strategies and focus on symmetric Markov Perfect Equilibria (MPE), where the state corresponds to the number of firms who hold the technology. The focus on symmetric (mixed-strategy) equilibria can appear restrictive. However, as Farrell and Saloner (1988) and Bolton and Farrell (1990) convincingly argue, decentralized coordination mechanisms involving anonymous players cannot be properly captured by asymmetric equilibria in which (asymmetric) roles are very well defined among players. In addition, play based on mixed strategies can be interpreted as play arising in a game in which each player has private information about some disturbance affecting her final payoff.[4] As pointed out by Cabral (1993), coordination failures occur under this interpretation not because of randomization but because players have incomplete information about others' payoffs.

Given our restriction on Markovian play, we use the following notation throughout: at the start of a period with k outsiders left to enter, we denote:

- the expected discounted profits of an insider by I_k
- the expected discounted profits of an outsider if she decides to enter by O_k

3 The dynamics of protection and imitation

In this section we mainly focus on the characterization of the equilibria, in the case where $\Pi_n > c_i$ (we consider the other case at the end of the section). To help the reader through the arguments, we first sketch the shape of the equilibrium. The finiteness of the pool of

[3] To avoid introducing several effects that would obscure the message of the paper, we assume that the flow profits earned do not depend on whether the protection cost was incurred or not. In other words, making the technology harder to reverse engineer does not directly affect the willingness to pay of consumers or production costs.

[4] This is the well-known purification argument in Harsanyi (1973).

potential imitators allows us to use backward induction when solving the infinite-horizon game, so we explain the reasoning by working backwards as well.

In the final subgames, when many firms are already active, the protection cost c_p appears large compared to the expected profits on the market. The next entrant will thus enter without any protective measure, thereby creating an incentive for the remaining imitators to delay imitation in the hope of free riding on the efforts of the next to enter. We actually find a critical number of outsiders J such that if the number of outsiders is strictly less than J, they engage in a waiting game and delay entry.

In earlier subgames with at least J outsiders, there is an incentive for imitators to enter quickly, preempt the others by protecting their technologies and benefit from the subsequent imitation delay. However, there is a risk of miscoordination were all imitators to enter simultaneously. This creates the conditions for a preemption game. In such a game, at least one outsider will enter right away (and several could in fact enter simultaneously). If the number of outsiders is still not below J following this wave of entry, another preemption game is played, and so on and so forth until the number of outsiders is finally smaller than J. Overall, we see that the pattern is a series of preemption games followed by a waiting game. Below, we make these arguments formal.

3.1 Solving the subgames with less than three outsiders

We note that in any subgame in which at least one of the insiders did not pay the protection cost upon entry, all outsiders immediately imitate the technology at no cost. Thus in the following discussion, we exclusively focus on subgames in which all insiders paid c_p upon entry.

The last entrant

We begin our analysis by considering those subgames in which just one imitator is left to enter the market. Since $\Pi_n > c_i$, the last outsider enters immediately without paying c_p. The expected profit of an insider in such a subgame is $I_1 = \Pi_n$. The expected profit of the outsider is $O_1 = \Pi_n - c_i$.

Two imitators left to enter

We now consider the subgames with only two outsiders. The outsider who enters first needs to incur cost c_i, but knows that, regardless of whether or not she pays the additional protection cost, the remaining outsider will enter immediately. It is then clear that action p (entering and paying the protection cost) is strictly dominated.

Therefore, the first entrant does not choose protection, and the second entrant incurs no imitation cost. This creates the conditions for a waiting game where both players mix between entering without paying the protection cost and waiting. Both players prefer to be the second entrant, but also do not want to wait excessively as they lose profits every period. As is standard in such games (if stationary), in the limit when Δ converges to zero, the entry time of each imitator converges to an exponential distribution.

Lemma 1 *In subgames with two outsiders, the only symmetric MPE is such that both outsiders mix between actions u (entering without paying the protection cost) and w (waiting another period). As Δ converges to zero, the entry time of each outsider converges to an exponential distribution with parameter λ_2, where $\lambda_2 \equiv r(\Pi_n - c_i)/c_i$. The expected profit of each outsider is $O_2 = \Pi_n - c_i$, whereas each of the insiders expects to gain $I_2 = \mu_2 \Pi_{n-2} + (1-\mu_2)\Pi_n$, where $\mu_2 \equiv r/(r+2\lambda_2)$.*

The expected payoff of an outsider at the beginning of these subgames is $O_2 = \Pi_n - c_i$ since she is indifferent between all entry times, including entering immediately. On the contrary, the insiders expect significant profits since they will earn per-period profits π_{n-2} until the time of first entry, which is exponentially distributed (with hazard rate $2\lambda_2$).[5]

Three imitators left to enter

Before studying the complete dynamics, it is useful to understand in detail the resolution of subgames with three outsiders left. All players know that in any period, if a single outsider enters and pays the protection cost, then the remaining two imitators will play a waiting game. In such a game, we established in Lemma 1 that insiders earn expected profits of $I_2 = \mu_2 \Pi_{n-2} + (1-\mu_2)\Pi_n$.

Thus, we first note that, if $I_2 - c_p - c_i \leq \Pi_n - c_i$, playing action p is (weakly) dominated by u, that is, outsiders will never pay the protection cost. The condition can be equivalently expressed as $c_p \geq c_2^* \equiv \mu_2(\Pi_{n-2} - \Pi_n)$. According to the same logic as in the previous section, the three imitators will then engage in a waiting game. We show in Lemma 2 that the individual entry time then follows an exponential distribution with parameter $\lambda_3 \equiv r(\Pi_n - c_i)/(2c_i)$.

On the contrary, if $c_p < c_2^*$, preemptively entering and paying the protection cost becomes very attractive if the two other outsiders do not enter. There is however a risk of coordination failure were all outsiders simultaneously to enter and pay c_p. This creates the conditions for a preemption game described in Lemma 2. As the time between two consecutive periods shrinks, outsiders mix between p and w, that is, between entering and paying the protection cost and waiting. Entry occurs almost instantaneously with probability one, and simultaneous entry of several outsiders occurs with positive probability.

[5] Indeed, the entry time of each individual imitator follows an exponential of parameter λ_2, and thus the time of first entry is an exponential of parameter $2\lambda_2$ since it is the minimum of two exponential random variables.

Lemma 2 *In subgames with three outsiders, as* Δ *converges to zero:*

(i) If $c_p \geq c_2^*$*, the three outsiders mix between actions* u *and* p*. The entry time of each outsider converges to an exponential distribution with parameter* λ_3*, where* $\lambda_3 \equiv r(\Pi_n - c_i)/(2c_i)$*. Furthermore, the expected profit of each outsider is* $O_3 = \Pi_n - c_i$*, whereas each of the insiders expects to gain* $I_3 = \mu_3 \Pi_{n-3} + (1 - \mu_3)\Pi_n$*, where* $\mu_3 \equiv r/(r + 3\lambda_3)$*.*

(ii) If instead $c_p < c_2^*$*, the three outsiders start playing a preemption game as soon as this subgame begins. The limiting distribution is such that outsiders play* w *and* p *with a probability bounded away from zero, and the payoff of the outsiders converges to* $O_3 = \Pi_n - c_i$*, whereas the payoff of the insiders converges to* $I_3 = \phi_3(1) I_2 + (1 - \phi_3(1))\Pi_n$*, where* $\phi_3(1)$ *is the probability of a single outsider entering.*[6]

Lemma 2 has a very natural interpretation. If the protection cost is relatively high, it will not be paid upon entry, and therefore all outsiders wait in the hope that one of them will move first without paying c_p. On the contrary, if the protection cost is low enough, it will be incurred upon entry. The problem is then one of coordination. All outsiders would like to be the only firm to enter and then enjoy payoff I_2 while the others engage in a waiting game, but no one has an interest in paying the protection cost if other outsiders choose to enter at the same time.

3.2 Subgames with more than three imitators left to enter

The ideas uncovered in the subgames with three outsiders partially extend to the subgames with a larger number of outsiders. In particular, if c_p is relatively large, the players will end up playing a waiting game.

In what follows, let $c_k^* \equiv \mu_k(\Pi_{n-k} - \Pi_n)$, where $\mu_k \equiv r/(r + k\lambda_k)$. We show in the following lemma that in the subgame with $k \geq 3$ outsiders, if $c_p \geq c_{k-1}^*$, players mix between waiting and entering without protection and the entry time is exponentially distributed. A key part of the induction argument is that $\{c_k^*\}_{k=2}^{n-1}$ is a monotonically increasing sequence.[7] This implies that, when $c_p \geq c_{k-1}^*$, if one outsider chooses to enter by paying the protection cost, the $k-1$ remaining outsiders will then engage in a waiting game, since $c_p > c_{k-2}^*$. Intuitively, the incentive to avoid paying the protection cost becomes more intense as fewer imitators remain inactive: as k decreases, the profit stream to be earned following entry becomes relatively smaller and the waiting game is expected to last less (note that Π_{n-k} is increasing in k, whereas $k\lambda_k$ is decreasing).

Lemma 3 *In the subgame with* $k \in \{3, ..., n-1\}$ *outsiders, if* $c_p \geq c_{k-1}^*$*, the* k *outsiders mix between actions* u *and* w*. The entry time of each outsider converges as* Δ *goes to*

[6] See expression (7) for the specific formula for $\phi_3(1)$.

[7] Note that $\mu_k = (k-1)c_i/(k\Pi_n - c_i)$ is increasing in k, since $c_i < \Pi_n$ implies that $d\mu_k/dk = c_i(\Pi_n - c_i)/(k\Pi_n - c_i)^2 > 0$. Taking into account that both μ_k and $\Pi_{n-k} - \Pi_n$ are positive, the fact that Π_{n-k} and μ_k are both increasing in k then yields that $c_2^* < c_3^* < ... < c_{n-1}^*$.

zero to an exponential distribution with parameter $k\lambda_k$*, where* $\lambda_k \equiv r(\Pi_n - c_i)/((k-1)c_i)$*. The expected profit of each outsider is* $O_k = \Pi_n - c_i$*, whereas each of the insiders expects to gain* $I_k = \mu_k \Pi_{n-k} + (1-\mu_k)\Pi_n$*, where* $\mu_k \equiv r/(r + k\lambda_k)$*.*

We now consider the more complex case with $k \geq 3$ outsiders and $c_p < c^*_{k-1}$. It is essential for our purposes to define J, the critical number of outsiders such that a waiting game is played if the number of outsiders is *strictly less* than J (i.e., in subgames in which the number of imitators left to enter equals $2, ..., J-1$). Formally, we have $J = \inf\{k \geq 3 : c_p < c^*_{k-1}\}$, where $J = n$ if it is not well-defined. Note that J is a step function of c_p ranging from 3 to n. We will now show that for $k \geq J$, a series of preemption games takes place. A priori, the players mix between the three available actions, w, p and u.[8]

Recall that we are interested in equilibria where players can react instantaneously to each others actions, i.e in situations where the time Δ between successive play is negligible.[9] In what follows, we will not be deriving the exact play in a symmetric equilibrium for small values of Δ but consider a continuous-time approximation of equilibrium play that is arbitrarily close to the true outcome.

More specifically, the approach will be the following. For a given period length Δ, let $\rho_{a,k}(\Delta) \geq 0$ be the probability with which each outsider plays action $a \in \{w, p, u)\}$ when k outsiders are left to enter. Also, let $V_{a,k}(\Delta)$ denote the outsider's payoff from choosing action a given that the $k-1$ other players are mixing over actions with probability $\rho_{a,k}(\Delta)$ in all subgames with k outsiders. In equilibrium, the mixing probabilities $\rho_{a,k}(\Delta)$ for $a \in \{w, p, u\}$ must be such that outsiders are indifferent between all three actions and such that these are indeed probabilities (i.e. $V_{p,k}(\Delta) = V_{u,k}(\Delta) = V_{w,k}(\Delta)$, $\rho_{a,k}(\Delta) \in (0,1)$ and $\sum_{a \in \{w,p,u\}} \rho_{a,k}(\Delta) = 1$). What we will do is to solve for the solution of this system for $\Delta = 0$, what we call the continuous-time approximation of the equilibrium outcome,[10] and we will show that this solution exists and is unique. Given that the value functions $V_{a,k}(\Delta)(a \in \{w, p, u\})$ are continuous in Δ and in the probabilities, this will be a close approximation of the equilibrium outcome for small enough values of Δ.

To illustrate further this method, consider the case of three players solved in the proof of Lemma 2. In that case we solved explicitly, for a small fixed value of Δ, for the probabilities $\rho_{a,3}(\Delta)$, $a \in \{w, p, u\}$ (see (5) and (6) in the appendix). In that case, we

[8] We will show when proving Lemma 4 that action u is chosen, but with vanishing probability as Δ goes to zero.

[9] As emphasized by Fudenberg and Tirole (1991) when dealing with preemption games, a continuous-time version of the game cannot be directly used, and one is forced either to use approximations based on discrete-time games or to properly expand strategy spaces to accommodate for such approximations, as done by Fudenberg and Tirole (1985).

[10] Formally, what we mean by (continuous-time) approximation of the equilibrium is a set of admissible mixing probabilities $\rho_{a,k}$ ($a \in \{p, u, w\}$) satisfying the following property: for any (small) $\epsilon > 0$, there exists Δ_ϵ such that $\Delta < \Delta_\epsilon$ implies that $|\rho_{a,k}(\Delta) - \rho_{a,k}| < \epsilon$, where $\rho_{a.k}(\Delta)$ is the exact equilibrium play.

see from the solution presented in the proof of Lemma 2, that taking the limit of all the probabilities as Δ converges to zero (as we did) leads to the same solution as directly solving the system consisting of equations (2)-(4) for $\Delta = 0$, as was to be expected due to the continuity of the system. From now on $\rho_{a,k}$ and $V_{a,k}$ denotes $\rho_{a,k}(\Delta)$ and $V_{a,k}(\Delta)$ for $\Delta = 0$.

We formally show in the proof of Lemma 4 below that the symmetric MPE can be approximated for small enough values of Δ by an equilibrium where the action of entering without protection is played with essentially zero probability, i.e., $\rho_{u,k} \approx 0$. Thus, in the approximation we consider, the players will essentially mix just between actions w and p. We denote $\rho_k \equiv \rho_{p,k}$ for the individual probability of entry (so we have $\rho_{w,k} \equiv 1 - \rho_k$). Given k outsiders, the payoff to choosing action p (gross of c_p and c_i) is given by

$$V_{p,k} = \sum_{l=0}^{k-1} C_{k-1}^{l} (\rho_k)^l (1 - \rho_k)^{k-1-l} I_{k-1-l},$$

where $C_{k-1}^{l} = \binom{k-1}{l}$ denotes the binomial coefficient indexed by $k-1$ and l. The value to an outsider of paying the protection cost when entering depends on how many other outsiders simultaneously enter. If l other outsiders enter, the outsider participates in the next period as an insider in a subgame with $k - 1 - l$ outsiders. Her expected gain in this case is thus I_{k-1-l} (the value of being an incumbent with $k - 1 - l$ outsiders).

Each of the k outsiders will mix between p and w so as to leave others indifferent between these two actions, which yields that

$$V_{p,k} - c_p - c_i = \Pi_n - c_i,$$

since it can be shown that an outsider's payoff to waiting is $V_{w,k} = \Pi_n - c_i$ for $\Delta = 0$. Letting $\overline{I}_{k-1-l} \equiv I_{k-1-l} - \Pi_n$ and

$$F_k(\rho) \equiv \sum_{l=0}^{k-1} C_{k-1}^{l} \, \rho^l (1 - \rho)^{k-1-l} \, \overline{I}_{k-1-l},$$

the indifference condition can be equivalently written as:

$$F_k(\rho_k) = c_p. \tag{1}$$

Thus, we have in subgames with k imitators left to enter (and such that $c_p < c_{k-1}^*$) that the approximate mixing probability (provided it exists) must solve $F_k(\rho_k) = c_p$. Largely inspired by Vettas (2000), we now exploit the recursive nature of the problem and the properties of $F_k(\cdot)$. We show that the symmetric MPE of the game can be approximated for small values of Δ by an equilibrium such that outsiders mix between actions p and w

with strictly positive probabilities. Furthermore, in this approximation, the probability of playing action p in equilibrium decreases as the number of outsiders decreases.

The main properties of the $F_k(\cdot)$ functions, for $k \in \{J, ..., n-1\}$, are presented in Figure 1.

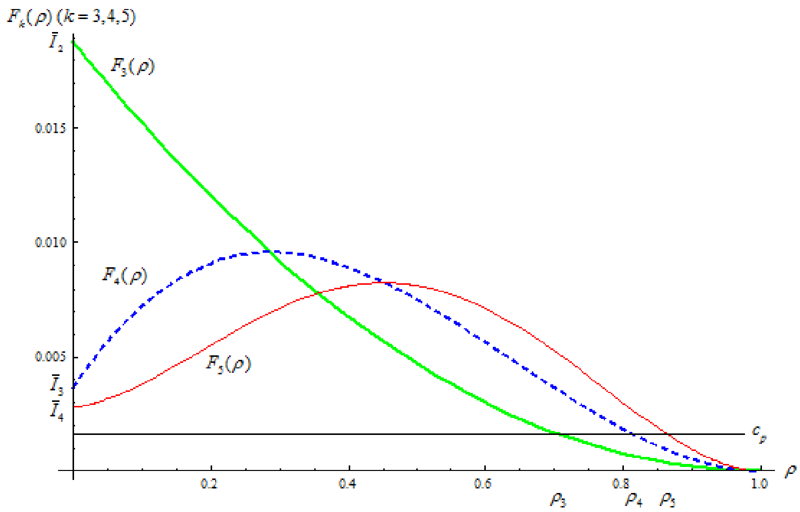

Figure 1: $F_5(\rho)$ (solid curve), $F_4(\rho)$ (dashed curve) and $F_3(\rho)$ (thick solid curve) plotted for $n = 6$, $c_p = 0.0017$, $c_i = 0.02$ and $\Pi_j = (j+1)^{-2}$ ($J = 3$ under these assumptions)

It holds that $F_J(\rho)$ is strictly decreasing in ρ, with $F_J(0) > c_p > F_J(1)$. There is thus clearly a unique solution to $F_J(\rho) = c_p$, namely ρ_J. This is intuitive: when $k = J$, following entry by at least one outsider, preemptive motives disappear and a waiting game is played thereafter (by definition of J). The length of such a waiting game is determined by the number of other outsiders who enter. Given our previous finding that the continuation payoff of an insider is lower in a waiting game played by fewer outsiders, the best scenario is if no one else enters ($\rho = 0$), whereas the worst scenario is if everyone else enters ($\rho = 1$). The randomization performed in equilibrium is somewhere in between.

For $k > J$, the pattern is slightly different. In these cases $F_k(\rho)$ is not everywhere decreasing in ρ. Unlike the case in which $k = J$, larger ρ does not make lower continuation payoffs more likely. Indeed, it can be shown (see proof of Lemma 4) that the continuation payoff of an insider (net of Π_n) has an inverted-U shape as a function of k: $\overline{I}_{n-1} < \overline{I}_{n-2} < ... < \overline{I}_J < \overline{I}_{J-1}$ and $\overline{I}_{J-1} > \overline{I}_{J-2} ... > \overline{I}_0 = 0$. So $F_k(\rho)$ also has an inverted-U shape as a function of ρ. Furthermore, we can show that $F_k(0) > c_p > F_k(1)$, for all $k > J$.

There is additional structure that can be exploited. In particular, $F_{k+1}(\cdot)$ starts off below $F_k(\cdot)$, reaches its maximum when crossing $F_k(\cdot)$ and then remains above $F_k(\cdot)$. A direct consequence is that the equilibrium ρ_k is increasing in k, an intuitive property. In these preemption games, players want to rush to enter to become one of the insiders during the waiting game that will likely follow. There is however a risk of excessive entry ex post. In a subgame where many players have already entered, and hence k is close to

J, this risk becomes particularly severe, and the players in equilibrium therefore chose to enter with a lower probability. The following lemma formalizes all these ideas.

Lemma 4 *In subgames with $k \in \{J, ..., n-1\}$ outsiders, if $c_p < c^*_{k-1}$, then, for small enough Δ, the symmetric MPE can be approximated by the following equilibrium:*

(i) Outsiders mix only between actions p and w, and the probability ρ_k of playing p is uniquely given by the solution to $F_k(\rho_k) = c_p$.

(ii) ρ_k is increasing in k.

(iii) Quasi-instantaneous entry by at least one outsider occurs.

We have therefore fully characterized the dynamics of imitation and protection in the case where $\Pi_n > c_i$. Our results are summarized in the following proposition where we describe the equilibrium path of imitation:

Proposition 1 *When $\Pi_n > c_i$, in the continuous time limit of the game, the symmetric MPE exhibits the following properties: there exists a number of entrants $J \in \{3, ..., n\}$ such that, if the innovator initially paid c_p:*

1. *At least J outsiders quasi-instantaneously imitate and pay the protection cost*
2. *The remaining outsiders, if there are more than one, delay imitation for a random length of time and do not pay for protection upon imitation*
3. *After entry of one of them, all the remaining outsiders immediately imitate*

We show below that these results partially extend to the case where $c_i \geq \Pi_n$, a situation that for large enough values of n approximates free entry. In particular, there exists a critical value J' such that if the number of outsiders is larger or equal to J', outsiders mix between actions p and w and at least J' quasi instantaneously enter. The main difference is that if the number of outsiders is less than J', no further entry takes place whereas in the case $c_i < \Pi_n$, all players played a waiting game. We derive the value of J' below.

In situations where $c_i \geq \Pi_n$, action u is always dominated by w if all insiders have paid c_p, since playing u yields payoff $\Pi_n - c_i \leq 0$. Letting $c'_k \equiv \Pi_{n-k} - c_i$ in what follows, J' is then defined by $J' \equiv \inf\{k \geq 3 : c_p < c'_{k-1}\}$ (with $J' = n$ if the definition is vacuous), so that subgames with $k \leq J' - 1$ outsiders exhibit no further entry.[11] Lemmas 1-4 are then directly applicable by simply letting $c_i = \Pi_n$ and redefining c^*_k and J as $c^*_k \equiv \Pi_{n-k} - c_i$ and J' respectively. Hence, the case in which $c_i \geq \Pi_n$ corresponds to that in which imitation delays in subgames without preemption features are infinitely long. Proposition 1 then applies accounting for this new notation and the fact that the imitation delay after the initial preemptive imitation phase is infinite.

[11] Since it holds that $\Pi_{n-(J'-1)} - c_i > c_p \geq \Pi_{n-(J'-2)} - c_i$ by definition of J'

4 When and if to patent

We can use the results of the previous section to draw important implications on incentives to innovate. We first show that our model and the mechanism we consider can explain why innovation flourished in certain sectors even when patents were not available. Second, by introducing explicitly the option to patent, we can provide an explanation for the variations across sectors in patenting rates.

4.1 High profits outside patents

Even though we consider an environment a priori very unfavorable to innovators, where in particular imitation is instantaneous, our first result shows that profits of innovators can be very high even when patents are not available.

Proposition 2 *In the continuous-time limit of the game, the equilibrium payoff of the innovator can be arbitrarily close to* $\Pi_1 - \Pi_2$*. This happens for technologies that are such that* $c_p \downarrow \Pi_2$ *and either* $c_i \geq \Pi_n$ *or* $c_i \uparrow \Pi_n$*.*

Interestingly, the maximum profit $\Pi_1 - \Pi_2$ is attained for a protection technology that is expensive ($c_p = \Pi_2$) and potentially does not perform very well (c_i close but less than Π_n).

The intuition behind this result for $c_i < \Pi_n$ is as follows: the fact that the protection technology is expensive ($c_p \geq \Pi_2$) means that, upon entry, imitators do not use it. Thus, as soon as the first imitator enters, the knowledge necessary to reproduce the technology enters the public domain and all remaining imitators enter for free. This creates a strong incentive for all imitators to try to free-ride on other imitators' efforts. The incentive to enter first converges to zero when c_i approaches Π_n and thus waiting is infinite and the innovator's payoff converges to Π_1. Of course, she has to pay a protection cost that, in the most favorable case, is equal Π_2. For such a technology, the payoff to the innovator is thus high even in environments with no legal protection.[12]

Our model can thus explain why innovation flourished in certain sectors where patent protection was not available. Consider the software industry. Until recently, software was not covered by patents and it was common for inventors to obfuscate the code: in other words, transforming the readable source code into code difficult to use directly. Today, various techniques are available and appear to be relatively cheap (low c_p). But this was not always the case and our theory could help explain why, even though patents did not apply, this was nevertheless an industry characterized by relentless innovation (see Boldrin and Levine (2007)).

[12] The case $c_i \geq \Pi_n$ is equivalent to $c_i \uparrow \Pi_n$ as in both cases the imitation delays are infinitely long.

There is unfortunately no data available on the level of c_p for different technologies. Note that potentially several techniques can simultaneously be used by firms. For instance, firms might not only invest in technologies making reverse engineering hard, but might also pay to keep their researchers from moving to other firms (something we explicitly consider in section 6). This might significantly raise protection costs and make it more likely that the conditions of Proposition 2 are satisfied.

4.2 Explaining variations in patenting rates across sectors

We discussed in the introduction the evidence showing large variations across sectors in the choice between patents and other means of protection, such as secrecy. Our theory provides a plausible explanation for these sectorial variations.

Let us first examine how the profits of the innovator vary with the characteristics of the protection technology when patents are not chosen. We plot in Figure 2, for a given value of c_i, how the innovator's payoff (net of the protection cost) varies with c_p. We will explain the intuition for the observed pattern for the case $c_i < \Pi_n$, the other case being equivalent given an appropriate notational change.

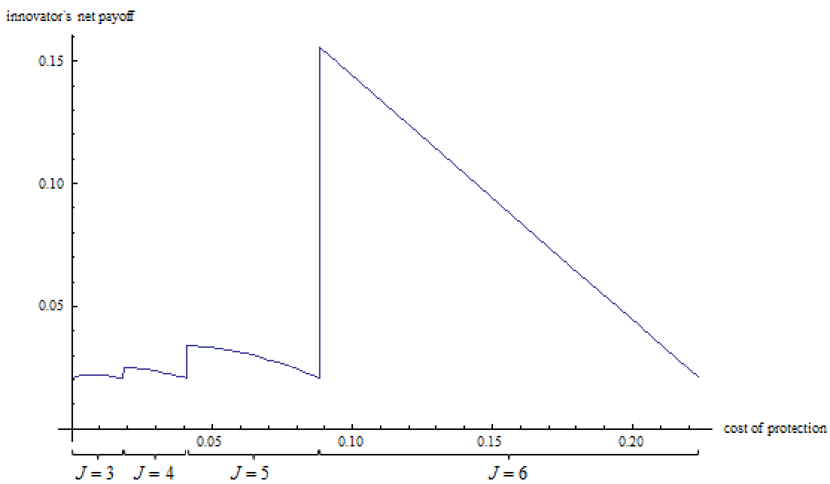

Figure 2: $I_5 - c_p$ plotted against c_p for $n = 6$, $c_i = 0.02$ and $\Pi_j = (j+1)^{-2}$

The pattern observed in Figure 2 is typically found for different specifications of parameters. First, when c_p is less than c_2^*, the net profits of the innovator initially increases and then decreases with c_p until c_p reaches c_2^*. Second, the behavior in the following intervals (c_{k-1}^*, c_k^*) (for $3 \leq k \leq n-2$) is the following: the profits of the innovator decrease and reach Π_n as c_p approaches the upper bound of the interval. Third, when c_p goes above that upper bound we observe a discrete upward jump. Finally, when c_p is above c_{n-2}^*, net profits are linearly decreasing in c_p.

The key to understanding these effects is to notice that within an interval (c_{k-1}^*, c_k^*), the value of J (the critical number of firms such that a waiting game starts) remains fixed. Within such an interval, an increase in c_p generates two opposing forces. On the one hand, clearly an increase in c_p directly decreases the payoff of the innovator as he has to pay a higher protection cost. On the other hand, an increase in c_p decreases the probability of excessive entry in the preemption phase.

The second effect can be understood as follows. When c_p is smaller than the upper bound of the interval, outsiders imitate in each period of the preemption phase with some probability. Excessive entry above J thus happens with positive probability, at a cost for the innovator. As c_p increases outsiders in any preemption subgame coordinate their actions better and better. As c_p gets close to the upper bound of the interval, the mixing probability in any subgame of the preemption phase converges to zero as the gains from entering before the others becomes small. The probability that at least two outsiders simultaneously enter in the same period converges even faster to zero. Thus, the outsiders perfectly coordinate their entry, and in equilibrium exactly J enter quasi instantaneously in a sequential manner.[13] So, as c_p increases, this second effect increases the profits of the innovator.

Figure 2 suggests that the first effect dominates only at the start of the first interval; in the other intervals, the second effect dominates. Note that if $c_p \geq c^*_{n-1}$, there is no preemption phase so only the first effect plays a role and profits linearly decrease with c_p. The previous discussion also allows us to explain why profits converge to Π_n at the upper bounds of the intervals (c^*_{k-1}, c^*_k). Because of the almost perfect coordination we highlighted when c_p approaches c^*_k, the gross profits of the innovator I_{n-1} are equal to the profits when k outsiders remain, I_k. Since $c^*_k = I_k - \Pi_n$, $I_k - c_p$ and hence $I_{n-1} - c_p$ both go to Π_n if c_p is close to the upper limit of the interval.

Figure 2 also clearly illustrates a discrete upward jump in the innovator's net payoff as c_p passes just above c^*_k. Although the value of c_p is quite similar on both sides of the threshold, the critical value J increases by one unit as soon as this threshold is crossed. This discretely increases I_{J-1}, which means that imitation delays involve more firms and hence being an insider is more valuable. This favors the innovator, but the downside is that the potential entrants find entry more attractive and choose to enter with higher probability than when c_p is just below c^*_k, thus leading to more miscoordination. However, as our previous discussion suggests, this faster rate of imitation does not completely dissipate profits.

Some of the results highlighted in Figure 2 are stated formally in the following lemma proved in the appendix:

Lemma 5 *Fix an interval* (c^*_{k-1}, c^*_k) *for* $2 \leq k \leq n-1$ *(with* $c^*_1 \equiv 0$ *and* $c^*_{n-1} \equiv \infty$*). Then the following properties hold:*

[13]Note that in Fudenberg and Tirole (1985) coordination failures do not occur on the equilibrium path even if players randomize independently, as in our case when c_p is close to the upper bound of the interval.

(i) As c_p converges to c_k^* from below ($2 \leq k \leq n-2$), net profits for the innovator converge to Π_n from above

(ii) As c_p converges to 0 from above, net profits for the innovator converge to Π_n from above

(iii) As c_p decreases starting from c_{n-2}^*, net profits for the innovator fall linearly

We are in now in a position to discuss the choice of whether to patent or not depending on the characteristics of protection technologies that vary by sector. To discuss this point we explicitly introduce a measure Π_P of profits under patents. Typically patents are imperfect (e.g., imitators can invent around the patented innovation), are limited in length, and take a long time to be obtained, so one would expect Π_P to be significantly lower than Π_1. Given a level of Π_P, we illustrate below in Figure 3 and Proposition 3 the set of values of c_p such that patenting is not pursued.

Proposition 3 *If $\Pi_P > \Pi_1 - \Pi_2$, firms will patent their innovations. If $\Pi_P < \Pi_1 - \Pi_2$, there exist values of c_i and c_p such that firms choose secrecy over patenting. Furthermore, this can take the form of a disconnected set of intervals.*

Our results suggest that variations in the popularity of patents across sectors might be explained by variations in protection technologies available. Admittedly, the results of Proposition 3 highlight discontinuities and suggest that empirical work should be done with particular care. This is illustrated in the graph below: if Π_P is high, patents are always preferred. For intermediate values of Π_P, patents will be chosen only for high values of c_p. However, as shown in the graph, for low Π_P, the values of c_p such that innovators choose not to patent forms a set of disconnected intervals.

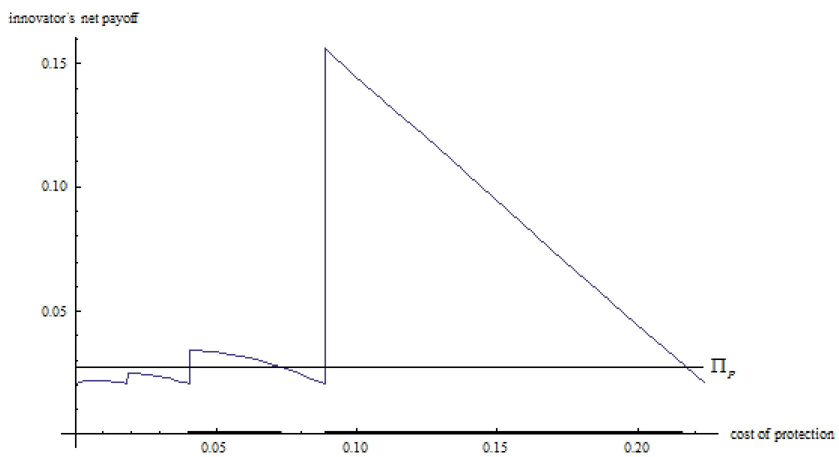

Figure 3: Patenting vs. Nonpatenting (thick portions of the horizontal axis represent values of c_p for which nonpatenting is preferred over patenting)

It is hard to provide even anecdotal evidence in support of the prediction of Proposition 3 since data on c_p and c_i does not exist. It is however interesting to pay attention to the sectors in which patents are judged to be least effective according to surveys of managers (Cohen et al. 2000), such as electronic components and semiconductors (the software industry is not part of the survey). These are also the sectors in which protection technologies can be most commonly observed. For instance, hardware obfuscation is a technique by which the description or the structure of electronic hardware is modified to intentionally conceal its functionality, making it significantly more difficult to reverse engineer.[14] We also observe technologies for sale that allow protection of integrated circuits from reverse engineering.[15] The fact that these sectors are characterized by lower rates of patenting is is coherent with our predictions if we consider that other sectors do not have access to such technologies. Note also that our results highlight that, even if the technologies previously mentionned appear not to be very technically efficient, they can still generate high level of profits for innovators.

5 Extension to stochastic protection

In the base model, we took an extreme view of protection technologies. If one of the initial entrants did not pay the protection cost c_p, the cost of imitation for the remaining outsiders fell to zero. We first note that assuming that the imitation cost falls not to zero but to an intermediate positive value, does not change qualitatively any of the conclusions. In this section we take a slightly different view of the role of protection. Not paying for protection allows the outsiders to know what research path to follow to reverse engineer the innovation, although reverse engineering remains costly. On the contrary, paying for protection, means that outsiders, if they choose to reverse engineer need to follow an uncertain research path that could potentially fail.

Specifically, we assume that, if all active firms have paid c_p, any of the remaining imitators has probability $q_i \in (0, 1)$ of successfully imitating by incurring cost c_i such that $c_i < q_i\Pi_n$. If at least one of the active firms did not pay c_p, then any imitator can imitate with probability one by just paying the imitation cost c_i. We assume that any firm that chooses to imitate and fails can never imitate afterwards, unless some subsequent imitator is successful at imitating and does not pay the protection cost (in which case the imitator who failed can imitate with probability one by just repaying c_i).[16] We assume that all firms observe whether a firm is successful at imitating or not whenever she tries to imitate.

[14] There are other techniques, some cryptography-based.

[15] An example is given by the United States patent 7128271, described as “a semiconductor integrated circuit having a reverse engineering protection part that can be easily implemented”.

[16] The case in which an imitator that fails at imitating can still try imitating at any later period can be shown to be equivalent to our base model (in fact, letting $q_i = 1$ is without loss of generality in terms of equilibrium strategies). Note that the assumption that an imitator that fails cannot imitate unless someone else does without protection can reflect the fact that imitators typically have preferred research paths and we implicitly assume that if the preferred path is not successful, other paths are too costly.

For tractability reasons, we consider the case $n = 3$, and we will let k denote the number of imitators who do not know their imitation capabilities yet. In turn, K will denote the current number of imitators that tried and failed to imitate in the past. As usual, equilibrium play in subgames in which some firm became active without paying c_p are trivial: all other imitators, regardless of whether they failed in the past at imitating, enter with probability one right after such a firm enters. Henceforth, we pay attention to subgames in which all active firms have paid c_p, and we work backwards. We obtain the following result, where we let $\bar{c}_1 = q_i(\Pi_2 - \Pi_3)$:

Proposition 4 *In the subgames where no outsider has yet attempted to reverse engineer ($k = 2$), we have:*

- *if $c_p > \bar{c}_1$, both outsiders mix between u and w every period and the first outsider to play u is not excluded from the market if he is not successful initially*
- *if $(1 - q_i)\bar{c}_1 \leq c_p < \bar{c}_1$, both outsiders mix between u and w and the first outsider to play u is excluded from the market if he is not successful initially*
- *if $c_p < (1 - q_i)\bar{c}_1$, both outsiders mix between p and w*

The intuition for this result is the following. First consider a subgame where a single imitator attempted to imitate and failed ($k = K = 1$). Clearly, the imitator who never tried to imitate in the past enters right away. She pays cost c_p if and only if $q_i\Pi_2 - c_p \geq q_i\Pi_3$, i.e $c_p < \bar{c}_1$. This condition reflects the idea that protection is only worth it if guaranteeing duopoly rather than triopoly profits in case of success, justifies paying the protection cost.

Working backwards, we find that if $c_p > \bar{c}_1$, both outsiders mix between u and w, knowing that the first to play u will, in the event that she initially fails, still have another chance if the later imitator is then successful since she will not pay for protection. If $(1 - q_i)\bar{c}_1 \leq c_p < \bar{c}_1$, we show in the appendix that both outsiders still mix between u and w but, in case of failure, the first imitator is excluded from the market. In both of these cases the innovator collects rents from the initial delay. Note that outsiders initially delay entry in the hope of free riding on the other outsider's reverse engineering effort if she is successful, a similar motivation as in our baseline model.

In the last case, $c_p < (1 - q_i)\bar{c}_1$, we show that firms also play a waiting game that benefits the innovator, but mix between p and w rather than between u and w. Intuitively, protection is somewhat costly and can be avoided if the other firm happens to (successfully) imitate first, so there is an incentive to wait so that the other firm incurs the protection cost in the first place (i.e., firms free-ride on the protection cost, not the imitation cost).

6 Paying for researchers to stay

In the previous sections we focused on cases where the innovator and the initial imitators could protect themselves from further imitation by making their technology hard to reverse engineer. However, knowledge of how to reproduce the technology is often embodied in the researchers who developed it. Another key dimension of protection is therefore offering wages sufficiently high so that these key researchers have less incentives to leave the firm. In the current section we examine the dynamics of wages in settings involving imitation. We show that this is a way of making the fixed protection cost c_p endogenous.

There is a growing literature examining the mobility of scientists and the associated diffusion of knowledge. Lewis and Yao (2006), in a situation where some ideas developed in one firm can be potentially more useful in another, show how allowing ex ante for mobility of researchers can be optimal from the innovating firm's point of view. Kim and Marschke (2005) examine a model where innovators can choose between patenting and secrecy and show, both theoretically and empirically, that patents become more attractive when there is a high risk of scientists leaving with firms secrets. Franco and Mitchell (2008) compare situations where clauses restraining worker mobility can be included in contracts to cases where this is illegal.[17] The novelty of our approach is that it allows us to study fully dynamic aspects of employment in situations where firms can imitate in two distinct ways: by doing in-house research or by poaching a scientist from a different firm.[18]

We modify our model in the following way to address the question. We suppose that there are $n+1$ firms. Firm j is considered to be a pair composed of a financier f_j and a researcher r_j whose value outside the industry under consideration is normalized to zero. As in our previous model, the game starts with a firm, labeled 1, having innovated. Researcher r_1 therefore has the knowledge of how to reproduce the invention. A firm who has not yet imitated and decides to do so can do it in two ways. She can hire a researcher of a previously successful firm or use her in-house researcher, who is uninformed and can develop the invention at cost c_i.

We consider a simple model for the hiring process. After a firm has successfully developed an invention or successfully imitated, a second-price auction for the researcher is run between this firm and all the remaining outsiders. The winning bidder hires the worker and pays the second-highest bid. We suppose furthermore that if the winning bidder is the current employer of the researcher, then this researcher will stay with the

[17] They find that the so-called "covenant not to compete" can explain the initial advantage of Massachusetts' Route 128 and the subsequent overtaking by the Sillicon Valley.

[18] Note that contrary to most papers in the literature, a notable exception being Franco and Mitchell (2008), we focus on a case where the creation of a spin-out by a researcher leaving the firm decreases total profits, i.e it does not create a differentiated product, although this case could be considered in our framework.

firm forever.[19] Thus, in that case, the firms not having yet imitated can only do so through in-house research at cost c_i.

We present the results in the case in which $c_i < c_2^*$ so as to illustrate the main intuition (results are similar for $c_i \geq c_2^*$).

Proposition 5 *Suppose that $c_i < c_2^*$. If the number of outsiders:*

(i) is at least as large as 4, one of the outsiders wins the auction and pays the researcher $c_2^* + c_i$

(ii) is equal to 3, one of the outsiders wins the auction and pays the researcher some amount between c_2^* and $c_2^* + c_i$

(iii) is equal to 2, the researcher stays with the current employer for an amount between 0 and c_2^* and the remaining two outsiders play a waiting game

(iv) is equal to 1, the researcher moves for a zero wage

Furthermore, the profits of the innovator are $\mu_2 \Pi_{n-2} + (1 - \mu_2)\Pi_n$, where $\mu_2 \equiv r/(r + 2\lambda_2)$, and the original researcher accumulates salaries of at least $c_2^* + (n-3)(c_2^* + c_i)$.

We first observe that the timing of imitation is the following: $n - 2$ firms quickly enter in a sequential manner by winning the auction and hiring the informed researcher. At this point, when only 2 imitators are left, the last firm having entered will pay a sufficiently high bonus so as to win the auction and keep the researcher within the firm. The remaining two imitators can thus only enter by conducting in-house research at cost c_i and will wait to do so in the hope that their competitor does it before them and that they can subsequently hire the informed researcher at a zero wage.

We note that this path of entry is very similar to the one identified in the general model of section 3 for the case $c_p < c_2^*$. In that case a series of preemption games were played and when two imitators were left, they played a waiting game with the same speed of entry λ_2. The main difference is that in the current setting we do not have the uncoordinated preemption phase. Outsiders still have an incentive to enter quickly, but the auction solves the mis-coordination problem characterizing preemption games, since the auction determines a unique winner.

Proposition 5 can be seen as providing a microeconomic foundation for our assumption of a fixed protection cost. Indeed, as long as the number of outsiders is greater than four, the imitators who enter pay a premium of c_2^* above the imitation cost c_i, premium that can be interpreted as the cost of protection. The intuition is the following: the

[19] A natural interpretation of this assumption is that the current firm is participating in the bidding to make the researcher sign a non-competition clause (a covenant not to compete).

early imitators, by paying the premium c_2^*, are purchasing the right to benefit from the imitation delay that arises in the waiting game. The order of entry among them does not influence their expected profits since with the auction there is no mis-coordination. All the initial entrants therefore pay the same price for entry $c_i + c_2^*$.

We characterize the level of bonuses on the equilibrium path. Naturally, as the number of remaining imitators decreases, the bonus that the researcher obtains decreases. We note that neither the innovator, nor the imitators, attempt to keep their researcher until only two imitators are left. The intuition is as follows: the innovator and the initial imitators are free riding on the protection effort of the antepenultimate entrant. In previous subgames, there is no point in trying to keep the researcher since the remaining imitators will in any case try to rush to enter, and if they cannot do it by hiring a researcher, they will do it through in house research. Keeping the researcher has even a negative effect on expected profits since it opens the way to a preemption game with excessive entry due to the risk of mis-coordination.

7 Conclusion

In this paper we show that considering the dynamics of investment in protection technologies can generate high rents for the innovators outside of patents and can help explain the observation that a large share of innovators choose secrecy over patenting. Surprisingly, the protection technologies that yield the highest returns for the innovator are expensive and do not protect very well. We also show that our model has implications for the large sectorial variations in patenting rates and for the patterns of employment in innovative industries.

We believe our model and results could be the basis for interesting empirical work. At the very least it underlines the need for more comprehensive data on two dimensions. First, little is still known on the cost of reverse engineering inventions, and how these costs vary by industry. Second, little information is available on protection technologies, their cost, the level of protection they confer. Although there is a large body of anecdotal evidence showing that technological protection is commonly used, there is no systematic measurement allowing for more detailed empirical analysis.

Finally, we want to suggest that our model can also contribute to the understanding of the path of diffusion of innovations. Starting with the seminal paper by Griliches (1957), numerous papers have documented the fact that the pattern of adoption of new technologies is typically S-shaped: slow initial adoption is followed by a quick acceleration and then slowing down.[20] If we view the process of imitation as a process of adoption

[20]There are numerous papers proposing a theoretical explanation for this pattern of adoption. Some are non strategic and often based on models of diffusion of information. Others consider firms that are strategic in their adoption decisions (Reinganum, Fudenberg and Tirole (1985), Ruiz Aliseda and Zemsky

of a technology, our paper provides a different theoretical foundation for the delay in adoption.[21] Firms wait to adopt in the hope that the technology will enter the public domain at some point. Of course the path is more a step function than a smooth S-shape. We could however imagine introducing uncertainties, for instance in the time needed to obtain an invention after having paid the imitation cost, that could generate a smoother path. This could be the object of interesting future work.

(2006)).

[21] Note that the empirical literature is not explicit about what is the process of adoption of a technology, whether it is purchasing from the inventor or whether it comes through imitation.

Appendix

Proof of Lemma 1. Fudenberg and Tirole (1991) show that the unique symmetric equilibrium of the discrete-time war of attrition with short period lengths converges to the unique symmetric equilibrium of the war of attrition in continuous time. This leads us to prove the result using the continuous-time version of the game directly.

Counting from the date at which the subgame is first reached, let us consider the expected payoff of an outsider if she chooses to imitate at time τ_2 given that the other outsider chooses her imitation time according to an atomless and gapless distribution $F_2(\cdot)$ with full support on $[0, \infty)$ and density $f_2(\cdot)$. Given that the other firm has made an unknown draw from $F_2(\cdot)$, a firm who enters at τ_2 expects to gain

$$\widehat{V}_2(\tau_2) = \int_0^{\tau_2} \Pi_n e^{-rs} dF_2(s) + \int_{\tau_2}^{\infty} (\Pi_n - c_i) e^{-r\tau_2} dF_2(s).$$

In a mixed-strategy Nash equilibrium, the firm should be indifferent among all possible imitation times, which formally means that we should have that $d\widehat{V}_2(\tau_2)/d\tau_2 = 0$ for all $\tau_2 \geq 0$. Straightforward differentiation using the fact that $\int_{\tau_2}^{\infty} dF_2(s) = 1 - F_2(\tau_2)$ yields that

$$\frac{d\widehat{V}_2(\tau_2)}{d\tau_2} = e^{-r\tau_2}[c_i f_2(\tau_2) - r(\Pi_n - c_i)(1 - F_2(\tau_2))].$$

Letting $h_2(\tau_2) \equiv f_2(\tau_2)/(1 - F_2(\tau_2))$ denote the hazard rate of $F_2(\cdot)$ and equating $d\widehat{V}_2(\tau_2)/d\tau_2$ to zero yields that the hazard rate is constant and equal to $h_2(\tau_2) = r(\Pi_n - c_i)/c_i$, so $F_2(\tau_2) = 1 - e^{-\lambda_2 \tau_2}$, where $\lambda_2 \equiv r(\Pi_n - c_i)/c_i$. Given that a probability distribution is exponential if and only if its hazard rate is constant, the individual entry time follows an exponential distribution with parameter $\lambda_2 = r(\Pi_n - c_i)/c_i$.

Furthermore, since a firm is indifferent among all the pure strategies played with positive density, the expected gain of an outsider converges to $O_2 = \Pi_n - c_i$ (payoff to imitating immediately).

We have shown that both outsiders make independent draws from an exponential distribution with the same hazard rate λ_2, so the time $\widehat{\tau}$ of first entry must be exponentially distributed with parameter $2\lambda_2$. The expected payoff for an insider is therefore given by:

$$I_2 = \int_0^{\infty} \left(\int_0^{\widehat{\tau}} \pi_{n-2} e^{-rs} ds + \int_{\widehat{\tau}}^{\infty} \pi_n e^{-rs} ds \right) 2\lambda_2 e^{-2\lambda_2 \widehat{\tau}} d\widehat{\tau}.$$

Integrating and letting $\mu_2 \equiv r/(r + 2\lambda_2)$ yields that:

$$I_2 = \mu_2 \Pi_{n-2} + (1 - \mu_2)\Pi_n.$$

■

Proof of Lemma 2. (i) As indicated in the main text, action p is weakly dominated if $c_p \geq c_2^*$, so the outsiders mix every period between u and w. Counting from the date at which the subgame is first reached, suppose that two outsiders draw their time of imitation with an unprotected technology using an atomless and gapless distribution function $F_3(\cdot)$ with full support on $[0, \infty)$. Denoting these (random) draws by s and s', we have that the expected payoff of an outsider if she imitates at time τ_3 with probability one (conditional upon no other outsider imitating earlier) is

$$\begin{aligned}\widehat{V}_3(\tau_3) &= \int_0^{\tau_3} \Pi_n e^{-rs} f_3(s)(1-F_3(s))ds + \int_0^{\tau_3} \Pi_n e^{-rs} f_3(s')(1-F_3(s'))ds' \\ &\quad +(1-F_3(\tau_3))^2(\Pi_n - c_i)e^{-r\tau_3}.\end{aligned}$$

Because it must hold that $d\widehat{V}_3(\tau_3)/d\tau_3 = 0$ for all $\tau_3 \geq 0$, straightforward computations show that we must have $h_3(\tau_3) \equiv f_3(\tau_3)/(1-F_3(\tau_3)) = r(\Pi_n - c_i)/(2c_i)$. Hence, $F_3(\tau_3) = 1 - e^{-\lambda_3 \tau_3}$, where $\lambda_3 \equiv r(\Pi_n - c_i)/(2c_i)$. Each outsider expects to gain $O_3 \equiv \Pi_n - c_i$ (since $\widehat{V}_3(\tau_3) = \Pi_n - c_i$ for $\tau_3 = 0$). In turn, the fact that the time at which imitation takes place is exponentially distributed with parameter $3\lambda_3$ yields that the payoff expected by the insiders is

$$I_3 = \mu_3 \Pi_{n-3} + (1-\mu_3)\Pi_n,$$

where $\mu_3 \equiv r/(r + 3\lambda_3)$.

(ii) We now consider the case $c_p < c_2^*$. In principle, firms will mix using the three actions available to each of them, namely w, p and u. We denote $\rho_{a,k} \geq 0$ the probability with which one of the outsiders plays action a when k outsiders remain to enter. We let $V_{a,k}$ denote the outsider's payoff when following action $a \in \{w, p, u\}$. In a mixed-strategy equilibrium in which outsiders play stationary strategies, we must have that $V_{w,3} = V_{p,3} = V_{u,3}$, where

$$V_{p,3} = \rho_{w,3}^2(\pi_{n-2}\Delta + I_2\delta^\Delta) + 2\rho_{w,3}(1-\rho_{w,3})(\pi_{n-1}\Delta + \Pi_n\delta^\Delta) + (1-\rho_{w,3})^2\Pi_n - c_i - c_p \quad (2)$$

$$V_{u,3} = \rho_{w,3}^2(\pi_{n-2}\Delta + \Pi_n\delta^\Delta) + 2\rho_{w,3}(1-\rho_{w,3})(\pi_{n-1}\Delta + \Pi_n\delta^\Delta) + (1-\rho_{w,3})^2\Pi_n - c_i \quad (3)$$

and

$$V_{w,3} = \rho_{w,3}^2(V_{w,3}\delta^\Delta) + 2\rho_{w,3}\rho_{p,3}O_2\delta^\Delta + (\rho_{w,3} + \rho_{p,3} + 1)\rho_{u,3}\Pi_n\delta^\Delta + \rho_{p,3}^2(\Pi_n - c_i)\delta^\Delta. \quad (4)$$

Because $V_{p,3} = V_{u,3}$, it holds after using the fact that $\rho_{w,3} \geq 0$ that

$$\rho_{w,3} = \sqrt{\frac{c_p}{(I_2 - \Pi_n)\delta^\Delta}}. \quad (5)$$

Using the working hypothesis that $c_p < c_2^* \equiv I_2 - \Pi_n$ yields that $\dfrac{c_p}{(I_2 - \Pi_n)\delta^\Delta} < \delta^{-\Delta}$, so $\rho_w < 1$ for $\Delta > 0$ close enough to zero.

Because $\rho_{u,3} = 1 - (\rho_{w,3} + \rho_{p,3})$ and $O_2 = \Pi_n - c_i$, the expression for $V_{w,3}$ can be rewritten as follows:

$$V_{w,3} = \frac{(1 - \rho_{w,3}^2)\Pi_n - \rho_{p,3}(\rho_{p,3} + 2\rho_{w,3})c_i}{\delta^{-\Delta} - \rho_{w,3}^2}.$$

Equating $V_{u,3}$ and $V_{w,3}$ yields the value for $\rho_{p,3} \geq 0$ after some manipulations:

$$\rho_{p,3} = \sqrt{\frac{c_i - (1 - \delta^{\Delta})B\Pi_n - \Delta\rho_{w,3}(1 - \delta^{\Delta}\rho_{w,3}^2)C}{\delta^{\Delta}c_i}} - \rho_{w,3}, \quad (6)$$

where: $B = \delta^{\Delta}(2 - \rho_{w,3})\rho_{w,3}^3 + (1 - \rho_{w,3})^2$ and $C = 2\pi_{n-1}(1 - \rho_{w,3}) + \rho_{w,3}\pi_{n-2}$

Using the fact that $I_2 - \Pi_n = \mu_2(\Pi_{n-2} - \Pi_n)$, we find for small $\Delta > 0$ that

$$\rho_{w,3} \approx \sqrt{\frac{c_p}{\mu_2(\Pi_{n-2} - \Pi_n)}},$$

$$\rho_{p,3} \approx 1 - \sqrt{\frac{c_p}{\mu_2(\Pi_{n-2} - \Pi_n)}},$$

and

$$\rho_{u,3} \approx 0,$$

that is action u is played with positive but vanishing probability.

We now determine payoffs. To make exposition notationally simpler, let us normalize to zero the date at which the subgame with three outsiders starts. Given m periods of play between time 0 and some fixed time $t > 0$, it holds that the probability that no outsider has imitated and protected her technology once time t has elapsed is $(\rho_{w,3})^{3m} = (\rho_{w,3})^{3t/\Delta}$ (since $m = t/\Delta$), which converges to zero as Δ converges to zero for any arbitrarily chosen $t > 0$. We then must have that there is probability one that at least one outsider will imitate and protect her technology (almost) instantaneously. In words, outsiders correlate their actions as Δ goes to zero even though they randomize independently.

We conclude the proof by characterizing the probability distribution over entry outcomes at (normalized) time 0 as well as equilibrium payoffs. Because the probability of no entry at any point in time is $(1 - \rho_{p,3})^3$, it holds that the probability that at least one outsider enters is $1 - (1 - \rho_{p,3})^3$. Conditional upon at least one outsider entering, we then have that

$$\phi_3(3) = (\rho_{p,3})^3/(1 - (1 - \rho_{p,3})^3),$$

$$\phi_3(2) = 3(1 - \rho_{p,3})(\rho_{p,3})^2/(1 - (1 - \rho_{p,3})^3),$$

and

$$\phi_3(1) = 3(1 - \rho_{p,3})^2\rho_{p,3}/(1 - (1 - \rho_{p,3})^3), \quad (7)$$

where $\phi_k(l)$ denotes the probability that $l \geq 1$ outsiders enter simultaneously at 0 given that there are $k \geq l$ of them. We finally observe that an outsider's continuation payoff at the beginning of these subgames is approximately $O_3 = \Pi_n - c_i$ (since $V_{p,3} = V_{u,3} = V_{w,3} \approx \Pi_n - c_i$ for small enough $\Delta > 0$). Since $I_1 = I_0 = \Pi_n$, the expected payoff earned by an insider is approximately

$$I_3 = \phi_3(1)I_2 + (1 - \phi_3(1))\Pi_n.$$

■

Proof of Lemma 3. We prove the result by induction. Lemma 2 established the result for $k = 3$, so it only remains to prove that it holds for $k \geq 4$ whenever it is true for $k - 1$. So suppose that the result holds for $k - 1$, and consider the subgames with k outsiders when $c_p \geq c^*_{k-1}$.

Let us focus on an outsider's incentive to play p. Since $c^*_j < c^*_{k-1}$ (see proof in main text) for all $j < k - 1$, he knows when choosing action p that p being simultaneously chosen by $l \geq 0$ other imitators will result in the remaining outsiders playing a waiting game (by the induction hypothesis). Clearly, the highest payoff that can be achieved is the one attained when no other outsider enters simultaneously, i.e., when $l = 0$. Thus, the highest payoff she can obtain by entering and paying the protection cost is $I_{k-1} - c_p - c_i = \mu_{k-1}\Pi_{n-k+1} + (1 - \mu_{k-1})\Pi_n - c_p - c_i$. Since $c_p \geq c^*_{k-1}$ implies $I_{k-1} - c_p - c_i < \Pi_n - c_i$, it then follows that no outsider must be willing to enter by paying the protection cost in subgames with k outsiders.

The k outsiders will therefore mix between waiting and entering without protection. Counting from the date at which the subgame is first reached, let us suppose that the outsiders draw their time of imitation with an unprotected technology using an atomless and gapless distribution function $F_k(\cdot)$ with full support on $[0, \infty)$. We then have that the expected payoff of an outsider if she imitates at time τ_k with probability one (conditional upon no other outsider imitating earlier) is

$$\widehat{V}_k(\tau_k) = (k-1)\int_0^{\tau_k} \Pi_n e^{-rs} f_k(s)(1 - F_k(s))ds + (1 - F_k(\tau_k))^{k-1}(\Pi_n - c_i)e^{-r\tau_k}.$$

In order for such an outsider to be indifferent between all the possible imitation times, it is easy to show that we must have that $F_k(\tau_k) = 1 - e^{-\lambda_k \tau_k}$, where $\lambda_k \equiv r(\Pi_n - c_i)/((k-1)c_i)$. Each of the outsiders expects to gain $O_k \equiv \Pi_n - c_i$ (since $\widehat{V}_k(\tau_k) = \Pi_n - c_i$ for $\tau_k = 0$). In addition, because the time at which the first imitation takes place is exponentially distributed with parameter $k\lambda_k$, the expected profit of an insider is given by

$$I_k = \mu_k \Pi_{n-k} + (1 - \mu_k)\Pi_n,$$

where $\mu_k \equiv r/(r + k\lambda_k)$. ■

Proof of Lemma 4. As explained in the main text, we solve for the approximation of the equilibrium outcome, taking directly the solution for a time period of length $\Delta = 0$. We show this result in a number of steps

Step 1: $\rho_{u,k} = 0$.

We show this result by induction. For $\Delta = 0$, we have

$$V_{u,J} = \Pi_n - c_i$$

and

$$\begin{aligned} V_{w,J} &= \Pr[X_{w,J} = J-1, X_{p,J} = 0, X_{u,J} = 0]\ V_{w,J} + \\ & \sum_{m=1}^{J-1} \sum_{l=0}^{J-1-m} \Pr[X_{w,J} = J-1-l-m, X_{p,J} = l, X_{u,J} = m]\ \Pi_n + \\ & \sum_{l=1}^{J-1} \Pr[X_{w,J} = J-1-l, X_{p,J} = l, X_{u,J} = 0]\ O_{J-l}, \end{aligned}$$

where $\Pr[X_{w,k}, X_{p,k}, X_{u,k}]$ denotes the probability that $X_{w,k}$ outsiders choose w, $X_{p,k}$ outsiders choose p and $X_{u,k}$ outsiders choose u. We know, that for all $k < J$, a waiting game is played and, according to Lemma 3, $O_k = \Pi_n - c_i$, so the system of equations can be rewritten as

$$V_{u,J} = \Pi_n - c_i$$

and

$$\begin{aligned} V_{w,J} &= \Pr[X_{w,J} = J-1, X_{p,J} = 0, X_{u,J} = 0]\ V_{w,J} + \\ & (1 - \Pr[X_{w,J} = J-1, X_{p,J} = 0, X_{u,J} = 0])\ \Pi_n - \\ & \sum_{l=1}^{J-1} \Pr[X_{w,J} = J-1-l, X_{p,J} = l, X_{u,J} = 0]\ c_i \end{aligned}$$

In a mixed-strategy equilibrium, an outsider must be indifferent between all actions played with positive probability, so we must have $V_{u,J} = V_{w,J}$, which implies that

$$\sum_{l=1}^{J-1} \Pr[X_{w,J} = J-1-l, X_{p,J} = l, X_{u,J} = 0] \ / \ (1-\Pr[X_{w,J} = J-1, X_{p,J} = 0, X_{u,J} = 0]) = 1.$$

This holds if and only if

$$\sum_{l=0}^{J-1} \Pr[X_{w,J} = J-1-l, X_{p,J} = l, X_{u,J} = 0] = 1,$$

hence we get that $\rho_{u,J} = 0$. Furthermore, this implies that $O_J = \Pi_n - c_i$, and the property is therefore true for $k = J$. The reasoning follows exactly the same lines for larger values of k. We can therefore use the notation adopted in the main text where $\rho_k \equiv \rho_{p,k}$

Step 2: ρ_k is the unique solution to $F_k(\rho_k) = c_p$.

Consider first the "last preemption game", i.e., the subgame where J outsiders are left to enter. As shown in the main text, the indifference between actions p and w is defined by

$$F_J(\rho_J) = c_p,$$

where

$$F_J(\rho) = \sum_{l=0}^{J-1} C_{J-1}^l \, \rho^l (1-\rho)^{J-1-l} \, \overline{I}_{J-1-l}.$$

Note that following entry by at least one outsider, a waiting game is played (by definition of J). The speed is determined by the number of other outsiders who enter. Note that according to Lemma 3, $\overline{I}_{J-1-l} = \mu_{J-1-l}(\Pi_{n-(J-1-l)} - \Pi_n) = c^*_{J-1-l}$. We showed previously that c^*_k is an increasing function of k. So we have $\overline{I}_{J-1} > \overline{I}_{J-2} > ... > \overline{I}_0$, and it can be immediately observed that $F_J(\rho)$ is a strictly decreasing function of ρ. Indeed, increasing ρ shifts the distribution to states where the payoff is lower.

Furthermore, $J = \inf\{k \geq 3 : c_p < c^*_{k-1}\}$ implies that $F_J(0) = \overline{I}_{J-1} = c^*_{J-1} > c_p$. Since $F_J(1) = \overline{I}_0 = 0$ and $F_J(\rho)$ is a continuous and strictly decreasing function, it then follows that the equation $F_J(\rho) = c_p$ has a unique solution $\rho_J \in (0, 1)$.

We now work recursively with $F_{k+1}(\rho)$ for $k \geq J$. We use the following key properties of $F_{k+1}(\rho)$ proven below:

- Property 1:
$$\frac{\partial F_{k+1}}{\partial \rho}(\rho) = (\frac{k}{1-\rho})(F_k(\rho) - F_{k+1}(\rho)).$$
- Property 2:
$$\frac{\partial F_{k+1}}{\partial \rho}(0) > 0.$$
- Property 3:
$$\overline{I}_k = F_{k+1}(\rho_k).$$

From Properties 1 and 2, we can conclude that $F_{k+1}(\rho)$ is increasing at zero, reaches a maximum when $F_{k+1}(\rho)$ and $F_k(\rho)$ cross and is then decreasing. Furthermore, we know that $F_{k+1}(1) = \overline{I}_0 = 0$. So to establish that $F_{k+1}(\rho) = c_p$ has a unique solution it is sufficient to show that $F_{k+1}(0) > c_p$. To prove it, note that we have $F_{k+1}(0) = \overline{I}_k$, and Property 3 implies that $\overline{I}_k = F_{k+1}(\rho_k)$, so it holds that $F_{k+1}(0) = F_{k+1}(\rho_k)$. Because $F_{k+1}(\rho)$ is increasing at zero according to Property 2, the unique maximum must be reached somewhere between 0 and ρ_k. According to Property 1, we know that $F_{k+1}(\rho) > F_k(\rho)$ for $\rho \geq \rho_k$, and therefore $F_{k+1}(\rho_k) > F_k(\rho_k)$. Taking into account that $F_{k+1}(0) = F_{k+1}(\rho_k)$, as we just showed, and that $F_k(\rho_k) = c_p$, it follows that $F_{k+1}(0) > c_p$.

Step 3: (i) follows directly from steps 1 and 2. We also showed above that $F_{k+1}(\rho) > c_p$ for $\rho \in (0, \rho_k)$, so we must that have $\rho_k < \rho_{k+1}$, which proves (ii). Finally (iii) can be shown as in the proof of Lemma 2.

To conclude the proof we show that properties 1-3 state above do hold:

Property 1 We have that

$$F_k(\rho) = \sum_{l=0}^{k-1} C_{k-1}^l \; (\rho)^l (1-\rho)^{k-1-l} \; \overline{I}_{k-1-l}$$

and

$$F_{k+1}(\rho) = \sum_{l=0}^{k} C_k^l \; (\rho)^l (1-\rho)^{k-l} \; \overline{I}_{k-l}. \tag{8}$$

So we can establish that

$$\begin{aligned} F_k(\rho) - F_{k+1}(\rho) &= \sum_{l=0}^{k-1} C_{k-1}^l \; (\rho)^l (1-\rho)^{k-1-l} \; \overline{I}_{k-1-l} - \sum_{l=0}^{k} C_k^l \; (\rho)^l (1-\rho)^{k-l} \; \overline{I}_{k-l} \\ &= \sum_{l=1}^{k} C_{k-1}^{l-1} \; (\rho)^{l-1} (1-\rho)^{k-l} \; \overline{I}_{k-l} - \sum_{l=1}^{k} C_k^l \; (\rho)^l (1-\rho)^{k-l} \; \overline{I}_{k-l} - (1-\rho)^k \; \overline{I}_k \end{aligned} \tag{9}$$

Consider

$$\begin{aligned} \frac{\partial F_{k+1}}{\partial \rho}(\rho) &= \sum_{l=0}^{k} C_k^l \; \left[l(\rho)^{l-1}(1-\rho)^{k-l} - (k-l)(\rho)^l (1-\rho)^{k-l-1} \right] \; \overline{I}_{k-l} \\ &= \sum_{l=0}^{k} C_k^l \; (\rho)^{l-1} (1-\rho)^{k-l-1} (l - k\rho) \; \overline{I}_{k-l} \\ &= \sum_{l=1}^{k} C_k^l \; (\rho)^{l-1} (1-\rho)^{k-l-1} (l - k\rho) \; \overline{I}_{k-l} - k(1-\rho)^{k-1} \overline{I}_k, \end{aligned} \tag{10}$$

so that

$$\frac{\partial F_{k+1}}{\partial \rho}(\rho) = \sum_{l=1}^{k} l C_k^l \; (\rho)^{l-1} (1-\rho)^{k-l-1} \; \overline{I}_{k-l} - k \sum_{l=1}^{k} C_k^l \; (\rho)^l (1-\rho)^{k-l-1} \; \overline{I}_{k-l} - k(1-\rho)^{k-1} \overline{I}_k.$$

Given that $C_{k-1}^{l-1} = l C_k^l / k$, using (9) yields:

$$\frac{\partial F_{k+1}}{\partial \rho}(\rho) = (\frac{k}{1-\rho})(F_k(\rho) - F_{k+1}(\rho)), \tag{11}$$

as claimed.

Properties 2 and 3 We have that

$$\begin{aligned}\frac{\partial F_k}{\partial \rho}(\rho) &= \sum_{l=0}^{k-1} C_{k-1}^l \ \left[l(\rho)^{l-1}(1-\rho)^{k-1-l} - (k-1-l)(\rho)^l(1-\rho)^{k-l-2}\right] \ \overline{I}_{k-1-l} \\ &= \sum_{l=1}^{k-1} C_{k-1}^l \ (\rho)^{l-1}(1-\rho)^{k-l-2} \left[l-(k-1)\rho\right] \ \overline{I}_{k-1-l} - (k-1)(1-\rho)^{k-2}\overline{I}_{k-1},\end{aligned}$$

so

$$\frac{\partial F_k}{\partial \rho}(0) = -(k-1)(\overline{I}_{k-1} - \overline{I}_{k-2}) \tag{12}$$

for $k \geq J+1$.

Denote now $\widehat{I}_k(\rho)$ for the expected payoff to an insider when there are k outsiders who choose to enter with probability ρ (the expectation being conditional upon at least one outsider entering). Then

$$\widehat{I}_k(\rho) = \sum_{l=1}^{k} C_k^l \ \frac{(\rho)^l(1-\rho)^{k-l}}{1-(1-\rho)^k} \ \overline{I}_{k-l}, \tag{13}$$

so straightforward manipulations yield:

$$\begin{aligned}(1-(1-\rho)^k) \ \widehat{I}_k(\rho) &= \sum_{l=1}^{k} C_k^l \ (\rho)^l(1-\rho)^{k-l} \ \overline{I}_{k-l} \\ &= \sum_{l=0}^{k} C_k^l \ (\rho)^l(1-\rho)^{k-l} \ \overline{I}_{k-l} - (1-\rho)^k \ \overline{I}_k \\ &= F_{k+1}(\rho) - (1-\rho)^k \ \overline{I}_k.\end{aligned}$$

If there existed a unique ρ_k satisfying $F_k(\rho_k) = c_p$, then we would have $\widehat{I}_k(\rho_k) = \overline{I}_k$, so using the previous equality for $\rho = \rho_k$ would yield

$$(1-(1-\rho_k)^k) \ \overline{I}_k = F_{k+1}(\rho_k) - (1-\rho_k)^k \ \overline{I}_k,$$

that is, an insider's expected payoff (net of Π_n) when k outsiders remain to enter would satisfy

$$\overline{I}_k = F_{k+1}(\rho_k) \tag{14}$$

if a unique ρ_k satisfying $F_k(\rho_k) = c_p$ existed.

Because we know that there exists a unique ρ_J satisfying $F_J(\rho_J) = c_p$, it simply remains to prove that $\frac{\partial F_k}{\partial \rho}(0) > 0$, that is, $\overline{I}_{k-1} < \overline{I}_{k-2}$ for $k \geq J+1$, which follows from working recursively on k as in Vettas (2000).[22] ∎

[22] Notice that expressions (4a), (5a) and (6)-(9) in Vettas (2000) are equivalent to expressions (1), (14), (8) for $\rho = 0$, (11), $F_k(1) = 0 < c_p$, and (12), respectively. Note that the expression that turns out to be

Proof of Proposition 1. Proposition 1 directly follows from Lemmas 1-4. ■

Proof of Proposition 2. Let $c_i < \Pi_n$. If $c_p \geq \Pi_2$, then $J = n$ and

$$I_{n-1} - c_p = \mu_{n-1}\Pi_1 + (1 - \mu_{n-1})\Pi_n - c_p.$$

Note that $I_{n-1} - c_p$ is decreasing in c_p for $c_p \geq \Pi_2$, whereas it increases in $c_i < \Pi_n$. In particular, $c_p \downarrow \Pi_2$ and $c_i \uparrow \Pi_n$ implies that $I_{n-1} - c_p$ converges to $\Pi_1 - \Pi_2$ from below (since $\mu_{n-1} \uparrow 1$). When $c_i \geq \Pi_n$, $c_p \geq \Pi_2$ implies that $J' = n$, so $I_{n-1} - c_p = \Pi_1 - c_p$, which implies that $I_{n-1} - c_p$ converges to $\Pi_1 - \Pi_2$ as $c_p \downarrow \Pi_2$. ■

Proof of Lemma 5. Let $c_p \in [c^*_{J-2}, c^*_{J-1})$ for some integer J between 3 and n, and consider the subgames with $k \geq J$ outsiders.

(i) Working backwards, we first show that $c_p \uparrow c^*_{J-1}$ implies that one, and only one, of the $k \geq J$ outsiders enters, even though each of them chooses action p with negligible probability (i.e., $\rho_k \downarrow 0$ as $c_p \uparrow c^*_{J-1}$ for all $k \geq J$). Given $k = J$ outsiders, the probability that one outsider enters conditional upon at least one of them entering equals

$$\phi_J(1) = \frac{J\rho_J(1-\rho_J)^{J-1}}{1-(1-\rho_J)^J}.$$

Since $\rho_J \downarrow 0$ as $c_p \uparrow c^*_{J-1}$, it follows from L'Hôpital's rule that

$$\lim_{c_p \uparrow c^*_{J-1}} \phi_J(1) = 1 - \frac{(J-1)\rho_J}{1-\rho_J} = 1,$$

so one, and only one, outsider (out of the J existing ones) enters as $c_p \uparrow c^*_{J-1}$. This also implies that $\overline{I}_J = \overline{I}_{J-1}$, so it also follows that $\rho_{J+1} \downarrow 0$ as $c_p \uparrow c^*_{J-1}$, with $\lim_{c_p \uparrow c^*_{J-1}} \phi_{J+1}(1) = 1$ and $\overline{I}_{J+1} = \overline{I}_J = \overline{I}_{J-1}$. Iteration then yields for all $k \geq J$ that $\rho_k \downarrow 0$ as $c_p \uparrow c^*_{J-1}$, with $\lim_{c_p \uparrow c^*_{J-1}} \phi_k(1) = 1$ and $\overline{I}_k = \overline{I}_{J-1}$, so outsiders enter quasi-instantaneously by paying c_p in a sequential and coordinated manner, with each having the same probability of entry as any other outsider in any subgame in which $k \geq J$. Because these results hold for any arbitrary value of J and $\overline{I}_k = \overline{I}_{J-1} = c^*_{J-1}$ for all $k \geq J$, we have that $\lim_{c_p \uparrow c^*_{J-1}}(\overline{I}_{n-1} - c_p) = 0$ regardless of the value taken by J, so $I_{n-1} - c_p = \Pi_n$.

Having shown that the innovator's net profits $I_{n-1} - c_p$ converge to Π_n as $c_p \uparrow c^*_{J-1}$ for any integer value of J between 3 and n, we now prove that the convergence is from above by showing that the innovator's net profit has a negative derivative as $c_p \uparrow c^*_{J-1}$. In subgames in which $k \geq J$, we showed (see (13) noticing that $\widehat{I}_k(\rho_k) = \overline{I}_k$) that an

equivalent in our setting to (10) in Vettas (2000) (namely, $F_{k+1}(0) > \overline{I}_k$) actually holds with equality, and hence it is redundant based on the expression in (8) evaluated at $\rho = 0$.

insider's continuation payoff (net of Π_n) satisfies the recursive equation

$$\overline{I}_k = \sum_{l=1}^{k} C_k^l \frac{(\rho_k)^l (1-\rho_k)^{k-l}}{1-(1-\rho_k)^k} \overline{I}_{k-l},$$

since at least one of them enters immediately. Noticing that ρ_k is an (implicit) function given by $F_k(\rho_k) = c_p$ and that $\rho_k \downarrow 0$ as $c_p \uparrow c^*_{J-1}$, we have that

$$\begin{aligned}\frac{\partial \overline{I}_k}{\partial c_p} &= \sum_{l=1}^{k} C_k^l \frac{(\rho_k)^l (1-\rho_k)^{k-l}}{1-(1-\rho_k)^k} \left(\frac{\partial \overline{I}_{k-l}}{\partial c_p}\right) + \\ &\sum_{l=1}^{k} C_k^l \Big(\frac{l(\rho_k)^{l-1}(1-\rho_k)^{k-1-l}}{1-(1-\rho_k)^k} - \frac{k(\rho_k)^l(1-\rho_k)^{k-1-l}}{(1-(1-\rho_k)^k)^2}\Big) \overline{I}_{k-l} \left(\frac{\partial \rho_k}{\partial c_p}\right).\end{aligned}$$

Making (repeated) use of L'Hôpital's rule, it follows that

$$\begin{aligned}\left.\frac{\partial \overline{I}_k}{\partial c_p}\right|_{c_p \uparrow c^*_{J-1}} &= C_k^1 \frac{1}{k} \left(\left.\frac{\partial \overline{I}_{k-1}}{\partial c_p}\right|_{c_p \uparrow c^*_{J-1}} \right) - \left(C_k^1 \frac{(k-1)}{2k} \overline{I}_{k-1} - C_k^2 \frac{1}{k} \overline{I}_{k-2} \right) \left(\left.\frac{\partial \rho_k}{\partial c_p}\right|_{c_p \uparrow c^*_{J-1}} \right) \\ &= \left(\left.\frac{\partial \overline{I}_{k-1}}{\partial c_p}\right|_{c_p \uparrow c^*_{J-1}} \right) - \frac{(k-1)(\overline{I}_{k-1} - \overline{I}_{k-2})}{2} \left(\left.\frac{\partial \rho_k}{\partial c_p}\right|_{c_p \uparrow c^*_{J-1}} \right),\end{aligned}$$

where

$$\left.\frac{\partial \rho_k}{\partial c_p}\right|_{c_p \uparrow c^*_{J-1}} = \left(\left.\frac{dF_k(\rho_k)}{d\rho_k}\right|_{\rho_k \downarrow 0} \right)^{-1} < 0,$$

since the derivative of $F_k(\rho)$ is negative for $\rho = \rho_k$.

Let $k = J$ and notice that $\partial \overline{I}_{J-l}/\partial c_p = 0$ for all $l \geq 1$ as well as that

$$\left.\frac{\partial \rho_J}{\partial c_p}\right|_{c_p \uparrow c^*_{J-1}} = -\frac{1}{(J-1)(\overline{I}_{J-1} - \overline{I}_{J-2})},$$

so

$$\left.\frac{\partial \overline{I}_J}{\partial c_p}\right|_{c_p \uparrow c^*_{J-1}} = \frac{1}{2}.$$

It follows that

$$\left.\frac{\partial (\overline{I}_J - c_p)}{\partial c_p}\right|_{c_p \uparrow c^*_{J-1}} = -\frac{1}{2} < 0.$$

Let now $k = J + 1$, so that

$$\left.\frac{\partial \overline{I}_{J+1}}{\partial c_p}\right|_{c_p \uparrow c^*_{J-1}} = \left.\frac{\partial \overline{I}_J}{\partial c_p}\right|_{c_p \uparrow c^*_{J-1}} - \frac{J(\overline{I}_J - \overline{I}_{J-1})}{2} \left(\left.\frac{\partial \rho_{J+1}}{\partial c_p}\right|_{c_p \uparrow c^*_{J-1}} \right).$$

Because

$$\left.\frac{\partial \overline{I}_J}{\partial c_p}\right|_{c_p \uparrow c^*_{J-1}} = \frac{1}{2}$$

and

$$\left.\frac{\partial \rho_{J+1}}{\partial c_p}\right|_{c_p \uparrow c^*_{J-1}} = \frac{1}{J(\overline{I}_J - \overline{I}_{J-1})},$$

it follows that

$$\left.\frac{\partial (\overline{I}_{J+1} - c_p)}{\partial c_p}\right|_{c_p \uparrow c^*_{J-1}} = -1.$$

Iteration yields that

$$\left.\frac{\partial \overline{I}_k}{\partial c_p}\right|_{c_p \uparrow c^*_{J-1}} = -\frac{J+1-k}{2},$$

so

$$\left.\frac{\partial (\overline{I}_k - c_p)}{\partial c_p}\right|_{c_p \uparrow c^*_{J-1}} < 0$$

for all $k \geq J$, which proves that the payoff achieved by the innovator if she pays the protection cost is decreasing in c_p whenever c_p is a bit smaller than c^*_{J-1} (where J can be any integer between 3 and n).

(ii) We now show that $I_{n-1} - c_p$ converges to Π_n from above as $c_p \downarrow 0$. To show this result, note that the fact that $\rho_k \uparrow 1$ as $c_p \downarrow 0$ yields after some straightforward manipulations that

$$\left.\frac{\partial \overline{I}_k}{\partial c_p}\right|_{c_p \downarrow 0} = -k\overline{I}_1 \left(\left.\frac{\partial \rho_k}{\partial c_p}\right|_{c_p \downarrow 0} \right).$$

Because

$$\left.\frac{\partial \rho_k}{\partial c_p}\right|_{c_p \downarrow 0} = \left(\left.\frac{dF_k(\rho_k)}{d\rho_k}\right|_{\rho_k \uparrow 1} \right)^{-1} = -\frac{1}{(k-1)\overline{I}_1},$$

we then have that

$$\left.\frac{\partial (\overline{I}_k - c_p)}{\partial c_p}\right|_{c_p \downarrow 0} = \frac{k}{k-1} - 1 > 0$$

for any arbitrary k. In particular, it holds for $k = n-1$, which shows that the payoff achieved by the innovator if she pays the protection cost is increasing in c_p for small values of c_p.

(iii) Follows trivially from the fact that $c_p \geq c^*_{n-2}$ implies $J = n$. ■

Proof of Proposition 3. The first part follows directly from Proposition 2. For the second part, an example is given in Figure 3, and more generally the result can be shown for small enough Π_P using Lemma 5. To this end, let $\Omega(\Pi_P) \equiv \{c_p > 0 : I_{n-1} - c_p > \Pi_P\}$ denote the set of values for c_p such that paying c_p is preferred by the innovator over a patent that yields payoff Π_P. Because ρ_k ($k = J, ..., n-1$) and $\overline{I}_{n-1} - c_p$ are continuous in c_p within any subinterval, it then holds from the properties stated in Lemma 5 that

$\Omega(\Pi_P)$ is nonconvex for values of Π_P close enough to Π_n. ■

Proof of Proposition 4. We start with the subgames in which $k = 1$ and $K \in \{0, 1\}$. Clearly, the imitator who never tried to imitate in the past enters right away. She pays cost c_p if and only if $q_i \Pi_{3-K} - c_p \geq q_i \Pi_3$. Letting $\overline{c}_K \equiv q_i(\Pi_{3-K} - \Pi_3)$, where $\overline{c}_1 > \overline{c}_0 = 0$, we have that $c_p \geq \overline{c}_K$ implies that the imitator enters without paying c_p, and hence the K imitators who attempted to imitate in the past (but failed) enter right away. The payoff expected by the imitator who never tried imitation in the past is $O_1(K) = q_i \Pi_3 - c_i$, whereas the payoff to insiders is $I_1(K) = q_i \Pi_3 + (1 - q_i)\Pi_{2-K}$. In turn, $c_p < \overline{c}_K$ implies that the imitator pays c_p when entering, and hence the K imitators that attempted to imitate in the past can never enter. The payoff expected by the imitator who never tried imitation in the past is $O_1(K) = q_i \Pi_{3-K} - c_p - c_i$, whereas the payoff to insiders is $I_1(K) = q_i \Pi_{3-K} + (1 - q_i)\Pi_{2-K}$.

We now examine the subgames in which $k = 2$ and hence $K = 0$. If any of the two imitators tries to imitate one of the protected technologies, she knows that the rival will try imitating with probability one right after she moves. We distinguish two situations, depending on whether $c_p - \overline{c}_1$ is nonnegative or not.

When $c_p \geq \overline{c}_1$, if one of the imitators fails in imitating, her continuation payoff will be $q_i(\Pi_3 - c_i)$, since the other one will try to immediately imitate without paying c_p and she will succeed with probability q_i. It is easy to show that firms can never mix between the three actions; indeed, the fact that $c_p > (1 - q_i)\overline{c}_1$ implies that p is strictly dominated by u.[23] It is then standard to show that the two imitators play a waiting game in which the hazard rate with which each chooses action u is

$$\begin{aligned}
\widehat{\lambda}_2 &= \frac{r[q_i(\Pi_3 - c_i) + (1 - q_i)q_i(\Pi_3 - c_i) - (1 - q_i)c_i]}{(1 - q_i)q_i c_i} \\
&= r(\frac{\Pi_3}{c_i} - 1)(\frac{1}{1 - q_i} + 1) - \frac{r}{q_i},
\end{aligned}$$

which is positive because $\dfrac{\Pi_3}{c_i} > \dfrac{1}{q_i}$. An imitator's expected payoff is $O_2(0) = q_i \Pi_3 + (1 -$

[23] Suppose that one of the imitators chooses actions u, p and w with respective probabilities ρ_u,ρ_p and ρ_w. The fact that $\rho_w = 1 - (\rho_p + \rho_u)$ implies then that the other imitator gains a higher payoff by choosing u rather than p for any (feasible) values of ρ_u,ρ_p and ρ_w. To prove this, note that the payoff to choosing action u is

$$V_u = q_i \Pi_3 + (1 - q_i)[\rho_u q_i(\Pi_3 - c_i) + \rho_w q_i(\Pi_3 - c_i)] - c_i,$$

whereas the payoff to choosing action p is

$$V_p = q_i[\rho_u(q_i\Pi_3 + (1 - q_i)\Pi_2) + \rho_p(q_i\Pi_3 + (1 - q_i)\Pi_2) + \rho_w(q_i\Pi_3 + (1 - q_i)\Pi_2)] + (1 - q_i)[\rho_u q_i(\Pi_3 - c_i) + \rho_w q_i(\Pi_3 - c_i)] - c_i - c_p.$$

$q_i)q_i(\Pi_3 - c_i) - c_i$, whereas an insider's expected continuation payoff is

$$\widehat{I}_2(0) = \Pi_1 - \frac{2\widehat{\lambda}_2 q_i(2-q_i)}{r+2\widehat{\lambda}_2}(\Pi_1 - \Pi_3).$$

When instead it holds that $c_p < \bar{c}_1$, if one of the imitators fails in imitating, her continuation payoff will be 0, since the other one will try to immediately imitate by paying c_p. It is easy to show that we cannot have an equilibrium in which imitators mix between p and u. So suppose first that they mix between u and w. Then it can be shown that they choose the timing at which to follow action u from an exponential distribution with hazard rate

$$\widehat{\lambda}_2' = \frac{r(q_i\Pi_3 - c_i)}{(1-q_i)(q_i\Pi_2 - c_p)}.$$

An imitator's expected payoff is $q_i\Pi_3 - c_i$, whereas an insider's continuation payoff is

$$\widehat{I}_2'(0) = \Pi_1 - \frac{2\widehat{\lambda}_2' q_i(2-q_i)}{r+2\widehat{\lambda}_2'}(\Pi_1 - \Pi_2) - \frac{2\widehat{\lambda}_2' q_i}{r+2\widehat{\lambda}_2'}(\Pi_2 - \Pi_3).$$

Choosing p yields $q_i[q_i\Pi_3 + (1-q_i)\Pi_2] - c_i - c_p$, which is smaller than $q_i\Pi_3 - c_i$ if and only if $(1-q_i)q_i(\Pi_2 - \Pi_3) < c_p$.

Suppose now that firms mix between p and w. Then they choose to take action p at a time drawn from an exponential distribution with hazard rate

$$\widehat{\lambda}_2'' = \frac{r(q_i(q_i\Pi_3 + (1-q_i)\Pi_2) - c_i - c_p)}{q_i c_p},$$

and each firm expects to earn $q_i[q_i\Pi_3 + (1-q_i)\Pi_2] - c_i - c_p$, whereas an insider earns continuation payoff

$$\widehat{I}_2''(0) = \Pi_1 - \frac{2\widehat{\lambda}_2'' q_i^2}{r+2\widehat{\lambda}_2''}(\Pi_2 - \Pi_3) - \frac{2\widehat{\lambda}_2'' q_i(2-q_i)}{r+2\widehat{\lambda}_2''}(\Pi_1 - \Pi_2).$$

Choosing u yields $q_i\Pi_3 - c_i$, which is smaller than $q_i[q_i\Pi_3 + (1-q_i)\Pi_2] - c_i - c_p$ if and only if $(1-q_i)q_i(\Pi_2 - \Pi_3) > c_p$.

To sum up, $c_p < (1-q_i)\bar{c}_1$ implies that imitators mix between p and w and there is delay from which the innovator benefits:[24] intuitively, protection is somewhat costly and can be avoided if the other firm happens to (successfully) imitate first, so there is an incentive to wait so that the other firm incurs the protection cost in the first place (i.e.,

[24] The fact that $c_p > 0$ can be easily shown to imply that conditional on the other imitator choosing p with probability one, the payoff to the other firm choosing w (i.e., $q_i(q_i\Pi_3 - c_i) + (1-q_i)(q_i\Pi_2 - c_p - c_i)$) is always greater than the payoff to choosing p (i.e., $q_i(q_i\Pi_3 + (1-q_i)\Pi_2) - c_i - c_p$). So there can be no symmetric equilibrium in which both imitators choose action p with probability one even if c_p is arbitrarily close to zero.

firms free-ride on the protection cost, not the imitation cost). Also, nobody can benefit from the innovation going to the public domain, unlike our base model. In equilibrium, the innovator's payoff is $\widehat{I}_2''(0) - c_p$, where

$$\widehat{I}_2''(0) = \Pi_1 - \frac{2\widehat{\lambda}_2'' q_i^2}{r + 2\widehat{\lambda}_2''}(\Pi_2 - \Pi_3) - \frac{2\widehat{\lambda}_2'' q_i(2 - q_i)}{r + 2\widehat{\lambda}_2''}(\Pi_1 - \Pi_2).$$

In turn, $(1 - q_i)\bar{c}_1 \leq c_p < \bar{c}_1$ implies that imitators mix between u and w knowing that the imitator that chooses u will be excluded from the market if she fails at imitating and the rival succeeds afterwards, whereas $\bar{c}_1 \leq c_p$ implies that firms mix between u and w knowing that the imitator that chooses u will not be excluded from the market if she fails and the rival succeeds afterwards. (What is remarkable is that this holds even if $c_i = 0$.) The innovator's payoff when $(1 - q_i)\bar{c}_1 \leq c_p < \bar{c}_1$ is $\widehat{I}_2'(0) - c_p$, where

$$\widehat{I}_2' = \Pi_1 - \frac{2\widehat{\lambda}_2' q_i}{r + 2\widehat{\lambda}_2'}(\Pi_2 - \Pi_3) - \frac{2\widehat{\lambda}_2' q_i(2 - q_i)}{r + 2\widehat{\lambda}_2'}(\Pi_1 - \Pi_2).$$

When $c_p \geq \bar{c}_1$, the innovator's payoff is $\widehat{I}_2(0) - c_p$, where

$$\widehat{I}_2(0) = \Pi_1 - \frac{2\widehat{\lambda}_2 q_i(2 - q_i)}{r + 2\widehat{\lambda}_2}(\Pi_1 - \Pi_3).$$

In short, for any value of c_p, the innovator can always benefit from an imitation delay, even if imitation is not costly, because imitators have an incentive either to free-ride on the protection cost or to benefit from the lack of protective measures by the other imitator. ■

Proof of Proposition 5. We solve the model by backwards induction, using the notation O_k for the expected payoff of an outsider in a subgame with k outsiders left, I_k for the payoff of an insider who entered in one of the previous subgames and I_k^l for the payoff of the insider who just entered (called the "last insider", hence the superscript). We work backwards as usual, so we consider the different subgames that may arise.

Suppose that $k = 1$. Then, regardless of the bidding outcome, the outsider will enter, so the insider bids his valuation, zero, and the outsider gets the researcher at a zero wage. Continuation payoffs are: $I_1 = I_1^l = \Pi_n$ and $O_1 = \Pi_n$.

Suppose that $k = 2$. The subgame starts with an auction between the last insider and the two outsiders. To determine the strategies in this auction, we need to determine the payoffs in the subsequent subgames. We first study the subgame following an outcome of the auction such that the last insider won the bidding in the previous subgame. In this class of subgames, all researchers have signed a non-competition clause, there are therefore two outsiders who can only enter by doing the research in-house by paying cost c_i. This leads to a war of attrition as in section 3 since $c_i < c_2^*$. Payoffs are then

$I_2 = I_2^l = \mu_2\Pi_{n-2} + (1-\mu_2)\Pi_n$ for the insiders and $O_2 = \Pi_n - c_i$ for outsiders. If an outsider won the bidding, then according to the previous step, the payoffs of all the players are Π_n.

We now examine the bidding strategies. The last insider gets $\mu_2\Pi_{n-2}+(1-\mu_2)\Pi_n - w$ if he wins with bid w, and Π_n if he loses. A weakly dominant strategy in a second-price auction is for the insider to bid his valuation c_2^* (where $c_2^* = I_2 - \Pi_n = \mu_2(\Pi_{n-2} - \Pi_n)$). An outsider gets $\Pi_n - w$ if he wins with bid w, but he gets $\Pi_n - c_i$ if he loses to the last insider and Π_n if he loses to the other outsider. Since $c_2^* > c_i$, the insider by bidding his valuation c_2^*, wins and pays price between 0 and c_i, payoffs are $I_2 = \mu_2\Pi_{n-2}+(1-\mu_2)\Pi_n$, $I_2^l \in [I_2 - c_i, I_2]$ and $O_2 = \Pi_n - c_i$.

Suppose that $k = 3$. We first study the subgame following an outcome of the auction such that the last insider won the bidding in the previous subgame. In that subgame, the outsiders can only enter by paying c_i (in-house research). When an outsider does imitate, he becomes the last insider in the next subgame and gets $I_2^l - c_i$, which is at least $I_2 - 2c_i$. If one of the competing outsiders enters, he gets $O_2 = \Pi_n - c_i$. Given that $c_2^* > c_i$ implies $I_2 - 2c_i > O_2 = \Pi_n - c_i$, this leads to a preemption motive and the expected payoff of the outsiders in such a subgame is $\Pi_n - c_i$. Furthermore, the insiders obtain an expected payoff strictly less than I_2 (since there is a risk of miscoordination). Thus, when we examine the bidding strategies, we see that the last insider knows that if he wins the bidding he gets a payoff strictly less than when he loses (and gets I_2). As a result, he always bids zero. The outsiders then bid their valuation v, that is, they bid $v = I_2^l - O_2 \in [c_2^*, c_2^* + c_i]$, so we have $I_3 = I_3^l = I_2$ and $O_3 = \frac{1}{3}(I_2^l - v) + \frac{2}{3}O_2 = O_2$.

Suppose that $k = 4$. We first study the subgame following an outcome of the auction such that the last insider won the bidding in the previous subgame. When an outsider imitates, he becomes the last insider in the next subgame and gets $I_2^l - c_i$, which equals $I_2 - 2c_i$. If one of the competing outsiders enters, he gets $O_2 = \Pi_n - c_i$. Given that $c_2^* > c_i$, this leads to a preemption motive and the expected payoff of the outsiders in such a subgame is $\Pi_n - c_i$ and the insiders obtain an expected payoff strictly less than I_2 (since there is a risk of miscoordination). Thus, when we examine the bidding strategies, we see that the last insider gets a payoff upon winning the auction that is strictly less than what he gets if he loses (namely, I_2). Thus, he always bids zero. In turn, the outsiders bid their valuation: $v' = I_3^l - O_3 = c_2^* + c_i$. We have for insiders $I_4 = I_4^l = I_2$, and for outsiders $O_4 = \frac{1}{4}(I_3^l - v') + \frac{3}{4}O_3 = O_3 = \Pi_n - c_i$.

The result can be easily shown by induction for $k > 4$. ■

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